

The Impact of a Bilingual Methodology on Mathematics Learning

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ABSTRACT

In the contemporary educational landscape, competency-based, problem-solving centered work in the domain of school mathematics exerts a significant influence on current educational practices. This influence is primarily manifested through its promotion of the development of mathematical activity processes, which serve as the foundation for problem-solving in diverse academic settings. The overarching objective of this research endeavor was to assess the impact of the implementation of the Content and Language Integrated Learning (CLIL) methodology in fostering mathematical competencies through problem-solving in primary school students enrolled at the public educational institution Normal Superior de Envigado. The research was grounded in a positivist paradigm, emphasizing quantitative analysis. It employed a hypothetical deductive approach, characterized by a trans-temporal and contemporary orientation, and an explanatory objective. The design was quasi-experimental, incorporating a pre-post test and intact groups for comparative analysis. The sample is non-probabilistic and consists of an experimental group of 39 students and a control group of 24 students in the fifth grade from two different institutions. The data collection technique employed was the survey, and the instrument applied to both groups was a questionnaire with closed questions as an initial evaluation (pre-test). A teaching unit was meticulously designed and implemented to enhance the mathematical aptitude of the experimental group, while the control group remained without intervention. A final evaluation, or post-test, was conducted to assess the performance of each group in both tests and to compare the results statistically. Subsequent to the implementation of statistical tests of normality and non-parametric tests on the results of both groups of students, it was demonstrated that the students in the experimental group exhibited a significant improvement in their results in the post-test, while the control group obtained similar results. The implementation of the CLIL methodology, as a strategy, has been demonstrated to contribute to the enhancement of mathematical competencies through problem-solving.

Keywords: Mathematical skills, problem solving, CLIL methodology, learning.

1 Introduction

At present, school mathematics is fundamental for the development of higher-order thinking skills. However, the prevailing emphasis on memorization of mathematical operations and formulas, or the mechanical execution of exercises, hinders the cultivation of higher-order mathematical competencies, particularly in problem-solving scenarios (Ministry of National Education, 2006). Consequently, within the classroom setting, students are identified as encountering challenges in comprehending problems, employing solution and verification strategies, and engaging with various mathematical perspectives. This hinders the development of mathematical concepts from diverse intellectual frameworks and impedes the cultivation of general mathematical processes.

Consequently, students are neither compelled nor obligated to cultivate diverse solution strategies, which hinder the enhancement of their competencies in operations and algorithms. Moreover, it impedes the development of their reasoning and communication skills, directly impacting their capacity to comprehend, interpret, and solve problem scenarios that are intimately associated with

their sociocultural context (MEN, 1998; Organization for Economic Cooperation and Development, 2013; 2019; 2023).

In contemporary education, the significance of acquiring a second language in various domains of social, political, economic, and cultural life is accentuated. The cultivation of communication competencies in a second foreign language facilitates an understanding of the multifaceted dynamics of the globalized world, fostering connections with diverse cultures and thought processes, and enabling cultural and economic exchanges, among other domains that enrich the knowledge of modern society (García et al., 2017).

Furthermore, it is imperative to underscore the implementation of the Content and Language Integrated Learning (CLIL) methodology in the development of learning and teaching environments that are integrated with the learning of a second language and mathematical content. This approach fosters the learning of English in a natural manner within a classroom environment, enabling students to enhance their mathematical aptitudes through problem-solving and concurrently interact with English as an additional element in school learning (Mehisto et al., 2008).

In accordance with the aforementioned points, research findings indicate that students demonstrate a progressive acquisition and development of competencies in mathematics and foreign language learning when the CLIL methodology is implemented. This methodology utilizes linguistic, didactic, and pedagogical resources, thereby fostering the development of lower- and higher-order thinking skills, creativity, reasoning, and critical thinking capacity through the resolution of mathematical problems. Additionally, it facilitates the integration of elements of a second language in school activities designed to teach subject matter across various academic disciplines (Cázares, 2015). As demonstrated in the works of San Román (2016); Zirilli (2019); Moreno (2020); Cancela (2021); Giraldo (2021); Hernández (2021); Olivares (2021); Salazar (2021); Bartosiewicz (2022); Martínez (2022); Duque (2022); and Segura (2022), the topic has been thoroughly researched and examined.

In summary, given the variables of this research and the analysis of the different doctoral research from its methodological and conceptual application and theoretical contributions, the research question of this study was: The present study seeks to examine the impact of the Content and Language Integrated Learning (CLIL) methodology on the development of mathematical aptitude through problem-solving in primary school students at the public educational institution Normal Superior de Envigado.

Objectives

To answer the research question, the following general objective was set::

- Evaluate the effect of implementing the Integrated Content and Foreign Language Learning methodology on the development of mathematical skills through problem solving in primary school students at the Normal Superior public educational institution in Envigado.

Thus, in order to achieve the overall objective, the following specific objectives were set:

- Measure performance levels in the development of mathematical skills, from problem solving to primary school students who make up the control and experimental groups, before implementing the CLIL methodology.
- Compare performance levels in the development of mathematical skills through problem solving among primary school students in the control group and the experimental group, before the implementation of the CLIL methodology.
- Compare performance levels in the development of mathematical skills through problem solving among primary school students in the control group and the experimental group, after the implementation of a teaching unit based on the CLIL methodology.

Theories that served as the basis for the study

AICLE Method

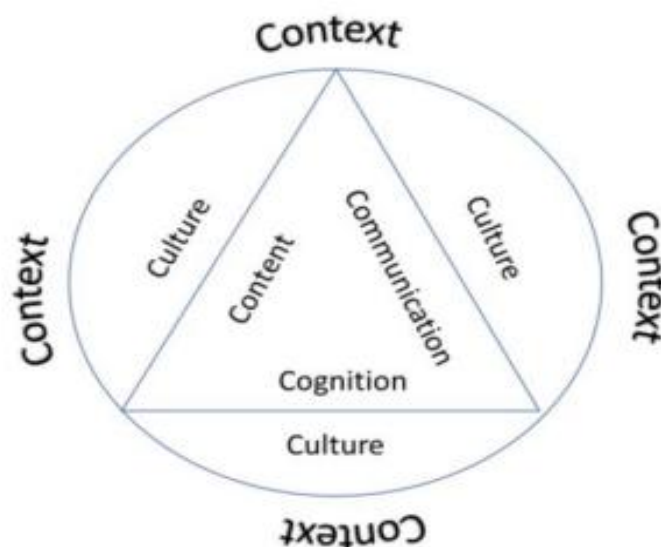
The CLIL methodology is conceptually based on a two-way educational approach, in that the teaching and learning strategies and activities implemented in the classroom simultaneously enable the natural learning of a second foreign language and the learning of academic content (Marsh, 2012). Likewise, this methodology is theoretically based on the principles proposed by Coyle et al. (2010), which are consistent with the following elements:

- *The content*: The teaching and learning process must ensure the development of the diverse and varied content of the areas of knowledge.
- *Communication*: A second language is used constantly in teaching and learning processes, which involves learning basic and advanced vocabulary and expressions in another language.
- *Cognition*: Teaching and learning processes should promote the development of skills in the area of knowledge and, in turn, the learning of a second language.
- *Culture*: activities should contribute to respect and recognition of all cultures, enriching knowledge and broadening sociocultural perspectives in school environments.
- *Context*: classroom processes should be cross-curricular and permeated by context to encourage participation and motivation in relation to content and a second language in school environments.

Figure 1 illustrates the model proposed by Coyle et al. (2010):

Figure 1

Framework proposed by Coyle et al. (2010).

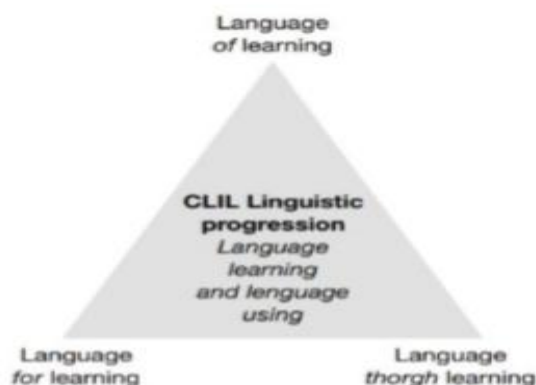


Source: Coyle et al. (2010).

Also, as a conceptual complement to the previous framework, Coyle et al. (2010) establish the relationship between content and language use from a cognitive perspective, proposing the following framework:

Figure 2

The linguistic triad proposed by Coyle et al. (2010).



Source: Coyle et al. (2010).

Based on the above conceptual framework, three fundamental elements are established that are important for planning and developing educational proposals. These elements relate to the use of language for learning new knowledge related to the non-linguistic content to be studied, language for learning, and language through learning, which together facilitate the learning of a second foreign language and the learning of content through the development of lower- and higher-order skills in teaching and learning processes (Coyle et al., 2010; Marsh, 2002; Marsh, 2012; Ball, 2014; and Attard and Walter, 2021).

Mathematical skills

Mathematical skills are defined, conceptually, as a student's ability to solve a problem situation using mathematical processes, including formulating and solving problems, modeling real-world processes and phenomena, communicating, reasoning, and formulating, comparing, and practicing problems and algorithms. which allows them to be skilled and efficient in logical thinking and in each of the mathematical concepts, thus recognizing random, metric, geometric, variational, and numerical thinking (MEN, 1998; 2006)

Murillo et al. (2010) address the concept of mathematical competence, stating that,

It consists of the ability to use and relate numbers, their basic operations, symbols, and forms of mathematical expression and reasoning, both to produce and interpret different types of information, and to expand knowledge about quantitative and spatial aspects of reality, and to solve problems that are closely related to everyday life and the world of work (p. 278).

In this sense, mathematical competence responds to the ways in which a mathematical problem is approached, both from the mathematical elements present, such as formulas, concepts, and procedures, and in the contextual relationship, which involves aspects of the student's social and cultural context. Furthermore, the OECD (2013) reinforces the definition of mathematical competence by relating it to the ability to analyze, interpret, and solve a mathematical problem linked to the real world.

Troubleshooting

Mathematical problem solving is defined based on the theoretical framework of Pólya (1965), who proposes four specific phases for solving a mathematical problem, which are:

- *Phase 1:* Students understand the problem, identifying the data and basic elements needed to address it.
- *Phase 2:* In addition to understanding the problem, students are able to design a plan or solution strategy that allows them to solve the problem.
- *Phase 3:* Students are able to understand the problem, design a plan, and execute it to correctly solve the problem at hand.
- *Phase 4:* Students are able to understand the problem, design a plan, apply it correctly, and carry out a verification or review process that allows them to validate it and be certain that it has been done correctly.

All phases are progressive, meaning that in order to reach phase 4, retrospection of the problem, a student must demonstrate mathematical processes and procedures that account for the previous phases.

Also, the MEN (2006) conceptualizes problem solving as focused on the general process of formulating, addressing, and solving problems, stating that “solving problems arising from a problem situation allows students to develop a persevering and inquisitive mental attitude, deploy a series of strategies to solve them, find results, verify and interpret their reasonableness, modify conditions, and generate other problems” (MEN, 2006, p. 52).

2 Method

Considering the objectives of this research, the study was conducted from a positivist paradigm and using the hypothetical-deductive method, employing quantitative techniques for data collection and analysis, where controlled measurements are made, a set of hypotheses and variables are determined, a design is established to test them, it is oriented toward the results of tests in the research, emphasis is placed on the reliability of the data, attempts to generalize the data, and assumes reality to be stable (Adler, 1964; Kuhn, 1992; Cook and Reichardt, 1995; Durán, 2002; Hernández et al., 2014; Norman et al., 2020).

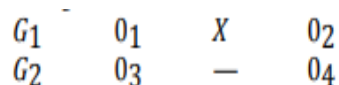
The type of research is explanatory, as it focused on explaining why a phenomenon occurs and observing whether or not there is a relationship between the variables involved (Hernández et al., 2014), which, for this study, will determine whether the implementation of the CLIL methodology has an effect on the development of mathematical skills in students.

Thus, a field design was developed to carry out the intervention process of the activities focused on the CLIL methodology, with a cross-sectional research design, since the information was collected at a specific point in time. In addition, the research was contemporary, in that it addressed fundamental elements in current educational processes, such as the learning of mathematics linked to the learning of English in a school context.

Based on the above, a quasi-experimental design with pre- and post-tests and intact groups was applied, in accordance with Hernández et al. (2014), where the researcher intervenes on the independent variable (implementation of the CLIL methodology) and observes its effect on the dependent variable (development of mathematical skills).

Thus, for this research, there was an experimental group, to which the CLIL methodology (experimental stimulus) was applied, and a control group, to which the aforementioned methodology was not applied (but rather the methodology they normally receive in class with their teacher).

The pre-test and post-test design with intact groups proposed by Hernández et al (2014) was as follows:

Figure 3*Diagram of the “Pre-Test, Post-Test, and Intact Groups” design”.**Source: Design proposed by Hernández et al. (2014)*

This diagram offers a comprehensive understanding of the procedure carried out, where the results of the initial tests (pre-test) allowed us to determine how equivalent the two groups were by comparing them. Following the implementation of the intervention, utilizing the CLIL methodology, the post-test was administered to both groups. This assessment was conducted to ascertain their performance and to ascertain whether there had been an improvement in the results obtained by the control and experimental groups. This would serve to determine whether there was an effect of the methodology on the development of mathematical skills.

The population of this research is determined by all fifth-grade students in the public educational institutions Normal de Envigado and Comercial de Envigado in the department of Antioquia in Colombia. The sample is comprised of 63 students, 39 of whom are enrolled at the I.E. Normal Superior and constitute the experimental group, and 24 of whom are enrolled at the I.E. Comercial and constitute the control group.

The students selected to apply the CLIL methodology were chosen intentionally (non-probabilistic sample) with the aim of evaluating the effect of this methodology with a specific group of students. This approach was taken to avoid the need to call students from other groups or interrupt classes in other areas, thereby facilitating the development of the methodology in mathematics classes.

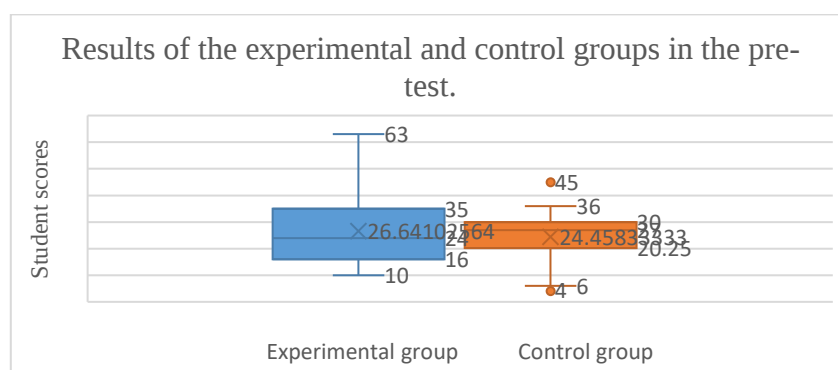
The methodology for data collection is outlined as follows: submission of a formal request to conduct the research in the two official educational institutions in the municipality of Envigado; selection of students for the experimental group and control group; submission of informed consent forms for the pre-test of both groups; general guidelines for the pre-test and, subsequently (at another time), the post-test; and finally, the administration of the pre-test to both groups. It is noteworthy that the pre-test is identical to the post-test and comprises a questionnaire containing 25 closed-ended questions, which evaluate mathematical aptitude based on the five mathematical thinking skills.

To ensure the development of the final version of the instrument, a content validation process was implemented. This process entailed a thorough review of the questionnaire by experts, who evaluated its design, content, and complexity level in relation to the students' educational level. Consequently, a coefficient of 0.90 was obtained, which, according to Hernández (2002), reflects a high degree of correspondence between the content of the instrument and the variable being assessed. In this case, the variable being assessed is mathematical skills based on problem solving.

3 Conclusions

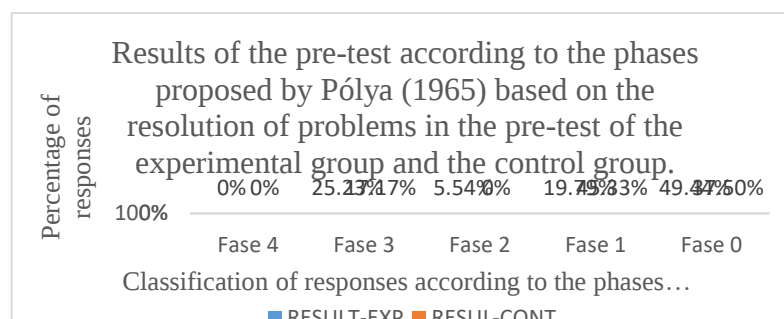
This section aligns with the outcomes observed in the pre-test and post-test of the experimental group and the control group, thereby addressing the research question that guided the methodological process and analysis of the results. In accordance with the aforementioned assertions, the subsequent section will address the research question by delineating the scope of each specific objective. This delineation was determined through a meticulous analysis of the results, employing graphical representations and rigorous statistical tests.

To illustrate the breadth and scope of the initial and secondary objectives, please refer to Tables 1 and 2. As illustrated in Table 1, the results are based on the scores obtained in the pre-test of the experimental and control groups. This analysis was used to determine the differences between the arithmetic means of both groups.

Table 1*Average results of the experimental group and the control group in the pre-test.**Source: own elaboration.*

In this regard, the figure 4 showed that the results of the experimental group in the pre-test were slightly better than those of the control group, with a small difference between the means of 2.19. However, based on the nonparametric Mann-Whitney U test for independent samples (given the non-normal distribution of the results obtained in the experimental group), it was established that, statistically, there are no significant differences between the medians of the results of both groups, and therefore, it was concluded that the results of the experimental group were similar to those of the control group (with equal medians).

On the other hand, Table 2 shows the results obtained by both groups, no longer based on their scores, but in relation to Pólya's phases (1965), where it was possible to determine the percentage of students who did not perform any written process (Phase 0), students who only managed to understand the problem but did not perform any further written processes (Phase 1), students who understood and applied a solution strategy but made errors in the processes applied and therefore gave an incorrect answer (Phase 2), students who managed to understand the problem, design a plan, and apply it appropriately, allowing them to obtain a correct answer to the problem posed (Phase 3); and finally, students who carried out the same processes as in Phase 3 and additionally carried out a problem verification process based on the mathematical competencies determined by the MEN (1998; 2006).

Table 2*Results of the experimental and control groups in the pre-test, taking into account the phases proposed by Pólya (1965).**Source: own elaboration.*

As illustrated in the preceding table, there was a higher percentage of responses from students in the experimental group that were classified in Phase 3 (8.05% more than the control group). These responses indicated an understanding of how to design and implement a plan that would enable them to respond correctly to the problems posed. In a similar vein, it was noted that the two groups exhibited

a percentage of incorrect responses that exceeded 70%, which corresponds to students who were incapable of comprehending, formulating, or implementing a plan that would enable them to solve a problem accurately based on the mathematical competencies stipulated by the MEN (1998; 2006).

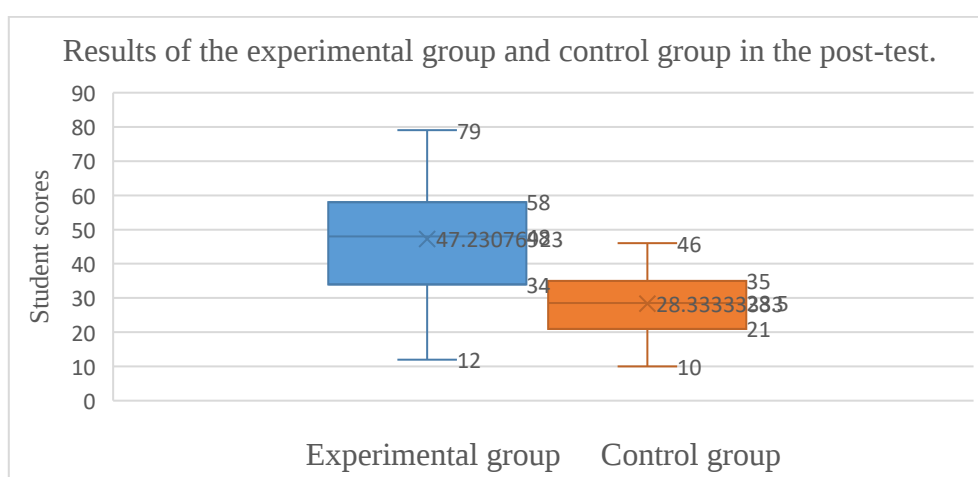
Consequently, as indicated by the findings presented in Tables 1 and 2, it was determined that a significant proportion of the experimental and control groups encountered challenges in comprehending a problem, formulating a plan, and implementing it in an appropriate manner. These challenges were particularly pronounced in the context of mathematical concepts, where processes and the interrelationships among concepts were assessed based on numerical, random, geometric, and metric principles, as delineated by the MEN (1998; 2006).

It was also concluded that no student carried out verification processes that allowed them to validate their answers. This finding suggests that, based on the methodologies implemented in both groups, there was no emphasis on carrying out retrospection processes of the answers. Such processes would guarantee that students analyze and verify the procedures and conceptual relationships used to answer the problems.

To illustrate the breadth of the aforementioned objective, please refer to Tables 3 and 4. As illustrated in Table 3, the subsequent post-test scores obtained by the students in the experimental group and the control group are presented.

Table 3

Average results of the experimental group and the control group in the post-test.



Source: own elaboration.

Therefore, following the implementation of the CLIL methodology in the experimental group and the traditional (or conventional) methodology in the control group, it was observed that the students in the experimental group exhibited a significant increase in their average score compared to their pre-test results, with an average improvement of 20.59 points. This observation is supported by the post-test results presented in Table 3. However, the results of the control group indicated a slight yet statistically insignificant improvement in post-test results, with an average increase of 3.88 points.

Subsequently, as a complement to the preceding analysis, several normality tests and nonparametric tests were applied (given the non-normal distribution of some of the results). Through these statistical tests, it was determined whether or not there were significant differences between the different results compared. Therefore, in order to compare the results of the experimental group and the control group in the post-test, the non-parametric Mann-Whitney U test for independent samples was performed. It was concluded that the medians were different, and therefore, statistically, there were significant differences (a difference of 18.9 points between their means) that show that the experimental group performed better.

In order to make a comparison between the results of the experimental group in the pre-test and post-test, the non-parametric Wilcoxon test for related samples was applied. It was concluded that there are statistically significant differences, which indicate that there was an improvement in the results in the post-test compared to the results obtained in the pre-test. The same test was administered to the control group, and it was concluded that, statistically, the results obtained in the pre-test were the same or similar to the results of the post-test. Therefore, no significant differences were observed (their medians were the same).

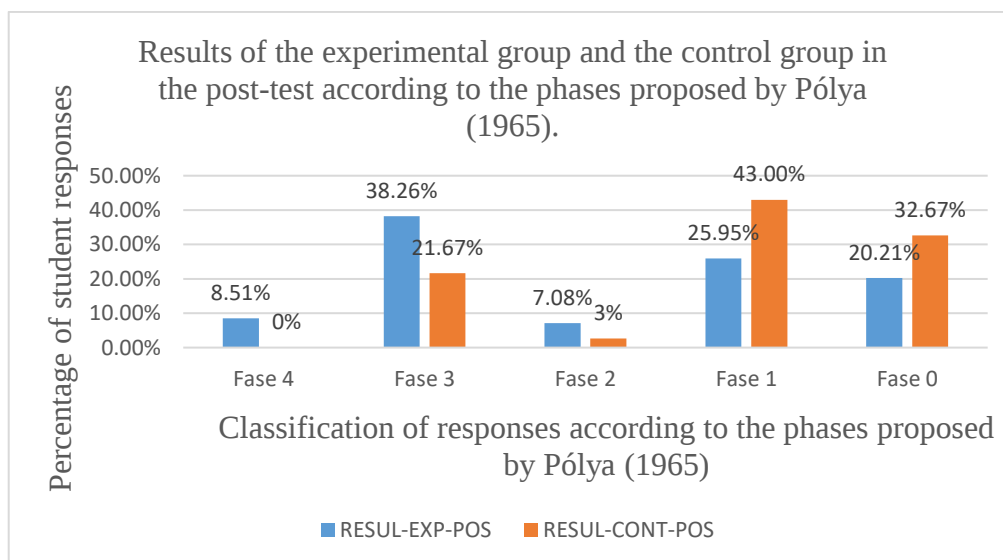
Subsequent to an examination of Table 3 and the nonparametric tests applied to the pre-test and post-test results of the experimental group and the control group, it is concluded that:

- After applying the CLIL methodology to the experimental group and the traditional (or conventional) methodology to the control group, the students in the experimental group obtained better results in the post-test than the control group.
- After applying the CLIL methodology to the experimental group, students obtained better results in the post-test than in the pre-test.
- After applying the traditional (or conventional) methodology to the control group, students obtained similar (statistically equal) results in the post-test compared to those obtained in the pre-test.

In line with the above, Table 4 shows, based on the stages proposed by Pólya (1965), the percentage of students who demonstrated an understanding of the problem, designed and implemented a plan correctly, and/or carried out a verification process for each of the problems proposed in relation to the competencies determined by the MEN (1998; 2006).

Table 4

Results of the experimental and control groups in the post-test, taking into account the phases proposed by Pólya (1965).



Source: own elaboration.

Table 4 shows that the percentage of responses from the experimental group classified in phases 3 and 4 was much higher than that of the control group, with a significant difference of 25.1%. It can also be determined, with regard to Table 2, that the experimental group improved in the percentage of responses classified in phases 3 and 4 in the post-test by 21.54%, while the control group improved by 4.5% (an insignificant percentage). It was also observed in the results of the experimental group that

the students in the post-test were able to carry out verification processes on the problems posed in the post-test (8.51% of the responses), unlike the pre-test, where no student carried out verification processes. However, the students in the control group did not demonstrate verification processes for the problems posed in either of the two tests.

From Tables 2 and 4, according to Polya's phases (1965), the following conclusions could be drawn:

- After applying the CLIL methodology to the experimental group and the traditional (or conventional) methodology to the control group, the students in the experimental group showed better performance in the post-test in terms of understanding the problem, designing and correctly applying the plan, and in the processes of retrospection or verification of the problem, compared to the processes evidenced in problem solving by the students in the control group in the post-test.
- After applying the CLIL methodology to the experimental group, the students in the post-test improved considerably in terms of understanding the problem, designing and correctly applying the plan, and in the processes of retrospection or verification of the problem, compared to the processes evidenced in problem solving in the pre-test.
- After applying the CLIL methodology to the control group, the students in the post-test did not improve significantly in terms of understanding the problem, designing and correctly applying the plan, and in the processes of retrospection or verification of the problem, compared to the processes evidenced in problem solving in the pre-test.

Finally, for this quantitative research, and using the previous analyses and conclusions regarding the scope of the three specific objectives as a reference, it was determined that:

- The overall objective was successfully achieved and the effect of implementing the CLIL methodology on the development of mathematical skills through problem solving in primary school students at the Normal Superior de Envigado public educational institution was evaluated based on the results obtained in the pre-test and post-test.
- The research question was answered, concluding, with the statistical analysis of the pre-test and post-test, that the implementation of the CLIL methodology had a positive effect on the development of mathematical skills in problem solving using metric, random, numerical, geometric, and variational thinking as determined by the MEN (1998; 2006), which were evident in the students' written records and processes when approaching a problem from the understanding, design, and application of a plan and the verification process developed from the phases proposed by Pólya. (1965).

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