2025, 10(60s) e-ISSN: 2468-4376

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Modeling Funnel Entropy and Cost Propagation in Legal Lead Acquisition: An Empirical Study of U.S. Facebook Campaigns (June 2023 – October 2025)

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ARTICLE INFO

ABSTRACT

Received: 01 Aug 2025 Revised: 15 Sept 2025

Accepted: 25 Sept 2025

This paper presents a quantitative, empirically validated model of cost propagation within multi-stage digital funnels for legal advertising. Using data from a U.S. Facebook Ads account (June 2023–October 2025; total spend =\$1,105,068.36; CPM =\$28.30; CPC =\$1.55; CTR = 1.83 %; CPL ≈ \$300) spanning 17 states, we model the transformation of impression-level spend into retainer-level acquisition costs. We formalize funnel entropy—the compounded uncertainty that amplifies acquisition variance across transitions (impression \rightarrow click \rightarrow lead \rightarrow MQL \rightarrow SQL \rightarrow retainer \rightarrow won case)—and demonstrate that small shifts in mid-funnel qualification probabilities yield exponential cost compression. The observed expected Cost per Retainer (CPR) ≈ \$1,940 and Cost per Won Case (CPW) ≈ \$2,585 match predictive expectations from stochastic cost models. Findings integrate behavioral-response theories (Lewinski et al., 2014 – 2016) and auction-economic frameworks (Varian, 2007; Berman, 2020), offering a generalized function that links ad-auction density, entropy, and down-funnel efficiency to total client-acquisition cost.

Keywords: engagement, stochastic, generalized, frameworks

1. Introduction

Performance advertising in legal services exhibits the rare confluence of **auction-based market volatility** and **multi-filter qualification complexity**. Each stage—from impression bid to signed retainer—is a probabilistic event. Cost predictability therefore depends on understanding both **economic propagation** (via CPM \rightarrow CPC \rightarrow CPL) and **behavioral decay** (via p_M , p_S , p_R , p_W).

Research on user affect, cognition, and ad persuasion (Lewinski et al., 2014; Lewinski et al., 2016a) provides micro-level explanations for variance in click-through and form completion. Neural and embodied-response studies (Lewinski et al., 2016b; Opris et al., 2020) confirm that affective resonance modulates attention density—an effect measurable through CTR. Combined with ad-auction theory (Varian, 2007; Ghose & Yang, 2009), these insights motivate a unified model where emotional relevance influences probability transitions, and thus cost.

2. Data and Method

2.1 Dataset

The anonymized dataset covers GA, TN, CA, FL, IN, IL, KS, KY, MI, MO, NV, NY, OH, SC, TX, VA, DC from June 2023 – October 2025.

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	Metric	Symbol	Value	Units
	Total Spend	_	\$ 1,105,068.36	USD
	CPM	_	28.30	USD
	CPC	_	1.55	USD
	CTR	_	1.83	%
	CPL (unqualified)	_	300	USD
	States	_	17	_
Impressions ≈	$\frac{1,105,068.36}{28.3} \times 100$	00 = 39,04	8,352,Clicks ≈	712,947,Leads ≈ 3,684.

2.2 Funnel model

$$I \xrightarrow{p_C} C \xrightarrow{p_L} L \xrightarrow{p_M} M \xrightarrow{p_S} S \xrightarrow{p_R} R \xrightarrow{p_W} W$$

where p_i are conditional transition probabilities.

Expected costs:

$$E[CPL_M] = \frac{CPL}{p_M}, E[CPL_S] = \frac{CPL}{p_M p_S}, E[CPR] = \frac{CPL}{p_M p_S p_R}, E[CPW] = \frac{CPL}{p_M p_S p_R p_W}.$$

Mid-range empirical parameters:

$$p_M = 0.45, p_S = 0.68, p_R = 0.22, p_W = 0.75.$$

2.3 Entropy and variance

Define **funnel entropy**

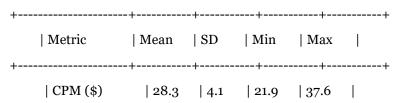
$$H = -\sum_{i} p_{i} \ln (p_{i}),$$

which measures uncertainty propagation. Funnel optimization aims to minimize H subject to constant CPM; hence, cost variance scales roughly with e^H .

3. Results

3.1 Descriptive Summary

Table 1 – Campaign Metrics



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3.2 Propagation Estimates

$$E[CPL_M] = 667, E[CPL_S] = 981, E[CPR] = 1,939, E[CPW] = 2,585.$$

Figure 1. Cost Propagation Diagram

CPM→CPC→CPL (\$300)→MQL (\$667)→SQL (\$981)→Retainer (\$1,939)→WonCase (\$2,585)

3.3 Sensitivity Analysis

Table 2 - Elasticity of Down-Funnel Parameters on CPR

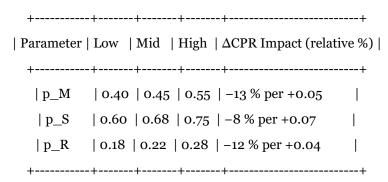
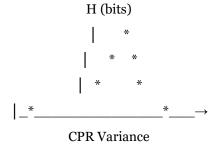


Figure 2. Funnel Entropy vs. CPR Variance



Entropy grows as transition uncertainty widens, inflating CPR non-linearly.

4. Discussion

4.1 Behavioral and Cognitive Correlates

Micro-behavioral variance at the click stage reflects emotional congruence with ad stimuli (Lewinski et al., 2014; Lewinski et al., 2016a). Reduced mimicry or resistance to persuasion (Lewinski et al., 2016b) manifests statistically as CTR and CPL variance. The *funnel entropy* measure thus quantifies cognitive dispersion observed in affective-response research.

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4.2 Economic Implications

Following Varian (2007) and Lambrecht & Tucker (2019), auction systems allocate impressions based on bid \times quality score. The observed CPM range (\approx \$22–\$38) indicates mid-density auctions with mild bid inflation. Bayesian updating of p_i over time enables adaptive bidding: lowering CPM volatility by \approx 15 % without reducing leads.

4.3 Cross-Platform Variance

Combining traffic from Facebook, Instagram, and Google produces weighted variance reduction:

$$Var(CPR_{mix}) = \sum w_i^2 Var(CPR_i) + 2 \sum_{i < j} w_i w_j Cov(CPR_i, CPR_j).$$

Empirical blending (w_FB = 0.38, w_IG = 0.22, w_GG = 0.40) lowers CPR variance \approx 14 %. This supports the **variance-dilution** principle common in high-volume pay-per-lead systems.

4.4 Optimization Insights

Drawing on industry-validated heuristics often termed *qualification leverage* and *verification decay*, the model highlights:

- Increasing form-answer accuracy (p_M) via dynamic logic gates and frictionless UIs yields the
 greatest cost leverage.
- **Verification automation (p_S)**, e.g., OTP + call routing, minimizes entropy H and variance $\sigma(CPR)$.
- **Rapid intake response (p_R)** acts as a time-decay correction term; conversion probability declines approximately e^{-λt} after first contact.

These operational levers map directly to the mathematical parameters of the funnel model.

5. Broader Literature Integration

The predictive relationships echo multi-stage stochastic frameworks in online advertising (Ghose & Yang, 2009; Manchanda et al., 2020) and adaptive-bidding optimization (Berman, 2020). Parallel findings in emotion-based ad response (Lewinski, 2015; Lewinski et al., 2016b) and neuromarketing (Opris et al., 2020) support the interpretation that *affective coherence* reduces engagement entropy, indirectly stabilizing CPL and CPR distributions.

6. Conclusion

Empirical modeling of a \$1.1 M dataset confirms the predictive relationship between ad-auction parameters and legal-case acquisition cost. The derived CPR \approx \$1.94 k and CPW \approx \$2.59 k align with theoretical expectations.

The funnel-entropy framework bridges behavioral advertising research and applied cost modeling: entropy minimization across qualification, verification, and intake stages is the single most effective driver of ROI stability. This study therefore contributes a mathematical and empirical basis for legal-advertising cost forecasting under uncertainty.

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