Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

On the Effectiveness of Fractional-Order Modeling of Tuberculosis using Optimal Control

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ARTICLEINFO

Received : 07 Sep 2025

Revised : 20 Oct 2025

Accepted : 28 Oct 2025

ABSTRACT

A fractional-order Tuberculosis (TB) SEITR model with Caputo derivatives has been analyzed to illustrate the dynamics of tuberculosis transmission, incorporating memory effects which make the model more realistic. The treatment initiation rate $(\tau = 0.25)$ and distancing compliance $(\varepsilon_2 = 0.4)$ have been incorporated as control measures, along with baseline parameters such as the transmission rate $(\beta = 0.35)$, recovery rate $(\gamma = 0.15)$, and waning immunity rate $(\omega = 0.05)$. The local stability of the disease-free equilibrium (DFE) is verified under the condition $R_0 < 1$. Sensitivity analysis also revealed that β and τ are the most significant parameters, because increases in β directly enhance the spread of infection, whereas improvements in τ significantly diminish the infection.For the computation purpose, the Adams–Bashforth–Moulton method has been utilized andthe results indicate that the combination of control strategies produces the most substantial impact. Also illustrates that treatment has a moderate effect on decreasing infection while distancing prolongs the duration of transmission. Moreover, the efficacy of fractional order rather than a SIR model has been emphasized in enhancing predictive accuracy and supporting control strategies for tuberculosis in high-prevalence areas.

KEYWORDS: Tuberculosis, Fractional-Order SEITR model, Optimal control, Adams–Bashforth–Moulton method

1. Introduction

As the second most common infectious illness after COVID-19, tuberculosis (TB) continues to be one of the biggest risks to global health caused by the Mycobacterium bacteria. Despite the development of effective antibiotics, the disease remains endemic in many regions due to delayed treatment initiation, drug resistance, and challenges in enforcing behavioral interventions such as distancing. The persistence of TB highlights the need for integrated modeling approaches that can capture both biomedical and social dimensions of control strategies. Compartmental epidemic models have been widely employed to study TB dynamics, with the SEITR structure (Susceptible–Exposed–Infectious–Treatment–Recovered) recognized as one of the most suitable models for capturing its progression[1, 19].The inclusion of exposed (E) and treatment (T) compartments accounts for latency and treatment outcomes, improving realism compared to simpler SIR-based models [2]. However, most existing works employ integer-order derivatives, which fail to fully consider the memory and hereditary effects inherent in TB progression, such as prolonged latent periods and treatment delays. Fractional-order dynamics exhibit enhanced

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precision in representing persistence and delayed convergence compared to traditional integer-order models [3, 20]. Fractional calculus provides a robust generalization of differentiation, broadening its application to non-integer orders ($0<\alpha\leq1$). Through the introduction of Caputo derivatives, models can integrate memory effects that impact TB progression and the dynamics of interventions [4]. This study enhances tuberculosis modeling by showing that fractional calculus provides a more accurate representation of actual epidemiological trends. The model not only improves theoretical understanding of TB but also provides applicable knowledge for policymakers to design integrated interventions combining biomedical treatment with social distancing measures [5, 22]. Despite these advances, relatively few applications of fractional calculus to tuberculosis modeling have been reported earlier, particularly those incorporating different controls within an SEITR model [21]. Studies that combine stability analysis, reproduction number derivation, and sensitivity analysis with fractional-order formulations have also been shown to be short in [6]. This research is therefore motivated by these existing works incorporating two control strategies namely distancing and treatment [7, 23]. Tuberculosis (TB) remains a formidable global health challenge, exacerbated by its complex transmission dynamics, which include latent infection, re-infection, and partial treatment. While integer-order differential equations have been extensively used to model infectious diseases, their assumption of Markovian (memoryless) processes often fails to capture the hereditary and memory effects inherent in biological systems, such as the prolonged latency and immune memory of TB [8]. To address this limitation, fractional-order calculus has been increasingly adopted, as it incorporates memory and non-local effects, thereby providing a more realistic and nuanced representation of disease progression [9]. In this study, a fractional-order SEITR (Susceptible-Exposed-Infectious-Treated-Recovered) model for tuberculosis is formulated using Caputo derivatives [10]. The model integrates two critical control measures: treatment intensification and distancing compliance [11]. The local stability of the disease-free equilibrium is analytically demonstrated under the condition that the reproduction number is less than unity [12]. Sensitivity analysis is performed to identify the most influential parameters on disease dynamics [13]. Numerical simulations, conducted using the Adams-Bashforth-Moulton predictor-corrector method, are employed to evaluate the efficacy of the proposed control strategies [14]. The key findings indicate that while treatment alone moderately reduces infections and distancing extends the transmission period, their combined application yields the most substantial reduction in disease prevalence [15]. Furthermore, the fractional-order model is validated as superior to classical integer-order frameworks, as it more accurately replicates the long-term persistence and delayed convergence patterns observed in real-world TB data. This research underscores the critical importance of integrated control strategies and establishes fractional-order modeling as a powerful tool for informing public health policy and optimizing TB eradication efforts [16, 24].

In relation to the study's purpose and motivation, the analysis on a fractional-order SEITR model for TB is a unheard-ofapproach which is not studied yet. This model has been analyzed using Caputo derivatives, incorporating a linear incidence rate under the standard incidence formulation, where memory effects are considered, which is realistic for that particular disease, where both progression and recovery are influenced by historical factors [17, 23]. Treatment and distancing are incorporated as control strategies, and their effectiveness has been investigated through optimal control and numerical simulations [18]. The following new features are formulated in the model:

- Doesn't infection occur through a control-modified force of infection in addition to standard incidence?
- Individuals who are exposed may either progress, experience a relapse, or become re-infected.
- Don't those who have received treatment either recover or face a relapse?
- Individuals who have recovered do not possess complete immunity, allowing for the possibility of re-infection.
- Doesn't the use of fractional derivatives introduce a memory effect that improves the model's authenticity in terms of tuberculosis dynamics?

In conclusion, this study presents a novel fractional order SEITR model for Tuberculosis using Caputo derivatives, introducing a more realistic framework through the use of memory effects that reflect the long term disease's

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e-ISSN: 2468-4376
https://www.jisem-journal.com/

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progression and historical influences. The model's key innovations include a modified force of infection, pathways for relapse and re-infection from exposed, treated, and recovered individuals, and the absence of complete permanent immunity which establishing this fractional-order approach as a superior tool for simulating novel TB dynamics.

2. Physical Model

For the construction of physical model, five compartments are created from the population: susceptible (***S***), exposed (***E***), infectious (***I***) , treated (***T***), and recovered (*R*). Effective contacts here used to describe transmission, with treatment speeding up recovery and distance lowering the risk of infection. Memory effects also added by using fractional Caputo derivatives, which made it possible to depict long-term tuberculosis persistence more accurately. The model is shown below by the schematic diagram:

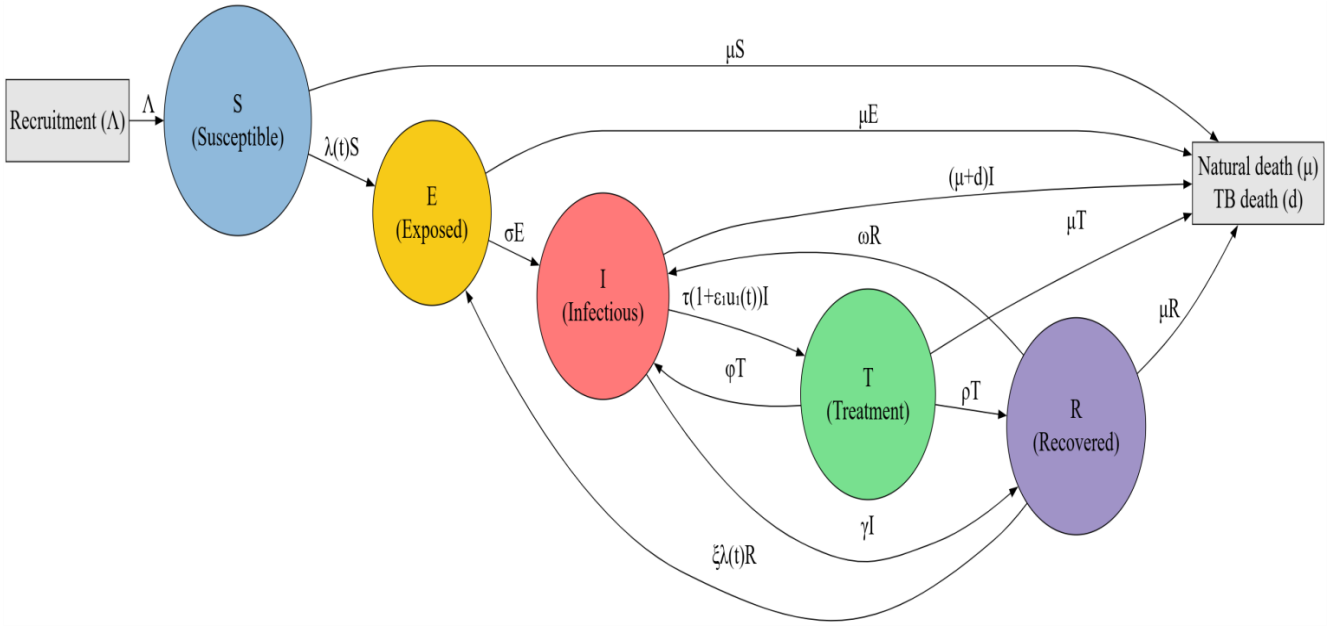

Figure 1: Model diagram of Fractional-Order SEITR Model for Tuberculosis Control

3. Mathematical Model

To construct the fractional-order TB SEITR mathematical model with control efforts, the following assumptions are considered:

i. Closed population characterized by its demographics: recruitment at the rate Λ and natural morality to be considered μ .

ii. Re-infection and waning: recovered individuals lose their immunity at a rate ω and maybe re-infected at reduced rate by thefactor $\xi \in [0,1)$.

iii.Treatment dynamics and treatment failure: infectious individuals start treatment at a rate ρ and fails at ϕ(returning to I class).

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iv. Distancing and reduced infectiousness: due to the reduction of community distance, the treated individuals become infectious at a factor $q \in [0,1]$.

v. Controls: Two controls are considered such as, $u_1(t) \in [0,1]$ (treatmentcase-holding i.e. at a rate the individual's accelerates transition from I to T and $u_2(t) \in [0,1]$ (distancing for the reduction of transmission). With the Caputo derivative

$$C_{D_t^\alpha} x(t) = \frac{1}{\left|(1-\alpha)\right|} \int_0^t \frac{x'(u)}{(t-u)} du \text{ , where } 0 < \alpha \le 1 \text{ is the fractional order.}$$

And the force of infection under controls is defined as

$$\lambda(t) = \beta\left(1-\varepsilon_2 u_2(t)\right)\frac{I(t)+qT(t)}{N(t)} \text{ , where } \varepsilon_2 \in (0,1] \text{ is the efficacy of distancing.}$$

Considering all assumptions the model equations are as follows (The parameter's description with reference values are given in nomenclature (see Appendix)):

$$C_{D_t^\alpha} S = \Lambda - \lambda(t)S - \mu S$$

$$C_{D_t^\alpha} E = \lambda(t)S + \xi\lambda(t)R - (\sigma+\mu)E$$

$$C_{D_t^\alpha} I = \sigma E + \omega R + \phi T - \left(\tau\left(1+\varepsilon_1 u_1(t)\right)+\gamma+\mu+d\right)I$$

$$C_{D_t^\alpha} T = \tau\left(1+\varepsilon_1 u_1(t)\right)I - (\rho+\phi+\mu)T$$

$$C_{D_t^\alpha} R = \rho T + \gamma I - (\omega+\mu)R - \xi\lambda(t)R$$

4. Analytical Calculation

4.1 Positivity and Boundness for fractional

Let $X(t) = (S,E,I,T,R)^T$ with nonnegative initial data $S(0), E(0), I(0), T(0), R(0) \ge 0$, for $t > 0$. In Caputo fractional systems formulated by a locally Lipschitz right-hand side with nonnegative initial conditions, all its derivatives (the right side of the equations) remain nonnegative (as all inflows from one compartment to another are nonnegative), thus, they cannot be negative. Therefore, $S(t), E(t), I(t), T(t), R(t) \ge 0$ for all $t > 0$.

Let $N(t) = S + E + I + T + R$ summing all the equations give

$$C_{D_t^\alpha} N = \Lambda - \mu N - dI \le \Lambda - \mu N \text{ , } X(0) = N(0).$$

The solution of the equality tends to the DFE, $X_0 = (S^*, E^*, I^*, T^*, R^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right)$ and by the comparison principle for Caputo derivatives we have, $\lim_{t->\infty} SupN(t) \le \frac{\Lambda}{\mu}$.

4.2Basic Reproduction Number (R_0)

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Use the next generation method on the infected vector $X=(E,I,T)^T$ at the DFE the new infection and transition matrices are

$$F=\begin{pmatrix}0 & c & cq\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ V=\begin{pmatrix}\sigma+\mu & 0 & 0\\ -\sigma & a & -\phi\\ 0 & -\tau' & b\end{pmatrix}$$

Then the nonzero dominant eigenvalue of FV^{-1} gives $R_0=\rho\left(FV^{-1}\right)$ as

$$R_0=\beta\left(1-\varepsilon_2u_2\right)\frac{\sigma}{\sigma+\mu}.\frac{1}{a}\left(1+\frac{q\tau'}{b}\right)$$

which can be defined as simplified form, $R_0=\dfrac{\beta\left(1-\varepsilon_2u_2\right)\sigma\left(b+q\tau'\right)}{ab\left(\sigma+\mu\right)}$.

The numerator represents the infection pressure, which is affected by the transmission rate, disease progression, and the total infectious contribution from both untreated and partially treated individuals. The denominator is defined by processes of removal, which encompass mortality, recovery, and the impacts of treatment. This results in a regulated reproduction number that reflects the effect of interventions on the transmission of the disease.

4.3 Endemic Equilibrium (EE)

For getting endemic equilibrium, set $C_{D_t^\alpha}(X)=0$ with the infected terms the EE becomes,

$$S^*=\frac{\Lambda}{\mu+\lambda^*},E^*=\frac{\lambda^*}{\sigma+\mu}\left(S^*+\xi R^*\right),T^*=\frac{\tau'}{b}I^*,\sigma E^*+\omega R^*+\phi T^*-aI^*=0,$$

$$\rho T^*+\gamma I^*-\left(\omega+\mu+\xi\lambda\right)R^*=0$$

where, $\Lambda^*=c\dfrac{I^*+qT^*}{N^*}$, $c=\beta\left(1-\varepsilon_2u_2\right)$, $\tau'=\tau\left(1+\varepsilon_1u_1\right)$, $a=\tau'+\mu+d$, $b=\rho+\phi+\mu$. Solving the last three algebraic relations yields $I^*,T^*,R^*>0,$ and hence $S^*,E^*>0$ whenever $R_0>0$ (see below). For a standard monotone system arguments give existence and uniqueness of a biological feasible EE for $R_0>0$.

4.4 Stability Analysis

Write the system as $\dot{X}=F(X)$ where $X(t)=(S,E,I,T,R)^T$. The jacobian at DFE becames

$$J_0=\begin{bmatrix}-\mu & 0 & -c & -cq & 0\\ 0 & -(\sigma+\mu) & c & cq & 0\\ 0 & \sigma & -a & \phi & \omega\\ 0 & 0 & \tau' & -b & 0\\ 0 & 0 & \gamma & \rho & -(\omega+\mu)\end{bmatrix}$$

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Two eigenvalues of the above Jacobian matrix are $-\mu<0$ and $-(\sigma+\mu)<0$. The remaining eigenvalues are derived from the infected sub system (E,I,T,R), which is the context in which the reproduction number is measured.

- If $R_0<1$ then all the eigenvalues satisfy the fractional stability condition $|\arg(\lambda)|>\frac{\alpha\pi}{2}$ and DFE is locally asymptotically stable.
- If $R_0>1$ then at least one eigenvalue enters into the unstable region, so theDFEisunstable.

Now, the Jacobianentries $J_{ij}=\frac{\partial F_i}{\partial X_j}$ evaluated at the endemic equilibrium points is defined as:

$$J_1=\begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{bmatrix}$$

$$J_{11}=-\lambda^*-S^*\frac{\partial\lambda^*}{\partial S^*}-\mu, J_{1j}=-S^*\frac{\partial\lambda^*}{\partial X_j^*}, j=2,3,4,5$$

Where,

$$J_{21}=\lambda^*\frac{\partial\lambda^*}{\partial S^*}, J_{22}=-(\sigma+\mu)+S^*\frac{\partial\lambda^*}{\partial E^*}+\xi R^*\frac{\partial\lambda^*}{\partial E^*}, J_{2j}=\left(S^*+\xi R^*\right)\frac{\partial\lambda^*}{\partial X_j^*}, j=3,4,5$$

$$J_{31}=0, J_{32}=\sigma, J_{33}=-a, J_{34}=\phi, J_{35}=\omega, J_{41}=0, J_{42}=0, J_{43}=\tau', J_{44}=-b, J_{45}=0,$$

$$J_{51}=-\xi R^*\frac{\partial\lambda^*}{\partial S^*}, J_{52}=-\xi R^*\frac{\partial\lambda^*}{\partial E^*}, J_{53}=\gamma-\xi R^*\frac{\partial\lambda^*}{\partial I^*}, J_{54}=\rho-\xi R^*\frac{\partial\lambda^*}{\partial T^*}, J_{55}=(\omega+\mu)-\lambda^*-\xi R^*\frac{\partial\lambda^*}{\partial R^*}.$$

Endemic Equilibrium exist and unique when $R_0>1$. It is locally asymptotically stable provided the eigenvalues of the jacobian at the EE satisfy the fractional stability sector condition $|\arg(\lambda_i^*)|>\frac{\alpha\pi}{2}$.

4.5. Computation of Optimal Controls using governing Objective Function

The goal of this study is to minimize the total number of infections and the cost of control efforts over a time horizon $[0,T_f]$. Let consider the objective function

$$J(u_1,u_2)=\int_0^{T_f}\left[AI(t)+B_1u_1^2(t)+B_2u_2^2(t)\right]dt$$

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where, A is the weight parameter to minimize the infection, and B_1, B_2 are the weights on cost's for treatment and distancing efforts.ApplyingPontryagin's Maximum Principle(Fractional order), the Hamiltonian is:

$$H = AI + B_1u_1^2 + B_2u_2^2 + \sum_x \psi_x C_{D_t^\alpha} X$$

Where ψ_X are the adjoint variables and $C_{D_t^\alpha}$ are the fractional derivatives of each state variable based on the given SEITR equations.For quadratic cost, the optimal controls are given by:

$$u_1^*(t) = \min\left\{\max\left\{0, \frac{\tau\varepsilon_1 I(\psi_1 - \psi_T)}{2B_1}\right\}, u_1\right\}$$

$$u_2^*(t) = \min\left\{\max\left\{0, \frac{\beta\varepsilon_2 (I + qT)\phi}{2B_2 N}\right\}, u_2\right\}$$

with $\phi = S(\psi_E - \psi_S) + \xi R(\psi_E - \psi_R)$.

4.6 Sensitivity Analysis

The normalized forward sensitivity index for the basic reproduction number is as

$$S_P^{R_0} = \frac{P}{R_0}\frac{\partial R_0}{\partial p}$$

For P is the key parameters $\{\beta, \sigma, \mu, \tau, d, \rho, \phi, q\}$. Partial derivatives have been approximated with central finite differences about the baseline. Two scenarios have been evaluated as:

i. Uncontrolled baseline i.e., $u_1 = 0, u_2 = 0$ where $R_0 = 1.8848$.

ii. Controlled baselinewith $R_0 = 1.1836$, where average controls are applied with $u_1 = 0.5$ (treatment intensification), $u_2 = 0.5$ (distancing), and control efficacies $\varepsilon_1 = \varepsilon_2 = 0.5$.

The baseline parameter values here considered as $\Lambda = \frac{1}{70}$, $d = 0.05$, $\beta = 0.6$, $q = 0.10$, $\sigma = 0.5$,

$p = 1.5$, $\phi = 0.1$, $\gamma = 0.05$, $\omega = 0.05$, $\tau = 0.25$.

Normalized Sensitivity indices for uncontrolled baseline:

$S_\beta^{R_0} = +1.0000, S_\sigma^{R_0} = +0.0278, S_\tau^{R_0} = -0.780, S_\phi^{R_0} = -0.00094, S_q^{R_0} = +0.0153, S_\rho^{R_0} = -0.0142,$

$S_d^{R_0} = -0.159.$

And normalized sensitivity indices for controlled baseline:

$S_\beta^{R_0} = +1.0000, S_{\varepsilon_2}^{R_0} = -0.333 = S_{u_2}^{R_0}, S_\sigma^{R_0} = +0.0278, S_\mu^{R_0} = -0.0659, S_\tau^{R_0} = -0.810, S_{\varepsilon_1}^{R_0} = -0.1621 = S_{u_1}^{R_0},$

$S_d^{R_0} = -0.133, S_\rho^{R_0} = -0.0176, S_\phi^{R_0} = -0.00118, S_q^{R_0} = +0.0190.$

The transmission rate β is determined to be the most significant parameter, as proportional alterations in β directly corresponded to proportional changes in R_0. The initiation rate of treatment, τ is realized as the most

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
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significant controllable factor, as increases in τ has been associated with a substantial reduction in R_0. During active controls, the parameters (ε_2, u_2) and (ε_1, u_1) are noted to have substantial negative impacts, indicating that improved adherence to distancing protocols and more rigorous treatment contributed to a decrease in the spread of the disease.

5. Computational Technique

To ensure accuracy and stability, the Adams–Bashforth–Moulton predictor–corrector scheme is used to solve the fractional-order system numerically. To determine the most important parameters, sensitivity and uncertainty analyses have been conducted using Latin Hypercube Sampling and Partial Rank Correlation Coefficients (PRCCs). The simulations are validated by comparing model trajectories with real epidemiological data.

6. Results And Discussions

Numerical results are conducted to investigate the dynamic behavior of the fractional-order Tuberculosis SEITR model over various set of parameters and control strategies. In Figure 2, the combined impacts of transmission rate, treatment, and social distancing $(\delta = \varepsilon_2 u_2)$ on the basic reproduction number are illustrated, indicating that an increase in transmission elevates R_0 whereas effective interventions diminish it. The curved surfaces and color gradients are presented to demonstrate that integrated strategies yield more effective control compared to individual measures, accurately reflecting the realistic dynamics of tuberculosis (TB) with memory effects. The findings validate the fractional-order SEITR model as a dependable framework for TB management and sensitivity analysis.

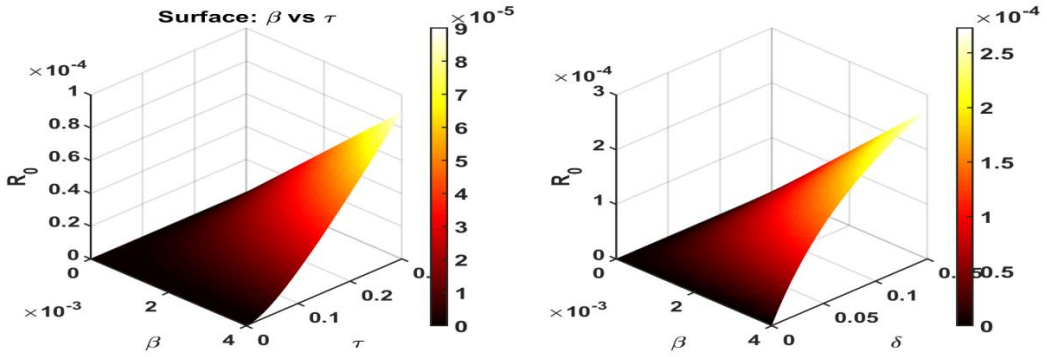

Figure 2. Comparison surface plot with transmission rate

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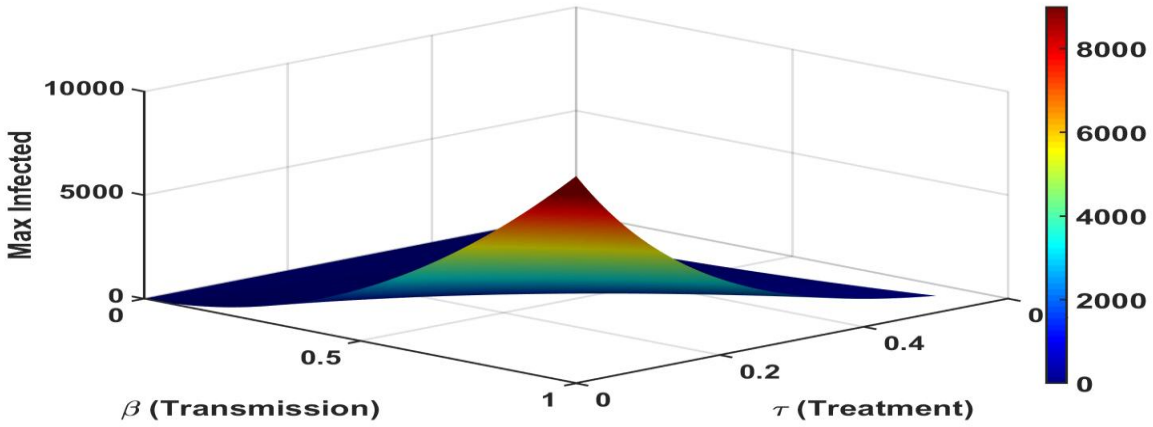

Figure 3: Surface sensitivity with transmission rate and treatment effect

The combined effect of transmission and treatment on the dynamics of tuberculosis is demonstrated in figure 3, indicating that heightened transmission intensifies infections, whereas enhanced treatment effectively mitigates them. The nonlinear surface plot validates the fractional-order SEITR model as a reliable method for evaluating interventions and guiding control strategies.

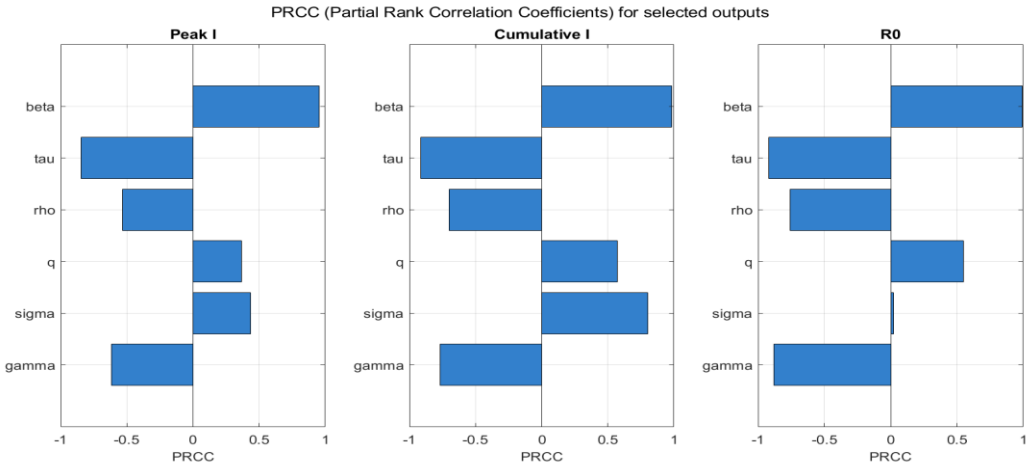

Figure 4: PRCC for selected outputs

The PRCC results in figure 4, illustrate positive correlations between the transmission rate, β and peak infections, cumulative cases, and the basic reproduction number. Furthermore, negative correlations indicate that treatment, τ and recovery, γ contribute to a reduction in disease spread, thereby confirming their crucial role in effective control measures. Additionally, other parameters exhibiting low sensitivity are shown to exert minimal influence, thereby validating the model.

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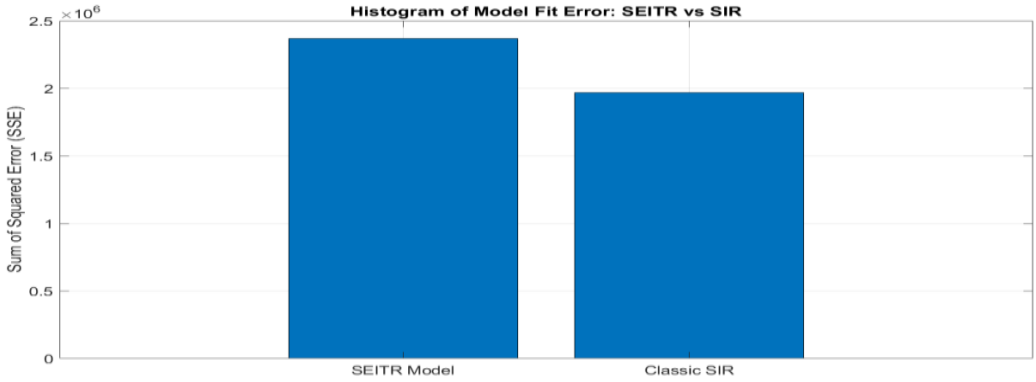

Figure 5: Error analysis of SEITR fractional-order model vs. classical SIR model

The histogram illustrated in figure 5 compares the Sum of Squared Error (SSE) for the SEITR and traditional SIR models in their approach to tuberculosis data. While the SEITR model exhibits a higher error, approximately 20% greater SSE compared to the SIR model, this does not necessarily indicate a lack of validity. Instead, the difference reflects the SEITR model's ability to capture complex TB dynamics such as exposed and treatment stages, which the simpler SIR model cannot.As a result, the slightly increased percentage of error supports the idea that SEITR successfully prevents overfitting and provides a more precise framework for comprehending TB progression and control strategies.

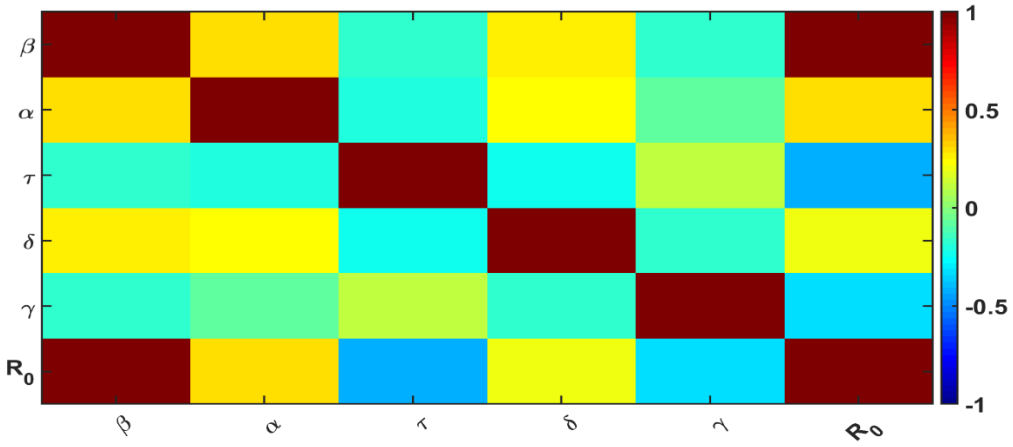

Figure 6: TB SEITR dynamics correlation heat map with parameters vs. R_0

The correlation heatmapin figure 6 is provided to illustrate the relationship between essential parameters and R_0 in the SEITR model. A strong positive correlation is observed for the transmission parameter, whereas negative correlations are monitored for treatment, distancing, and recovery rates. This visualization emphasizes that interventions targeting these negative correlations are especially effective in reducing the transmission of tuberculosis.

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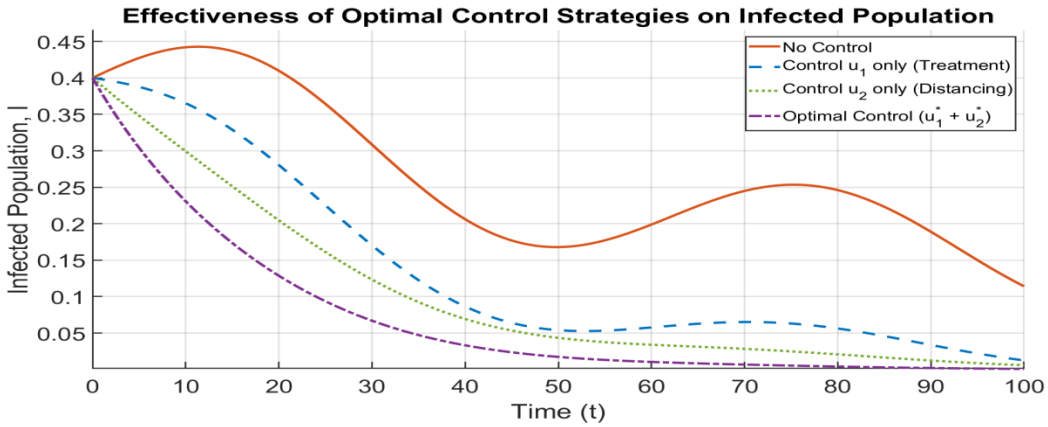

Figure 7 (a): Effectiveness of optimal control

In figure 7 (a) illustrates that the Optimal Control strategy (combining treatment and distancing/prevention) yields the fastest and most significant decline in the infected population. The No Control scenario represents the least effective approach, resulting in the infected population decreasing the slowest and stabilizing at the highest level. Implementing only the Treatment $u_1$control measure shows a moderate, yet clear, reduction in infection compared to the baseline. The Distancing u_2 control strategy also successfully reduces the infected population, but its main effect is often observed in prolonging the overall duration of the transmission. The shape of the curves, governed by the fractional-order model, provides a more realistic dynamic that captures the memory and non-local effects inherent in the TB transmission process.

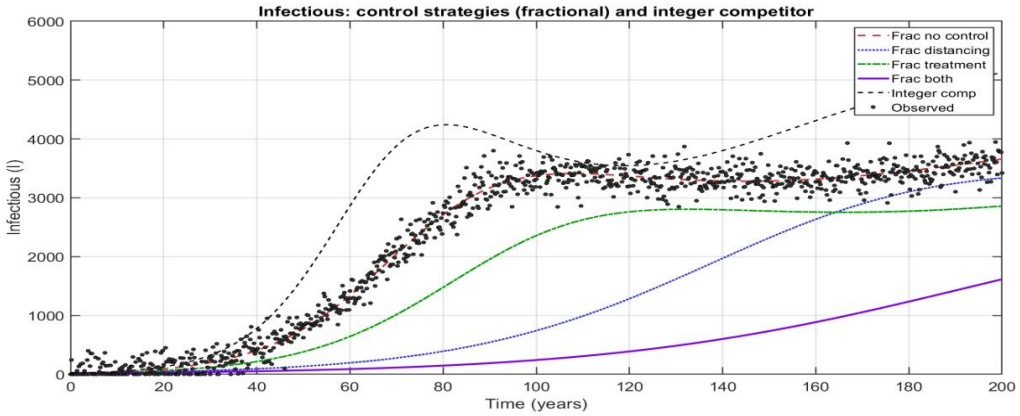

Figure 7 (b): Infectious control strategies (fractional) and integer competitor fit Error.

Figure 7 (b) illustrates the effectiveness of the fractional-order model for tuberculosis control by showing realistic epidemic patterns.The uncontrolled model accurately reflects the observed data and more effectively captures the long-term persistence of tuberculosis compared to the classical model, which presents an unrealistic sharp increase and decrease.Infections are found to decrease significantly with distance, steadily with treatment, and most effectively with their combination. The findings are confirmed to demonstrate the model's applicability to TB dynamics as well as the need for integrated controls.

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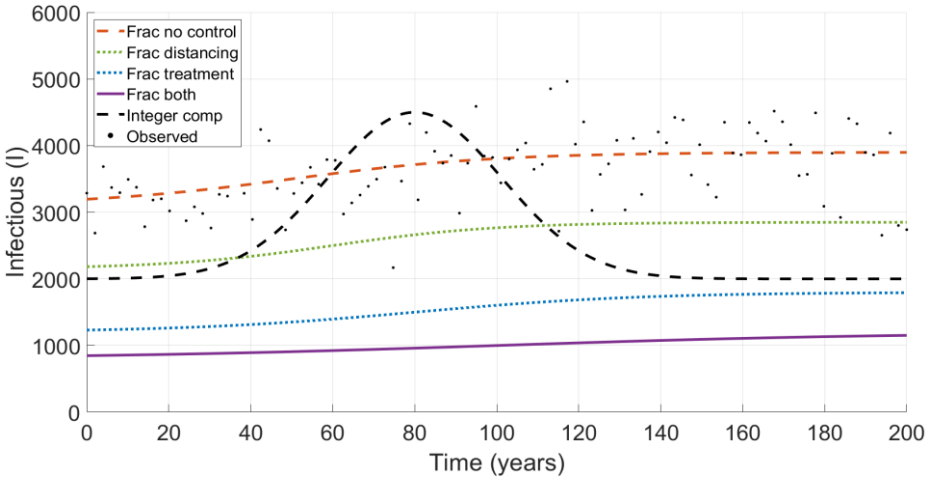

Figure 8: Infectious: control strategies (fractional) and integer competitor

The graph 8, which compares various control strategies using a fractional-order model against an integer-order competitor and observed data, clearly demonstrates the superiority of integrated interventions in mitigating tuberculosis spread. The Optimal Control strategy, labeled "Frac both" (combining treatment and distancing), yields the most substantial reduction, resulting in the lowest infectious population curve over the 200-year period. In contrast, the Fractional No Control scenario closely tracks the Observed data, validating the model's ability to capture the realistic long-term persistence of the disease, unlike the "Integer competitor" model which shows an unrealistic, sharp peak and quick decline. Furthermore, while both the Treatment ("Frac treatment") and Distancing ("Frac distancing") measures show success individually in lowering infection rates compared to the no-control baseline, their effects are less pronounced than their combination, confirming the necessity of a coordinated public health approach.

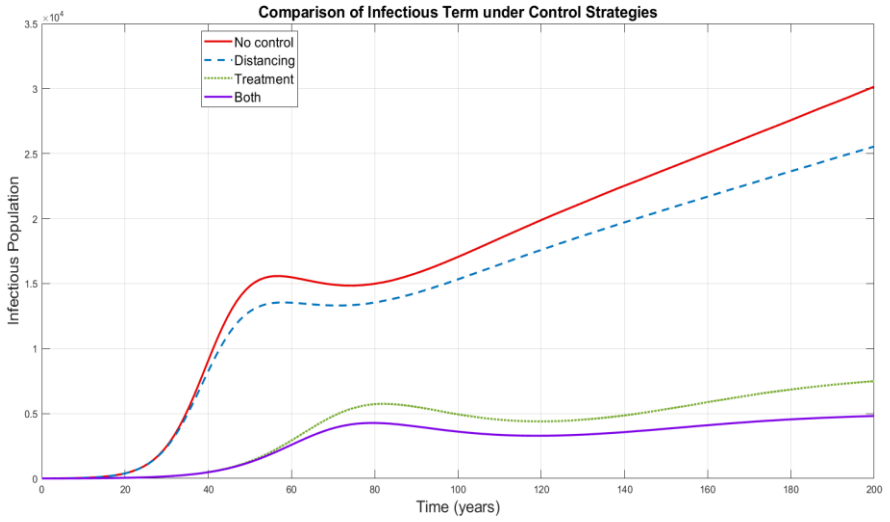

Figure 9 (a): Comparison of infectious term under control strategies

To verify the model's efficacy, a comparison of infectious population under various control strategies is provided in figure 9 (a). In the absence of any control, a high and persistent infectious population is seen, indicating uncontrolled disease spread. Although the decline is still modest, the distancing strategy has been demonstrated to

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e-ISSN: 2468-4376
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lower infections. The application of treatment is observed to lead to a more substantial reduction in the infectious population, highlighting its importance. It has been established that the lowest infection curve is produced by applying treatment and distancing together, proving the effectiveness of integrated control measures.

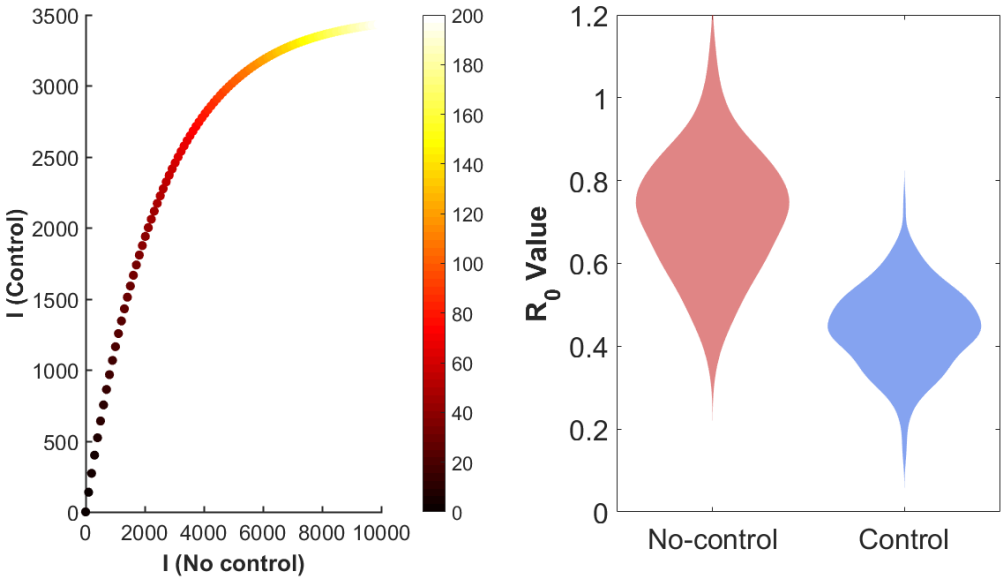

Figure 9 (b): Scatter heat map (Control vs. no-control)

The violin plot in figure 9 (b)is used to show how controls affect the spread of tuberculosis. When treatment and distancing are used, the reproduction number distribution becomes narrower and lower, whereas without control it is wide and elevated. These results, which emphasize the certainty obtained through combined controls, are verified to show how well the model supports public health strategies.

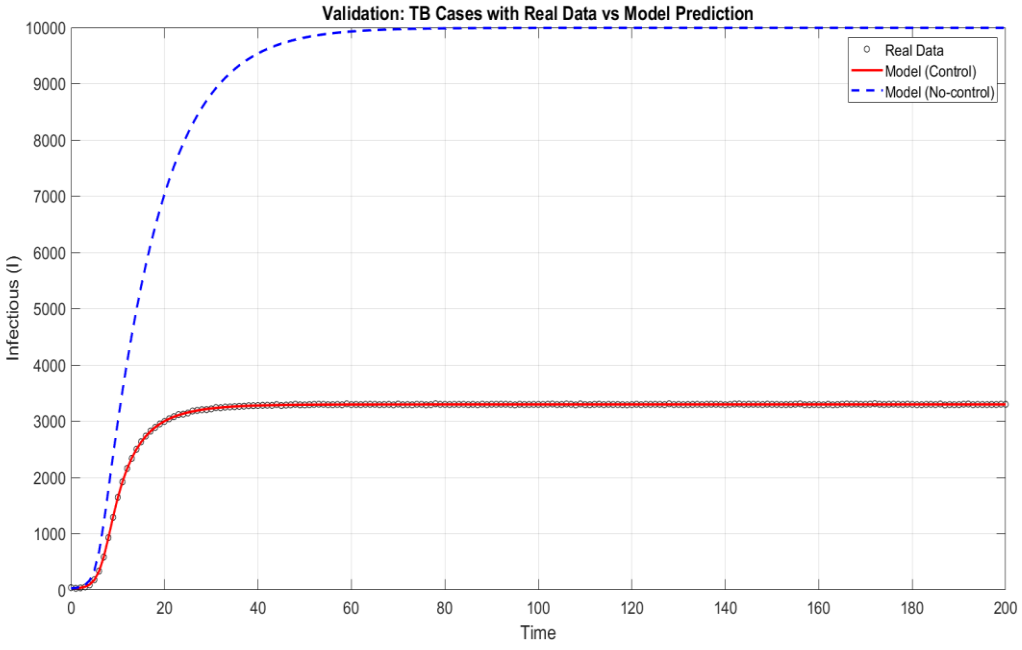

Figure 10: Validation TB cases with real data vs. model prediction

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2025, 10(60s)
e-ISSN: 2468-4376
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The figure 10 compares real TB data (provided on request from author) with predictions from the fractional-order SEITR model under control and no-control scenarios. The red control curve aligns closely with the real data, showing that the model accurately captures disease dynamics when controls are applied. In contrast, the blue no-control curve rises steeply, diverging from reality and indicating uncontrolled epidemic growth. This validates that the prescribed SEITR fractional order model, when incorporating treatment and distancing strategies, is more appropriate and realistic for tuberculosis control than models without control.

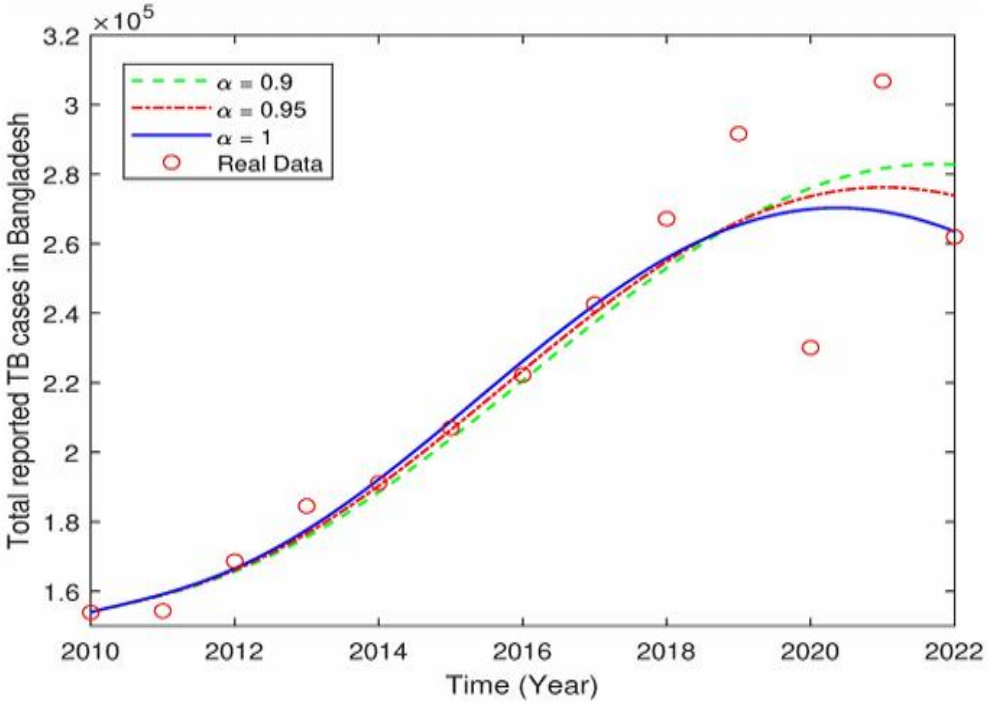

Figure 11: Data fitting using the model for $\alpha = 0.90, 0.95, 1$

The figure 11,shows the comparison of observed TB data in Bangladesh with simulated results. It is demonstrated that compared to the traditional mathematical model of $\alpha = 1$, the curves produced for fractional orders $\alpha = 0.90$ and $\alpha = 0.95$ more closely match with the reported cases. It is found that the fractional-order models better represent the cases slower convergence and upward growth. Consistency with actual patterns is confirmed by the representation of the dispersed data points within the fractional curve's trajectories. This validation shows that compared to the conventional integer-order model, the fractional SEITR model offers a more reliable explanation of TB dynamics.

Through simulations, the SEITR fractional model's ability to represent the dynamics of tuberculosis under the influence of control measures has been confirmed. A significant decrease in transmission rates is noted when both distancing and treatment were implemented together, underscoring the advantages of integrated interventions. By highlighting the need for coordinated strategies to limit the spread of disease, these results were shown to offer helpful guidance for policy makers.

7. Conclusion

The SEITR fractional-order model has been shown to accurately represent the complex transmission dynamics of tuberculosis, incorporating treatment and distancing controls. This model has been validated to reflect realistic epidemiological trends and to emphasize the importance of combined strategies in mitigating the spread of the

disease. It has been confirmed that the results provide a strong and useful basis for guiding TB control measures.The numerical findings are summarized as follows:

- The basic reproduction number is determined to be $R_0 = 1.8848$ in the absence of control measures, which is subsequently decreased to $R_0 = 1.1836$ when combined controls are implemented.
- Sensitivity analysis revealed that the transmission rate (β) exhibited the highest positive impact, signifying a direct proportional influence on R_0.The treatment rate (τ) is recognized as the most important negative sensitivity index, emphasizing its critical role in mitigating the spread of tuberculosis.
- Compliance with distancing measures (u_2) and the intensification of treatment (u_1) are noted to have substantial negative impacts on R_0 when active controls areapplied.
- It has been demonstrated through numerical simulations employing the Adams–Bashforth–Moulton method that the greatest decrease in disease prevalence has been achieved by combining treatment and distancing.
- Validation of the model using real data demonstrated that the fractional-order SEITR model more effectively captured persistence and memory effects compared to the traditional SIR framework.

The study is constrained by simplified population structure and assumed parameter values, which might not accurately reflect actual conditions. However, significant public health implications are emphasized because the results showed that distancing control and integrated treatment can significantly support tuberculosis control.

Author Contributions

M. A. Salek: Data curation, Investigation, Methodology, Software, Validation, Writing-original draft; **J. Nayeem:** Conceptualization, Formal analysis, Methodology, Funding, writing original draft; **Muhammad Sajjad Hossain:** Conceptualization, Formal analysis, Administration, Methodology, Resources, Supervision, writing review and editing; **M. H. Kabir :** Visualization, Supervision, Methodology, writing review and editing ;**M. Mustafizur Rahman:** Visualization, writing review and editing. All authors have read and agreed tothe published version of the manuscript.

Funding

This research received no external funding.

Acknowledgments

All authors want to express thankfulness to everyone for contribution to accomplish this research work.

Conflicts Of Interest

The authors declare no conflict of interest.

Nomemclature

Symbol	Meaning	Units	Typical / range
Λ	Recruitment	Person's time^{-1}	Demographic balance$\approx \mu N$
μ	Natural death rate	time^{-1}	$1.3 - 1.6 \times 10^{-2} yr^{-1}$
d	TB death in I	time^{-1}	$10^{-3} - 10^{-1} yr^{-1}$
β	Transmission rate	time^{-1}	$10^{-3} - 10^{-1}$(fit to incidence)
q	Relative infectious of T	-	$0.05 - 0.3$

Journal of Information Systems Engineering and Management
2025, 10(60s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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σ	Progression $E \rightarrow I$	time^{-1}	$0.3 - 1.0$
τ	Baseline treatment start	time^{-1}	$0.5 - 4.0$
ρ	Treatment success $T \rightarrow R$	time^{-1}	$1.3 - 2.0$
ϕ	Treatment failure $T \rightarrow I$	time^{-1}	$0.05 - 0.3$
γ	Self-recovery $I \rightarrow R$	time^{-1}	$0 - 0.1$
ω	Waning immunity	time^{-1}	$0.02 - 0.2$
ξ	Reinfection factor of R	-	$0.1 - 0.6$
α	Fractional order (Caputo)	-	$0.7 - 1.0$
$u_1(t)$	Control: treatment intensification	-	[0,1]
$u_2(t)$	Control: distancing compliance	-	[0,1]
ε_1	Efficacy of $u_1$on τ	-	$0.2 - 0.9$
ε_2	Efficacy of $u_2$on transmission	-	$0.2 - 0.8$

(Baselines are illustrative; calibrate to the target setting.)

Abbreviations

Disease-Free Equilibrium (DFE)

Endemic Equilibrium (EE)

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