

# Bipedal Robot Walking and Locomotion, The Intersection of Robotics and Biomechanics by Oscillatory Solutions

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## ABSTRACT

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The study of bipedal robot locomotion lies at the fascinating crossroads of robotics and biomechanics. This field aims to emulate human walking patterns in robots by applying the principles of human gait to robotic systems. The specific method in this area involves oscillatory solutions, which utilize rhythmic patterns to create stable and efficient walking mechanisms. Here some oscillatory solutions for first-order neutral delay difference equations have been developed, contributing to the stability and effectiveness of bipedal robotic movement. When these oscillatory solutions are integrated into bipedal robots within the framework of 3D printing, a novel and innovative fusion of robotics and biomechanics emerges. This integration not only broadens the functional capabilities of bipedal robots but also improves the accuracy and efficiency of 3D printing technology.

### Keywords:

Neutral,Delay,DifferenceEquations

Robot,Bipedal,Stability,OscillatorySolutions,Firstorder,

## 1.1 INTRODUCTION

Bipedal robots are amazing inventions in the field of robotics that mimic a basic feature of human anatomy the capacity to walk on two legs. These humanoid-looking robots, which can walk on two feet, are the result of the fusion of biomechanics, artificial intelligence, and engineering. An overview of bipedal robots is given in this introduction, along with information on their importance, unique qualities, and variety of uses. Anthropomorphic in nature, bipedal robots usually have two legs, a torso, and frequently arms that mimic the upper limbs of humans. This design decision is not random, rather it is motivated by the goal of building robots that can function in human-designed surroundings, communicate using human tools and interfaces, and eventually improve human-robot cooperation. The capacity of bipedal robots to walk on two legs and imitate human locomotion is fundamental to their identity. Their capacity to traverse a wide variety of situations, including those with uneven terrain, stairs, and obstacles, distinguishes them from robots with wheels or tracks. Because of their adaptability in movement, bipedal robots are especially desirable for a range of applications.

Bipedal robots are now used in many different fields, extending beyond the confines of research labs. They are being used more and more in the medical field, where they help with physical therapy and rehabilitation and provide support to people who want to get their mobility back. Moreover, bipedal robots have a role in both entertainment and education, enthralling viewers with their realistic motions and instructional initiatives. Bipedal robots are a driving force behind innovative research. These robots are used by scientists and robotics engineers to investigate areas like biomechanics, artificial intelligence, control theory, and sensor integration. Research on bipedal robotics has yielded insights that go well beyond the area and aid in the creation of increasingly complex and intelligent robotic systems. Bipedal robots have enormous potential in the future. The possibilities are virtually

limitless when it comes to how they may improve human-robot interaction, support emergency response, transform healthcare, and even go into extraterrestrial exploration.

Integrating oscillatory solutions into bipedal robots within the context of 3D printing creates a unique and groundbreaking synergy between robotics and biomechanics. This combination enables robots to move in a way that closely mimics natural human walking patterns, allowing for greater precision and adaptability in various tasks, including 3D printing. The rhythmic, oscillatory control mechanisms allow bipedal robots to move smoothly and efficiently. This natural, human-like gait is crucial for tasks that require high levels of precision, such as 3D printing. The robot's ability to navigate different terrains while maintaining balance and stability enhances its functionality in diverse environments.

Unlike traditional 3D printers that are stationary, a bipedal robot equipped with these oscillatory solutions can move across a larger area, printing structures directly on-site. This mobility opens up new possibilities for large-scale manufacturing and construction projects, where the robot can autonomously print complex designs that would be challenging with fixed machinery. The integration of oscillatory solutions allows the robot to adapt its movements in real-time, ensuring that the 3D printing process remains accurate even on uneven or changing surfaces. This adaptability is essential for maintaining the quality of the printed output, regardless of the environment.

### **1.2 Central Pattern Generators**

Robots that walk on two legs, like humans or animals, are known as bipedal robots. In robots, achieving stable and effective walking is a challenging task. Oscillatory solutions of the difference equation can be used to regulate leg movements and guarantee balanced and fluid gait for bipedal robots. Central Pattern Generators are neural networks or control algorithms that generate rhythmic patterns of activity, and they are designed using oscillatory solutions. Oscillatory solutions in CPGs offer a means of controlling different gaits. Natural and human-like walking is made possible for bipedal robots by the application of oscillatory solutions in CPGs. Aesthetically beautiful and biomechanically effective movement is achieved by robots by mimicking the periodicity and coordination of human leg movements. Real-time adjustments can be made to oscillatory solutions to accommodate shifting circumstances. For stability to be maintained during movement, this adaptability is essential. In CPGs, oscillatory solutions provide a way to regulate. Robots are more adaptable to a variety of jobs and environments because they may change between walking, running, climbing, and other locomotion patterns by modifying the oscillations' parameters. It's crucial for bipedal robots to keep their leg movements in rhythm. To improve overall locomotion efficiency and prevent tripping, oscillatory solutions in CPGs make sure that the robot's legs move in unison.

### **1.3 Significance of Stability:**

Ensuring the safety of the robot and its environment is the main priority in bipedal robotics. In order to avoid falls that could harm the robot and endanger nearby objects or people, stability is essential. Walking steadily reduces energy use. Stability allows a robot to use its energy more effectively, resulting in longer operating hours and less frequent refuelling or charging. Robots that are stable can adapt better to a variety of surfaces and settings. Their versatility in a range of applications stems from their ability to climb stairs, handle uneven surfaces, and effectively adapt to unforeseen disturbances. Neural networks or control algorithms known as CPGs are used to produce rhythmic motor activity patterns. These oscillatory solutions regulate the time and synchronisation of leg motions, which is a vital part of stability maintenance. They offer a balanced and rhythmic gait. Bipedal robots are furnished with various sensors, including force, gyro, and accelerometers. Real-time data from these sensors is used by feedback control systems to continuously modify the robot's movements. The control system can intervene to restore stability if the robot begins to tilt or lose its equilibrium. A few bipedal robots use algorithms for predictive control. These algorithms anticipate and adapt to changes in the environment and status of the robot.

An essential factor in bipedal robotics is stability. Stability must be attained and maintained for movement to be safe, effective, and flexible. By utilising control algorithms, feedback mechanisms, adaptive tactics, and suitable mechanical construction, bipedal robots may manoeuvre intricate surroundings with minimal chance of falls and guarantee secure functioning in an extensive array of uses.

### 1.4 Oscillatory Solutions

Here some criteria for the oscillation of first order neutral delay difference equation is obtained. Consider the Neutral Delay difference equation,

$$\Delta(B_\theta + p_\theta B_{\theta-1})^\emptyset + q_\theta f(B_{\theta-\gamma}) = 0 \quad (1.1)$$

Where  $\theta \in \mathbb{N}_0 = \{n_0, n_0 + 1, n_0 + 2, \dots\}$ ,  $\emptyset > 0$ ,  $\{p_\theta\}, \{q_\theta\}$  are the positive sequences.

$f: \mathbb{R} \rightarrow \mathbb{R}$  is continuous and  $xf(x) > 0$ ,  $f'(x) \geq 0$  for  $x \neq 0$ . Sequence  $\{B_\theta\}$  satisfies Eq.(1.1) for all  $\theta$ . An eventual positive or negative nontrivial solution  $\{B_\theta\}$  is considered non oscillatory, while an oscillatory solution falls in between. The importance of difference equation application in robotic engineering has increased as a result of the widespread adoption of this behaviour by existing schemes. Novel technologies, autonomous vehicles, biological systems, distribution networks, social interaction, and communication systems are some of the topics that have become independent fields of study. Being the discrete analogue of differential equations, first order Neutral Delay Difference Equations are becoming more and more popular. Numerous studies on the oscillation of solutions to neutral delay difference equations have been published in recent years. The issue of the oscillation of a neutral differential equation with positive and negative coefficients is covered in detail by Ozkan Ocalan[1]. Trajectory Planning of Flexible Walking for Biped Robots Using Linear Inverted Pendulum Model and Linear Pendulum Model is studied by **Long Li, Zhongqu Xi, Xiang Luo**, [3]. Joris Verhagen[6], established the results on Bipedal Robot Locomotion: Reflex Inspired Compensation on Planned and Unplanned Downsteps, Grace and Lalli [8] explained the Oscillation theorems for second order delay and neutral difference equations. Tanaka. S[9] talked about different approaches to solving first-order oscillatory neutral delay differential equations. Here we established some solutions for the oscillations for first order difference equation which improves the previously established results.

### 1.5 Theorem

All of the solutions to equation (1.1) exhibit oscillations, if the following condition holds.

$A_1: f_{nm} < f_{nm}$ ,

$A_2: \sum_{u=\epsilon}^{\infty} \frac{u}{f(u)} < \infty$ , for some  $\epsilon > 0$ .

$A_3: \lim_{s \rightarrow \infty} \sup \sum_{s=N_0}^n \rho_{s+1} q_s f(1 - \mu_{\theta-\gamma}) = \infty$ , where  $\rho$  be a function such that  $\Delta \rho_s \geq 0$ ,

for  $\forall n \in \mathbb{N}(n_0)$

**Proof :** Assume that  $B_\theta$  represents a non-oscillatory solution to (1.1). We can presume that  $B_\theta$  is ultimately positive without losing generality.

$$\mu_\theta = B_\theta + p_\theta B_{\theta-1}$$

Now equation (1.1) becomes, (1.2)

$$(\Delta \mu_\theta)^\emptyset + q_\theta f(\mu_{\theta-\gamma} - p_{\theta-\gamma} B_{\theta-\gamma-1}) = 0$$

From  $A_1$ ,  $(\Delta \mu_\theta)^\emptyset > -q_\theta f(1 - \mu_{\theta-\gamma}) f(\mu_{\theta-\gamma})$  (1.3)

Define  $\sigma_\theta = \rho_\theta \mu_\theta / f(\mu_{\theta-\gamma})$ , then  $\sigma_\theta > 0$ .

Equation (1.3) becomes ,

$$(\Delta \mu_\theta)^\emptyset > -q_\theta f(1 - \mu_{\theta-\gamma}) f(\mu_{\theta-\gamma})$$

$$(\Delta \mu_\theta) > -q_\theta f(1 - \mu_{\theta-\gamma}) f(\mu_{\theta-\gamma})$$

$$\Delta(\sigma_\theta f(\mu_{\theta-\gamma})) / \rho_\theta > -q_\theta f(1 - \mu_{\theta-\gamma}) f(\mu_{\theta-\gamma})$$

$$\{\rho_\theta \Delta(\sigma_\theta f(\mu_{\theta-\gamma})) - \sigma_\theta f(\mu_{\theta-\gamma}) \Delta \rho_\theta\} / \rho_\theta \rho_{\theta+1} > -q_\theta f(1 - \mu_{\theta-\gamma}) f(\mu_{\theta-\gamma})$$

$$\begin{aligned} & \{\rho_{\theta}(\sigma_{\theta}f(\mu_{\theta-\gamma+1}) - \sigma_{\theta}f(\mu_{\theta-\gamma})) + \rho_{\theta}((\sigma_{\theta+1}f(\mu_{\theta-\gamma}) - \sigma_{\theta}f(\mu_{\theta-\gamma})) - \sigma_{\theta}f(\mu_{\theta-\gamma})\Delta\rho_{\theta})/\rho_{\theta}\rho_{\theta+1} > -q_{\theta}f(1-\mu_{\theta-\gamma})f(\mu_{\theta-\gamma}) \\ & \rho_{\theta}f(\mu_{\theta-\gamma})\Delta\sigma_{\theta} > -q_{\theta}f(1-\mu_{\theta-\gamma})f(\mu_{\theta-\gamma})\rho_{\theta}\rho_{\theta+1} + f(\mu_{\theta-\gamma})\sigma_{\theta}\rho_{\theta} - \rho_{\theta}f(\mu_{\theta-\gamma+1})\sigma_{\theta} + \sigma_{\theta}f(\mu_{\theta-\gamma})\Delta\rho_{\theta} \\ & \Delta\sigma_{\theta} > -\rho_{\theta+1}q_{\theta}f(1-\mu_{\theta-\gamma}) + [\mu_{\theta}/f(\mu_{\theta-\gamma})][f(\mu_{\theta-\gamma})\rho_{\theta} - \rho_{\theta}f(\mu_{\theta-\gamma+1}) + f(\mu_{\theta-\gamma})\Delta\rho_{\theta}]/ \\ & f(\mu_{\theta-\gamma}) \end{aligned}$$

$$\Delta\sigma_{\theta} > -\rho_{\theta+1}q_{\theta}f(1-\mu_{\theta-\gamma}) + [\mu_{\theta}]\{-\rho_{\theta}f(\mu_{\theta-\gamma+1}) + \rho_{\theta+1}f(\mu_{\theta-\gamma})\}/f(\mu_{\theta-\gamma})^2\}$$

Since  $\{\Delta\mu_{\theta}^0\}$  is a decreasing sequence, we have Sequence,  $\Delta\mu_{\theta} \leq \Delta\mu_{\theta-\gamma}$ .

$$\Delta\sigma_{\theta} > -\rho_{\theta+1}q_{\theta}f(1-\mu_{\theta-\gamma}) + [\mu_{\theta}][(-\rho_{\theta}f(\mu_{\theta-\gamma+1})/f(\mu_{\theta-\gamma})^2) + \rho_{\theta+1}/f(\mu_{\theta-\gamma})]$$

$$-\rho_{\theta+1}q_{\theta}f(1-\mu_{\theta-\gamma}) < \Delta\sigma_{\theta} - [\mu_{\theta}](-\rho_{\theta}\frac{f(\mu_{\theta-\gamma+1})}{f(\mu_{\theta-\gamma})^2}) + \rho_{\theta+1}/f(\mu_{\theta-\gamma})$$

$$\rho_{\theta+1}q_{\theta}f(1-\mu_{\theta-\gamma}) > -\Delta\sigma_{\theta} - [\mu_{\theta}](-\rho_{\theta}\frac{f(\mu_{\theta-\gamma+1})}{f(\mu_{\theta-\gamma})^2}) + \rho_{\theta+1}/f(\mu_{\theta-\gamma})$$

$\lim_{s \rightarrow \infty} \sup \sum_{s=N_0}^n \rho_{s+1} q_s f(1-\mu_{\theta-\gamma}) > \infty$ , which contradicts  $A_3$ . As a result, all the solutions of equation (1.1) oscillates. Similar reasoning applies when  $B_{\theta}$  finally turns negative.

**1.6 Conclusion:** The exploration of bipedal robot walking and locomotion at the intersection of robotics and biomechanics, particularly through the use of oscillatory solutions, represents a significant leap forward in the development of advanced robotic systems. By drawing inspiration from the natural rhythms and dynamics of human gait, these solutions enable robots to walk more efficiently, stably, and adaptively. Integrating oscillatory solutions into bipedal robots within the framework of 3D printing represents a significant advancement that combines the precision of additive manufacturing with the adaptability of human-like movement. This fusion enables robots to perform complex printing tasks over larger and more varied surfaces, while maintaining stability and accuracy. The result is a new level of flexibility and efficiency in 3D printing, opening up possibilities for on-site construction, large-scale manufacturing, and other innovative applications. As this integration continues to evolve, it will likely lead to more versatile and capable robotic systems that redefine the boundaries of what can be achieved in both robotics and 3D printing.

## REFERENCES

- [1] Ozkan Ocalan, Oscillation of Neutral Differential Equation with positive and Negative coefficients, *J. Math. Anal. Appl.* 331(1) (2007), 644-654..
- [2] Dallmann CJ, Dickerson BH, Simpson JH, Wyart C, Jayaram K. Integr. MechanoSensory Control of Locomotion in Animals and Robots: Moving Forward. *Integr Comp Biol.* 2023 Aug 23;63(2):450-463. doi: 10.1093
- [3] Long Li, Zhongqu Xi, Xiang Luo, Trajectory Planning of Flexible Walking for Biped Robots Using Linear Inverted Pendulum Model and Linear Pendulum Model, *Sensors*, 2021, 21(4), 1082.
- [4] R.P. Agarwal and P.J.Y. Wong, *Advanced Topics in Difference Equations*, Kluwer, Dordrecht, 1997.
- [5] John R. Graef, Oscillation of First order Neutral Delay Difference Equations, *Electric Journal of Qualitative of differential equations*, *proc. 7th coll. QTDE, NO.* (2004) 12, 1-11.
- [6] Joris Verhagen, Xiaobin Xiong, Aaron D. Ames, and Ajay Seth, From Human Walking to Bipedal Robot Locomotion: Reflex Inspired Compensation on Planned and Unplanned Downsteps, *arXiv:2209.02995v1 [cs.RO]* 7 sep 2022.
- [7] Jörn Malzahn, Navvab Kashiri, Monica A. Daley and Nikos Tsagarakis, Advances in Mechatronics and Biomechanics Towards Efficient Robot Actuation, *Frontiers in Robotics and AI*, 2019 Mar 26. doi: 10.3389/frobt.2019.00019,
- [8] S.R. Grace and B.S. Lalli, Oscillation theorems for second order delay and neutral difference equations, *Utilitas Math.* 45 (1994), 197-211.
- [9] Tanaka. S, Oscillation of solution of First order Neutral Delay Differential Equations, *Hiloshima Mathematical Journal*, (2002) 32, 73-85.