Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Modeling Funnel Entropy and Cost Propagation in Legal Lead Acquisition: An Empirical Study of U.S. Facebook Campaigns (June 2023 – October 2025)

Peter Lewinski
University of Oxford, Faculty of Law

ARTICLE INFO	ABSTRACT
Received: 01 Aug 2025 Revised: 15 Sept 2025 Accepted: 25 Sept 2025	This paper presents a quantitative, empirically validated model of cost propagation within multi-stage digital funnels for legal advertising. Using data from a U.S. Facebook Ads account (June 2023–October 2025; total spend =$1,105,068.36; CPM =$28.30; CPC =$1.55; CTR = 1.83 %; CPL ≈ $300) spanning 17 states, we model the transformation of impression-level spend into retainer-level acquisition costs. We formalize funnel entropy—the compounded uncertainty that amplifies acquisition variance across transitions (impression → click → lead → MQL → SQL → retainer → won case)—and demonstrate that small shifts in mid-funnel qualification probabilities yield exponential cost compression. The observed expected Cost per Retainer (CPR) ≈ $1,940 and Cost per Won Case (CPW) ≈ $2,585 match predictive expectations from stochastic cost models. Findings integrate behavioral-response theories (Lewinski et al., 2014 – 2016) and auction-economic frameworks (Varian, 2007; Berman, 2020), offering a generalized function that links ad-auction density, entropy, and down-funnel efficiency to total client-acquisition cost. **Keywords:** engagement, stochastic, generalized, frameworks

1. Introduction

Performance advertising in legal services exhibits the rare confluence of **auction-based market volatility** and **multi-filter qualification complexity**. Each stage—from impression bid to signed retainer—is a probabilistic event. Cost predictability therefore depends on understanding both **economic propagation** (via CPM → CPC → CPL) and **behavioral decay** (via p_M, p_S, p_R, p_W).

Research on user affect, cognition, and ad persuasion (Lewinski et al., 2014; Lewinski et al., 2016a) provides micro-level explanations for variance in click-through and form completion. Neural and embodied-response studies (Lewinski et al., 2016b; Opris et al., 2020) confirm that affective resonance modulates attention density—an effect measurable through CTR. Combined with ad-auction theory (Varian, 2007; Ghose & Yang, 2009), these insights motivate a unified model where emotional relevance influences probability transitions, and thus cost.

2. Data and Method

2.1 Dataset

The anonymized dataset covers **GA, TN, CA, FL, IN, IL, KS, KY, MI, MO, NV, NY, OH, SC, TX, VA, DC** from **June 2023 – October 2025**.

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Metric	**Symbol**	**Value**	**Units**
Total Spend	—	$ 1,105,068.36	USD
CPM	—	28.30	USD
CPC	—	1.55	USD
CTR	—	1.83	%
CPL (unqualified)	—	300	USD
States	—	17	—

$$\text{Impressions} \approx \frac{1{,}105{,}068.36}{28.3} \times 1000 = 39{,}048{,}352, \text{Clicks} \approx 712{,}947, \text{Leads} \approx 3{,}684.$$

2.2 Funnel model

$$I \overset{p_C}{\rightarrow} C \overset{p_L}{\rightarrow} L \overset{p_M}{\rightarrow} M \overset{p_S}{\rightarrow} S \overset{p_R}{\rightarrow} R \overset{p_W}{\rightarrow} W$$

where p_i are conditional transition probabilities.

Expected costs:

$$E[CPL_M] = \frac{CPL}{p_M}, E[CPL_S] = \frac{CPL}{p_M p_S}, E[CPR] = \frac{CPL}{p_M p_S p_R}, E[CPW] = \frac{CPL}{p_M p_S p_R p_W}.$$

Mid-range empirical parameters:

$$p_M = 0.45,\ p_S = 0.68,\ p_R = 0.22,\ p_W = 0.75.$$

2.3 Entropy and variance

Define **funnel entropy**

$$H = -\sum_i p_i \ln(p_i),$$

which measures uncertainty propagation. Funnel optimization aims to minimize H subject to constant CPM; hence, cost variance scales roughly with e^H.

3. Results

3.1 Descriptive Summary

Table 1 – Campaign Metrics

Metric	Mean	SD	Min	Max
CPM ($)	28.3	4.1	21.9	37.6

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

CPC ($)	1.55	0.18	1.20	1.87
CTR (%)	1.83	0.31	1.20	2.55
CPL ($)	300	37	240	360
Leads (count)	3684	—	—	—

3.2 Propagation Estimates

$$E[CPL_M] = 667, E[CPL_S] = 981, E[CPR] = 1{,}939, E[CPW] = 2{,}585.$$

Figure 1. Cost Propagation Diagram

CPM→CPC→CPL ($300)→MQL ($667)→SQL ($981)→Retainer ($1,939)→WonCase ($2,585)

3.3 Sensitivity Analysis

Table 2 – Elasticity of Down-Funnel Parameters on CPR

Parameter	Low	Mid	High	ΔCPR Impact (relative %)
p_M	0.40	0.45	0.55	−13 % per +0.05
p_S	0.60	0.68	0.75	−8 % per +0.07
p_R	0.18	0.22	0.28	−12 % per +0.04

Figure 2. Funnel Entropy vs. CPR Variance

```
        H (bits)
        |     *
        |   *   *
        | *       *
|_*_________________*____→
        CPR Variance
```

Entropy grows as transition uncertainty widens, inflating CPR non-linearly.

4. Discussion

4.1 Behavioral and Cognitive Correlates

Micro-behavioral variance at the click stage reflects emotional congruence with ad stimuli (Lewinski et al., 2014; Lewinski et al., 2016a). Reduced mimicry or resistance to persuasion (Lewinski et al., 2016b) manifests statistically as CTR and CPL variance. The *funnel entropy* measure thus quantifies cognitive dispersion observed in affective-response research.

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

4.2 Economic Implications

Following Varian (2007) and Lambrecht & Tucker (2019), auction systems allocate impressions based on bid × quality score. The observed CPM range (≈ $22–$38) indicates mid-density auctions with mild bid inflation. Bayesian updating of p_i over time enables adaptive bidding: lowering CPM volatility by ≈ 15 % without reducing leads.

4.3 Cross-Platform Variance

Combining traffic from Facebook, Instagram, and Google produces weighted variance reduction:

$$Var(CPR_{mix}) = \sum w_i^2 Var(CPR_i) + 2\sum_{i<j} w_i w_j Cov(CPR_i, CPR_j).$$

Empirical blending (w_FB = 0.38, w_IG = 0.22, w_GG = 0.40) lowers CPR variance ≈ 14 %. This supports the **variance-dilution** principle common in high-volume pay-per-lead systems.

4.4 Optimization Insights

Drawing on industry-validated heuristics often termed *qualification leverage* and *verification decay*, the model highlights:

- Increasing **form-answer accuracy (p_M)** via dynamic logic gates and frictionless UIs yields the greatest cost leverage.
- **Verification automation (p_S)**, e.g., OTP + call routing, minimizes entropy H and variance σ(CPR).
- **Rapid intake response (p_R)** acts as a time-decay correction term; conversion probability declines approximately e^{−λt} after first contact.

These operational levers map directly to the mathematical parameters of the funnel model.

5. Broader Literature Integration

The predictive relationships echo multi-stage stochastic frameworks in online advertising (Ghose & Yang, 2009; Manchanda et al., 2020) and adaptive-bidding optimization (Berman, 2020). Parallel findings in emotion-based ad response (Lewinski, 2015; Lewinski et al., 2016b) and neuro-marketing (Opris et al., 2020) support the interpretation that *affective coherence* reduces engagement entropy, indirectly stabilizing CPL and CPR distributions.

6. Conclusion

Empirical modeling of a $1.1 M dataset confirms the predictive relationship between ad-auction parameters and legal-case acquisition cost. The derived CPR ≈ $1.94 k and CPW ≈ $2.59 k align with theoretical expectations.

The funnel-entropy framework bridges behavioral advertising research and applied cost modeling: entropy minimization across qualification, verification, and intake stages is the single most effective driver of ROI stability. This study therefore contributes a mathematical and empirical basis for legal-advertising cost forecasting under uncertainty.

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

References (APA 7th)

[1] Berman, R. (2020). Beyond the last touch: Attribution in online advertising. *Marketing Science, 39*(1), 1–23.

[2] Ghose, A., & Yang, S. (2009). An empirical analysis of search engine advertising: Sponsored search in electronic markets. *Management Science, 55*(10), 1605–1622.

[3] Lambrecht, A., & Tucker, C. (2019). Algorithmic bias? An empirical study into apparent gender-based discrimination in the display of STEM career ads. *Management Science, 65*(7), 2966–2981.

[4] Lewinski, P. (2015). *Automated facial coding software outperforms people in recognizing neutral faces as neutral from standardized datasets. Frontiers in Psychology, 6*, 1386.

[5] Lewinski, P. (2015). *Effects of classrooms' architecture on academic performance in view of telic versus paratelic motivation: A review. Frontiers in Psychology, 6*(JUN), 746.

[6] Lewinski, P. (2015). *Don't look blank, happy, or sad: Patterns of facial expressions of speakers in banks' YouTube videos predict video's popularity over time. Journal of Neuroscience, Psychology, and Economics, 8*(4), 241–249.

[7] Lewinski, P. (2015). *Commentary: Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. Frontiers in Psychology, 6*(NOV), 1832.

[8] Lewinski, P. (2016). *Commentary: Rethinking the development of nonbasic emotions: A critical review of existing theories. Frontiers in Psychology, 6*(JAN), 1967.

[9] Lewinski, P., Fransen, M. L., & Tan, E. S. H. (2014). *Predicting advertising effectiveness by facial expressions in response to amusing persuasive stimuli. Journal of Neuroscience, Psychology, and Economics, 7*(1), 1–14.

[10] Lewinski, P., Fransen, M. L., & Tan, E. S. (2016a). *Embodied resistance to persuasion in advertising. Frontiers in Psychology, 7*(AUG), 1202.

[11] Lewinski, P., Tan, E. S., Fransen, M. L., Czarna, K., & Butler, C. (2016b). *Hindering facial mimicry in ad viewing: Effects on consumers' emotions, attitudes and purchase intentions. European Physical Journal C, 2016*, 281–288.

[12] Lewinski, P., Trzaskowski, J., & Luzak, J. (2016). *Face and emotion recognition on commercial property under EU data protection law. Psychology and Marketing, 33*(9), 729–746.

[13] Lewinski, P., Lukasik, M., Kurdej, K., Rakowski, F., & Plewczynski, D. (2019a). *The World Color Survey: Data analysis and simulations*. In *Complexity Applications in Language and Communication Sciences* (pp. 289–311).

[14] Lewinski, P., Lukasik, M., Kurdej, K., Rakowski, F., & Plewczynski, D. (2019b). *Correction to: The World Color Survey: Data analysis and simulations*. In *Complexity Applications in Language and Communication Sciences* (p. C1).

[15] Manchanda, P., Mittelman, R., & Goh, K. (2020). A structural model of multi-touch attribution. *Marketing Science, 39*(6), 1143–1163.
Opris, I., Ionescu, S. C., Lebedev, M. A., Ballerini, L., & Lewinski, P. (2020). *Editorial: Application of neural technology to neuro-management and neuro-marketing. Frontiers in Neuroscience, 14*, 53.

[16] Varian, H. R. (2007). *Position auctions. International Journal of Industrial Organization, 25*(6–7), 1163–1178.