

# Sustainable Oyster Aquaculture in Coastal Ecosystems for Optimal Management of Nutrient-Loading

Worku T. Bitew <sup>1\*</sup>, Richard Vogel <sup>2</sup>

<sup>1</sup> State University of New York, Department of Mathematics, Farmingdale, NY, USA. Email: [biteww@farmingdale.edu](mailto:biteww@farmingdale.edu)

<sup>2</sup> State University of New York, School of Business, Farmingdale, NY, USA. Email: [vogelrm@farmingdale.edu](mailto:vogelrm@farmingdale.edu)

\*Corresponding Author: Worku T. Bitew

ARTICLE INFO	ABSTRACT
Received: 11 Mar 2025	<p>Excess nitrogen and other pollutants have been a concern in New York and Connecticut’s coastal waters for nearly 40 years. The waters in the Long Island Sound (LIS) area are affected by water quality concerns from nitrogen inputs, algal blooms, and eutrophication. New York and Connecticut established a plan to reduce nitrogen pollution in LIS by 58.5 percent. The plan, called the Total Maximum Daily Load (TMDL), was approved in 2001 and updated in 2015 with input from stakeholders. Commercial bioremediation represents a new potential path forward to remedy nitrogen eutrophication, changing the incentive structure from one rooted in government subsidization and directed actions to reduce nitrogen pollution into the sound to one that relies on private producers engaged in a for-profit enterprise. The commercial cultivation of various oyster species, like oyster native to the waters of the LIS, has been suggested as one possible strategy to address the nitrogen issue. Utilizing a dynamic optimization model, our research presents a feasible path to address nitrogen eutrophication at targeted reduction levels while balancing government expenditures and subsidization against the potential for-profit aquaculture production by private producers.</p>
Revised: 11 May 2025	
Accepted: 20 May 2025	
<p><b>Keywords:</b> Nitrogen Pollution, Oyster Aquaculture, Optimal Control, Space Allocation.</p>	

## INTRODUCTION

Municipal wastewater and agricultural runoff are the principal sources of eutrophying nutrients in many coastal water ecosystems [9]. Excess nitrogen and other pollutants have been a concern in areas such as the coastal waters of New York and Connecticut for over 40 years. New York and Connecticut, for example, established a plan in 2001 to reduce nitrogen pollution in the Long Island Sound (LIS) by 58.5 percent, further revising this plan in 2015. The plan aims to reduce nitrogen pollution, improve water quality, and make LIS safe for wildlife and people. The commercial cultivation of various algae and oyster species (both traditional and non-traditional) has been identified as one possible strategy to address the nitrogen issue. However, for commercial bio-extraction to be successful, it must be profitable. Utilizing a dynamic optimization model, our research presents a possible path to address nitrogen eutrophication at targeted reduction levels while balancing government expenditures and subsidization against the potential for-profit aquaculture production by private producers.

Studies by [8], [6], and [16] all point to the value of eastern oysters (*Crassostrea virginica*) for use in bioextraction. This oyster species has a high nitrogen uptake level and immediate commercial value (see [19] for more details) and has been cultivated using traditional and modern floating bed processes. While oyster aquaculture can be profitable, traditional marine-based activities in regions such as Long Island have declined over the past century due in part to the economic development of other sectors of the economy and declines in water quality, and pathogens that have affected commercial oysters such as clams and scallops [16, 18, 11].

Following studies such as those by [14], [7], [4], [5], and [1], we model bioextraction within the framework of an optimal control model with a reserved area. Regional fisheries and environmental policymakers can establish targeted bioextraction levels and reserve areas for commercial oyster aquaculture. Aquaculture for bioextraction

takes place within the reserve area. Other marine-based activities are assumed to take place outside of the reserve area. Improved water quality resulting from nutrient bioextraction is expected to have multiple positive spillovers from increasing recreational uses of the marine area, improving the general habitat for other species, which may increase recreational fishing and tourism yields.

The rest of this paper is organized as follows. The optimal control model is initially described and developed in Section 2. Section 3 presents an analysis of the steady state. In Section 4, we create an optimal management strategy for commercial bioextraction, including a simulated numerical solution to the problem. The conclusions of the analysis are presented in Section 5.

## MODEL FORMULATION

### Nitrogen Dynamics

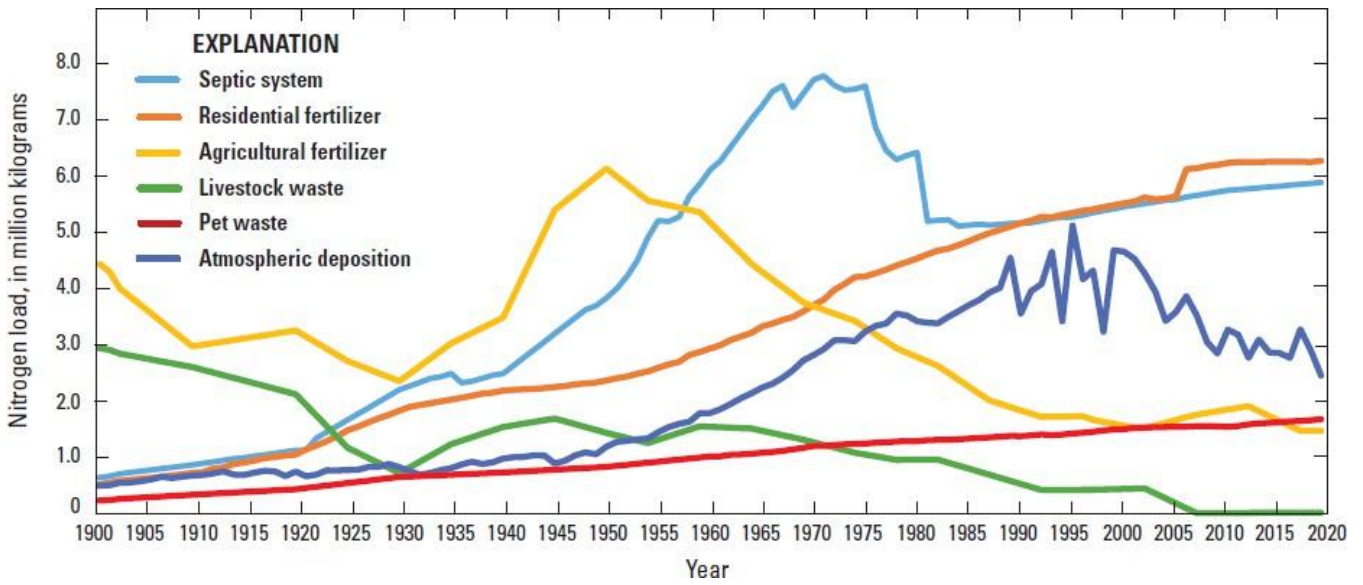
Suppose the total area under consideration, Long Island Sound, is  $T$  square kilometers with carrying capacity  $K$ . We assume a reserved area, and oyster aquaculture development will be in the reserve. Let  $m$  be the reserve size and  $a$  be the portion of the reserved area dedicated to oyster aquaculture. Then, we make the following assumptions.

1. The natural decay rate of nitrogen in the area is  $\alpha n$ , where  $\alpha$  is the decay rate nitrogen other than oysters. According to [20], 60% of the Nitrogen delivered to LIS is either buried in sediments or lost through denitrification.
2. Oyster habitats, both wild reefs and aquaculture sites, enhance denitrification rates (oysters host denitrifying bacteria in their bodies and shells). Moreover, studies have shown that oyster tissues and shells contain a significant percentage of nitrogen, ranging from 7 – 9.3% in the tissues and around 0.2 – 0.3% in the shell [17]. Let the amount of nitrogen removed due to the denitrification of oysters in the aquaculture and aquaculture harvest be  $B(a) = s_1\theta_1 p_a a + \beta\theta_2 p_a a$ , where  $p_a$  per square kilometer aquaculture production (in pounds),  $\theta_1$  is the portion of oyster in the aquaculture (around one-year-old) capable of denitrifying nitrogen,  $s_1$  is per pound denitrification rate of oysters in the aquaculture,  $\beta$  is the percentage of nitrogen contained in each pound of oysters or the amount of the pollutant removed in per pound of harvested oysters, and  $\theta_2$  is the portion of the aquaculture harvested each year.
3. The nitrogen loading function is  $L(g, w) = w - g$ , where  $w$  is the total amount of nitrogen that has the potential of reaching the water body given current Best Management Practices (BMPs) and other pollution control practices implemented and  $g$  is a control variable that measures the government agency's effort to reduce nitrogen further by  $g$  amount before or after it reaches the water body using additional BMPs. These practices include a range of strategies and modern technologies designed to control the quantity of pollutants released into the marine environment. It includes minimizing pollution released from Nonpoint Sources such as septic systems, residential fertilizer, agricultural fertilizer, livestock waste, pet waste, and atmospheric deposition, and Point Sources such as industrial wastewater, municipal wastewater treatment plants, and stormwater systems. According to the Long Island Sound Office of the U.S. Environmental Protection Agency report, the total loading from Point and Nonpoint Sources to Long Island Sound in 2022 is around 200.872 million pounds of nitrogen annually.

We describe the dynamics of nitrogen over the area by

$$\frac{dn}{dt} = L(g, w) - B(a) - \beta\rho\sigma Ex - s_2\rho x - \alpha n, n(0) = n_0 > 0,$$

where  $\rho$  is the average ratio of oysters from the total harvest (most oysters in Long Island Sound come from aquaculture rather than wild harvest),  $h(x, E) = \sigma Ex$  is the harvest rate from the capture fishery for stock size  $x$ , effort level,  $E$ , and catchability coefficient,  $\sigma$ , and  $s_2$  is the denitrification rate of wild oysters, larger in most cases than the aquaculture denitrification rate,  $s_1$  [6].



**Figure 1:** Nitrogen Load Estimates (a total of 14.92 million kilograms per year in 2019) From Six Nonpoint Sources on Long Island, New York, from 1900 to 2019, copied from [13].

### Capture Stock dynamics

In our study, we consider the pollution caused by nitrogen loading, which contributes to environmental degradation and creates an algal bloom. Algal blooms hinder the flow of sunlight and cause a decline in the dissolved oxygen level in the water. It causes many marine animals to suffocate and die, creating “dead zones” [2, 3]. We assume oyster aquaculture reverses the impact, improves the environment, and improves fish biomass’s growth rate. We presume recovery of the fish stock’s growth rate depends on the aquaculture size. Let  $r(a) = r + \varepsilon a$ , for  $0 \leq r(a) \leq r_0$ , is the per capita growth rate, where  $r_0$  is the intrinsic growth rate,  $r$  is the measure of the growth rate in the polluted environment, and  $\varepsilon$  is the measure of the impact of aquaculture on the stock’s growth rate. Because the fraction,  $m$ , of the total area of LIS is dedicated to the reserve, we assume the carrying capacity of the remaining area outside the reserve is  $K - \gamma m$ , where  $\gamma$  is a conversion factor.

We describe the transition equation of the stock as

$$\frac{dx}{dt} = r(a) \left(1 - \frac{x}{K - \gamma m}\right) - h(E, x), x(0) = x_0 > 0.$$

The growth rate is positive provided  $r + \varepsilon a > \sigma E$ .

### Aquaculture Dynamics

The oyster aquaculture expansion rate depends on its relative size to the reserve and the government policy on the maximum portion of the reserved area that can be available for the aquaculture sector,  $v$ . We set up the transition dynamics by the equation:

$$\frac{da}{dt} = w \left(v - \frac{a}{m}\right), a(0) = a_0 > 0.$$

where,  $a \leq \gamma m$ ,  $v$  is the exogenous control variable that the government imposes, and  $w$  is a conversion factor.

In steady state

$$\frac{dx}{dt} = \frac{dn}{dt} = \frac{da}{dt} = 0,$$

implies sustainable aquaculture size

$$a_s(v, m) = wm. \quad (1)$$

From Eq. (1), the size of the aquaculture area depends on the government policy and reserve size. The sustainable stock size is

$$x_s(E, m) = (K - \gamma m) \left( 1 - \frac{\sigma E}{r + \varepsilon a_s} \right) = (K - \gamma m) \left( 1 - \frac{\sigma E}{r + \varepsilon vm} \right). \quad (2)$$

From Eq.(2), the fish stock outside the reserve benefits from the increment in the oyster aquaculture production, and it decreases with wild catch effort.

The corresponding stable steady-state nitrogen concentration is

$$n_s(E, m, g) = \frac{1}{\alpha} \left( w - g - (\beta \theta_2 + s_1 \theta_1) p_a vm - \rho (K - \gamma m) \left( 1 - \frac{\sigma E}{r + \varepsilon vm} \right) (s_2 + \beta \sigma E) \right). \quad (3)$$

From Eq. (3), we can observe that the nitrogen concentration in the area decreases when the per-area production of oyster aquaculture and the rate of the external effort increase. The level of nitrogen decreases with the increment in the natural decay rate and the population of oysters in the area.

### OPTIMIZED MANAGEMENT STRATEGY

In this section, we consider the optimized version of our problem. We set up the problem as a dynamically efficient utilization of the resources and space by considering  $E$  and  $m$  as control variables. We aim to maximize the long-run profit from wild-caught fish, oyster aquaculture harvest, and the revenue from tourism activities.

#### The Objective Functional

We consider three profit functions related to incomes generated from the wild catch outside the marine reserve, aquaculture production in marine reserves, and tourism activities. Let the unit price of captured fish be constant,  $p_1$ , the unit price of wild oysters be  $p_2$ , and  $c$  is the per unit cost of the combined effort,  $E$ . Then, the profit from the wild catch is

$$\Pi_W(E) = p_1(1 - \rho)\sigma E x + p_2\rho\sigma E x - cE. \quad (4)$$

Then, we consider the presence of aquaculture production in the reserved area, suppose the production function for aquaculture is linear  $Z(a) = p_a a$ , where  $p_a$  is the per unit area production of farmed fish. Let the total cost of production also be linear,  $\Phi(a) = c_a p_a a$ , where  $c_a$  is the per-unit production cost. We assume that buyers cannot distinguish between a species of oysters that are farmed or caught in the wild (i.e., the price of the wild oyster is the same as the farmed oyster). Thus, the net profit from aquaculture at time  $t$  is

$$\Pi_A(a) = p_2 Z(a) - \Phi(a) \quad (5)$$

We also assume the introduction of tourism activities in the marine reserve. It includes a finite number of non-consumptive activities such as swimming, kayaking, scuba diving, mammal-watching tours, sailing trips, and recreational fishing (catch and release fishing with littering and pollution potential) [10, 15]. Even though non-extractive activities can affect the marine ecosystem, we assume minimal damage. In our analysis, we didn't include the dynamics of the stock in the reserve and the impact of these activities on the stock in the reserve. On the other hand, revenue from non-consumptive activities is a significant fraction of the operating budget and a source of income for the surrounding community [10, 15]. Let the number of tourists attracted to the marine reserve be based on a Cobb–Douglas production function, depending directly on the size of the reserved area (i.e.,  $m$ ), and inversely to the nitrogen level,  $n$ .

$$R(m, n) = \lambda(\kappa m)^\eta(n)^\mu \quad (6)$$

where  $\lambda, \kappa$  are positive parameters,  $\mu < 0$ , and  $0 < \eta < 1$  [12]. Hence, the net profit from tourism activities using the estimated average net profit per tourist,  $n_1$ , is given by:

$$\Pi_R(m, n) = n_1 \lambda(\kappa m)^\eta(n)^\mu = \tau(m)^\eta(n)^\mu, \quad (7)$$

where  $\tau = n_1 \lambda \kappa^\eta$ .

Therefore, the total net profit from all these activities becomes:

$$MP(E, a, m, n) = \Pi_W(E) + \Pi_A(a) + \Pi_R(m, n), \quad (8)$$

where  $\Pi_W(E)$ ,  $\Pi_A(a)$  and  $\Pi_R(m, n)$  are given by Eqs.(4),(5) and (7), respectively.

If all future costs and benefits are discounted at a positive social discount rate of  $\delta$ , the overall present value of the stream of surpluses given by Eq. (8) is

$$J(E, m, g) = \int_0^\infty (MP(E, a, m, n) + C_1(a) - C_2(g)) e^{-\delta t} dt,$$

where  $C_1(a) = c_1 a$ ,  $c_1$  per unit area leasing cost,  $C_2(g) = lg$  cost of removing  $g$  amount of the pollutant with per unit cost  $l$ . The objective of the social planner is to maximize

$$\max_{E, m, g} J(E, m, g) = \max_{E, m, g} \int_0^\infty (MP(E, a, m, n) + C_1(a) - C_2(g)) e^{-\delta t} dt,$$

Subject to

$$\frac{dn}{dt} = L(g, w) - B(a) - \beta \rho \sigma E x - s_2 \rho x - \alpha n, n(0) = n_0 > 0,$$

$$\frac{dx}{dt} = r(a) \left(1 - \frac{x}{K - \gamma m}\right) - h(E, x), x(0) = x_0 > 0,$$

$$\frac{da}{dt} = w \left(v - \frac{a}{m}\right), a(0) = a_0 > 0.$$

The current value Hamiltonian corresponding to this problem is

$$H(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = MP(E, a, m, n) - C_1(a) - C_2(g) + \lambda_n \left( L(g, w) - B(a) - \beta \rho \sigma E x - s_2 \rho x - \alpha n \right) + \lambda_x \left( r(a) \left(1 - \frac{x}{K - \gamma m}\right) - h(E, x) \right) + \lambda_a \left( w \left(v - \frac{a}{m}\right) \right)$$

where  $\lambda_x$  is the shadow value of the stock in the open access area,  $\lambda_n$  is the shadow value of the nitrogen, and  $\lambda_a$  is the shadow value of the aquaculture.

Using Pontryagin's maximum principle for an infinite-horizon autonomous system, the necessary conditions for optimality are

$$H_E(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = H_m(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = H_g(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = 0 \quad (9)$$

where  $H_E$ ,  $H_m$ , and  $H_g$  are partial derivatives of the Hamiltonian function for the control variables,  $E$ ,  $m$ , and  $g$ , which implies

$$\begin{aligned}\lambda_n &= l, \\ \lambda_x &= \frac{-c}{\sigma x} + \rho(p_2 - \beta l) + p_1(1 - \rho), \\ \lambda_a &= c_1 + \frac{m}{wa} \left( \gamma m \left( \frac{x}{K - \gamma m} \right)^2 \left( \frac{-c}{\sigma x} + \rho(p_2 - \beta l) + p_1(1 - \rho) \right) - \frac{\tau m}{n} \right).\end{aligned}\quad (10)$$

The co-state equations are

$$\begin{aligned}\frac{d\lambda_n}{dt} - \delta\lambda_n &= -H_n(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \frac{d\lambda_x}{dt} - \delta\lambda_x &= -H_x(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \frac{d\lambda_a}{dt} - \delta\lambda_a &= -H_a(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g),\end{aligned}\quad (11)$$

where  $H_x$ ,  $H_n$ , and  $H_a$  are partial derivatives of the Hamiltonian function with respect to the state variables,  $x$ ,  $n$ , and  $a$ , respectively.

At steady state

$$\frac{d\lambda_n}{dt} = \frac{d\lambda_x}{dt} = \frac{d\lambda_a}{dt} = 0,\quad (12)$$

implying

$$\begin{aligned}\delta\lambda_n &= H_n(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \delta\lambda_x &= H_x(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \delta\lambda_a &= -H_a(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g),\end{aligned}\quad (13)$$

Therefore, at the steady state, we have

$$\begin{aligned}\frac{dn}{dt} = \frac{dx}{dt} = \frac{da}{dt} &= 0, \\ H_E(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) &= H_m(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = H_g(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g) = 0, \\ \delta\lambda_n &= H_n(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \delta\lambda_x &= H_x(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g), \\ \text{and} \\ \delta\lambda_a &= H_a(x, n, a, \lambda_n, \lambda_x, \lambda_a, E, m, g).\end{aligned}\quad (14)$$

### Numerical Simulations: Optimal Steady-State Solutions

In this subsection, since the closed-form solutions for the above system, Eq. (14), in terms of the parameters are very complicated, we numerically solve for the optimal steady-state solutions of the state and control variables,  $n^*$ ,  $x^*$ ,  $a^*$ ,  $g^*$ ,  $E^*$ , and  $m^*$ . First, we substitute the values for  $\lambda_n$ ,  $\lambda_x$ , and  $\lambda_a$  from Eq. (10) into Eq.(13). Then we solve the system of equations in Eq.(12) and Eq.(13) for the optimal steady-state solutions by assigning parameter values given in Table 1, some of the values taken from [14] and [4]. We also perform a sensitivity analysis of the optimal solutions by



changing the values of the exogenous control variable,  $v$ , and other biological and economic parameters. The results are summarized in Table 2.

**Table 1:** Parameters and their values used for numerical simulations

Parameter	Description	Value
$T$	Total area of Long Island Sound	3056 Km <sup>2</sup>
$K$	Carrying Capacity of Long Island Sound	610,680,466 Lbs
$r$	Growth rate parameter	1.8
$w$	Nonpoint sources nitrogen loading rate to Long Island Sound per year	32,892,970 Lbs
$p_a$	Average per square kilometer of aquaculture production	6,175,000 Lbs
$p_2$	Price of a pound of medium-sized farmed Oyster	\$12.95
$c_a$	Cost per pound of farmed Oyster	\$4.16
$p_1$	Price of a pound of captured fish	\$16
$c$	Cost per unit effort	\$400
$\delta$	Positive social discount rate	0.05
$\alpha$	Natural decay rate other than Oyster Aquaculture	0.60
$s_1=s_2$	Pounds of nitrogen removed per pound of oysters through denitrification	0.01984, 0.02584
$\beta$	Pounds of nitrogen removed in each pound of harvested oysters	0.01375
$\varepsilon$	The measure of the benefit of the oyster's aquaculture to the growth rate	0.003
$\tau$	A measure of net profit from tourism	34000
$\gamma$	Conversion factor	7993.2
$l$	A measure of the per-pound cleaning cost of nitrogen	\$6.6, \$8.6
$v$	Percentage of the reserved area allowed for Oyster farming	0.25, 0.30
$c_1$	Per unit aquaculture area leasing cost	\$500

**Table 2:** Optimal steady-state solution for different scenarios

$l$	$\tau$	$v$	$s$	$n^*$	$x^*$	$a^*$	$g^*$	$E^*$	$m^*$
6.6	34000	0.15	0.01984	2539.74	$292.009 \times 10^6$	122.081	$20.363 \times 10^6$	73.8575	813.871
6.6	34000	<b>0.20</b>	0.01984	2790.74	$291.326 \times 10^6$	196.528	$13.0833 \times 10^6$	81.3021	982.638
6.6	34000	0.15	<b>0.02584</b>	2542.04	$292.02 \times 10^6$	122.302	$18.723 \times 10^6$	73.8744	815.346
<b>8.6</b>	34000	0.15	0.01984	2229.84	$292.07 \times 10^6$	122.623	$20.304 \times 10^6$	73.9065	817.486

From the numerical results summarized in Table 2:

- i. As we reserve 26.6319% of the space and dedicate 15% of the reserve for oyster aquaculture, the nitrogen level can be maintained at 2539.74, which increases the overall growth rate of stock by 0.366242, and the stock of fish grows to 47.8169%.
- ii. When we increase the restriction on the area allocation for the reserve from 15% to 20%, it allows for more flexibility in resource management. Farmers might invest in larger operations and make oyster farming more economically viable, allowing for more competitive production levels. This resulted in reducing government efforts, saving the public millions of dollars.
- iii. It is observed that government investment in environmental protection measures and the scale of oyster farming operations depend on nitrogen denitrification and the removal factors of oysters.
- iv. If per-unit nitrogen reduction or removal effort becomes more expensive, we increase the area dedicated to aquaculture production. Increasing the area for aquaculture could potentially offset the higher costs by allowing for greater production capacity and the removal of nitrogen in improving water quality.

### **ANALYSIS SUMMARY**

#### **Environmental Benefits**

For a square kilometer oyster farm, the annual amount of nitrogen cleaned and removed in pounds is 179, 090, which saves the public, by the least estimate, 1.182 million. Even though the reserve takes away the fishing area, the contribution of the aquaculture in the reserved area positively impacts the environment. More areas dedicated to farming can improve efficiency and possibly lead to better ecosystem and environmental sustainability.

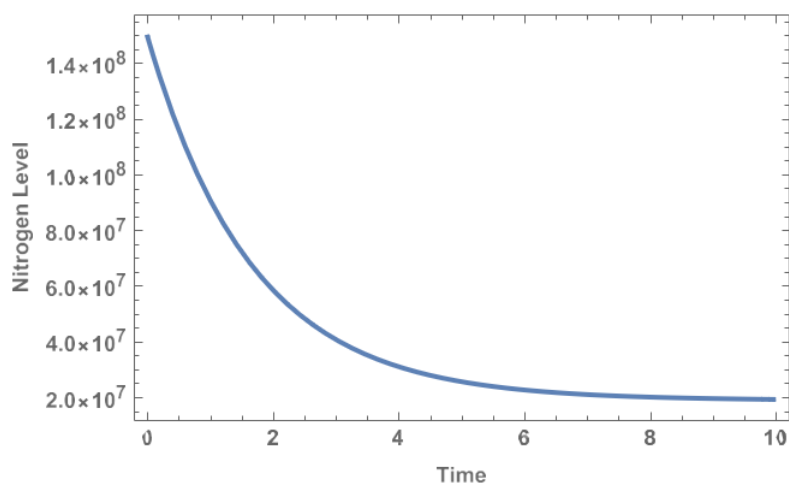
#### **Economic Valuation**

The annual net income per square kilometer of oyster aquaculture is around \$28.6108 million. The reserved area also attracts around 10, 896 in recreational fishing trips and generates \$5.44774 million per year.

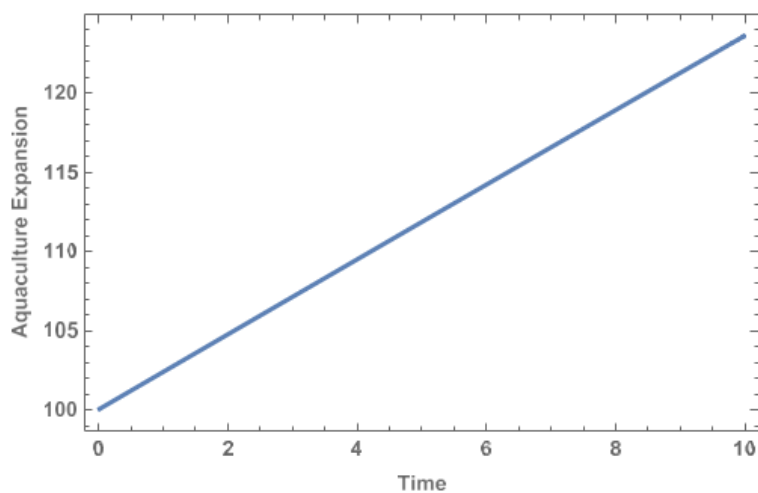
#### **Strategic Planning or Transition Dynamics**

After computing the overtime optimal steady-state solutions given in Table 2, it is natural to determine the trajectories of the states toward close-to-equilibrium solutions or the target nitrogen concentration level over a finite interval of time. For example, we can plan to achieve the desired goal in ten or twenty years. This determines the expansion rate of aquaculture and public allocations. The Total Maximum Daily Load (TMDL) agreement, established in 2001, mandated a 58.5% reduction in nitrogen loads from human sources (wastewater treatment plants, stormwater, septic systems, etc.) to improve water quality and dissolved oxygen levels in LIS. The 2017 target to reduce nitrogen loads discharged into LIS from wastewater treatment plants (a point source) by 58.5% has been met. Oyster cultivation helps control the remaining nitrogen discharged from the ocean from water treatment plants and other Point and Nonpoint sources. The current nitrogen level in LIS is around 0.18 mg/L or 150 million pounds. In Figures 2 and 3, we presented the optimal aquaculture expansion and the corresponding reduction of nitrogen by assuming a 21% government contribution and a ten-year BMP plan. Consequently, we can reduce the concentration to 0.0228 mg/L (equivalent to 19 million in pounds), less than the recommended 0.1 level. Achieving this goal requires increasing the expansion rate of aquaculture by 10 times the optimal trajectory derived from the optimal steady-state solution.



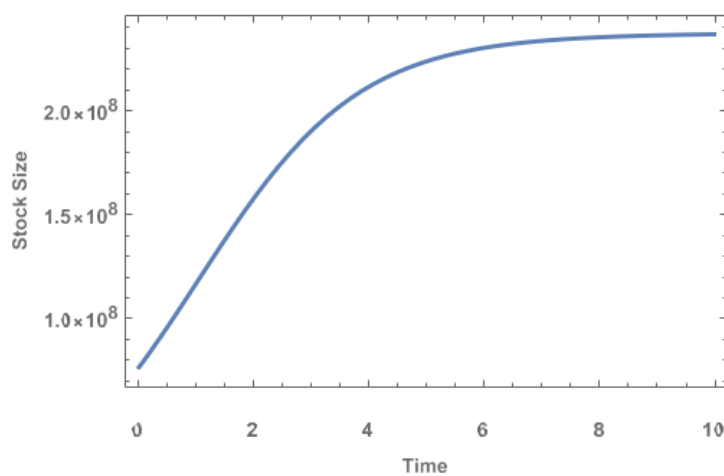


**Figure 2:** The trajectory of the optimal nitrogen level relative to aquaculture expansion.



**Figure 3:** Ten times the trajectory of the optimal aquaculture expansion rate.

Under the above expansion rate of the aquaculture, Figure 3, and corresponding recovery of the environment, the stock growth is given in Figure 4.



**Figure 4:** The trajectory of the recovery of the stock.

In summary, the results of the optimal control analysis illustrate the possibilities and trade-offs that policymakers will face as they seek to implement market-based practices, e.g., reliance on commercial aquaculture to address excess nitrogen. It will take time for any local community to follow this policy. For commercial crops such as oysters, the grow-out period before harvest can be 18 to 36 months before they are commercially viable. Spillovers from other marine-based activities, such as increased tourism, will further enhance the outcome and enable communities to find other potential sources of income to support the implementation of this strategy.

## REFERENCES

- [1] Akpalu, W. and Bitew, W. T. (2014). Optimum reserve size, fishing induced change in carrying capacity, and phenotypic diversity. *Journal of Bioeconomics*, pages 289–304.
- [2] BIELLO, D. (2008). Fertilizer Runoff Overwhelms Streams and Rivers—Creating Vast “Dead Zones”. *Scientific American*.
- [3] Changjiang, L., Fei, Z., Xiangyu, G., Xianlong, Z., Ngai weng, C., and Yaxiao, Q. (2020). Measurement of Total Nitrogen Concentration in Surface Water Using Hyperspectral Band Observation Method. *MDPI*, pages 1–18.
- [4] Getahun, T. B., Bitew, W. T., Ayele, T. G., and Zawka, S. D. (2024). Optimal effort, fish farming, and marine reserve in fisheries management. *Aquaculture and Fisheries*, 9(6):975–980.
- [5] Hailu, F. F., Bitew, W. T., Ayele, T. G., and Zawka, S. D. (2023). Marine Protected Areas for Resilience and Economic Development. *Aquatic Living Resources*, 36:289–304.
- [6] Humphries, A. T., Ayyazian, S. G., Carey, J. C., Hancock, B. T., Grabbert, S., Cobb, D., Strobel, C. J., and Fulweiler, R. W. (2016). Directly measured denitrification reveals oyster aquaculture and restored oyster reefs remove nitrogen at comparable high rates. *Frontiers in Marine Science*, 3:1–10.
- [7] Ibrahim, M. and Benyah, F. (2017). An application of optimal control to the effective utilization of a renewable resource. *Afrika Statistika*, 12(2):1313 – 1331.
- [8] Julie M., R., J. Stephen, G., Suzanne, B., Mark J., B., Allison, C., Lora, H., Eric, K., Alix, L., Nathaniel, H. M., Tammy B., M., Joshua, R., Johnny, S., Kurt, S., Seth, T., Dan, W., and Robinson W., F. (2021). Opportunities and Challenges for Including Oyster-Mediated Denitrification in Nitrogen Management Plans. *Estuaries and Coasts*, 44:2041–2055.
- [9] Laukkanen, M. and Huhtala, A. (2008). Optimal management of an eutrophied coastal ecosystem: balancing agricultural and municipal abatement measures. *Environ Resource Econ*, 39:139–159.
- [10] Leachman, S. (2023). Illuminating the benefits of marine protected areas for ecotourism, and vice versa. *Science + Technology*, University of California, Santa Barbara.
- [11] Li, S. and Vogel, R. and Viswanathan, N. (2016). Economic growth and revitalization on Long Island: the role of recreational fishing and marine economy. *Academy of Economics and Finance*, 7:29–34.
- [12] Merino, G., Maynou, F., and Boncoeur, J. (2009). Bioeconomic model for a three-zone marine protected area: a case study of medes islands (northwest mediterranean). *ICES Journal of Marine Science*, 66(1):147– 154.
- [13] Monti, J. J., Walter, D. A., and Jahn, K. L. (2024). Nitrogen Load Estimates from Six Nonpoint Sources on Long Island, New York, From 1900 To 2019. *National Water Quality Program*, Scientific Investigations Report 2024–5047:1–50.
- [14] Mykoniatis, N. and Ready, R. (2020). The potential contribution of oyster management to water quality goals in the chesapeake bay. *Water Resources and Economics*, 32:100167.
- [15] Ramiro, A.-A., Fabio, F., Joy A., K., Victoria, J.-E., Adan ´L., M.-C., and Aburto-Oropeza, O. (2021). Diving tourism in Mexico – Economic and conservation importance. *Marine Policy*, 126.
- [16] Ray, N. E., Hancock, B., Brush, M. J., Colden, A., Cornwell, J., Labrie, M. S., Maguire, T. J., Maxwell, T., Rogers, D., Stevick, R. J., Unruh, A., Kellogg, M. L., Smyth, A. R., and Fulweiler, R. W. (2021). A review of how we assess denitrification in oyster habitats and proposed guidelines for future studies. *Limnology and Oceanography: Methods*, 19(10):714–731.
- [17] Reitsma, J., Murphy, D. C., Archer, A. F., and York, R. H. (2017). Nitrogen extraction potential of wild and cultured bivalves harvested from nearshore waters of cape cod, usa. *Marine Pollution Bulletin*, 116(1):175–181.
- [18] Rose, J. M., Bricker, S. B., Tedesco, M. A., and Wikfors, G. H. (2014). A Role for Shellfish Aquaculture in Coastal Nitrogen Management. *Environ. Sci. Technol.*, 48:2519–2525.

- [19] Shore, A., Park, P. J., Ebru, U., Viswanathan, N., Zhang, X., Vogel, R., and Clifford, M. C. (2024). Economic Feasibility of Commercial Nutrient Bioextraction in Long Island Sound. *NEIWPCC Guides and Reports*, 48:1 180.
- [20] Vlahos, P., Whitney, M. M., Menniti, C., Mullaney, J. R., Morrison, J., and Jia, Y. (2020). Nitrogen budgets of the Long Island Sound estuary. *Estuarine, Coastal and Shelf Science*, 232:106493.