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Research Article

Hinge Bidomination of a Graph

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ABSTRACT

Domination in graphs is a fundamental concept in graph theory with numerous applications across various fields. The introduction of many variants of domination stems from the diverse applications and the need to address different real-world problems more effectively. In this paper, we introduce a new variant of domination namely hinge bidomination in graphs. We find the hinge bidomination values of certain graphs and few results about the existence of hinge bidomination in graphs.

Keywords: Domination, hinge domination, Pendant vertex, bidomination, Book Graph, Dumbell Graph.

Introduction

The study of domination in graphs is a cornerstone of graph theory, with numerous applications ranging from network design to optimization problems. Traditional domination concepts focus on selecting a subset of vertices such that every vertex in the graph is either in this subset or adjacent to a vertex in the subset. However, as the complexity of real-world problems increases, so too does the need for more sophisticated models of domination. One such refinement is the concept of hinge bidomination. In network design and reliability, hinge bidomination can be crucial. Networks often need to ensure that even if some connections or nodes fail, the overall functionality remains intact. By identifying hinge bidominating sets, one can design networks that are resilient to failures by ensuring there are alternative pathways or critical redundancies. From a theoretical perspective, hinge bidomination enriches the understanding of graph properties and relationships. It provides a deeper insight into how domination concepts can be extended and applied to more complex scenarios, contributing to the broader field of combinatorial optimization and graph theory.

Hinge bidomination

Let G be a graph with vertex set V and Edge set E. A subset D of the vertex set V of a graph G is said to be Hinge Bidominating set if it satisfies the following properties:

i) *Every vertex in D is adjacent to exactly two vertices in V-D.*
ii) *For each $u \in V - D$, there exists $v \in D, w \in V - D$ such that $(u, v), (u, w) \in E$ but (v, w) is not in E.*

A hinge bidominating set D is said to be minimal if no subset of D is hinge bidominating set.Minimum number of vertices in a minimal hinge bidominating set is called hinge bidomination number of a graph G and it is denoted by $\gamma_{hb}(G)$.

Proposition 1: Any graph containing pendant vertex does not contain hinge bidominating set.

Proof: Suppose D be a hinge bidominating set of a graph G(V,E) and let u be a pendant vertex of the graph G. Since every vertex in D must be adjacent to exactly two vertices in V-D, and u has degree one, u cannot be in D. So $u \in V - D$. But the condition ii) of the definition says that u must be adjacent to one dominating and one non dominating vertex. For that, the degree of u must be atleast 2. This is a contradiction. So G does not have hinge bidominating set.

Remark: Hence, graphs containing pendant vertices such as trees, paths, star graphs etc do not contain hinge bidominating set.

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Proposition 2: For any graph G, if $\gamma_{hb}(G)$ exists, then $\gamma(G) \leq \gamma_h(G) \leq \gamma_{hb}(G)$

Proof: Let S be the hinge bidominating set of the graph G with minimum number of vertices. So $|S| = \gamma_{hb}(G)$. But since S is hinge bidominating set, S must be a hinge dominating set as well as dominating set. So domination number and hinge domination number $\gamma(G), \gamma_h(G)$ must be atmost $|S|$. Hence $\gamma(G) \leq \gamma_h(G) \leq \gamma_{hb}(G)$.

Proposition 3: *The hinge bidomination number of cycle graph with 3n vertices* C_{3n} *is* n.

Hinge bidomination number does not exist for the cycle graph C_m *if* $m \equiv 1(mod 3)$ *or* $m \equiv 2(mod 3)$

Proof: We start with choosing any random vertex of the cycle C_{3n} and include it in the hinge bidominating set S. Let the vertices of C_{3n} be $v_1, v_2, v_3, \dots, v_{3n}$. So without loss of generality, we start with $v_1 \in S$. Now since we want each nondominating vertex to be adjacent to one dominating vertex and one nondominating vertex, which inturn are not adjacent to each other for hinge property to be satisfied, we leave the next two vertices and choose v_4.So we get $v_1, v_4 \in S$. Similarly, we leave the next two vertices and so on. So finally, we get that

$S = \{v_1, v_4, v_7, \dots, v_{3n-2}\}$

This set has exactly n number of vertices and also it satisfies the bidominating property because every vertex v_{3k-2} for $k \geq 1$ is adjacent to exactly two nondominating vertices v_{3k-3}, v_{3k+1}. So S becomes hinge bidominating set and hence $\gamma_{hb}(C_{3n}) = n$. Hinge bidominating sets of C_6, C_9 are shown in figures 1a and 1b.

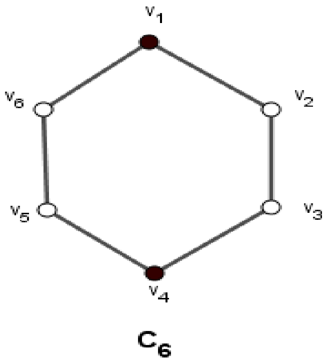

fig 1a

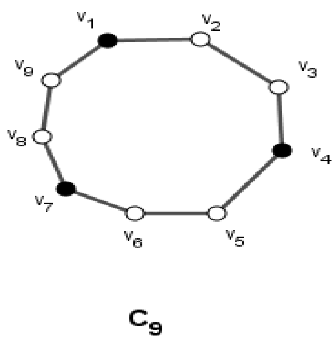

fig 1b

Note that, in the above set S, between every two dominating vertices, there are exactly two nondominating vertices. For $m \equiv 1(mod 3)$, in C_m, we get $v_1, v_m \in S$ and this violates bidomination because both these vertices dominate only one vertex each. So there does not exist hinge bidominating set for C_m if $m \equiv 1(mod 3)$. Similarly, for $m \equiv 2(mod 3)$, we get that $v_1, v_{m-1} \in S$. For example, in C_8, we get $S = \{v_1, v_4, v_7\}$ and so $v_1, v_7 \in S$. So the vertex v_m is adjacent to both dominating vertices v_1, v_{m-1} and not adjacent to any nondominating vertex. So hinge property is not satisfied. Hence there does not exist hinge bidominating set for C_m if $m \equiv 2(mod 3)$.

Proposition 4: The Hinge bidomination number of book graph is $\gamma_{hb}(B_m) = m$

Proof: Consider a book graph B_m with m number of pages. We take that the inward vertices of the book graph be u_1, u_2 and the outward vertices of i^{th} page be a_i, b_i where all the $a_i's$ are adjacent to u_1 and all $b_i's$ are adjacent to u_2.

Then the set $S = \{a_1, b_2, a_3, b_4, \dots, a_m / b_m\}(a_m\ for\ m\ odd\ and\ b_m\ for\ m\ even.)$ becomes a hinge bidominating set for the book graph B_m. Because each a_k in S is adjacent to exactly two non-dominating vertices $u_1 and\ b_k$. Also each b_k in S is adjacent to exactly two non-dominating vertices $u_1 and\ a_k$. This gives bidomination property. Now, each nondominating vertex b_i is adjacent to a dominating vertex a_i and a non-dominating vertex u_2 and these two vertices a_i, u_2 are not adjacent. Similarly, each nondominating vertex a_i is adjacent to a dominating vertex b_i and a non-dominating vertex u_1 and these two vertices b_i, u_1 are not adjacent. This gives the hinge property.

The figures 2a and 2b represent the hinge bidominating sets for the book graphs $B_3 and\ B_4$ respectively.

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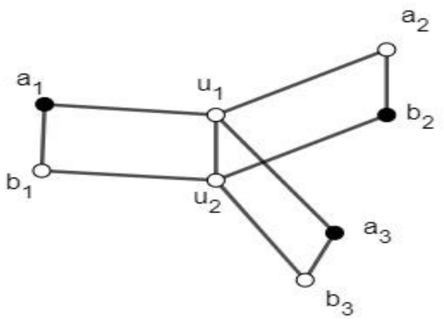

Fig 2a

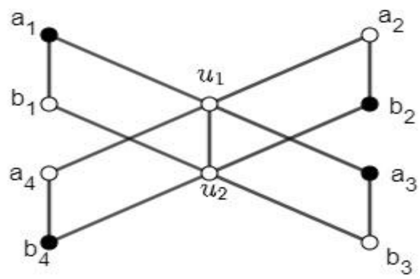

Fig 2b

Proposition 5: The Hinge bidomination number of dumbbell graph is $\gamma_{hb}\big(D(3m,3n)\big) = m + n\ where\ m \geq 2, n \geq 2.$. The dumbbell graph $D(p,q)$ does not have hinge bidominating set if either 3 does not divide p or 3 does not divide q.

Proof: Consider a dumbbell graph $D(3m,3n)$. So it have two cycles C_{3m}, C_{3n} namely $C_{3m} = u_1u_2 \ldots u_{3m}u_1\ and\ v_1v_2 \ldots v_{3n}v_1$ respectively being connected by an edge, say u_1v_1.

Referring to the proof of proposition 3, we take the set $S = \{u_1, u_4, \ldots, u_{3m-2}, v_1, v_4, \ldots, v_{3n-2}\}$. We show that this is hinge bidominating set. As we already saw from proposition 3, the sets $\{u_1, u_4, \ldots, u_{3m-2}, \}and\ \{v_1, v_4, \ldots, v_{3n-2}\}$ form hinge bidominating sets for C_{3m}, C_{3n} respectively. Every nondominating vertex is adjacent to a dominating vertex and a non-dominating vertex both of which are not adjacent. Also every dominating vertex dominates exactly two nondominating vertices. We just note that the vertices of the bridge edge must be included in the dominating set. The hinge bidomination sets for $D(6,6) and\ D(6,9) are\ shown\ in\ figures\ 3a\ and\ 3b$.

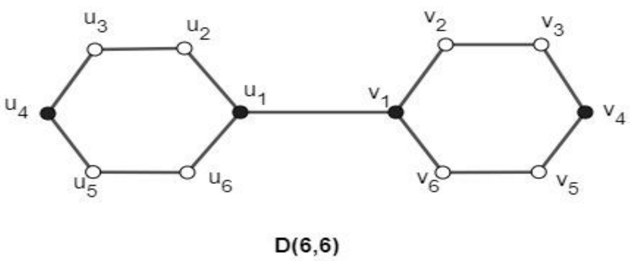

Fig 3a

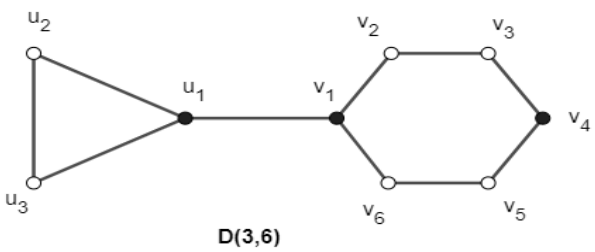

Fig 3b

Conclusion and Future Scope:

The paper dealt with the hinge bidomination values of various families of graphs and provided certain conditions on the existence of hinge bidomination of a graph. Further, there is scope to find the bounds for hinge bidomination number of a graph and can be extended to other domination types which can be introduced using hinge bidomination.

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