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Research Article

On the Modification of Simpson Method and their Application to Solve Some Problems from the Enviromental

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ABSTRACT

Recently, many scientists have been engaged in research differential equations, which have used in solving some problems from the environment. The increase in pollution volumes has recently ensures the relevance of research in this area. Therefore here consider investigation of numerical solution to solve initial-value problems for Ordinary Differential Equations by the well-known Simpson method. This method is stable and has the order of accuracy and the number of mesh points used in its construction equal to 3(three). However, the stability Region of this method consists of one point, which is called the origin of the coordinate system. Therefore, the expansion of the stability region for this method is relevant. Her have constructed Simpson-type methods, which have an extended region of stability. For the construct, more exact methods are suggested to use advanced or forward-jumping methods. And also have shown that by using an advanced method, one can been construct A-stable methods with the order of accuracy , in the case when the number of mesh points is equal to three.

Keywords: equations, constructed, Simpson-type methods, SME's, accuracy.

INTRODUCTION:

As is known, one of the popular tasks in Natural sciences is the initial-value problem for the Ordinary differential Equation（ODE）of the first order, which can be presented as follows:

(1)

If we take into account that the problem (1) has been studied by scientists for more than three centuries, then it is obvious that research in this direction is difficult. Recently, some of these method have been successfully applied in solving cerdtion environmental problems. For this aim have been suggest some known methods.

The investigation of the problem (l), supposed that the right-hand side of the equation (1) has the continuous partial derivatives to some , inclusively. The problem (1) has the unit solution , which is defined in the segment , and has the continuous derivatives up to , inclusively.

To define the numerical solution of the problem (1), let us designate the exact values of the solution of problem (1). And designate by the by - the exact values of the function at the point and the corresponding approximate values by the at the mesh-point . And the mesh points designate as the . Here, -is step size, which divides the segment into equal parts.

Let us consider the following famous methods:

(2)

(3)

(4)

(5)

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(6)

These methods are successfully applied to solve of initial value problem (1) and also to the calculation of definite integrals (see for example [1]-[19]).

By using that the methods (4) and (6) more exact, let us to consider the investigation of the Simpson method.

§1. On some presentation of Simpson method.

As is known, there are some methods for solving problem (1), one of which is the Multistep Methods with constant coefficients, and the other is the explicit Runge-Kutta methods. The explicit Runge-Kutta methods are studied very well. There are many popular methods in the class of explicit Runge-Kutta methods. One of them is called as the Runge method and presented as follows: (7)

here

Let us suppose that the function is independent of the variable . In this case, from the method (7) is follows the method: (8)

If here replace by then from (8) it follows: (9)

In our case receive that methods (9) and (4) as the same. Thus established a direct link between the classes of Runge-Kutta and Multistep Methods. Multistep method usually presented as the follows: (10)

As is known, the Numerical Methods comprise by using the conception of Stability and Degree, which can be formulated in the following form:

DEFINITION 1. Method (10) called as the stable, if the roots of the following polynomial

located in the unite circle on the boundary of which there is not multiply roots.

DEFINITION 2. The integer value is called as the degree for the method (10), if the following asymptotic equality is holds:

(11)

Method (4) is stable and has the degree of . By the Dahlquist's rule, receive that if method (10) is stable, then . It follows from here that if method (10) is stable, then there is a stable method with the degree for the and . Note that methods with the getting for values and don't match. Stable method with degree for the is the Simpson method, and method with degree for the , can been presented as:

(12)

Some authors, this method call as the Simpson's rule. As was noted, the region of stability for the Simpsons method consists from the one point which called as the origin coordinate system. By using the predictor-corrector method, one can expand the region of stability. For example, let us consider to following predictor-corrector method:

(13)

(14)

Note that the local truncation error for this methods can presented as follows:

. (15)

By above described predictor-corrector method the region of stability for the Simpson method (6), extended as .

The method (5) is one of the popular method with the fractional step-size, in the application of which arises some difficult in the calculation of the values of type . If the - rational number, then in calculation of values is not any arises difficult. But -irrational number then arises some difficult in the calculation of the value: (-is irrational). For the simplicity of this, let us consider case, when -rational number. The Simpson method let us write in the following form (see for examplr [2],[4],[5],[6],[10],[11],[13]):

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(16)

here have used the hybrid point.

For the receiving more exact results one can be used the following predictor-corrector method:

Noted that method (16), has been received by using the chaining step-size by the method. By the above description, it is clear that the predictor-corrector method can be applied to using method (16). In this case, received the following:

(17)

Here for the calculation of the value one can be used the method (12) or (13). Method (12) is implicit, but the method (13) is explicit. In the method (17) let us replace the in , then one can be presented:

(18)

Similar studies have been investigated by many authors (see for example [35]-[47]). Note that, to solve some problems by using method (17) is preferable to using method (18), because method (17) is one step method. If the values and are given, then by using method (18), one can be calculated the values . But for using method (17), at each step must be calculated the values and .

As was noted above the following method:

(19)

is also stable and has the degree . Note that this method receive from the method (8) as the partial case for the .

In the work [48] by using this and some others method's one can construct the L-stable methods with degree and .

One of this method can be presented as follows:

, (20)

here

The local truncation error for the method (20) can be presented as the . And now let us consider the construction A-stable method by using the advanced (forward-jumping) method.

2. Construction and investigation of the Advanced methods.

By using Dahlquist law, receive that the exactness for the stable multistep methods are bounded (see for example [49]-[61]). Therefor, for the construction stable methods recommended to use the stable advanced methods. Advanced methods in one version can been written as:

(21)

In formal one can be say that the method (8) can been received from the (21) as the partial case. If in the method (21) to put in this case receive the known Multistep method. For the case of the objectivity, let us note that the main properties of these methods differ in that, must be satisfies (which satisfies because).

Numerical methods as the type (21) have constricted by some known scientists as Laplas, Steklov and etc. The advanced methods constructed by the known scientists, which have obeyed the law of Dahlquist. In the work [38] has constructed the following method:

(22)

And prove that the method (22) is stable and has degree of p=5. It follows that if method (21) is stable, then it will be exact and stable, it is preferable to method (2). But method (21) has some disadvantages, for example in the application of this method to solve some problem is arises the nesserty calculation of the value , which participate in the method (21). In our case, it is arises the calculation of the value before calculation . Here, suggest to use some method for calculation the value . And want noted that properties of the receiving method dependents from the properties of the method, which has applied to calculation of the value y_(n+k-j) (j≤m). For the illustration of this, let us to use the following method:

(23)

with the local truncation error:

After using method (21) in the method (22), receive:

(24)

This method is L-stable and has the degree .

CONCLUSIONS:

The pice very thing in the world is in order. However, we don't always notice them we have considering the investigation of the initial-value problem for the ODEs of the first order. For this aim have used the Numerical solution of named problem. As is known, with the studies of this problem, many famous scientists were involved. Recent research has shown that there are some unresolved issues. One of them is to find finding reliable values for the solution of the problem under study. To find a reliable solution of investigated problem here it is proposed to use bilateral (two-sided) numerical methods. As is known, the analytical bilateral numerical methods was constructed by chapligin, which here has applied to solve problem (1).

Keywords: Initial - value problem, Ordinary Differential Equation, Multistep Methods, Degree and Stability, Runge-Kutta methods.

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Conflict to Interest

The authors state express that there is no conflict of interest misunderstanding between them.

We hereby confirm that all the methods in this manuscript are ours.

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