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Production Inventory Model for Crumbling Items With Time Dependent Production and Demand with Deficiency

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ABSTRACT

This study develops a novel Economic Production Quantity (EPQ) model for deteriorating items where both production and demand rates vary over time, following power patterns. Unlike traditional models that assume constant production or demand rates, our model accommodates realistic variations production and demand may increase or decrease over a cycle, depending on the chosen power exponents. Demand rate is expressed as λ(t)=g·m·t^{m−1}, and production rate as R(t)=r·n·t^{n−1}, with deterioration occurring at an exponential rate q. Shortages are allowed and fully backlogged. We derive closed-form differential equations to describe inventory dynamics across four phases: production build-up, depletion under demand and deterioration, shortage accumulation, and subsequent backlog replenishment. From these, we obtain expressions for cycle-end inventory level (S), production downtime (t_1), uptime (t_3), and production quantity (Q). Incorporating holding, setup, production, and shortage costs, we construct the total cost function K(t_1, t_2, t_3) and determine optimal policies by minimizing with respect to t_1 (and thus t_3 and Q). Numerical examples illustrate how variability in deterioration, cost, production, and demand parameters significantly affects optimal production schedules and costs. Sensitivity analysis highlights that deterioration rate and demand growth parameters most strongly influence optimal Q, t_1, and t_3. Importantly, the model generalizes various existing EPQ formulations, and can be extended to account for time dependent pricing and demand relationships.

Keywords: Economic Production Quantity, deteriorating items, traditional models, cost production, demand

1.1 INTRODUCTION

An inventory model for crumbling items is developed with the assumption that the production rate is a function of time and demand rate is constant. This model is suitable for some situations where the demand remains to be constant. But in many practical situations arising at places like food processing industries where the demand is a function of time. Several authors developed various inventory models assuming that the demand is a function of time. Dave U (1981) is developed the inventory model for deteriorating items with time proportional demand'. Ritchi e (1984) developed the exact solution for a linearly increasing demand. A survey of literature on inventory models for deteriorating items was given by Raafat (1991). Datta and Pal (1992) studied the inventory model with demand as a function of time. Goyal and Giri (2001), Ruxian et al. (2010) have reviewed the inventory models for deteriorating items. Srinivasa Rao, et; al., (2011) developed a production inventory system with demand rate as a function of production quantity. Kaliraman, Raj, Chandra and Chaudhry (2015) and Ardak, and Borade, (2017) studied an EPQ Inventory Model For deteriorating items with Weibull deterioration under stock dependent demands. Recently Sujata Saha, and Chakrabarti, (2018) studied an EPQ model for deteriorating items with probabilistic demand and variable production rates. Khurana , Tayal, and Singh (2018) EPQ model for deteriorating items with variable demand rates and allowable shortages. Majumder, Bera and Maiti (2019) developed an EPQ model for deteriorating items under trade credit policy. Aruna Kumari developed a model for

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deteriorating items with time dependent production having production quantity dependent demand (2017) Janrdan Rao et; al., was developed Economic Lot Size Model with Weibull Deterioration and on-hand inventory demand under allowable delay in payments, Solid state technology (2020) Dari, S., Sani, B. (2020) are EPQ model for delayed deteriorating items with quadratic demand and linear holding cost. Malumfashi (2021) are developed the EPQ model for delayed deteriorating items with variable production rate, two-phase demand rates and shortages. M. L. Malumfashi1 (2022) developed the Deteriorating Items with Two-Phase Production Period, Exponential Demand Rate and Linear Holding Cost. S. Sindhuja1 (2023) studied an inventory model for deteriorating products under preservation technology with time-dependent quality demand. Jitendra Kaushik (2024) developed an inventory model for deteriorating items with ramp type demand pattern: a stock- dependent approach. Very little work has been reported in literature regarding inventory models for deteriorating items with demand as a function of time and production rate is time dependent. The time dependent demand can be well characterized by a power pattern demand which is of the form $\lambda(t)=\frac{gt^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}$, where g is the total demand, λ (t) is demand rate at time t, m is the index parameter, and T is the cycle length. For different values of m this function includes various patterns of demand. If m =1, this reduces to the constant rate of demand. This also includes increasing or decreasing rates of demand. Hence, in this chapter an economic production quantity (EPQ) model for deteriorating item is developed with the assumption that the production and demand rates are functions of time and follows power pattern.

Using the differential equations, the instantaneous state of inventory with shortages is derived and analyzed. With suitable cost considerations the total cost function is derived. By minimizing the total cost function, the optimal production quantity, production downtime and production uptime are derived. The sensitivity of the model with respect to the parameters and costs is studied. As a limiting case the inventory model with time dependent production rate and time dependent demand without shortages is also studied.

1.2 ASSUMPTIONS OF THE MODEL

For developing the model the following assumptions are made.

(i) The demand rate $\lambda(t)=\frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}$, is a power function of time

where, g is the total demand, m is the index parameter.

If m =1 then $\lambda(t)=\frac{g}{T}$, which gives constant demand.

If m = 0.5 than $\lambda(t)=\frac{g\,t}{mT^{2}}$, which is linear function of time

For different values of m it includes various patterns of demand.

(ii) The rate of production R (t) is time dependent and follows a power pattern.

$R(t)=\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$, where, r is the total production and n is the index parameter

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If n =1, $R(t)=\frac{r}{T}$, which includes the finite rate of production

If n ≠ 1, it includes increasing or decreasing rates of production.

(iii) Lead time is zero

(iv) Cycle length is known and fixed say T

(v) Shortages are allowed and fully back locked

(vi) A deteriorated unit is lost, and there is no repair on replacement of the deteriorated unit

(vii) The life time of the commodity is a random variable and follows an exponential distribution. Then the instantaneous rate of deterioration is θ

1.3 PRODUCTION LEVEL INVENTORY MODEL WITH DEFICIENCY

In this model the stock level is zero at time t = 0. The stock level increases during the period $(0, t_1)$ due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3 the production again starts and fulfils the backlog after satisfying the demand during (t_3, T), the inventory level increasing.

Let I (t) be inventory level of the system at time t $(0 \le t \le T)$.

The differential equations governing the instantaneous state of inventory over the cycle length T are

$$\frac{d}{dt}I(t)+\theta I(t)=\frac{r\,t^{\frac{1}{n}-1}}{T^{\frac{1}{n}}n}-\frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}, \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{d}{dt}I(t)+\theta I(t)=-\frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}, \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{d}{dt}I(t)=-\frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}, \qquad t_2 \le t \le t_3 \qquad (3)$$

$$\frac{d}{dt}I(t)=\frac{r\,t^{\frac{1}{n}-1}}{T^{\frac{1}{n}}n}-\frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}, \qquad t_3 \le t \le T \qquad (4)$$

with the initial conditions

$I(t_1) = S$, $I(t_2) = 0$, and $I(T) = 0$

Solving the differential equations (4.3.1) to (4.3.4) we get the on hand inventory at time t as

$$I(t)=e^{-\theta t}\left[Se^{\theta t_1}-\int_t^{t_1}\left(\frac{r\,u^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}-\frac{g\,u^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}\right)e^{\theta u}du\right], \qquad 0 \le t \le t_1 \qquad (5)$$

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$$I(t)=e^{-\theta t}\left[Se^{\theta t_1}-\frac{g}{mT^{\frac{1}{m}}}\int_t^{t_1}u^{\frac{1}{m}-1}e^{\theta u}du\right], \qquad t_1 \le t \le t_2 \tag{6}$$

$$I(t)=\frac{g}{T^{\frac{1}{m}}}\left(t_2^{\frac{1}{m}}-t^{\frac{1}{m}}\right), \qquad t_2 \le t \le t_3 \tag{7}$$

$$I(t)=g\left(1-\frac{t^{\frac{1}{m}}}{T^{\frac{1}{m}}}\right)+\frac{r}{T^{\frac{1}{n}}}\left(t^{\frac{1}{n}}-T^{\frac{1}{n}}\right), \qquad t_3 \le t \le T \tag{8}$$

The stock loss due to deterioration in the interval (0, t) is

$$L(t)=\int_0^t R(t)dt-\int_0^t \frac{g\,t^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}dt-I(t)$$

This implies

$$L(t) = \frac{r\,t^{\frac{1}{n}}}{T^{\frac{1}{n}}}-\frac{g\,t^{\frac{1}{m}}}{T^{\frac{1}{m}}}-e^{-\theta t}[Se^{\theta t_1}-\int_t^{t_1}(\frac{r\,u^{\frac{1}{n}-1}}{T^{\frac{1}{n}}n}-\frac{gu^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}})e^{\theta u}du], \qquad 0 \le t \le t_1$$

$$= \frac{r\,t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}}-\frac{g\,t^{\frac{1}{m}}}{T^{\frac{1}{m}}}-e^{-\theta t}(Se^{\theta t_1}-\frac{g}{mT^{\frac{1}{m}}}\int_{t_1}^{t}u^{\frac{1}{m}-1}e^{\theta u}du), \qquad t_1 \le t \le t_2$$

Therefore the stock loss deterioration in the cycle length T is

$$L(T)=\frac{r\,t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}}-\frac{g\,t_2^{\frac{1}{m}}}{T^{\frac{1}{m}}}.$$

The production quantity Q in the cycle of length T is

$$Q=\int_0^{t_1}R(t)dt+\int_{t_3}^{T}R(t)dt$$

$$=\frac{r}{T^{\frac{1}{n}}}[t_1^{\frac{1}{n}}+T^{\frac{1}{n}}-t_3^{\frac{1}{n}}]. \tag{9}$$

From the equation (5) and using the initial condition I (0) = 0, we get the value of S as

$$S=e^{-\theta t_1}\int_t^{t_1}\left(\frac{r\,u^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}-\frac{g\,u^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}}\right)e^{\theta u}du.$$

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Using the Taylor's series expansion for small values of θ and ignoring higher order terms of θ, we get

$$S = e^{-\theta t_1}\left[\frac{r}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+1}}{n+1}\right) - \frac{g}{T^{\frac{1}{m}}}\left(t_1^{\frac{1}{m}} + \frac{\theta t_1^{\frac{1}{m}+1}}{m+1}\right)\right] \quad . \tag{10}$$

From equation (6) and using the condition I (t_2) = 0, implies

$$Se^{\theta t_1} - \frac{g}{mT^{\frac{1}{m}}}\int_{t_1}^{t_2} u^{\frac{1}{m}-1} e^{\theta u} du = 0.$$

$$t_2\left(1+\frac{\theta t_2}{m+1}\right)^m = \left(\frac{T^{\frac{1}{m}}}{g} Se^{\theta t_1} + t_1^{\frac{1}{m}} + \frac{\theta t_1^{\frac{1}{m}+1}}{m+1}\right)^m \tag{11}$$

Substituting the value of S from equation (10) in equation (11) we get t_2 from the equation

$$t_2\left(1+\frac{\theta t_2}{n+1}\right)^m = \left[\frac{rT^{\frac{1}{m}}}{gT^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+1}}{n+1`}\right)\right]^m$$

$$t_2(1+\frac{\theta t_2}{m+1})^m = [B(t_1)]^m$$

$$\text{where,} \quad B(t_1) = \frac{r\, T^{\frac{1}{m}}}{T^{\frac{1}{n}} d}\left(t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+1}}{n+1}\right) . \tag{12}$$

Considering the bionamial expansion of $\left(1+\frac{\theta t_2}{m+1}\right)^m$ and ignoring the higher order terms of θ, This implies

$$t_2\left(1+\frac{m\,\theta t_2}{m+1}\right) = [B(t_1)]^m$$

$$t_2 = \frac{m+1}{2m\theta}\left[\sqrt{1+\frac{4m\theta}{m+1}B^m(t_1)} - 1\right] = D(t_1) \quad (\text{say}) \quad . \tag{13}$$

Taking t = t_3, in the equations (7) and (8) and equating these we get

$$t_3 = \left[\frac{g}{r} T^{\frac{1}{n}}\left(\frac{t_2^{\frac{1}{m}}}{T^{\frac{1}{m}}} - 1\right) + T^{\frac{1}{n}}\right]^n \quad . \tag{14}$$

Substituting the value of t_2 in equation (14) we get

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$$t_3 = \left[\frac{g}{r} T^{\frac{1}{n}} \left(\frac{[D(t_1)]^{\frac{1}{m}}}{T^{\frac{1}{m}}} - 1 \right) + T^{\frac{1}{n}} \right]^n \quad . \qquad (15)$$

This implies $t_3 = (H(t_1))^n$ (16)

$$\text{where,} \quad H(t_1) = \frac{g\, T^{\frac{1}{n}}}{r} \left(\frac{[D(t_1)]^{\frac{1}{m}}}{T^{\frac{1}{m}}} - 1 \right) + T^{\frac{1}{n}} \quad . \qquad (17)$$

Let $K(t_1, t_2, t_3)$ be the total cost per unit time. Since the total cost is sum of the setup cost, cost of the units, the inventory holding cost and shortage cost, the

$K(t_1, t_2, t_3)$ can be obtained as

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt \right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt \right] \quad . \qquad (18)$$

Substituting the values of I (t) and Q from equations (5), (.6), (7), (8) and (9) in equation (18)

we get

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C\,r}{T^{\frac{n+1}{n}}}\left(T^{\frac{1}{n}} + t_1^{\frac{1}{n}} - t_3^{\frac{1}{n}} \right) + \frac{h}{T}\left\{ \int_0^{t_1} e^{-\theta t}\left[Se^{\theta t_1} - \int_t^{t_1} \left(\frac{r}{nT^{\frac{1}{n}}} u^{\frac{1}{n}-1} - \frac{g\, u^{\frac{1}{m}-1}}{mT^{\frac{1}{m}}} \right) e^{\theta u} du \right] dt \right.$$
$$\left. + \int_{t_1}^{t_2} e^{-\theta t}\left[Se^{\theta t_1} - \frac{g}{mT^{\frac{1}{m}}} \int_{t_1}^{t} u^{\frac{1}{m}-1} e^{\theta u} du \right] dt \right\} + \left\{ \frac{\pi}{T} \int_{t_2}^{t_3} \frac{g}{T^{\frac{1}{m}}}\left(t^{\frac{1}{m}} - t_2^{\frac{1}{m}} \right) dt \right.$$
$$\left. + \int_{t_3}^{T} \left[g\left(\frac{t^{\frac{1}{m}}}{T^{\frac{1}{m}}} - 1 \right) + \frac{r}{T^{\frac{1}{n}}}\left(T^{\frac{1}{n}} - t^{\frac{1}{n}} \right) \right] dt \right\} \quad . \qquad (19)$$

Substituting the values of S, t_2, and t_3 from equations (10), (13) and (16) in equation (19), $K(t_1, t_2, t_3)$ becomes $K(t_1)$,

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$$K(t_1)=\frac{A}{T}+\frac{C\,r}{T^{\frac{n+1}{n}}}\left(T^{\frac{1}{n}}+t_1^{\frac{1}{n}}-H(t_1)\right)+\frac{h}{T}\left\{\frac{r}{T^{\frac{1}{n}}\theta}\left(t_1^{\frac{1}{m}}+\frac{\theta\, t_1^{\frac{1}{m}+1}}{m+1}\right)\left(e^{-\theta\, t_1}-e^{-\theta\, D(t_1)}\right)+\frac{r\,n}{T^{\frac{1}{n}}(n+1)}\right.$$

$$\left.\left(t_1^{\frac{1}{n}+1}-\frac{n\,\theta\, t_1^{\frac{1}{n}+2}}{(2n+1)}-\frac{\theta^2\, t_1^{\frac{1}{n}+3}}{(3n+1)}\right)-\frac{m\,g}{(m+1)T^{\frac{1}{m}}}\left((D(t_1))^{\frac{1}{m}+1}-\frac{m\,\theta\,(D(t_1))^{\frac{1}{m}+2}}{(2m+1)}-\frac{\theta^2\,(D(t_1))^{\frac{1}{m}+3}}{(3m+1)}\right)\right\}$$

$$+\frac{\pi}{T}\left\{\frac{g}{(m+1)T^{\frac{1}{m}}}\left((D(t_1))^{\frac{1}{m}+1}-(m+1)(D(t_1))^{\frac{1}{m}}(H(t_1))^{n}\right)+g\left((H(t_1))^{n}-\frac{T}{m+1}\right)\right.$$

$$\left.+\frac{r\,n}{T^{\frac{1}{n}}(n+1)}\left(\frac{1}{n}T^{\frac{1}{n}+1}+(H(t_1))^{n+1}-\left(\frac{n+1}{n}\right)T^{\frac{1}{n}}(H(t_1))^{n}\right)\right\}. \qquad (20)$$

1.4 OPTIMAL POLICIES OF THE MODEL

In this section the optimal policies of the inventory system are derived. To find the optimal value of t_1, we minimize the total cost per unit time with respect to t_1. The conditions for optimal value of t_1 are

$$\frac{\partial K(t_1)}{\partial t_1}=0 \text{ and } \frac{\partial^2 K(t_1)}{\partial t_1^{\,2}}>0 ,$$

Differentiating K (t_1) with respect to t_1 and equate to zero, we get

$$\frac{C\,r}{nT^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}-1}-(H(t_1))^{1-n}\,G(t_1)\right]+h\left\{\frac{r}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}}+\frac{\theta\, t_1^{\frac{1}{n}+1}}{(n+1)}\right)\left(e^{-\theta\,(D\,(t_1))}F(t_1)-e^{-\theta\, t_1}\right)+\frac{r}{nT^{\frac{1}{n}}}\left(e^{-\theta\, t_1}-e^{-\theta\, D\,(t_1)}\right)\right.$$

$$\left.\left(\frac{t_1^{\frac{1}{n}-1}}{\theta}+t_1^{\frac{1}{n}}\right)+\frac{r}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}}-\frac{\theta\, n}{(n+1)}t_1^{\frac{1}{n}+1}-\frac{\theta^2}{(n+1)}t_1^{\frac{1}{n}+2}\right)-\frac{g\,F\,(t_1)}{T^{\frac{1}{n}}}\left((D\,(t_1))^{\frac{1}{m}}-\frac{m\,\theta}{(m+1)}(D\,(t_1))^{\frac{1}{m}+1}-\frac{\theta^2}{(m+1)}(D\,(t_1))^{\frac{1}{m}+2}\right)\right\}$$

$$+\frac{\pi}{T}\left\{\frac{g}{T^{\frac{1}{m}}}\left[\frac{1}{m}(D\,(t_1))^{\frac{1}{m}}\,F\,(t_1)-(D\,(t_1))^{\frac{1}{m}}\,G\,(t_1)-\frac{1}{m}(H\,(t_1))^{n}\,(D\,(t_1))^{\frac{1}{m}-1}F(t_1)\right]\right.$$

$$\left.+\,g\,G\,(t_1)+\frac{r}{T^{\frac{1}{n}}}G\,(t_1)\left[H(t_1)-T^{\frac{1}{n}}\right]\right\}=0 \quad . \qquad (21)$$

where, $$F(t_1)=\frac{dt_2}{dt_1}=\frac{m\,r}{T^{\frac{1}{n}}n\,d}T^{\frac{1}{m}}\left(1+\frac{4\,m\,\theta}{m+1}B(t_1)\right)^{-\frac{1}{2}}B^{m-1}(t_1)\left(t_1^{\frac{1}{n}-1}+\theta\, t_1^{\frac{1}{n}}\right)$$

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and $$G(t_1)=\frac{dt_3}{dt_1}=\frac{n\,g\,T^{\frac{1}{n}}}{r\,m\,T^{\frac{1}{m}}}(H\,(t_1))^{n-1}(D\,(t_1))^{\frac{1}{m}-1}F(t_1)\quad ,$$

B (t_1), D (t_1) and H (t_1) are as given in equations (12), (13) and (17) respectively.

of t_1 is determined

Solving the equation (21) by using numerical methods, the optimal production downtime t_1 * of
for t_1 * of t1. Similarly, the optimal time t_3*t_3*t_3*is obtained for t_3 * of t_3." At which the production uptime is obtained by substituting the optimal value of t_1 in equation (16)

Therefore, the optimal time at which the production to be started is

$$t_3^*=\left[\frac{gT^{\frac{1}{n}}}{r}\left(\left(\frac{m+1}{2m\theta T}\left[\sqrt{1+\frac{4m\theta T}{m+1}\left(\frac{r}{T^{\frac{1}{n}}g}\right)^m\left(t_1^{*\frac{1}{n}}+\frac{\theta t_1^{*\frac{1}{n}+1}}{n+1}\right)}-1\right]\right)^{\frac{1}{m}}-1\right)+T^{\frac{1}{n}}\right]^n . \qquad (22)$$

The optimum production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1

and t_3 in equation (9)

Therefore the optimal production quantity is

$$Q^*=\frac{r}{T^{\frac{1}{n}}}\left[t_1^{*\frac{1}{n}}-\frac{gT^{\frac{1}{n}}}{r}\left[\left(\frac{m+1}{2m\theta T}\left(1+\frac{4m\theta T}{m+1}\left(\frac{r}{T^{\frac{1}{n}}d}\right)^m\left(t_1^{*\frac{1}{n}}+\frac{\theta t_1^{*\frac{1}{n}+1}}{(n+1)}\right)=1\right)\right)^{\frac{1}{m}}-1\right]\right]. \qquad (23)$$

1.5

NUMERICAL ILLUSTRATION

In this section, consider the case of deriving the optimal production quantity, production downtime and production uptime of an industry. Here it is assumed that the product is of deteriorating nature and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the deteriorating parameter θ is considered to vary as 0.3, 0.4, 0.5, 0.6 and 0.7. The values of other parameters and costs associated with the model are

A = 1000, 1500, 2000, 2500, 3000; C = 20, 21, 22, 23, 24

h = 2.0, 2.5, 2.8, 3.1, 3.4 ; π = 4, 5, 6, 7, 8

n = 2.0, 2.5, 2.8, 3.1, 3.4 ; r = 150, 155, 160, 165, 170

θ = 0.3, 0.4, 0.5, 0.6, 0.7 ; m = 2.0, 2.1, 2.2, 2.3, 2.4

g = 100, 101, 102, 103, 104 ; T = 12

Substituting these values the optimal production quantity Q^*, production downtime t_1^*, production uptime t_3^* and optimal cost of production K are computed and presented in Table 1.1. From Table 1.1 it is observed that the deterioration parameter and production parameters have a tremendous influence on the optimal values of the other

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parameters. As the deterioration parameter θ varies from 0.3 to 0.7, then the optimal production quantity Q^* decreases from 131.397 to 122.29, the optimal value of production down time t_1^* decreases from 3.986 to 2.506, the optimal values of production uptime t_3^* decreases from 8.66 to 6.632 and the total cost of production per unit time K increases from 328.404 to 331.154 units.

As the production parameter r increases from 150 units to 170 units, the optimal production quantity Q^* increases from 131.397 to 140.175 units, this increase is marginal. The optimal values of production downtime t_1^* increases from 3.986 to 4.262, the production uptime t_3^* also increases from 8.660 to 10.163, this increase is marginal. The total cost for a production per unit time K increases from 328.404 to 402.360 units. The indexing parameter of the production rate n increases from 2.0 to 2.8 then the optimal value of production quantity increases from 131.397 to 135.286. The optimal production downtime t_1^* decreases from 3.986 to 3.738 and the optimal value of production uptime t_3^* decreases from 8.660 to 8.121, the optimal value of production cost per unit time K increases from 328.404 to 359.255.

The demand parameter g increases from 100 to 104 units then the optimal values of production downtime t_1^* decreases from 3.986 to 3.895 and production uptime t_3^* decreases from 8.660 to 8.155 and production quantity Q^* increases from 131.397 to 134.126, this increase is marginal. The cost parameter C increases from 20 units to 24 units, the optimal values of production quantity Q^* increases from 131.397 to 132.918 units, this increase is marginal. The optimal values of production down time t_1^* increases from 3.986 to 4.205, the production uptime t_3^* increases from 8.660 to 8.945, the total cost of production per unit time K increases from 328.404 to 374.190 units.

Table-1.1

OPTIMAL VALUES OF t_1^*, t_3^*, Q^*, AND K

C	r	n	h	θ	g	m	π	A	T	t_1^*	t_3^*	Q^*	K
20	150	2.0	2.0	0.3	100	2.0	4	1000	12	3.986	8.66	131.397	328.404
21	150	2.0	2.0	0.3	100	2.0	4	1000	12	4.043	8.734	131.797	339.805
22	150	2.0	2.0	0.3	100	2.0	4	1000	12	4.098	8.806	132.180	351.219
23	150	2.0	2.0	0.3	100	2.0	4	1000	12	4.152	8.876	132.553	362.687
24	150	2.0	2.0	0.3	100	2.0	4	1000	12	4.205	8.945	132.918	374.190
20	**155**	2.0	2.0	0.3	100	2.0	4	1000	12	4.062	9.072	133.554	343.514
20	**160**	2.0	2.0	0.3	100	2.0	4	1000	12	4.133	9.459	135.736	360.744
20	**165**	2.0	2.0	0.3	100	2.0	4	1000	12	4.199	9.821	137.940	380.255
20	**170**	2.0	2.0	0.3	100	2.0	4	1000	12	4.262	10.163	140.175	402.360
20	150	**2.5**	2.0	0.3	100	2.0	4	1000	12	3.800	8.274	133.916	351.577
20	150	**2.6**	2.0	0.3	100	2.0	4	1000	12	3.768	8.197	134.384	355.504
20	150	**2.7**	2.0	0.3	100	2.0	4	1000	12	3.738	8.121	134.842	359.255
20	150	**2.8**	2.0	0.3	100	2.0	4	1000	12	3.709	8.045	135.286	362.805
20	150	2.0	**2.5**	0.3	100	2.0	4	1000	12	3.768	8.375	129.845	332.525
20	150	2.0	**2.8**	0.3	100	2.0	4	1000	12	3.667	8.242	129.111	335.034
20	150	2.0	**3.1**	0.3	100	2.0	4	1000	12	3.580	8.128	128.472	337.476
20	150	2.0	**3.4**	0.3	100	2.0	4	1000	12	3.506	8.030	127.923	339.961
20	150	2.0	2.0	**0.4**	100	2.0	4	1000	12	3.412	7.887	128.001	328.578
20	150	2.0	2.0	**0.5**	100	2.0	4	1000	12	3.021	7.352	125.599	329.385
20	150	2.0	2.0	**0.6**	100	2.0	4	1000	12	2.732	6.951	123.766	330.315
20	150	2.0	2.0	**0.7**	100	2.0	4	1000	12	2.506	6.632	122.290	331.154
20	150	2.0	2.0	0.3	**101**	2.0	4	1000	12	3.964	8.534	132.081	326.765
20	150	2.0	2.0	0.3	**102**	2.0	4	1000	12	3.941	8.408	132.760	325.247

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20	150	2.0	2.0	0.3	**103**	2.0	4	1000	12	3.918	8.281	133.442	323.881
20	150	2.0	2.0	0.3	**104**	2.0	4	1000	12	3.895	8.155	134.126	322.657
20	150	2.0	2.0	0.3	100	**2.1**	4	1000	12	3.917	8.728	131.008	324.687
20	150	2.0	2.0	0.3	100	**2.2**	4	1000	12	3.850	8.786	130.601	320.969
20	150	2.0	2.0	0.3	100	**2.3**	4	1000	12	3.784	8.833	130.174	317.228
20	150	2.0	2.0	0.3	100	**2.4**	4	1000	12	3.718	8.870	129.726	313.448
20	150	2.0	2.0	0.3	100	2.0	**5**	1000	12	3.971	8.640	131.292	329.313
20	150	2.0	2.0	0.3	100	2.0	**6**	1000	12	3.955	8.619	131.179	330.228
20	150	2.0	2.0	0.3	100	2.0	**7**	1000	12	3.939	8.599	131.066	331.187
20	150	2.0	2.0	0.3	100	2.0	**8**	1000	12	3.923	8.578	130.953	332.191
20	150	2.0	2.0	0.3	100	2.0	4	**1500**	12	3.986	8.660	131.397	370.070
20	150	2.0	2.0	0.3	100	2.0	4	**2000**	12	3.986	8.660	131.397	411.737
20	150	2.0	2.0	0.3	100	2.0	4	**2500**	12	3.986	8.660	131.397	453.404
20	150	2.0	2.0	0.3	100	2.0	4	**3000**	12	3.986	8.660	131.397	495.070

The holding cost h increases from 2.0 to 3.4, the optimal production quantity Q^* decreases from 131.397 to 127.923, the optimal value of production down time t_1^* decreases from 3.986 to 3.506, the optimal value of production uptime t_3^* decreases from 8.66 to 8.03, and the total cost of production per unit time K increases from 328.404 to 339.961 units. The shortage cost π increases from 4 to 8, then the optimal production quantity Q^* decreases from 131.397 to 130.953, the optimal value of the production downtime t_1^* decreases from 3.986 to 3.923, the optimal value of production uptime t_3^* decreases from 8.66 to 8.578 and the total cost of production per unit time K increases from 328.404 to 332.191 units. When as the setup cost A increases from 1000 to 3000 there is no effect of change in optimal production quantity Q^*, production down time t_1^*, and production uptime t_3^*, where as the total cost function K is increasing from 328.404 to 495.07 units, this increase is marginal.

1.6 SENSITIVITY ANALYSIS OF THE MODEL

A sensitivity analysis is carried out to explore the effect of changes in parameters and costs on the optimal policies by varying each parameter (-15%, -10%, -5%, 5%, 10%, 15%) at a time and all parameters together for the model under study. The results are obtained and presented in Table 1.2.The relationship between the parameters, cost on the optimal values of the production schedule are shown in Figure 4.1. From Table 4.2 it is observed that the variation in the deterioration parameter θ and the demand parameter g, m have significant influence on optimal production quantity Q^*.

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Table – 1.2

SENSITIVITY ANALYSIS OF THE MODEL WITH RESPECT TO PARAMETERS & COSTS

Variation in parameters	Optimal policies	Percentage Change in Parameters						
		-15	-10	-5	0	5	10	15
C	t_1^*	3.807	3.869	3.928	3.986	4.043	4.098	4.152
	t_3^*	8.426	8.507	8.584	8.660	8.734	8.806	8.876
	Q^*	130.126	130.569	130.988	131.397	131.797	132.18	132.553
	K	294.494	305.78	317.061	328.404	339.805	351.219	362.681
r	t_1^*	3.571	3.726	3.863	3.986	4.098	4.199	4.292
	t_3^*	6.453	7.259	7.992	8.660	9.268	9.821	10.326
	Q^*	122.072	125.117	128.224	131.397	134.641	137.94	141.301
	K	281.337	293.87	309.322	328.404	351.847	380.255	414.414
n	t_1^*	4.134	4.081	4.032	3.986	3.944	3.905	3.868
	t_3^*	8.900	8.819	8.740	8.660	8.582	8.504	8.427
	Q^*	311.432	317.381	323.043	328.404	333.531	338.412	343.032
	Q	129.837	130.356	130.879	131.397	131.918	132.434	132.939
h	t_1^*	4.161	4.098	4.040	3.986	3.937	3.890	3.847
	t_3^*	8.887	8.806	8.730	8.660	8.596	8.535	8.478
	Q^*	132.615	132.18	131.776	131.397	131.052	130.719	130.412
	K	325.99	326.792	327.603	328.404	329.255	330.051	330.888
θ	t_1^*	4.353	4.220	4.099	3.986	3.883	3.786	3.697
	t_3^*	9.147	8.971	8.810	8.660	8.522	8.392	8.273
	Q^*	133.506	132.747	132.052	131.397	130.798	130.229	129.705
	K	329.011	328.726	328.562	328.404	328.364	328.313	328.361
g	t_1^*	4.331	4.215	4.101	3.986	3.872	3.757	3.642
	t_3^*	10.566	9.931	9.295	8.66	8.029	7.404	6.788
	Q^*	121.587	124.776	128.055	131.397	134.812	138.279	141.801
	K	381.641	356.554	339.583	328.404	321.568	317.858	316.45
m	t_1^*	4.222	4.136	4.059	3.986	3.917	3.850	3.784
	t_3^*	8.400	8.495	8.583	8.660	8.728	8.786	8.833
	Q^*	132.47	132.127	131.774	131.397	131.008	130.601	130.174
	K	339.918	335.961	332.172	328.404	324.687	320.969	317.228
π	t_1^*	3.996	3.993	3.990	3.986	3.983	3.980	3.977

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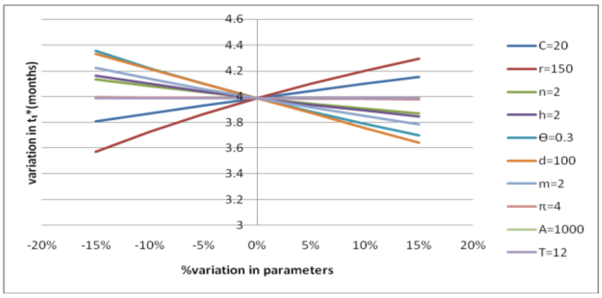
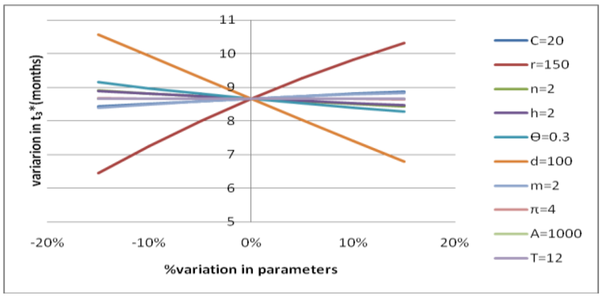
	t_3^*	8.673	8.669	8.665	8.660	8.656	8.652	8.648
	Q^*	131.468	131.447	131.426	131.397	131.376	131.355	131.334
	K	327.918	328.092	328.267	328.404	328.582	328.763	328.944
A	t_1^*	3.986	3.986	3.986	3.986	3.986	3.986	3.986
	t_3^*	8.660	8.660	8.660	8.660	8.660	8.660	8.660
	Q	131.397	131.397	131.397	131.397	131.397	131.397	131.397
	K	315.904	320.07	324.237	328.404	332.57	336.737	340.904

As θ increases the production quantity Q* is decreasing and the production down time and production uptime are also decreasing when other parameters remain fixed. As d increases the optimal production quantity Q* is increasing and the production down time and up time are decreasing. When demand parameter m is increasing the production quantity Q^* is increasing and the production downtime t_1^* time is decreasing. This decrease in both Q^* and t_1^* marginal. Whereas the production uptime t_3^* is increasing when m increases.

It is further observed that the costs are having a significant influence on the optimal production quantity and production schedules. As the penalty cost π increases the optimal values of the production quantity, production down time and production uptime are decreasing. This decrease is very low. When the cost per unit time C is increasing the optimal values of t_1^* and t_3^* are increasing. When the holding cost h increases the optimal production quantity Q^*, the optimal production down time and production up time are decreasing. There is no effect of change in set up cost on the optimal values of Q^*, t_1^* and t_3^*. However the total cost is increasing when A increases. The production rate parameters have significant influence on the optimal values of the production quantity Q^* and production down time t_1^* and production uptime t_3^*. As r increases the values of Q^*, t_1^* and t_3^* are increasing. This increase is rapid, as n increases the optimal values of t_1^* and t_3^* are decreasing. This decrease is marginal.





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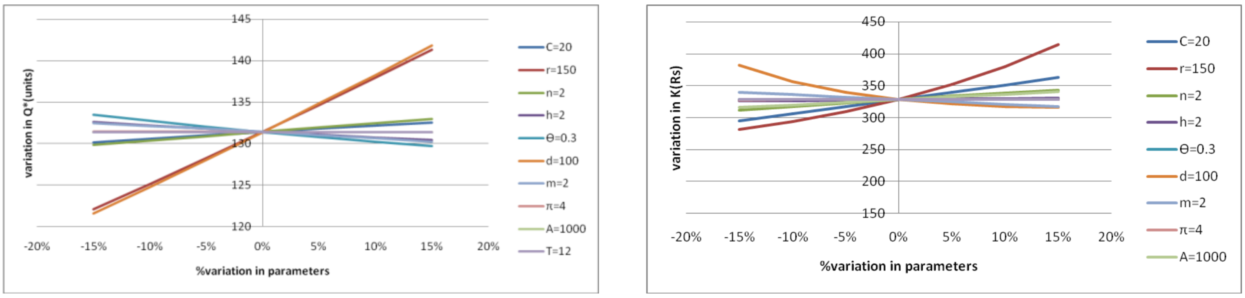

With Deficiency Fig. 1.2 The graphical representation of sensitivity analysis of production and demand – with deficiency

1.6. Conclusions

This study deals with a novel EPQ model with time -dependent production rate. Here it is assumed that the production follows a power pattern. The power pattern production rate includes constant/increasing/decreasing rates of production .Assuming that shortages are allowed and fully backlogged the instantaneous inventory state is derived. Through numerical illustration it was observed that the deteriorating rate, production rate and demand rate have a significant influence on optimal production schedule and quantity. This model also includes some earlier models as particular cases for the specific or discretionary values of the parameters. This model can be extended by considering demand as a function of time and selling price which will be discussed elsewhere.

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Old abstract

In this study an economic production quantity (EPQ) model consider that crumbling items with time dependent production and demand with deficiency. It is further assumed that the lifetime of item follows an exponential distribution. The total cost function was derived with suitable cost considerations. By minimizing the function the optimal production schedule and quantity were derived. The sensitivity of the model with reference to the costs and parameters were studied.