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Research Article

Mixture Generalized-Gamma Distribution for Sea-Clutter and its Parameter Estimation Method

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ABSTRACT

The paper proposes the mixture generalized gamma (G-gamma) model for sea clutter. Such distribution is useful, because the mixture gamma, the mixture Weibull and the mixture Nakagami distributions are special cases. A hybrid estimation method based on integer order moments and simplex search algorithm is derived in order to reduce the shape parameters finding complexity in four dimensions (4D) instead of 7D. To this end, a linear equation of second order is obtained to solve the probability variable as a function of the estimated shape parameters. The scale parameter is then calculated as a function of the first order moment and the estimated probability and shape parameters. The latter are computed by means of the recursive Least Square (LS) method with the simplex search algorithm. Modeling of IPIX (Intelligent PIxel X-bande radar) sea clutter is investigated using both single and mixture G-gamma distributions. The convergence of the proposed hybrid estimation approach is confirmed in terms of several scenes of IPIX database. It is also observed that the mixture G-gamma distribution exhibits the best fit to real data in most cases.

Keywords: Mixture distribution, G-gamma model, Sea-clutter, Parameter estimation, Hybrid estimator.

INTRODUCTION

Doppler spectrum and intensity statistics are the effective tools to describe and analyze sea clutter. The research on the distribution of sea clutter is crucial for the development of CFAR (Constant False Alarm Rate) radar detection schemes and the acquisition of ocean information [1]. Sea clutter Doppler spectrum contains abundant sea surface information, which can reflect the complex modulation effects of sea area waves on radar signals. When sea conditions suffer complex influences, such as wind speed, wind currents, topography, temperature and humidity, as well as the influence of radar parameters, such as the modern radar high resolution and small grazing angles, sea clutter has extremely non-stationary properties [2].

Based on the analysis of experimental data of IPIX radar which cover almost sea clutter conditions, a large number of distributions are carried out in terms of range resolutions, antennas polarizations and range cell numbers [3, 4]. In this context, the purpose is to utilize a general distribution which consists of several heavy tailed models and determine which parametric model is suitable for a given set of data. Compound Gaussian (CG) distributions, General compound (GC) distribution, mixtures distributions and composite distributions can produce a goodness of fit to real data [5-8]. A two parameter *K*, Pareto type 2, CG-IG, CG-LNT are CG models with gamma, inverse gamma, inverse Gaussian and log-normal texture variates respectively. A five parameter GC model is more general than the CG models, because the speckle and the texture components follow the generalized gamma distribution. In order to obtain improved fit to empirical data, the estimated thermal noise power is incorporated into the speckle component of both CG and GC models [9]. On the other hand, the mixture distributions are created to characterize complex sea or land clutter data with additive outliers where the nature of their CCDF curves contain some inflexion points [10]. They are given as a sum of two weighted distributions with different scale and shape parameters values. Recently, composite distributions are proposed for complex data fitting and have different forms than the above models. They are constructed by the conjunction of two distributions in the same interval of the amplitude or the normalized threshold variables [8].

Obviously, without parameter estimation procedures, we cannot study the statistical properties of sea clutter as well as the adaptation of detection thresholds. Also, it is not possible to obtain information on sea or ocean environment. That is why, numerous research works about the estimation methods are developed for statistical distributions parameters [11-15]. Matching of moments, MLE (Maximum Likelihood Estimator), CMLE (Censored MLE), Bayes, PCFE (Parametric Curve Fitting Estimator), etc have shown to be effective estimation approaches of CG, GC, mixture and composite distributed clutter parameters.

In this paper, the heavy tailed mixture G-gamma model for sea clutter is analyzed through real IPIX database. Such distribution is very interesting, since the mixture gamma, the mixture Weibull and the mixture Nakagami distributions are particular cases. A hybrid estimation method based on the first two integer order moments and simplex search algorithm is developed in order to simplify the shape parameters finding complexity in 4D instead of 7D. To this effect, a linear equation of second order is obtained to solve the probability variable in terms of estimated shape parameters. The scale parameter is calculated as a function of the first order moment, the estimated probability and the estimated shape parameters. The latter are computed by the recursive LS method based on the simplex search algorithm. Modeling of IPIX sea clutter is investigated using both single and mixture G-gamma distributions. The convergence of the proposed hybrid estimation approach is preserved in terms of numerous scenarios of IPIX database. It is observed that the mixture G-gamma distribution exhibits the best fit to real data in most cases.

The paper is organized as follows. Section 2 presents the mixture G-gamma pdf (probability density function) with their CCDF and moments expressions. Section 3 outlines the proposed hybrid estimation method for the mixture G-gamma model parameters. Section 4 examines the modeling of IPIX data using both single and mixture G-gamma pdfs. Finally, Section 5 cites the main remarks of this work.

MIXTURE G-GAMMA DISTRIBUTION

The mixture G-gamma distribution is a flexible probability distribution that includes the mixture gamma, the mixture Weibull and the mixture Nakagami distributions. Based on [6, 7, 10], the mixture G-gamma pdf for the amplitude, $X > 0$, is given by

$$p(x) = \sum_{i=1}^{n} k_i \frac{b_i}{a_i \Gamma(\nu_i)} \left(\frac{x}{a_i}\right)^{b_i \nu_i - 1} \exp\left(-\left(\frac{x}{a_i}\right)^{b_i}\right) \quad (1)$$

Where $\Gamma(.)$ is the gamma function, $a_i > 0$ are scale parameters, $0 < k_i < 1$ are probabilities, $b_i > 0$ and $\nu_i > 0$ are shape parameters. Values of b_i and ν_i which lie between '0' and 'infinity' are responsible to the spikiness degrees of sea clutter. The corresponding CCDF (Cumulative Distributed Function) is obtained from (1) as

$$CCDF(T) = \sum_{i=1}^{n} k_i \left[1 - \frac{b_i}{a_i \Gamma(\nu_i)} \int_0^T \left(\frac{x}{a_i}\right)^{b_i \nu_i - 1} \exp\left(-\left(\frac{x}{a_i}\right)^{b_i}\right) dx\right] \quad (2)$$

Where, T is the normalized threshold. If we set, $t = (x/a_i)^{b_i}$ and $\frac{dt}{dx} = \frac{b_i}{a_i}(x/a_i)^{b_i - 1}$, (2) becomes

$$CCDF(T) = \sum_{i=1}^{n} k_i \left[1 - \gamma\left((T/a_i)^{b_i}, \nu_i\right)\right] \quad (3)$$

Where $\gamma(.,.)$ is the lower incomplete gamma function.

Using (1), the expression of moments of order, r is given by

$$\langle x^r \rangle = \sum_{i=1}^{n} a_i^r \frac{\Gamma(\nu_i + r/b_i)}{\Gamma(\nu_i)} \quad (4)$$

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Where $\langle . \rangle$ denotes expectation which is equivalent to the following sample moment of order, r

$$\langle x^r \rangle \approx \hat{\mu}_r = \frac{1}{M}\sum_{i=1}^{M} x_i^r \tag{5}$$

Where x_i, i = 1, ..., M are independent and identically distributed (iid) samples of sea echoes.

PROPOSED ESTIMATOR

In this section, we consider the estimation of a mixture of two G-gamma distributed parameters. Thus, for n = 2, (4) is given by

$$\langle x^r \rangle = ka_1^r \frac{\Gamma(\nu_1 + r/b_1)}{\Gamma(\nu_1)} + (1-k)a_2^r \frac{\Gamma(\nu_2 + r/b_2)}{\Gamma(\nu_2)} \tag{6}$$

Note that, we need 7 equations from (6) to resolve the estimation problem of mixture G-gamma clutter parameters. The estimation complexity can be reduced if we take, $a_1 = a_2 = a$. As discussed in [10], this choice does not affect the modeling performances of sea clutter in most cases. Based on (6), moments of orders 1 and 2 will be

$$\mu_1 = a\left[k\frac{\Gamma(\nu_1 + 1/b_1)}{\Gamma(\nu_1)} + (1-k)\frac{\Gamma(\nu_2 + 1/b_2)}{\Gamma(\nu_2)}\right] \tag{7}$$

and

$$\mu_2 = a^2\left[k\frac{\Gamma(\nu_1 + 2/b_1)}{\Gamma(\nu_1)} + (1-k)\frac{\Gamma(\nu_2 + 2/b_2)}{\Gamma(\nu_2)}\right] \tag{8}$$

Then, it is possible to simplify the estimation problem in 4D instead of 6D by manipulating (7) and (8). The latter are equivalent to

$$\begin{cases} \mu_1 = a[kA + (1-k)B] \\ \mu_2 = a^2[kC + (1-k)D] \end{cases} \tag{9}$$

Where

$$\begin{cases} A = \dfrac{\Gamma(\nu_1 + 1/b_1)}{\Gamma(\nu_1)} \\ B = \dfrac{\Gamma(\nu_2 + 1/b_2)}{\Gamma(\nu_2)} \\ C = \dfrac{\Gamma(\nu_1 + 2/b_1)}{\Gamma(\nu_1)} \\ D = \dfrac{\Gamma(\nu_2 + 2/b_2)}{\Gamma(\nu_2)} \end{cases} \tag{10}$$

Combination of (9) leads to eliminate the scale parameter a so that

$$\frac{\mu_2}{\mu_1^2} = \frac{k(C-D)+D}{k^2(A-B)^2 + 2(A-B)Bk + B^2} \tag{11}$$

Setting, $E = \frac{\mu_2}{\mu_1^2}$, (11) is transformed to the 2nd order linear equation with an unknown probability variable, k between 0 and 1. Hence

$$E(A-B)^2k^2 + (2E(A-B)B - C + D)k + EB^2 - D = 0 \quad (12)$$

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With the new coefficients, α, β and φ, (12) is a linear equation of order 2

$$\alpha k^2 + \beta k + \varphi = 0 \tag{13}$$

Where:

$$\begin{cases} \alpha = \frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_1+1/b_1)}{\Gamma(\nu_1)} - \frac{\Gamma(\nu_2+1/b_2)}{\Gamma(\nu_2)}\right]^2 \\ \beta = 2\frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_1+1/b_1)}{\Gamma(\nu_1)} - \frac{\Gamma(\nu_2+1/b_2)}{\Gamma(\nu_2)}\right]\frac{\Gamma(\nu_2+1/b_2)}{\Gamma(\nu_2)} \\ \quad -\frac{\Gamma(\nu_1+2/b_1)}{\Gamma(\nu_1)} + \frac{\Gamma(\nu_2+2/b_2)}{\Gamma(\nu_2)} \\ \varphi = \frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_2+1/b_2)}{\Gamma(\nu_2)}\right]^2 - \frac{\Gamma(\nu_2+2/b_2)}{\Gamma(\nu_2)} \end{cases} \tag{14}$$

It is evident that (13) has two different solutions for, k

$$k = \begin{cases} \frac{-\beta + \sqrt{\beta^2 - 4\alpha\varphi}}{2a} \\ or \\ \frac{-\beta - \sqrt{\beta^2 - 4\alpha\varphi}}{2a} \end{cases} \tag{15}$$

Note that, the solutions given by (15) as a function of shape parameters are accepted if and only if, $0 < k < 1$. Below, we can obtain several forms of (14) for cases of mixture gamma, mixture Weibull, and mixture Nakagami distributed sea clutter.

a) Mixture gamma clutter case ($b_1 = b_2 = 1$):

$$\begin{cases} \alpha = \frac{\mu_2}{\mu_1^2}(\nu_1 - \nu_2)^2 \\ \beta = 2\frac{\mu_2}{\mu_1^2}(\nu_1 - \nu_2)\nu_2 - \nu_1(\nu_1+1) + \nu_2(\nu_2+1) \\ \varphi = \frac{\mu_2}{\mu_1^2}\nu_2^2 - \nu_2(\nu_2+1) \end{cases} \tag{16}$$

b) Mixture Weibull clutter case ($\nu_1 = \nu_2 = 1$):

$$\begin{cases} \alpha = \frac{\mu_2}{\mu_1^2}[\Gamma(1+1/b_1) - \Gamma(1+1/b_2)]^2 \\ \beta = 2\frac{\mu_2}{\mu_1^2}[\Gamma(1+1/b_1) - \Gamma(1+1/b_2)]\Gamma(1+1/b_2) \\ \quad -\Gamma(1+2/b_1) + \Gamma(1+2/b_2) \\ \varphi = \frac{\mu_2}{\mu_1^2}[\Gamma(1+1/b_2)]^2 - \Gamma(1+2/b_2) \end{cases} \tag{17}$$

c) Mixtue Nakagami clutter case ($b_1 = b_2 = 2$):

$$\begin{cases} \alpha = \frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_1+1/2)}{\Gamma(\nu_1)} - \frac{\Gamma(\nu_2+1/2)}{\Gamma(\nu_2)}\right]^2 \\ \beta = 2\frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_1+1/2)}{\Gamma(\nu_1)} - \frac{\Gamma(\nu_2+1/2)}{\Gamma(\nu_2)}\right]\frac{\Gamma(\nu_2+1/2)}{\Gamma(\nu_2)} \\ \quad -\nu_1 + \nu_2 \\ \varphi = \frac{\mu_2}{\mu_1^2}\left[\frac{\Gamma(\nu_2+1/2)}{\Gamma(\nu_2)}\right]^2 - \nu_2 \end{cases} \tag{18}$$

Due to the mathematical complexity, the non-linear estimation search for the shape parameters, b_1, ν_1, b_2 and ν_2 is executed numerically using the LS method based on the Nelder-Mead simplex algorithm [16]. It involves the

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matching of real and theoretical CCDFs. The fitness function is the sum of residuals (i.e., squared errors) between empirical CCDF ($CCDF_{emper}$) and theoretical CCDF ($CCDF_{theor}$) given by (2)

$$Fitness(b_1, \nu_1, b_2, \nu_2) = \sum_{i=m_1}^{m_2} \left(CCDF_{emper}(T_i) - CCDF_{there}(T_i) \right)^2 \tag{19}$$

Where m_1 and m_2 describe the adequate region of the two CCDFs as shown in Fig. 1.

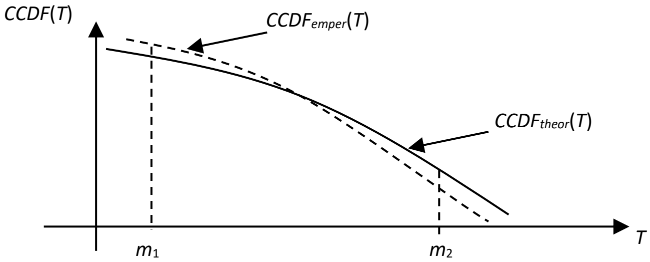

Fig. 1 Regression by the LS method between empiricaland theoretical CCDFs.

During the search of underling variables, the probability k and the scale parameter a are computed using (15) and (7) respectively. Once the different shape parameters are obtained, k is estimated by (15) and the first order moment is used to estimate a so that

$$\hat{a} = \frac{\hat{\mu}_1}{\hat{k}\dfrac{\Gamma(\hat{\nu}_1 + 1/\hat{b}_1)}{\Gamma(\hat{\nu}_1)} + (1-\hat{k})\dfrac{\Gamma(\hat{\nu}_2 + 1/\hat{b}_2)}{\Gamma(\hat{\nu}_2)}} \tag{22}$$

RESULTS

In this section, the mixture G-gamma model given by (1) is assessed through real IPIX data. For comparison purposes, the G-gamma model is considered using the zlog(z) estimator (see Appendix A). Characteristics details of IPIX data is highlighted in [17, 18]. For a finite number of independent samples, M = 60 000 were employed to estimate shape and scale parameters of G-gamma and mixture G-gamma distributions.

Our first experimental study concerns the case of HH antennas polarization, 17th range cell and resolution 3m as shown in Figs. 2 (a) and (b). It is seen from the resulting CCDfs and moments curves that the mixture G-gamma model is more accurate than that of the G-gamma model. This scenario of the data should have a mixture of Gaussian and non-Gaussian clutter. As the experimental results shown in Figs. 3 (a) and (b) for the case of VV polarization, 9th range cell and resolution 3m, the fitting errors are small if the proposed mixture G-gamma distribution is used. From the modeling results given in Figs. 4 (a) and (b), for the case of HH polarization, resolution 15m and 22nd range cell, it is evident that the CCDFs and moments curves become closer when the cell resolution decreases. Figs. 5 (a) and (b) report the fitting results obtained for VV polarization, resolution 15m and 2nd range cell. From the figures it is obvious that with the use of a mixture G-gamma model, it is possible to model IPIX data having inflection points that are not visible in some scenarios of the data. Figs. 6 (a) and (b) lay out the comparisons between the empirical and the estimated theoretical CCDFs and moments for cases of HH polarization, resolution 30m and 13th range cell. The mixture G-gamma distribution maintains the same modeling performances as before. From the fitting results shown in Figs. 7 (a) and (b), there are no considerable differences between the empirical curves and the estimated curves (i.e., mixture G-gamma and G-gamma distributions). This implies that this scenario of the data follows the G-gamma model. Table. 1 present the MSE values obtained for each case of the above experimental studies (i.e., Figs. 2-7).

In conclusion, the improvement in sea clutter modeling is justified by the use of the proposed general mixture model that includes the mixture gamma, mixture Weibull, and mixture Nakagami distributions.

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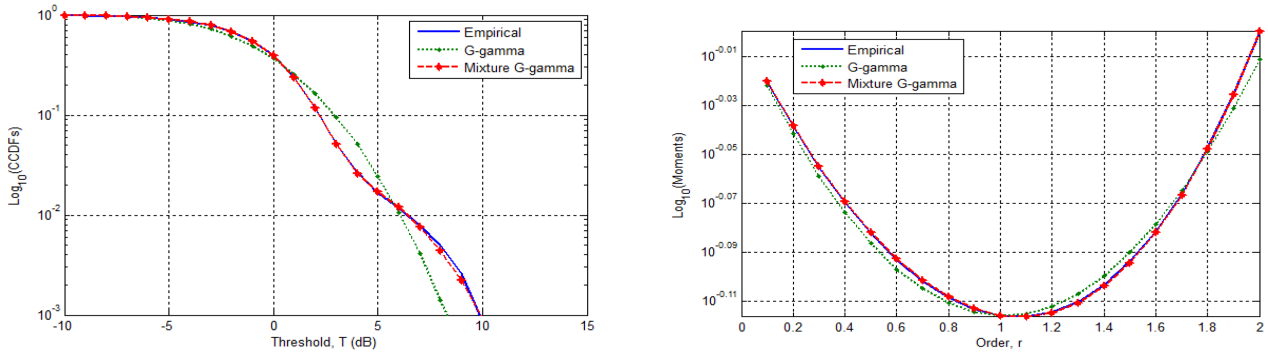
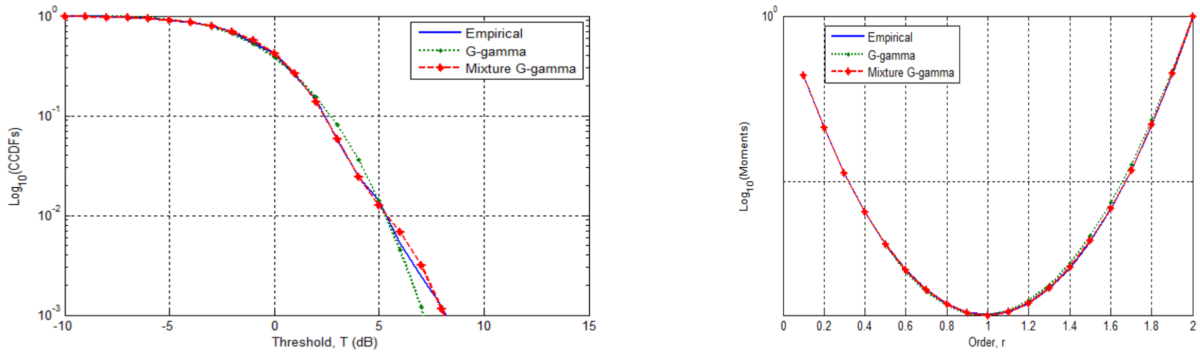


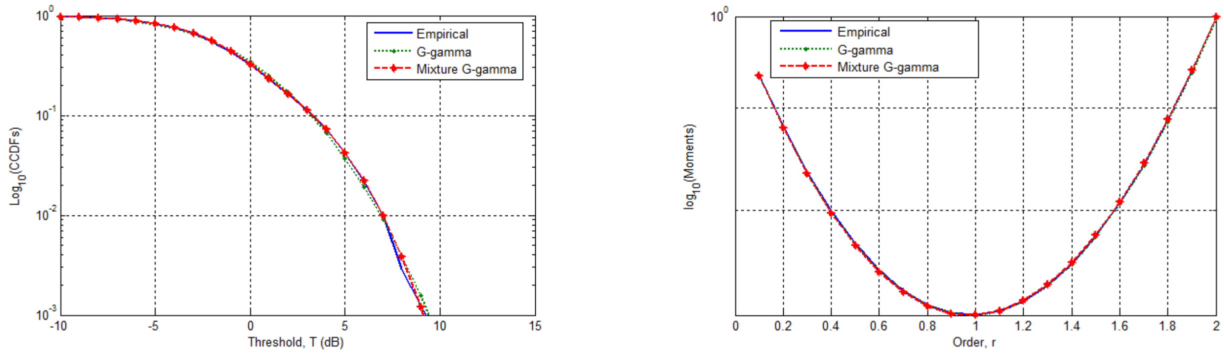
(a) (b)

Fig. 2 IPIX data modeling for the case of HH polarization, resolution 3m and 17th range cell (a) Fitted CCDFs, (b) Fitted moments.



(a) (b)

Fig. 3 IPIX data modeling for the case of VV polarization, resolution 3m and 9th range cell, (a) Fitted CCDFs, (b) Fitted moments



(a) (b)

Fig. 4 IPIX data modeling for the case of HH polarization, resolution 15m and 22nd range cell, (a) Fitted CCDFs, (b) Fitted moments

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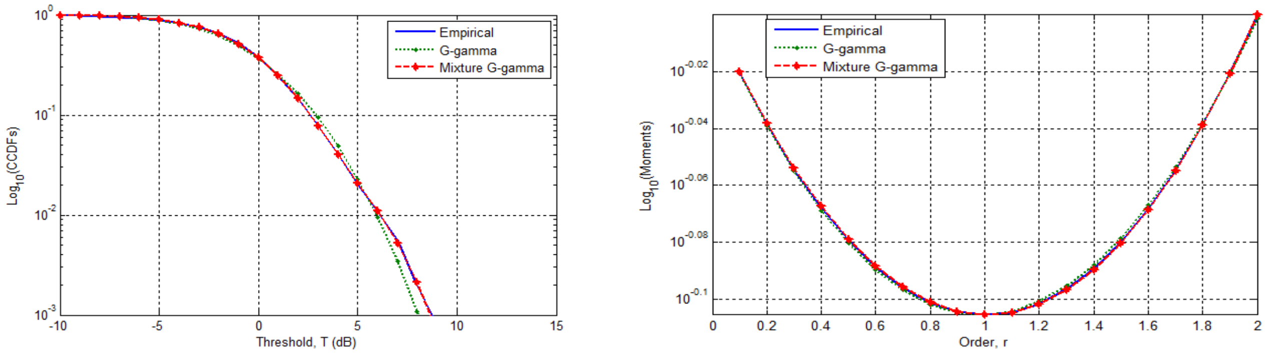
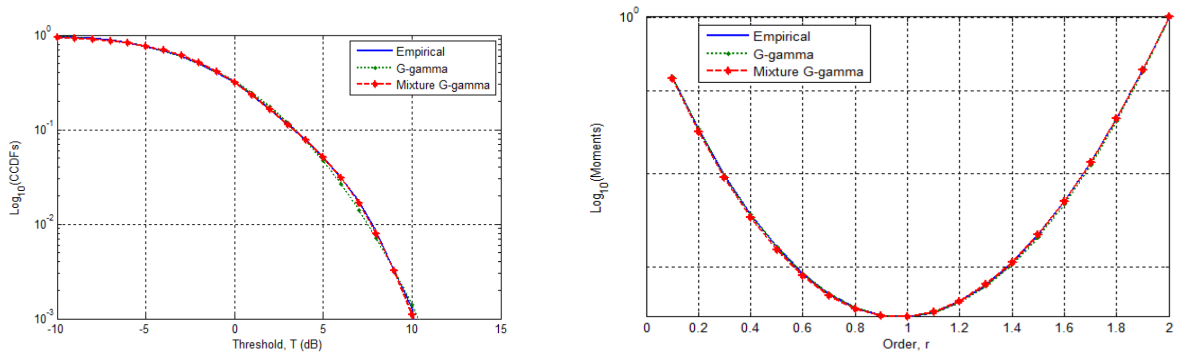


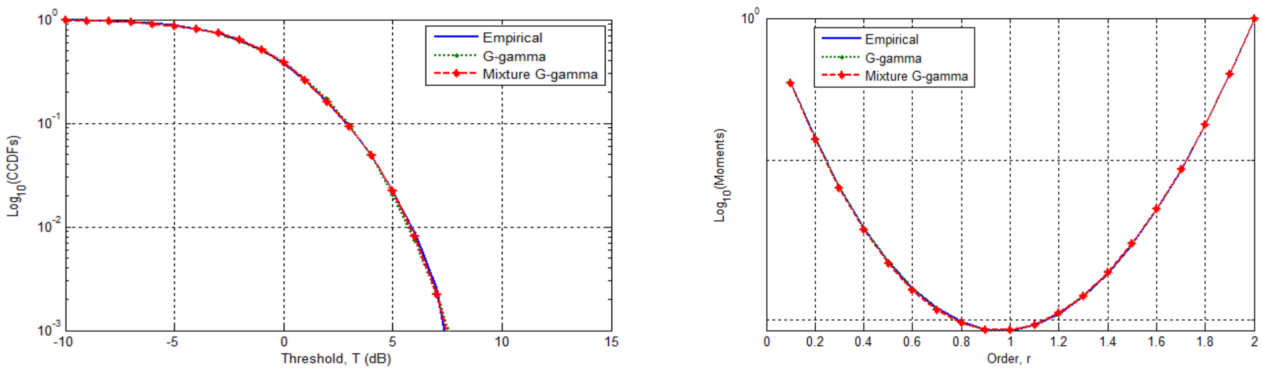
(a) (b)

Fig. 5 IPIX data modeling for the case of VV polarization, resolution 15m and 2nd range cell, (a) Fitted CCDFs, (b) Fitted moments



(a) (b)

Fig. 6 IPIX data modeling for the case of HH polarization, resolution 30m and 13th range cell, (a) Fitted CCDFs, (b) Fitted moments



(a) (b)

Fig. 7 IPIX data modeling for the case of VV polarization, resolution 30m and 8th range cell, (a) Fitted CCDFs, (b) Fitted moments

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Table. 1 MSE metric test of G-gamma and mixture G-gamma models

IPIX real data	G-gamma model	Mixture G-gamma model
HH, 3m and 17th cell	0.00071829	0.00000548
VV, 3m and 9th cell	0.00016674	0.00006940
HH, 15m and 22nd cell	0.00009834	0.00002467
VV, 15m and 2nd cell	0.00012604	0.00000834
HH, 30m and 13th cell	0.00003421	0.00007453
VV, 30m and 8th cell	0.00001853	0.00001608

CONCLUSION

This paper firstly outlines the mixture G-gamma model and the proposed estimator of the shape and scale parameters using a hybrid method based on moments and least square approaches (NOT CLEAR). They were obtained numerically by means of the simplex search algorithm. Experimental results, based on the analysis of many files at different range resolutions of the Grimsby data set, recorded by IPIX radar, reveal that the mixture G-gamma pdf provides a good-fit to the data for all polarizations with resolutions of 3m, 15m and 30m. It was pinpointed out that the increasing of the range resolution up to 15m, the mixture G-gamma and G-gamma pdfs consistently yield good-fit in several scenes. Moreover, it was noticed that when the empirical CCDF contains inflection points, the mixture G-gamma model was well adapted compared to the standard G-gamma pdf.

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Appendix A

zlog(z) estimator:

The zlog(z) method for the G-gamma clutter parameters is obtained from the moments expression of order, r. The matching of log-moments are considered so that, the corresponding derivative with respect to, r is calculated as follows

$$\begin{aligned}\frac{\partial\langle x^r\rangle}{\partial r} &= \langle x^r \log(x)\rangle \\ &= \frac{a^r \log(a)\Gamma(v+r/b)+a^r\Gamma(v+r/b)\psi(v+r/b)/b}{\Gamma(v)}\end{aligned} \tag{A.1}$$

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For r = 0 and r = 1, the following system is obtained

$$\begin{cases} \langle \log(x) \rangle = \log(a) + \dfrac{\psi(\nu)}{b}, & r = 0 \\ \langle x\log(x) \rangle = a\dfrac{\Gamma(\nu + 1/b)}{\Gamma(\nu)}\left[\log(a) + \dfrac{\psi(\nu + 1/b)}{b}\right], & r = 1 \end{cases} \tag{A.2}$$

After combining, $\langle x \rangle$, $\langle x^2 \rangle$, $\langle \log(x) \rangle$ and $\langle x\log(x) \rangle$, we get

$$\begin{cases} \dfrac{\langle x^2 \rangle}{\langle x \rangle^2} = \dfrac{\Gamma(\hat{\nu})\Gamma(\hat{\nu} + 2/\hat{b})}{\Gamma(\hat{\nu} + 1/\hat{b})^2} \\ \dfrac{\langle x\log(x) \rangle}{\langle x \rangle} - \langle \log(x) \rangle = \dfrac{\psi(\hat{\nu} + 1/\hat{b}) - \psi(\hat{\nu})}{\hat{b}} \end{cases} \tag{A.3}$$

Numerical solutions are required to obtain, $\hat{\nu}$ and $\hat{b}$ simultaneously. The scale parameter, a is then computed as

$$\hat{a} = \frac{\mu_1 \Gamma(\hat{\nu})}{\Gamma(\hat{\nu} + 1/\hat{b})} \tag{A4}$$