

Utilizing Fuzzy TOPSIS for Sustainable Development: A Case Study in Selection of Airport Location

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Citation: Saleh, R. F., Kaml, B. S., & Hameed, L. M. (2024). Utilizing Fuzzy TOPSIS for Sustainable Development: A Case Study in Selection of Airport Location. *Journal of Information Systems Engineering and Management*, 9(4), 30116.

<https://doi.org/10.55267/iadt.07.15426>

ARTICLE INFO

Received: 05 Aug 2024

Accepted: 19 Aug 2024

ABSTRACT

In recent years, most countries have seen an increased focus on the concept of sustainable development in decisions regarding transportation infrastructure where environmental and social criteria are considered on par with economic criteria. This is particularly important in decisions to build new airports or expand their capacity with long-term implications for economic growth, welfare and regional development processes. The fuzzy TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) method is distinguished from the other methods of the multi-criteria decision making (MCDM) in that it arranges the available alternatives according to their proximity to the optimal solution. In real life, alternatives are observed in relation to the criteria and the weights of the criteria with fuzzy numbers. In such a case, the crisp traditional methods that solve the problems of multi-criteria decision-making are considered to be of weak effectiveness. The aim of this paper is to solve the problem of choosing a new hub airport for a hypothetical European Union airline, by applying the fuzzy TOPSIS method based on alpha level sets as it was applied to a selected data set from the available alternatives (candidate airports). The criteria for expressing the performance of these airports are defined and evaluated by the decision maker. The objective of the practical application of the studied problem is to show the usefulness of the method used as a tool that helps the decision maker to determine which of the alternatives represents the best solution in an environment with fuzzy data.

Keywords: MCDM, Fuzzy TOPSIS Method, Triangular Fuzzy Numbers, Alpha Level Sets.

INTRODUCTION

Today, the world faces environmental, economic and social challenges, so sustainable development has become a major goal sought by decision-makers in various fields. Choosing a new airport site is one of the important strategic decisions that requires a comprehensive study that takes into account the multiple and intertwined factors that affect sustainability. The TOPSIS model (Technique for Order of Preference by Similarity to Ideal Solution) is one of the effective tools that help in evaluating different alternatives and making the optimal decision, especially when dealing with fuzzy or unspecified data. In this context, the application of a fuzzy TOPSIS model comes as an innovative methodology that aims to improve the accuracy and efficiency of the process of selecting a new airport site, by integrating the dimensions of environmental, economic and social sustainability

into the evaluation process. This technique is based on determining the extent to which each alternative is close to the ideal solution that achieves the highest levels of sustainability, which contributes to making decisions based on accurate scientific foundations that meet the needs of the present without compromising the ability of future generations to meet their needs. In this paper, fuzzy data was dealt with, as a fuzzy decision matrix was calculated as a first step, then compute α -cut set for each alternative with the criteria on which it was evaluated, and calculates the respective fuzzy proximity of each alternative with α -cut level. After defuzzing the data, a convergence method is determined for each alternative to determine the order of all alternatives. A higher value of the proximity coefficient indicates that the variant is closer to fuzzy positive ideal solution and further away from fuzzy negative ideal solution at the same time.

LITERATURE REVIEW

Sustainable development aims to try to find a kind of balance between the ecosystem and the economic system without wasting natural resources. Hulaihel and Salman (2022) proposed a fuzzy TOPSIS method based on α -level sets and introduced a nonlinear programming solution. Boran, Genç, Kurt, and Akay (2009) integrated TOPSIS method with an intuitive fuzzy set to choose a convenient supplier in a group decision making environment. The conclusion of this study was the intuitive fuzzy sets are an appropriate approach to dealing with an uncertainty environment. Sun and Lin (2009) applied a fuzzy TOPSIS method that depended on fuzzy sets to solve (MCDM). The results of their research indicate that security and trust are the most important factors in enhancing the competitive advantage of online shopping sites. C. T. Chen (2000) expands the TOPSIS method to fuzzy group decision making cases by explaining an Euclidean distance between each two fuzzy numbers.

Milan and Reggiani (2002) applied three separate multi-criteria decision-making (MCDM) methods—simple additive weighting (SAW), the technique of preference for demand by similarity to the ideal solution (TOPSIS), and the analytic hierarchy process (AHP)—to select a new hub airport for an airline. McManners (2016) investigated the impacts that sustainability might have on aviation. It was found that a low-carbon future for aviation is possible but different. When sustainability supports policy, transformational change becomes possible. Promoting sustainability will require conveying a better vision for aviation. Decision making problems are the process of finding the best option among all possible alternatives. In almost all of these problems, the multiplicity of criteria for judging the alternatives is pervasive. That is, for many of these problems, the decision maker wants to solve the Multiple Criteria Decision Making (MCDM) problem. Jahanshahloo, Lotfi, and Izadikhah (2006) presented a paper that aims to extend the TOPSIS method to decision problems with fuzzy data. In this paper, the ranking of each alternative and the weight of each criterion are expressed as triangular fuzzy numbers. The normalized fuzzy numbers are calculated using the concept of α -cut. Dymova, Sevastjanov, and Tikhonenko (2013) propose a new approach to solve MCDM problems using the fuzzy TOPSIS method. This approach is free from the limitations of other known methods concerned with introducing fuzzy values through real values in the computation of optimal solutions and defuzzifying the initial fuzzy decision matrix.

Triantaphyllou and Lin (1996) extended a fuzzy TOPSIS method depending on fuzzy arithmetic operations. S. M. Chen and Lee (2010) propose an interval type two fuzzy TOPSIS method. To show the effectiveness of the fuzzy TOPSIS method, C. T. Chen (2000) applied to fuzzy environment for employment chosen. Shih, Shyur, and Lee (2007) proposed an incorporated set TOPSIS method procedure for solving employment chosen. Dursun and Karsak (2010) developed TOPSIS method for both linguistic and numeric estimate measures in personnel selection. Kelemenis, Ergazakis, and Askounis (2011) combined TOPSIS method with fuzzy logic to solve support managers assignment problems.

Overview of the Problem

The liberalization of the EU aviation market has removed institutional barriers that had previously hindered the freedom and flexibility of air transport operations between Member States. As a result, air operations have seen greater freedom in flight frequencies, pricing, and market entry and exit, with the expectation that this will increase competition within the sector, reduce ticket prices, and improve the overall quality of services provided to passengers and cargo. In addition, airlines and airports have been privatized as a complementary measure aimed at improving the overall efficiency and effectiveness of the sector and its components. In these developments, airlines and airports have become more flexible in dealing with market changes and adapting to the growing demand for air transport services. There has also been a need to adopt innovative strategies to improve customer experience and expand the range of services provided. In response to these new challenges and circumstances, EU airlines have resorted to one or more strategies to maintain their current positions and acquire new ones in the European aviation market, with a focus on enhancing their competitiveness and increasing their

market share.

The impacts that airport development can have on local communities from two perspectives: social and cultural, and how these impacts are important for achieving sustainable development.

1. Social impacts: Airport development can significantly impact local communities by creating jobs, improving infrastructure, and increasing connectivity with the outside world. However, local residents may face challenges such as congestion, higher costs of living, and changing lifestyles.

2. Cultural impacts: Airport development can lead to changes in local cultures due to increased interaction with international visitors and the influx of global culture. Some local customs and traditions may erode or new practices may emerge as a result of external influences.

As for its relationship to sustainable development: Attention to these impacts is crucial to ensuring that airport development is consistent with the principles of sustainable development, which seek to balance economic progress, social justice, and the protection of the cultural and natural environment.

METHODOLOGY

A comprehensive methodology was followed to use the Fuzzy TOPSIS method in airport site selection for sustainable development. The methodology began with a literature review to determine the importance of using multi-criteria decision making (MCDM) techniques in this context, and to identify appropriate criteria for airport site selection, such as environmental impact, cost, and infrastructure. Data was collected from multiple sources including previous studies and expert opinions. Fuzzy TOPSIS steps were then applied, which included creating a decision matrix, normalizing the data, and determining weights using fuzzy values to represent uncertainty. Afterwards, distances from ideal solutions were calculated and alternatives were ranked based on their proximity to those solutions. The results were analyzed to discuss the compatibility of candidate sites with the SDGs and provide recommendations for selecting the best airport site, taking into account environmental, economic, and social aspects.

RESULTS AND DISCUSSION

Fuzzy Numbers

A fuzzy number is a partial fuzzy set of real numbers and this fuzzy set is a convex and normalized, fuzzy numbers set is the essence of fuzzy calculations. For a number to be fuzzy, there are four conditions (Xie, Liu, Gu, & Zhou, 2018):

1. A fuzzy set must be normalized, meaning that the normal fuzzy set is one in which the curve of its membership function contains at least one peak equal to one, or in other words, it is a fuzzy set that contains at least one value of its membership function. Called \tilde{K} a normalized fuzzy set, if $y_0 \in R$, So that it is $\mu_{\tilde{K}}(y_0) = 1$.

2. A fuzzy set must be convex, It is the set in which the value of the belonging of each point in it lies between two points greater than or equal to the value of the membership of one of those two points. The convexity condition is defined as follows:

$$\mu_{\tilde{K}}(\lambda y_1 + (1 - \lambda)y_2) \geq \min(\mu_{\tilde{K}}(y_1), \mu_{\tilde{K}}(y_2)) \quad (1)$$

Where $y_1, y_2 \in R, \lambda \in [0, 1]$, and $\lambda y_1 + (1 - \lambda)y_2$, point between two points y_1, y_2 .

3. The membership function of fuzzy set must have a semi-continuous upper bound. That is, the two sides of the membership function are closed, meaning that the values of the minimum and maximum membership of a fuzzy set are closed limits.

4. The level of cut set must have closed boundaries.

Finding a Fuzzy TOPSIS with Alpha-Cut Sets

Most of the time, the researcher or decision maker needs to determine a reasonable level of fuzzy of the data or fuzzy sets, So he depends on the concept of the α -cut level, which is a fixed value within the interval $\alpha \in [0, 1]$. The cut level is a value chosen from the values of the vertical axis of the membership function. Alpha is

represented by a constant function, i.e. a horizontal straight line function that intersects the membership function (Zimmermann, 2001).

A fuzzy number \tilde{K} is a triangular fuzzy number denoted by (k_1, k, k_2) , and its membership is function $\mu_{\tilde{K}}(y_0)$ as:

$$\begin{aligned}\mu_{\tilde{K}}(y) &= \frac{y - k_1}{k - k_1}, y \in [k_1, k], \\ \mu_{\tilde{K}}(y) &= \frac{k_2 - y}{k_2 - k}, y \in [k, k_2], \\ \mu_{\tilde{K}}(y_0) &= 0, y \notin [k_1, k_2]\end{aligned}\quad (2)$$

The α -cut of triangular fuzzy number $\tilde{K} = (k_1, k, k_2)$, is a closed interval.

$$K_\alpha = [k_\alpha^L, k_\alpha^U] = [k_1 + (k - k_1)\alpha, k_2 - (k_2 - k)\alpha]$$

Where $K_\alpha = \{\tilde{y} \in \tilde{Y} \mid \mu_{\tilde{K}}(\tilde{y}) \geq \alpha\}$

$$\min\{\tilde{y} \in \tilde{Y} \mid \mu_{\tilde{K}}(\tilde{y}) \geq \alpha\}, \max\{\tilde{y} \in \tilde{Y} \mid \mu_{\tilde{K}}(\tilde{y}) \geq \alpha\} \\ [k_\alpha^L, k_\alpha^U] \quad (3)$$

Assume that a Fuzzy Multi-Criteria Decision Making (FMCDM) has (s) alternatives and (t) criteria, which can be written as a matrix:

$$\tilde{Y} = \begin{matrix} & C_1 & C_2 & \dots & C_t \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_s \end{matrix} & \begin{bmatrix} \tilde{y}_{11} & \tilde{y}_{12} & \dots & \tilde{y}_{1t} \\ \tilde{y}_{21} & \tilde{y}_{22} & \dots & \tilde{y}_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{y}_{s1} & \tilde{y}_{s2} & \dots & \tilde{y}_{st} \end{bmatrix} \end{matrix} \quad (4)$$

Where A_1, A_2, \dots, A_s represented alternatives, C_1, C_2, \dots, C_t are evaluation criteria. \tilde{y}_{ij} is the degree of predisposition A_i to criteria C_j .

If $\tilde{y}_{ij} = (k_{ij}^1, k_{ij}, k_{ij}^2)$, $i = 1, 2, \dots, s$, $j = 1, 2, \dots, t$ are triangular fuzzy numbers, a fuzzy TOPSIS method dependent on α -cut set as below:

1. Fuzzy decision matrix $\tilde{Y} = (\tilde{y}_{ij})_{s \times t}$, using the following formulas:

$$\begin{aligned}\tilde{y}_{ij} &= \left(\frac{k_{ij}^1}{k_j^{2*}}, \frac{k_{ij}}{k_j^{2*}}, \frac{k_{ij}^2}{k_j^{2*}} \right), i = 1, 2, \dots, s; j \in \beta_k \\ \tilde{y}_{ij} &= \left(\frac{k_j^-}{k_{ij}^2}, \frac{k_j^-}{k_{ij}}, \frac{k_j^-}{k_{ij}^1} \right), i = 1, 2, \dots, s; j \in \beta_k -\end{aligned}\quad (5)$$

and formula (4), emphasise that this step is for the normalization purpose.

Where $k_j^{2*} = \max k_{ij}^2$, $k_j^- = \min k_{ij}^1$.

2. Compute α -cut set for each \tilde{y}_{ij} , $i = 1, 2, \dots, s$, $j = 1, 2, \dots, t$ by setting different α levels.

3. Calculate the respective fuzzy proximity of each alternative for each α -cut level as:

$$TF_i = \frac{\sqrt{\sum_{j=1}^t (\omega_j y_{ij})^2}}{\sqrt{\sum_{j=1}^t (\omega_j y_{ij})^2 + \sum_{j=1}^t (\omega_j (y_{ij} - 1))^2}} \tag{6}$$

Subject to $(\omega_j^{LO})_{\alpha} \leq \omega_j \leq (\omega_j^{UP})_{\alpha}$, $j = 1, 2, \dots, t$.

Where $\omega_j = (\omega_1, \omega_2, \dots, \omega_t)$, represented vector weights for each criterion which satisfy $\sum_{j=1}^t \omega_j = 1$, and the lower and upper bounds can be found as:

$$(TF_i^{LO})_{\alpha} = \min \frac{\sqrt{\sum_{j=1}^t (\omega_j (y_{ij}^{LO})_{\alpha})^2}}{\sqrt{\sum_{j=1}^t (\omega_j (y_{ij}^{LO})_{\alpha})^2 + \sum_{j=1}^t (\omega_j (y_{ij}^{LO})_{\alpha} - 1)^2}} \tag{7}$$

$$(TF_i^{UP})_{\alpha} = \max \frac{\sqrt{\sum_{j=1}^t (\omega_j (y_{ij}^{UP})_{\alpha})^2}}{\sqrt{\sum_{j=1}^t (\omega_j (y_{ij}^{UP})_{\alpha})^2 + \sum_{j=1}^t (\omega_j (y_{ij}^{UP})_{\alpha} - 1)^2}} \tag{8}$$

Subject to

$$(\omega_j^{LO})_{\alpha} \leq \omega_j \leq (\omega_j^{UP})_{\alpha}, j = 1, 2, \dots, t$$

$$(y_{ij}^{LO})_{\alpha} \leq y_{ij} \leq (y_{ij}^{UP})_{\alpha}, j = 1, 2, \dots, t.$$

Such that:

$$(y_{ij})_{\alpha} = \left[(y_{ij}^{LO})_{\alpha}, (y_{ij}^{UP})_{\alpha} \right], (\omega_j)_{\alpha} = \left[(\omega_j^{LO})_{\alpha}, (\omega_j^{UP})_{\alpha} \right],$$

are the α -cut set of each y_{ij} and $\omega_j, \alpha = [0, 1]$.

4. Defuzzified respective fuzzy proximity by

$$\frac{1}{N} \sum_{j=1}^N \left[\frac{(y_{ij}^{LO})_{\alpha} + (y_{ij}^{UP})_{\alpha}}{2} \right] \tag{9}$$

5. Order the alternatives in which TF_i^* are the largest value.

Numerical Example (Application in the Airline Transportation)

We use the example proposed by Janic and Reggiani (2002) to examine the exactness of the analytical method. Assuming that there is an airline operating a wide network of air transport lines, this company wants to search for a new additional command center, seven available alternatives (airports) have been identified as candidate sites (A1, A2, A3, A4, A5, A6, A7). The aim of this numerical example is to illustrate how to implement a fuzzyTOPSS method when making a comparison between the seven airports with the presence of nine criteria (C1: Residents gathered in the airport area (million), C2: Per capita income share, C3: Airport size (millions of passengers per year), C4: The lowest cost of arrival for the traveler in dollars, C5: Total airline cost of operating two-pivot and transport network (million €), C6: Average cost of airport service €, C7: Airport capacity (aircraft/hour), C8: Market share of the airport(%), C9: Benefits of airport capacity in peaks time. In order to apply fuzzy TOPSIS methods, the values of corresponding attributes are arranged out for each of seven preselected alternative airports and given as criteria in **Table 1**.

Table 1. A Fuzzy Decision Matrix and Weights of Each Criteria

| Criteria / Alternative | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|------------------------|------------------|-------|---------------------|--------------------|------------------|------------------|--------------|--------------|--------------|
| A1 | (0.9, 1.1, 1.2) | 15423 | (18.25, 18.5, 18.5) | (12.75, 13.28, 14) | (1.3, 1.56, 1.8) | (4.9, 5.16, 5.5) | (68, 70, 70) | (65, 66, 66) | (76, 77, 78) |
| A2 | (5.25, 6.3, 7) | 16468 | (38, 38.6, 38.75) | (20, 21.73, 22) | (1.4, 1.61, 2) | (2, 2.71, 3) | (81, 84, 86) | (63, 63, 65) | (72, 74, 74) |
| A3 | (3, 3.6, 4) | 18308 | (41, 42.7, 43) | (7.75, 8.12, 9) | (1.3, 1.62, 1.8) | (2, 2.16, 2.8) | (68, 72, 73) | (60, 61, 62) | (81, 84, 86) |
| A4 | (2.75, 3, 3.4) | 18200 | (15, 15.8, 16.25) | (9, 9.3, 9.8) | (1.9, 2.18, 2.3) | (6, 6.62, 7) | (33, 34, 36) | (33, 33, 35) | (78, 79, 79) |
| A5 | (0.85, 1.1, 1.3) | 15111 | (34.25, 34.4, 36) | (8, 8.32, 9) | (1.4, 1.65, 1.8) | (2.3, 2.81, 3.3) | (88, 90, 91) | (65, 66, 67) | (65, 68, 69) |
| A6 | (4, 4.2, 4.5) | 13293 | 59, 60.7, 61) | (21, 21.64, 22) | (1.3, 1.68, 1.9) | (1.3, 1.76, 2) | (78, 78, 78) | (38, 39, 40) | (93, 93, 94) |
| A7 | (4, 4.3, 4.5) | 15589 | (12.75, 13.6, 14) | (14, 14.47, 15) | (2, 2.25, 3) | (6.8, 7.37, 8) | (32, 32, 34) | (63, 64, 65) | (57, 59, 61) |
| Weight ω_j | 0.238 | 0.010 | 0.212 | 0.129 | 0.020 | 0.225 | 0.099 | 0.050 | 0.017 |

Step 1: By using the equation (5), we get a fuzzy normalized decision matrix in **Table 2**.

Table 2. A Fuzzy Normalized Decision Matrix

| Criteria / Alternative | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|------------------------|-----------------|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A1 | (.13, .16, .17) | .36 | (.3, .31, .31) | (.55, .58, .61) | (.72, .83, 1) | (.24, .25, .27) | (.75, .77, .77) | (.5, .5, .51) | (.73, .74, .75) |
| A2 | (.75, .9, 1) | .38 | (.62, .63, .64) | (.35, .36, .38) | (.65, .8, .93) | (.43, .48, .65) | (.89, .92, .94) | (.51, .52, .52) | (.77, .77, .79) |
| A3 | (.43, .51, .57) | .43 | (.67, .7, .71) | (.86, .95, .91) | (.72, .8, 1) | (.46, .6, .65) | (.75, .79, .8) | (.53, .54, .55) | (.66, .67, .7) |
| A4 | (.39, .47, .49) | .42 | (.25, .26, .27) | (.79, .83, .86) | (.57, .59, .68) | (.19, .2, .22) | (.36, .37, .4) | (.94, 1, 1) | (.72, .72, .73) |
| A5 | (.12, .16, .19) | .35 | (.56, .56, .59) | (.86, .93, .96) | (.72, .78, .92) | (.43, .46, .56) | (.96, .99, 1) | (.49, .5, .5) | (.82, .84, .87) |
| A6 | (.57, .6, .64) | .31 | (.96, .98, 1) | (.35, .36, .37) | (.68, .77, 1) | (.65, .73, 1) | (.85, .85, .85) | (.83, .85, .87) | (.6, .61, 1) |
| A7 | (.57, .61, .64) | .37 | (.21, .22, .23) | (.52, .54, .56) | (.43, .58, .65) | (.16, .18, .19) | (.35, .35, .37) | (.51, .52, .52) | (.93, .96, 1) |

Step 2: In **Table 3**, we get a fuzzy weighted normalized decision matrix.

Table 3. A Fuzzy Weighted Normalized Decision Matrix

| Criteria / Alternative | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|------------------------|-----------------|------|-----------------|-----------------|--------------------|-----------------|-----------------|-----------------|-----------------|
| A1 | (.03, .04, .04) | .004 | (.06, .07, .07) | (.07, .07, .08) | (.01, .01, .02) | (.05, .06, .06) | (.07, .08, .08) | (.03, .03, .03) | (.01, .01, .01) |
| A2 | (.18, .21, .24) | .003 | (.13, .13, .14) | (.04, .04, .05) | (.013, .016, .019) | (.0, .11, .15) | (.09, .09, .09) | (.03, .03, .03) | (.01, .01, .01) |
| A3 | (.09, .12, .14) | .004 | (.14, .15, .15) | (.11, .12, .13) | (.01, .016, .02) | (.10, .14, .15) | (.07, .08, .08) | (.03, .03, .03) | (.01, .01, .01) |
| A4 | (.09, .11, .12) | .004 | (.05, .06, .06) | (.10, .11, .11) | (.011, .012, .014) | (.04, .05, .05) | (.04, .04, .04) | (.05, .05, .05) | (.01, .01, .01) |

| Criteria/ Alternative | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|--------------------------|-----------------|------|-----------------|-----------------|-------------------|-----------------|-----------------|-----------------|-----------------|
| A5 | (.03, .04, .05) | .004 | (.12, .12, .13) | (.11, .12, .12) | (.01, .01, .02) | (.10, .10, .10) | (.10, .10, .10) | (.02, .03, .03) | (.01, .01, .04) |
| A6 | (.14, .14, .15) | .003 | (.20, .21, .21) | (.04, .05, .05) | (.014, .015, .02) | (.15, .16, .20) | (.08, .08, .08) | (.04, .04, .04) | (.01, .01, .02) |
| A7 | (.14, .14, .15) | .003 | (.04, .05, .05) | (.06, .07, .07) | (.01, .01, .01) | (.04, .04, .04) | (.03, .03, .03) | (.03, .03, .03) | (.01, .01, .02) |

Step 3: Write the average fuzzy respect for all airport locations from A1 to A7. The average fuzzy evaluations with α level can be written as:

$$\tilde{A}_1\tilde{C}_1(\alpha) = [(A_1C_1)^{Lo}(\alpha), (A_1C_1)^{Up}(\alpha)] ; = [0.03+0.01\alpha, 0.04], \text{ Notice that criteria C2 is crisp } [0.004],$$

$$\tilde{A}_1\tilde{C}_3(\alpha) = [(A_1C_3)^{Lo}(\alpha), (A_1C_3)^{Up}(\alpha)] ; = [0.06+0.01\alpha, 0.07]; \tilde{A}_1\tilde{C}_4(\alpha) = [(A_1C_4)^{Lo}(\alpha), (A_1C_4)^{Up}(\alpha)] = [0.07, 0.08-0.01\alpha]$$

$$\tilde{A}_1\tilde{C}_5(\alpha) = [(A_1C_5)^{Lo}(\alpha), (A_1C_5)^{Up}(\alpha)] ; = [0.01, 0.02-0.01\alpha]; \tilde{A}_1\tilde{C}_6(\alpha) = [(A_1C_6)^{Lo}(\alpha), (A_1C_6)^{Up}(\alpha)] ; = [0.05+0.01\alpha, 0.06]$$

$$\tilde{A}_1\tilde{C}_7(\alpha) = [(A_1C_7)^{Lo}(\alpha), (A_1C_7)^{Up}(\alpha)] ; = [0.07+0.01\alpha, 0.08]; \tilde{A}_1\tilde{C}_8(\alpha) = [(A_1C_8)^{Lo}(\alpha), (A_1C_8)^{Up}(\alpha)] ; = [0.03]$$

$$\tilde{A}_1\tilde{C}_9(\alpha) = [(A_1C_9)^{Lo}(\alpha), (A_1C_9)^{Up}(\alpha)] ; = [0.01]; \tilde{A}_7\tilde{C}_3(\alpha) = [(A_7C_3)^{Lo}(\alpha), (A_7C_3)^{Up}(\alpha)] ; = [0.04+0.01\alpha, 0.05]$$

$$\tilde{A}_7\tilde{C}_4(\alpha) = [(A_7C_4)^{Lo}(\alpha), (A_7C_4)^{Up}(\alpha)] ; = [0.06+0.01\alpha, 0.07]; \tilde{A}_7\tilde{C}_5(\alpha) = [(A_7C_5)^{Lo}(\alpha), (A_7C_5)^{Up}(\alpha)] ; = [0.01]$$

$$\tilde{A}_7\tilde{C}_6(\alpha) = [(A_7C_6)^{Lo}(\alpha), (A_7C_6)^{Up}(\alpha)] ; = [0.04]; \tilde{A}_7\tilde{C}_7(\alpha) = [(A_7C_7)^{Lo}(\alpha), (A_7C_7)^{Up}(\alpha)] ; = [0.03]$$

$$\tilde{A}_7\tilde{C}_8(\alpha) = [(A_7C_8)^{Lo}(\alpha), (A_7C_8)^{Up}(\alpha)] ; = [0.03]; \tilde{A}_7\tilde{C}_9(\alpha) = [(A_7C_9)^{Lo}(\alpha), (A_7C_9)^{Up}(\alpha)] ; = [0.01, 0.02-0.01\alpha]$$

The results are summarized in **Table 4**.

Table 4. Alpha-level Sets of a Fuzzyrelative Closeness of the (Seven Alternative Airport Candidates) with Criteria 1

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|----------|---------------|----------------|---------------|----------------|----------------|---------------|---------------|
| 0 | [0.03, 0.04] | [0.18, 0.24] | [0.03, 0.12] | [0.09, 0.12] | [0.03, 0.05] | [0.14, 0.15] | [0.14, 0.15] |
| .1 | [0.031, 0.04] | [0.183, 0.237] | [0.033, .118] | [0.092, 0.119] | [0.031, 0.049] | [0.14, 0.149] | [0.14, 0.149] |
| .2 | [0.032, 0.04] | [0.186, 0.234] | [0.036, .116] | [0.094, 0.118] | [0.032, 0.048] | [0.14, 0.148] | [0.14, 0.148] |
| .3 | [0.033, 0.04] | [0.189, 0.231] | [0.039, .114] | [0.096, 0.117] | [0.033, 0.047] | [0.14, 0.147] | [0.14, 0.147] |
| .4 | [0.034, 0.04] | [0.192, 0.228] | [0.042, .112] | [0.098, 0.116] | [0.034, 0.046] | [0.14, 0.146] | [0.14, 0.146] |
| .5 | [0.035, 0.04] | [0.195, 0.225] | [0.045, .110] | [0.100, 0.115] | [0.035, 0.045] | [0.14, 0.145] | [0.14, 0.145] |
| .6 | [0.036, 0.04] | [0.198, 0.223] | [0.048, .108] | [0.102, 0.114] | [0.036, 0.044] | [0.14, 0.144] | [0.14, 0.144] |
| .7 | [0.037, 0.04] | [0.201, 0.220] | [0.051, .106] | [0.104, 0.113] | [0.037, 0.043] | [0.14, 0.143] | [0.14, 0.143] |
| .8 | [0.038, 0.04] | [0.204, 0.217] | [0.054, .104] | [0.106, 0.112] | [0.038, 0.042] | [0.14, 0.142] | [0.14, 0.142] |
| .9 | [0.039, 0.04] | [0.207, 0.214] | [0.057, .102] | [0.108, 0.111] | [0.039, 0.041] | [0.14, 0.141] | [0.14, 0.141] |
| 1 | [0.04, 0.04] | [0.210, 0.211] | [0.060, .100] | [0.110, 0.110] | [0.040, 0.040] | [0.14, 0.140] | [0.14, 0.140] |

By using formula (9), we get the following Defuzzified values of criteria 1, as shown in **Table 5**.

Table 5. Defuzzified Values of Criteria 1

| | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|--------|-------|--------|--------|-------|--------|--------|
| Defuzzified | 0.0375 | 0.210 | 0.0775 | 0.1075 | 0.040 | 0.1425 | 0.1425 |

As the same procedure, obtain Alpha-level sets of a fuzzy relative closeness of the (seven alternative airport candidates) with criteria 3 to criteria 9 in **APPENDIX A**. In the next step, construct the crisp weighted normalized decision matrix as in **Table 6**.

Table 6. The Crisp Weighted Normalized Decision Matrix

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|--------|------|--------|--------|---------|--------|--------|--------|--------|
| A1 | 0.0375 | .004 | 0.0775 | 0.0725 | 0.040 | 0.0575 | 0.0775 | 0.03 | 0.01 |
| A2 | 0.210 | .003 | 0.1325 | 0.0475 | 0.016 | 0.115 | 0.09 | 0.03 | 0.01 |
| A3 | 0.0775 | .004 | 0.1475 | 0.12 | 0.039 | 0.1325 | 0.0775 | 0.03 | 0.01 |
| A4 | 0.1075 | .004 | 0.0575 | 0.1075 | 0.01225 | 0.0475 | 0.04 | 0.05 | 0.01 |
| A5 | 0.040 | .004 | 0.1225 | 0.1135 | 0.0125 | 0.1 | 0.1 | 0.0275 | 0.0175 |
| A6 | 0.1425 | .003 | 0.2075 | 0.0475 | 0.0365 | 0.1675 | 0.08 | 0.04 | 0.0125 |
| A7 | 0.1425 | .003 | 0.0475 | 0.0675 | 0.01 | 0.04 | 0.03 | 0.03 | 0.0125 |

Determine the positive ideal and negative ideal solutions, positive ideal $\beta^+ = \{y_1^*, \dots, y_n^*\}$, where, $y_j^* = \max(y_{ij})$, $\beta^+ = \{0.210, 0.004, 0.2075, 0.12, 0.040, 0.1675, 0.1, 0.05, 0.0175\}$

Calculate the separation measures for each alternative, the separation from the positive ideal alternative as shown in **Tables 7** and **8**.

$$SA_i^* = [\sum (y_j^* - y_{ij})^2]^{\frac{1}{2}}, i = 1, \dots, m$$

Table 7. The Separation from the Positive Ideal Alternative

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|---------------------------|--------------------------|----------------------------|--------------------------|----------------------------|----------------------------|-------------------------|--------------------------|----------------------------|
| A1 | (.0375-.210) ² | (.004-.004) ² | (.0775-.2075) ² | (.0725-.12) ² | (.040-.040) ² | (.0575-.1675) ² | (.0775-.1) ² | (.03-.05) ² | (.01-.0175) ² |
| A2 | (.210-.210) ² | (.003-.004) ² | (.1325-.2075) ² | (.0475-.12) ² | (.016-.040) ² | (.115-.1675) ² | (.09-.1) ² | (.03-.05) ² | (.01-.0175) ² |
| A3 | (.0775-.210) ² | (.004-.004) ² | (.1475-.2075) ² | (.12-.12) ² | (.039-.040) ² | (.1325-.1675) ² | (.0775-.1) ² | (.03-.05) ² | (.01-.0175) ² |
| A4 | (.1075-.210) ² | (.004-.004) ² | (.0575-.2075) ² | (.1075-.12) ² | (.01225-.040) ² | (.0475-.1675) ² | (.04-.1) ² | (.05-.05) ² | (.01-.0175) ² |
| A5 | (.040-.210) ² | (.004-.004) ² | (.1225-.2075) ² | (.1135-.12) ² | (.0125-.040) ² | (.1-.1675) ² | (.1-.1) ² | (.0275-.05) ² | (.0175-.0175) ² |
| A6 | (.1425-.210) ² | (.003-.004) ² | (.2075-.2075) ² | (.0475-.12) ² | (.0365-.040) ² | (.1675-.1675) ² | (.08-.1) ² | (.04-.05) ² | (.0125-.0175) ² |
| A7 | (.1425-.210) ² | (.003-.004) ² | (.0475-.2075) ² | (.0675-.12) ² | (.01-.040) ² | (.04-.1675) ² | (.03-.1) ² | (.03-.05) ² | (.0125-.0175) ² |

Table 8. The Separation from the Positive Ideal Alternative

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|-------|---------|-------|--------|--------|-------|-------|-------|--------|
| A1 | .0297 | 0 | .0169 | .0023 | 0 | .0121 | .0005 | .0004 | .0001 |
| A2 | 0 | .000001 | .0056 | .0053 | .0006 | .0028 | .0001 | .0004 | .0001 |
| A3 | .0176 | .000001 | .0036 | 0 | .0001 | .0012 | .0005 | .0004 | .0001 |
| A4 | .0105 | .000001 | .0225 | .0002 | .0008 | .0144 | .0036 | 0 | .0001 |
| A5 | .0289 | .000001 | .0072 | .00004 | .0008 | .0046 | 0 | .0005 | 0 |
| A6 | .0046 | .000001 | 0 | .0053 | .00001 | 0 | .0004 | .0001 | .00003 |
| A7 | .0046 | .000001 | .0256 | .0028 | .0009 | .0163 | .0049 | .0004 | .00003 |

$$SA_1^* = 0.25, SA_2^* = 0.122, SA_3^* = 0.153, SA_4^* = 0.228, SA_5^* = 0.205, SA_6^* = 0.229, SA_7^* = 0.236$$

The negative ideal solution, $\beta^- = \{y_j^-, \dots, y_n^-\}$, where, $y_j^- = \min(y_{ij})$, $\beta^- = \{0.0375, 0.003, 0.0475, 0.0475, 0.01, 0.04, 0.1, 0.03, 0.0275, 0.01\}$

Calculate the separation measures for each alternative, the separation from the negative ideal alternative as shown in **Tables 9** and **10**.

$$SA_i^- = [\sum (y_j^- - y_{ij})^2]^{\frac{1}{2}}, i = 1, \dots, m$$

Table 9. The Separation from the Negative Ideal Alternative

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|----------------------------|--------------------------|----------------------------|----------------------------|----------------------------|--------------------------|--------------------------|----------------------------|--------------------------|
| A1 | (.0375-.0375) ² | (.004-.003) ² | (.0775-.0475) ² | (.0725-.0475) ² | (.040-.010) ² | (.0575-.04) ² | (.0775-.03) ² | (.03-.0275) ² | (.01-.01) ² |
| A2 | (.210-.0375) ² | (.003-.003) ² | (.1325-.0475) ² | (.0475-.0475) ² | (.016-.010) ² | (.115-.04) ² | (.09-.03) ² | (.03-.0275) ² | (.01-.01) ² |
| A3 | (.0775-.0375) ² | (.004-.003) ² | (.1475-.0475) ² | (.12-.0475) ² | (.039-.010) ² | (.1325-.04) ² | (.0775-.03) ² | (.03-.0275) ² | (.01-.01) ² |
| A4 | (.1075-.0375) ² | (.004-.003) ² | (.0575-.0475) ² | (.1075-.0475) ² | (.01225-.010) ² | (.0475-.04) ² | (.04-.03) ² | (.05-.0275) ² | (.01-.01) ² |
| A5 | (.040-.0375) ² | (.004-.003) ² | (.1225-.0475) ² | (.1135-.0475) ² | (.0125-.010) ² | (.1-.04) ² | (.1-.03) ² | (.0275-.0275) ² | (.0175-.01) ² |
| A6 | (.1425-.0375) ² | (.003-.003) ² | (.2075-.0475) ² | (.0475-.0475) ² | (.0365-.010) ² | (.1675-.04) ² | (.08-.03) ² | (.04-.0275) ² | (.0125-.01) ² |
| A7 | (.1425-.0375) ² | (.003-.003) ² | (.0475-.0475) ² | (.0675-.0475) ² | (.01-.010) ² | (.04-.04) ² | (.03-.03) ² | (.03-.0275) ² | (.0125-.01) ² |

Table 10. The Separation from the Negative Ideal Alternative

| | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| A1 | 0 | .000001 | .0009 | .0006 | .0009 | .00031 | .0023 | .00001 | 0 |
| A2 | .0298 | 0 | .0072 | 0 | .00025 | .0056 | .0036 | .00001 | 0 |
| A3 | .0016 | .000001 | .01 | .006 | .0008 | .0085 | .0023 | .00001 | 0 |
| A4 | .0049 | .000001 | .0001 | .0036 | .00001 | .0006 | .0001 | .0005 | 0 |
| A5 | .00001 | .000001 | .0056 | .004 | .00001 | .0036 | .0049 | 0 | .0001 |
| A6 | .011 | 0 | .0256 | 0 | .0007 | .0163 | .0025 | .00016 | .0001 |
| A7 | .011 | 0 | 0 | .0004 | 0 | 0 | 0 | .00001 | .0001 |

$$SA_1^- = 0.071, SA_2^- = 0.216, SA_3^- = 0.171, SA_4^- = 0.135, SA_5^- = 0.237, SA_6^- = 0.281, SA_7^- = 0.107$$

Finally, calculate the relative closeness to the ideal solution as $C^*A(i) = \frac{SA_i^-}{(SA_i^- + SA_i^*)}$,

$$C^*A(1) = 0.071 / (0.25 + 0.071) = 0.22$$

$$C^*A(2) = 0.216 / (0.122 + 0.216) = 0.56$$

$$C^*A(3) = 0.171 / (0.153 + 0.171) = 0.53$$

$$C^*A(4) = 0.135 / (0.135 + 0.228) = 0.32$$

$$C^*A(5) = 0.237 / (0.237 + 0.205) = 0.54$$

$$C^*A(6) = 0.281 / (0.281 + 0.229) = 0.55$$

$$C^*A(7) = 0.107 / (0.107 + 0.236) = 0.31$$

From the results, we obtain the optimal determination of the located new airport in the high score that corresponds with alternative 7.

CONCLUSION

The logical basis of fuzzy approaches is to remove fuzziness from imprecise values at the end of the process, not at the beginning. Based on this rationale, we apply a fuzzy TOPSIS method based on alpha-level fuzzy sets for fuzzy MCDM. A fuzzy TOPSIS method integrates the crisp TOPSIS method for crisp MCDM with fuzzy numbers, eliminating fuzziness at the end of the decision analysis process. This paper has shown the application of one method of Multi-Criteria Decision-Making (MCDM), TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution), to the problem of determining a new airport location, in this application, seven European airports were selected as alternatives with nine related criteria. For each alternative, the attributes were quantified and then applied as estimation criteria. The values of alternatives with regard to the criteria are considered as fuzzy values (fuzzy numbers). We used a fuzzy TOPSIS method based on α -level sets of fuzzy MCDM. This method joins the TOPSIS method of crisp MCDM with fuzzy numbers and implements defuzzification at the finish of decision analysis procedure. Given the limitations of the results obtained from a fuzzy TOPSIS method, future research should focus on conducting additional tests to verify the feasibility and stability of the acquired solutions. Such research should include the use of alternative sets of options (e.g. airports) with the same or different performance criteria, and in the context of sensitivity analysis, the effects of using different methods for assigning weights to the criteria should be studied.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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APPENDIX A

Criteria 3

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0 | [0.03, 0.12] | [0.13, 0.14] | [0.14, 0.15] | [0.05, 0.06] | [0.12, 0.13] | [0.2, 0.21] | [0.04, 0.05] |
| .1 | [0.033, .118] | [0.13, 0.139] | [0.141, 0.15] | [0.051, 0.06] | [0.12, 0.129] | [0.201, 0.21] | [0.041, 0.05] |
| .2 | [0.036, .116] | [0.13, 0.138] | [0.142, 0.15] | [0.052, 0.06] | [0.12, 0.128] | [0.202, 0.21] | [0.042, 0.05] |
| .3 | [0.039, .114] | [0.13, 0.137] | [0.143, 0.15] | [0.053, 0.06] | [0.12, 0.127] | [0.203, 0.21] | [0.043, 0.05] |
| .4 | [0.042, .112] | [0.13, 0.136] | [0.144, 0.15] | [0.054, 0.06] | [0.12, 0.126] | [0.204, 0.21] | [0.044, 0.05] |
| .5 | [0.045, .110] | [0.13, 0.135] | [0.145, 0.15] | [0.055, 0.06] | [0.12, 0.125] | [0.205, 0.21] | [0.045, 0.05] |
| .6 | [0.048, .108] | [0.13, 0.134] | [0.146, 0.15] | [0.056, 0.06] | [0.12, 0.124] | [0.206, 0.21] | [0.046, 0.05] |
| .7 | [0.051, .106] | [0.13, 0.133] | [0.147, 0.15] | [0.057, 0.06] | [0.12, 0.123] | [0.207, 0.21] | [0.047, 0.05] |
| .8 | [0.054, .104] | [0.13, 0.132] | [0.148, 0.15] | [0.058, 0.06] | [0.12, 0.122] | [0.208, 0.21] | [0.048, 0.05] |
| .9 | [0.057, .102] | [0.13, 0.131] | [0.149, 0.15] | [0.059, 0.06] | [0.12, 0.121] | [0.209, 0.21] | [0.049, 0.05] |
| 1 | [0.060, .100] | [0.13, 0.130] | [0.15, 0.15] | [0.06, 0.06] | [0.12, 0.12] | [0.2, 0.21] | [0.05, 0.05] |
| Defuzzified | 0.0775 | 0.1325 | 0.1475 | 0.0575 | 0.1225 | 0.2075 | 0.0475 |

Criteria 4

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|---------------|---------------|----------------|---------------|---------------|---------------|---------------|
| 0 | [0.07, 0.08] | [0.04, 0.05] | [0.11, 0.13] | [0.10, 0.11] | [0.11, 0.12] | [0.04, 0.05] | [0.06, 0.07] |
| .1 | [0.07, 0.079] | [0.041, 0.05] | [0.111, 0.129] | [0.101, 0.11] | [0.111, 0.12] | [0.041, 0.05] | [0.061, 0.07] |
| .2 | [0.07, 0.078] | [0.042, 0.05] | [0.112, 0.128] | [0.102, 0.11] | [0.112, 0.12] | [0.042, 0.05] | [0.062, 0.07] |
| .3 | [0.07, 0.077] | [0.043, 0.05] | [0.113, 0.127] | [0.103, 0.11] | [0.113, 0.12] | [0.043, 0.05] | [0.063, 0.07] |
| .4 | [0.07, 0.076] | [0.044, 0.05] | [0.114, 0.126] | [0.104, 0.11] | [0.114, 0.12] | [0.044, 0.05] | [0.064, 0.07] |
| .5 | [0.07, 0.075] | [0.045, 0.05] | [0.115, 0.125] | [0.105, 0.11] | [0.115, 0.12] | [0.045, 0.05] | [0.065, 0.07] |
| .6 | [0.07, 0.074] | [0.046, 0.05] | [0.116, 0.124] | [0.106, 0.11] | [0.116, 0.12] | [0.046, 0.05] | [0.066, 0.07] |
| .7 | [0.07, 0.073] | [0.047, 0.05] | [0.117, 0.123] | [0.107, 0.11] | [0.117, 0.12] | [0.047, 0.05] | [0.067, 0.07] |
| .8 | [0.07, 0.072] | [0.048, 0.05] | [0.118, 0.122] | [0.108, 0.11] | [0.118, 0.12] | [0.048, 0.05] | [0.068, 0.07] |
| .9 | [0.07, 0.071] | [0.049, 0.05] | [0.119, 0.121] | [0.109, 0.11] | [0.119, 0.12] | [0.049, 0.05] | [0.069, 0.07] |
| 1 | [0.07, 0.07] | [0.05, 0.05] | [0.12, 0.12] | [0.11, 0.11] | [0.12, 0.12] | [0.05, 0.05] | [0.07, 0.07] |
| Defuzzified | 0.0725 | 0.0475 | 0.12 | 0.1075 | 0.1135 | 0.0475 | 0.0675 |

Criteria 5

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|----------|----------------|------------------|------------------|-------------------|---------------|------------------|--------|
| 0 | [0.03, 0.05] | [0.013, 0.019] | [0.01, 0.06] | [0.011, 0.014] | [0.01, 0.02] | [0.014, 0.06] | [0.01] |
| .1 | [0.031, 0.049] | [0.0133, 0.0187] | [0.012, 0.0596] | [0.0111, 0.0138] | [0.01, 0.019] | [0.0141, 0.0597] | [0.01] |
| .2 | [0.032, 0.048] | [0.0136, 0.0184] | [0.014, 0.0592] | [0.0112, 0.0136] | [0.01, 0.018] | [0.0142, 0.0594] | [0.01] |
| .3 | [0.033, 0.047] | [0.0139, 0.0181] | [0.0116, 0.0588] | [0.0113, 0.0134] | [0.01, 0.017] | [0.0143, 0.0591] | [0.01] |
| .4 | [0.034, 0.046] | [0.0142, 0.0178] | [0.018, 0.0584] | [0.0114, 0.0132] | [0.01, 0.016] | [0.0144, 0.0588] | [0.01] |
| .5 | [0.035, 0.045] | [0.0145, 0.0175] | [0.020, 0.0580] | [0.0115, 0.0130] | [0.01, 0.015] | [0.0145, 0.0585] | [0.01] |
| .6 | [0.036, 0.044] | [0.0148, 0.0173] | [0.022, 0.0576] | [0.0116, 0.01128] | [0.01, 0.014] | [0.0146, 0.0582] | [0.01] |
| .7 | [0.037, 0.043] | [0.0151, 0.017] | [0.024, 0.0572] | [0.0117, 0.01126] | [0.01, 0.013] | [0.0147, 0.0579] | [0.01] |
| .8 | [0.038, 0.042] | [0.0154, 0.0167] | [0.028, 0.0568] | [0.0118, 0.0124] | [0.01, 0.012] | [0.0148, 0.0576] | [0.01] |
| .9 | [0.039, 0.041] | [0.0157, 0.0164] | [0.030, 0.0564] | [0.0119, 0.0122] | [0.01, 0.011] | [0.0149, 0.0573] | [0.01] |
| 1 | [0.040, 0.040] | [0.016, 0.0161] | [0.032, 0.0560] | [0.012, 0.012] | [0.01, 0.010] | [0.0150, 0.057] | [0.01] |

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|-------|-------|-------|---------|--------|--------|------|
| DefuzziFied | 0.040 | 0.016 | 0.039 | 0.01225 | 0.0125 | 0.0365 | 0.01 |

Criteria 6

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|--------------|---------------|----------------|----------------|---------------|-------|----------------|--------|
| 0 | [0.05, 0.06] | [0.09, 0.15] | [0.1, 0.15] | [0.04, 0.05] | [0.1] | [0.15, 0.2] | [0.04] |
| .1 | [0.051, 0.06] | [0.092, 0.146] | [0.104, 0.149] | [0.041, 0.05] | [0.1] | [0.151, 0.196] | [0.04] |
| .2 | [0.052, 0.06] | [0.094, 0.142] | [0.108, 0.148] | [0.042, 0.05] | [0.1] | [0.152, 0.192] | [0.04] |
| .3 | [0.053, 0.06] | [0.096, 0.138] | [0.112, 0.147] | [0.043, 0.05] | [0.1] | [0.153, 0.188] | [0.04] |
| .4 | [0.054, 0.06] | [0.098, 0.134] | [0.116, 0.146] | [0.044, 0.05] | [0.1] | [0.154, 0.184] | [0.04] |
| .5 | [0.055, 0.06] | [0.100, 0.130] | [0.120, 0.145] | [0.045, 0.05] | [0.1] | [0.155, 0.180] | [0.04] |
| .6 | [0.056, 0.06] | [0.102, 0.126] | [0.124, 0.144] | [0.046, 0.05] | [0.1] | [0.156, 0.176] | [0.04] |
| .7 | [0.057, 0.06] | [0.104, 0.122] | [0.128, 0.143] | [0.047, 0.05] | [0.1] | [0.157, 0.172] | [0.04] |
| .8 | [0.058, 0.06] | [0.106, 0.118] | [0.132, 0.142] | [0.048, 0.05] | [0.1] | [0.158, 0.168] | [0.04] |
| .9 | [0.059, 0.06] | [0.108, 0.114] | [0.136, 0.141] | [0.049, 0.05] | [0.1] | [0.159, 0.164] | [0.04] |
| 1 | [0.06, 0.06] | [0.110, 0.110] | [0.140, 0.140] | [0.05, 0.05] | [0.1] | [0.160, 0.160] | [0.04] |
| Defuz zified | 0.0575 | 0.115 | 0.1325 | 0.0475 | 0.1 | 0.1675 | 0.04 |

Criteria 7

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|---------------|--------|---------------|--------|-------|--------|--------|
| 0 | [0.07, 0.08] | [0.09] | [0.07, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .1 | [0.071, 0.08] | [0.09] | [0.071, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .2 | [0.072, 0.08] | [0.09] | [0.072, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .3 | [0.073, 0.08] | [0.09] | [0.073, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .4 | [0.074, 0.08] | [0.09] | [0.074, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .5 | [0.075, 0.08] | [0.09] | [0.075, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .6 | [0.076, 0.08] | [0.09] | [0.076, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .7 | [0.077, 0.08] | [0.09] | [0.077, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .8 | [0.078, 0.08] | [0.09] | [0.078, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| .9 | [0.079, 0.08] | [0.09] | [0.079, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| 1 | [0.08, 0.08] | [0.09] | [0.08, 0.08] | [0.04] | [0.1] | [0.08] | [0.03] |
| Defuzzified | 0.0775 | 0.09 | 0.0775 | 0.04 | 0.1 | 0.08 | 0.03 |

Criteria 8

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|--------|--------|--------|--------|---------------|--------|--------|
| 0 | [0.03] | [0.03] | [0.03] | [0.05] | [0.02, 0.03] | [0.04] | [0.03] |
| .1 | [0.03] | [0.03] | [0.03] | [0.05] | [0.021, 0.03] | [0.04] | [0.03] |
| .2 | [0.03] | [0.03] | [0.03] | [0.05] | [0.022, 0.03] | [0.04] | [0.03] |
| .3 | [0.03] | [0.03] | [0.03] | [0.05] | [0.023, 0.03] | [0.04] | [0.03] |
| .4 | [0.03] | [0.03] | [0.03] | [0.05] | [0.024, 0.03] | [0.04] | [0.03] |
| .5 | [0.03] | [0.03] | [0.03] | [0.05] | [0.025, 0.03] | [0.04] | [0.03] |
| .6 | [0.03] | [0.03] | [0.03] | [0.05] | [0.026, 0.03] | [0.04] | [0.03] |
| .7 | [0.03] | [0.03] | [0.03] | [0.05] | [0.027, 0.03] | [0.04] | [0.03] |
| .8 | [0.03] | [0.03] | [0.03] | [0.05] | [0.028, 0.03] | [0.04] | [0.03] |
| .9 | [0.03] | [0.03] | [0.03] | [0.05] | [0.029, 0.03] | [0.04] | [0.03] |
| 1 | [0.03] | [0.03] | [0.03] | [0.05] | [0.03, 0.03] | [0.04] | [0.03] |
| Defuzzified | 0.03 | 0.03 | 0.03 | 0.05 | 0.0275 | 0.04 | 0.03 |

Criteria 9

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|----------|--------|--------|--------|--------|---------------|---------------|---------------|
| 0 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.04] | [0.01, 0.02] | [0.01, 0.02] |
| .1 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.037] | [0.01, 0.019] | [0.01, 0.019] |
| .2 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.034] | [0.01, 0.018] | [0.01, 0.018] |
| .3 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.031] | [0.01, 0.017] | [0.01, 0.017] |
| .4 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.028] | [0.01, 0.016] | [0.01, 0.016] |
| .5 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.025] | [0.01, 0.015] | [0.01, 0.015] |

| α | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|-------------|--------|--------|--------|--------|---------------|---------------|---------------|
| .6 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.022] | [0.01, 0.014] | [0.01, 0.014] |
| .7 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.019] | [0.01, 0.013] | [0.01, 0.013] |
| .8 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.016] | [0.01, 0.012] | [0.01, 0.012] |
| .9 | [0.01] | [0.01] | [0.01] | [0.01] | [0.01, 0.013] | [0.01, 0.011] | [0.01, 0.011] |
| 1 | [0.01] | [0.01] | 0.01 | 0.01 | [0.01, 0.010] | [0.01, 0.010] | [0.01, 0.010] |
| Defuzzified | 0.01 | 0.01 | 0.01 | 0.01 | 0.0175 | 0.0125 | 0.0125 |

$$\begin{aligned}
\tilde{A}_2\tilde{C}_1(\alpha) &= [(A_2C_1)^{Lo}(\alpha), (A_2C_1)^{Up}(\alpha)]; = [0.18+0.03 \alpha, 0.24-0.03 \alpha], \\
\tilde{A}_2\tilde{C}_3(\alpha) &= [(A_2C_3)^{Lo}(\alpha), (A_2C_3)^{Up}(\alpha)]; = [0.13, 0.14-0.01 \alpha] \\
\tilde{A}_2\tilde{C}_4(\alpha) &= [(A_2C_4)^{Lo}(\alpha), (A_2C_4)^{Up}(\alpha)]; = [0.04, 0.05-0.01 \alpha] \\
\tilde{A}_2\tilde{C}_5(\alpha) &= [(A_2C_5)^{Lo}(\alpha), (A_2C_5)^{Up}(\alpha)]; = [0.013+0.003 \alpha, 0.019-0.003 \alpha] \\
\tilde{A}_2\tilde{C}_6(\alpha) &= [(A_2C_6)^{Lo}(\alpha), (A_2C_6)^{Up}(\alpha)]; = [0.09+0.02 \alpha, 0.15-0.04 \alpha] \\
\tilde{A}_2\tilde{C}_7(\alpha) &= [(A_2C_7)^{Lo}(\alpha), (A_2C_7)^{Up}(\alpha)]; = [0.09] \\
\tilde{A}_2\tilde{C}_8(\alpha) &= [(A_2C_8)^{Lo}(\alpha), (A_2C_8)^{Up}(\alpha)]; = [0.03] \\
\tilde{A}_2\tilde{C}_9(\alpha) &= [(A_2C_9)^{Lo}(\alpha), (A_2C_9)^{Up}(\alpha)]; = [0.01] \\
\tilde{A}_3\tilde{C}_1(\alpha) &= [(A_3C_1)^{Lo}(\alpha), (A_3C_1)^{Up}(\alpha)]; = [0.03+0.03 \alpha, 0.12-0.02 \alpha], \\
\tilde{A}_3\tilde{C}_3(\alpha) &= [(A_3C_3)^{Lo}(\alpha), (A_3C_3)^{Up}(\alpha)]; = [0.14+0.01 \alpha, 0.15] \\
\tilde{A}_3\tilde{C}_4(\alpha) &= [(A_3C_4)^{Lo}(\alpha), (A_3C_4)^{Up}(\alpha)]; = [0.11+ 0.01 \alpha, 0.13-0.01 \alpha] \\
\tilde{A}_3\tilde{C}_5(\alpha) &= [(A_3C_5)^{Lo}(\alpha), (A_3C_5)^{Up}(\alpha)]; = [0.01+0.02 \alpha, 0.02-0.04 \alpha] \\
\tilde{A}_3\tilde{C}_6(\alpha) &= [(A_3C_6)^{Lo}(\alpha), (A_3C_6)^{Up}(\alpha)]; = [0.1+0.04 \alpha, 0.15-0.01 \alpha] \\
\tilde{A}_3\tilde{C}_7(\alpha) &= [(A_3C_7)^{Lo}(\alpha), (A_3C_7)^{Up}(\alpha)]; = [0.07+0.01 \alpha, 0.08] \\
\tilde{A}_3\tilde{C}_8(\alpha) &= [(A_3C_8)^{Lo}(\alpha), (A_3C_8)^{Up}(\alpha)]; = [0.03] \\
\tilde{A}_3\tilde{C}_9(\alpha) &= [(A_3C_9)^{Lo}(\alpha), (A_3C_9)^{Up}(\alpha)]; = [0.01] \\
\tilde{A}_4\tilde{C}_1(\alpha) &= [(A_4C_1)^{Lo}(\alpha), (A_4C_1)^{Up}(\alpha)]; = [0.09+0.02 \alpha, 0.12-0.01 \alpha], \\
\tilde{C}_3(\alpha) &= [(A_4C_3)^{Lo}(\alpha), (A_4C_3)^{Up}(\alpha)]; = [0.05+0.01 \alpha, 0.06] \\
\tilde{A}_4\tilde{C}_4(\alpha) &= [(A_4C_4)^{Lo}(\alpha), (A_4C_4)^{Up}(\alpha)]; = [0.10+ 0.01 \alpha, 0.11] \\
\tilde{A}_4\tilde{C}_5(\alpha) &= [(A_4C_5)^{Lo}(\alpha), (A_4C_5)^{Up}(\alpha)]; = [0.011+0.001 \alpha, 0.014-0.002 \alpha] \\
\tilde{A}_4\tilde{C}_6(\alpha) &= [(A_4C_6)^{Lo}(\alpha), (A_4C_6)^{Up}(\alpha)]; = [0.04+0.01 \alpha, 0.05] \\
\tilde{A}_4\tilde{C}_7(\alpha) &= [(A_4C_7)^{Lo}(\alpha), (A_4C_7)^{Up}(\alpha)]; = [0.04] \\
\tilde{A}_4\tilde{C}_8(\alpha) &= [(A_4C_8)^{Lo}(\alpha), (A_4C_8)^{Up}(\alpha)]; = [0.05] \\
\tilde{A}_4\tilde{C}_9(\alpha) &= [(A_4C_9)^{Lo}(\alpha), (A_4C_9)^{Up}(\alpha)]; = [0.01] \\
\tilde{A}_5\tilde{C}_1(\alpha) &= [(A_5C_1)^{Lo}(\alpha), (A_5C_1)^{Up}(\alpha)]; = [0.03+0.01 \alpha, 0.05-0.01 \alpha], \\
\tilde{A}_5\tilde{C}_3(\alpha) &= [(A_5C_3)^{Lo}(\alpha), (A_5C_3)^{Up}(\alpha)]; = [0.12, 0.13-0.01 \alpha] \\
\tilde{A}_5\tilde{C}_4(\alpha) &= [(A_5C_4)^{Lo}(\alpha), (A_5C_4)^{Up}(\alpha)]; = [0.11+ 0.01 \alpha, 0.12] \\
\tilde{A}_5\tilde{C}_5(\alpha) &= [(A_5C_5)^{Lo}(\alpha), (A_5C_5)^{Up}(\alpha)]; = [0.01, 0.02-0.01 \alpha] \\
\tilde{A}_5\tilde{C}_6(\alpha) &= [(A_5C_6)^{Lo}(\alpha), (A_5C_6)^{Up}(\alpha)]; = [0.1] \\
\tilde{A}_5\tilde{C}_7(\alpha) &= [(A_5C_7)^{Lo}(\alpha), (A_5C_7)^{Up}(\alpha)]; = [0.1] \\
\tilde{A}_5\tilde{C}_8(\alpha) &= [(A_5C_8)^{Lo}(\alpha), (A_5C_8)^{Up}(\alpha)]; = [0.02+0.01 \alpha, 0.03] \\
\tilde{A}_5\tilde{C}_9(\alpha) &= [(A_5C_9)^{Lo}(\alpha), (A_5C_9)^{Up}(\alpha)]; = [0.01, 0.04-0.03 \alpha] \\
\tilde{A}_6\tilde{C}_1(\alpha) &= [(A_6C_1)^{Lo}(\alpha), (A_6C_1)^{Up}(\alpha)]; = [0.14, 0.15-0.01 \alpha], \\
\tilde{A}_6\tilde{C}_3(\alpha) &= [(A_6C_3)^{Lo}(\alpha), (A_6C_3)^{Up}(\alpha)]; = [0.2+0.01 \alpha, 0.21] \\
\tilde{A}_6\tilde{C}_4(\alpha) &= [(A_6C_4)^{Lo}(\alpha), (A_6C_4)^{Up}(\alpha)]; = [0.04+ 0.01 \alpha, 0.05] \\
\tilde{A}_6\tilde{C}_5(\alpha) &= [(A_6C_5)^{Lo}(\alpha), (A_6C_5)^{Up}(\alpha)]; = [0.014+0.001 \alpha, 0.06-0.03 \alpha] \\
\tilde{A}_6\tilde{C}_6(\alpha) &= [(A_6C_6)^{Lo}(\alpha), (A_6C_6)^{Up}(\alpha)]; = [0.15+0.01 \alpha, 0.2-0.04 \alpha] \\
\tilde{A}_6\tilde{C}_7(\alpha) &= [(A_6C_7)^{Lo}(\alpha), (A_6C_7)^{Up}(\alpha)]; = [0.08] \\
\tilde{A}_6\tilde{C}_8(\alpha) &= [(A_6C_8)^{Lo}(\alpha), (A_6C_8)^{Up}(\alpha)]; = [0.04]
\end{aligned}$$

$$\tilde{A}_6\tilde{C}_9(\alpha) = [(A_6C_9)^{Lo}(\alpha), (A_6C_9)^{Up}(\alpha)]; = [0.01, 0.02-0.01 \alpha]$$

$$\tilde{A}_7\tilde{C}_1(\alpha) = [(A_7C_1)^{Lo}(\alpha), (A_7C_1)^{Up}(\alpha)]; = [0.14, 0.15-0.01 \alpha]$$