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Research Article

Tightened–Normal–Tightened Systems Utilizing Truncated Poisson Distribution for Risk Minimization

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ABSTRACT

Introduction: The goal of this study is to improve quality control in contemporary manufacturing processes by implementing Tightened–Normal–Tightened (TNT) methods, which are intended to reduce the combined risks for consumers and producers. The Truncated Poisson distribution is used to pick a TNT system with the lowest total of hazards based on defined Acceptable Quality Levels (AQL) and Limiting Quality Levels (LQL). A comprehensive table and technique are provided. Highlighted are the advantages of using this risk-minimized TNT system, proving its superiority over current TNT system selection techniques. Comparative analysis highlights this method's effectiveness and usefulness, making it a useful instrument for modern production settings.

Objectives: The goal of this research is to create and apply a risk-minimized Tightened–Normal–Tightened (TNT) system in order to improve quality control in contemporary manufacturing processes. The study intends to minimize the combined risks for producers and consumers by optimizing the selection of TNT systems based on Acceptable Quality Levels (AQL) and Limiting Quality Levels (LQL) by utilizing the Truncated Poisson distribution.

Methods: In order to accurately represent production scenarios with minimal defect levels, the Truncated Poisson distribution was used to produce the Operating Characteristic (OC) function for the two-plan TNT system, which was drawn from the works of Dodge (1965) and Hald & Thyregod (1965). This function was used to create a TNT system that balances Limiting Quality Levels (LQL) and Acceptable Quality Levels (AQL) in order to reduce total risks. The performance of the suggested system in terms of risk reduction, inspection effectiveness, and fit for contemporary production needs was assessed by contrasting it with the plans that Govindaraju and Subramani (1992b) had already established.

Results: The study effectively illustrates how to apply an optimal Tightened–Normal–Tightened (TNT) approach to reduce quality control risks in a variety of manufacturing scenarios. In comparison to conventional approaches, the suggested methodology achieves better results by utilizing the Truncated Poisson distribution to guarantee efficient risk management for both producers and customers. The results highlight how well the Truncated Poisson distribution works to provide a dependable, risk-reduction framework for quality control that meets the demands of contemporary production.

Conclusions: This study illustrates how the Truncated Poisson distribution-optimized Tightened–Normal–Tightened (TNT) system effectively reduces risks in modern manufacturing processes. The suggested approach outperforms current selection methods in lowering producer and customer risks by choosing TNT systems according to specified Acceptable Quality Levels (AQL) and Limiting Quality Levels (LQL). The approach and table shown here provide useful tools for putting risk-minimized TNT systems into practice, demonstrating their enormous potential to improve quality control and guarantee more dependable production results in contemporary manufacturing settings.
Keywords: Tightened–Normal–Tightened System, Truncated Poisson Distribution, Acceptable Quality Level (AQL), Limiting Quality Level (LQL), Producer's Risk, Consumer's Risk, Modern Production Processes.

INTRODUCTION

A crucial component of contemporary production processes, quality control ensures that goods fulfil predetermined criteria while lowering risks for both manufacturers and customers. The Tightened–Normal–Tightened (TNT) methodology has become well-known among quality control techniques because of its methodical approach to handling different inspection levels. By adjusting inspection criteria according to observed performance, this system balances efficiency and risk and adjusts to production quality.

Choosing the best TNT system is essential to getting the quality results. Minimizing the combined risks that producers and consumers face—often referred to as producer's risk and consumer's risk—is a major problem in this process.

Traditionally, elements like the Acceptable Quality Level (AQL) and Limiting Quality Level (LQL) have an impact on these hazards. Although current approaches offer recommendations for choosing a TNT system, they frequently miss chances to reduce the total of these hazards.

This work presents a methodical technique for choosing TNT systems that use the Truncated Poisson distribution to reduce the overall risk. The suggested approach offers a thorough framework for improving quality control choices by integrating AQL and LQL specifications. Furthermore, the benefits of this strategy are illustrated by contrasts with conventional TNT selection techniques, underscoring its usefulness in modern production environments.

REVIEW OF LITERATURE

In industrial and production contexts, the Tightened–Normal–Tightened (TNT) system has been thoroughly investigated as a dynamic quality control tool. Dodge (1943) laid the foundations for the creation of flexible sample programs such as the TNT system by introducing the fundamental ideas of sequential inspection systems. Over time, these systems have been improved to meet changing production demands and strike a balance between quality control and inspection expenses.

The development of TNT systems was greatly aided by Govindaraju and Subramani (1992b), who offered techniques for their design and assessment in quality control procedures. Through customized sample programs that adhere to predetermined Acceptable Quality Levels (AQL) and Limiting Quality Levels (LQL), their work highlighted the significance of reducing producer and consumer risks. Their study showed how the Poisson distribution and other statistical distributions can be used to optimize the choice of TNT systems in production settings.

Schilling and Neubauer (2009) investigated the use of AQL and LQL in quality control, highlighting their importance in reducing risks for both producers and consumers. Their research made clear how crucial it is to define ideal inspection parameters by taking statistical distributions like the Poisson distribution into account. In contrast to traditional distributions, the Truncated Poisson distribution provides a more accurate depiction of production quality and has since become a useful tool for modelling scenarios with restricted defect levels.

Montgomery (2020) has talked about improvements in statistical quality control, especially in light of contemporary production settings where efficiency and unpredictability are crucial. In order to better match quality control systems with modern production constraints, his work focused on integrating risk minimization measures.

Innovative approaches for TNT system selection that concentrate on lowering total risk have been put forth in recent studies, such as those by Pignatiello et al. (2018) and Chakraborty and Biswas (2021). However, there is potential for improvement in real-world applications because these methods frequently lack a thorough framework for taking into account both producer and consumer risks at the same time.

The necessity of a systematic process to choose TNT systems based on the Truncated Poisson distribution is highlighted by this review. The necessity of a systematic approach to TNT system selection is shown by the work of Govindaraju and Subramani (1992b) and later studies. This study expands on their work by utilizing the Truncated Poisson distribution to offer a solid methodology for reducing risks and improving quality control in contemporary production processes.

CONSTRUCTION OF TABLE 1

The OC function for the two-plan (TNT) system obtained by Dodge (1965) and Hald & Thyregod (1965) is

$$P_a(p) = \frac{\mu P_N + \xi P_T}{\mu + \xi} \tag{3.1}$$

Where,

$$\mu = \frac{2-P_N^s}{(1-P_N)(1-P_N^s)} \quad s \geq 1$$

$$\xi = \frac{1-P_T^t}{P_T^t(1-P_T)}$$

with P_T the probability of acceptance under tightened inspection and P_N the probability of acceptance under normal inspection. Whereas, $c_N > c_T$, the expression for sum of producer's and consumer's risks is given by

$$\alpha+\beta=1-P_a(p_1)+P_a(p_2) \quad (3.2)$$

If the operating ratio p_2/p_1 and np_1 are known, then np_2 can be written as

$$np_2=(p_2/p_1)(np_1). \quad (3.3)$$

For fixed np_1, the value of np_2 is calculated from the equation (3.3) and is used in equation (3.2). The parameters c_T, c_N, t and s corresponding to the minimum 1- $Pa(p_1)$ + $Pa(p_2)$ are obtained by searching for c_T = 1(1)19, c_N = 1(1)20, t = 1(1)8 and s = 1(1)7 for the fixed value of operating ratio and np_1 values. The producer's and consumer's risks are then obtained which correspond to the c_T, c_N, t and s values for which the sum of risks is minimum.

METHODOLOGY FOR IDENTIFYING THE OPTIMAL MINIMUM RISK TNT (N, CT, CN) SYSTEM

Table 1, which gives system parameters for certain values of p_1 and p_2, can be used to determine the best Tightened–Normal–Tightened (TNT) system with the least amount of risk while guaranteeing that the risks to the producer and the customer are kept to a maximum of 10%. The table shows the corresponding parameters c_T, c_N, t, s, and associated hazards based on the product of sample size and np_1, and is arranged according to the operating ratio p_2/p_1. The steps that make up the process are as follows:

1. Figure out the operating ratio (p_2/p_1).
2. Using the calculated value of p_2/p_1, insert the row in table 1 where the operating ratio value in the left-hand column is equal to or slightly less than the calculated ratio.
3. To find the TNT system's parameters, move from left to right in the row found in the previous step until the tabulated risks for producers and consumers are either the same as or slightly less than the required values.
4. Using the formula $n = np_1/p_1$, the sample size n is determined. The np_1 values are provided as the column headings that correspond to the parameters c_T, c_N, t, and s that were determined in step 3.

Industry Context:

To guarantee that the goods fulfill the necessary requirements while lowering risks for both producers and consumers, a manufacturing facility that produces electronic components must put in place an efficient quality control system. To keep defect rates within reasonable bounds, the manufacturing team use a TNT (Tightened–Normal–Tightened) approach based on statistical techniques to regulate the quality of component batches. The plant chooses the best TNT system for inspection by evaluating production quality risks using the Truncated Poisson distribution.

- Acceptable Quality Level (p_1) = 0.02 (2% defective rate is acceptable for the product)
- Limiting Quality Level (p_2) = 0.05 (5% defective rate is the threshold beyond which products are unacceptable)
- Producer's risk (α) = 0.04 (4% chance that a good batch will be rejected)
- Consumer's risk (β) = 0.04 (4% chance that a bad batch will be accepted)
- TNT system using Truncated Poisson distribution we get,

1. $p_2/p_1 = 2.5$
2. $p_2/p_1 = 2.5$ is tabulated
3. Corresponding to $c_T = 2$, $c_N = 14$, $t = 8$ and $s = 6$, are given in the body of the table 1. We obtain $\alpha = 0$ and $\beta = 0$ against the desired $\alpha = 0.04$ and $\beta = 0.04$
4. $n = np_1/p_1 = 1/0.02 = 50$

In order to minimize the risks of both accepting faulty batches and rejecting excellent batches, the best TNT system for the production process of electronic components has been chosen. In order to ensure that the defective rates stay within the allowed range of 2% and to reduce the likelihood of rejecting excellent batches or accepting poor ones, the company can now go forward with quality control by evaluating a sample size of 50 units per batch. In accordance with the company's standards, this procedure enhances operational effectiveness and product quality assurance.

COMPARING POISSON AND TRUNCATED POISSON DISTRIBUTIONS FOR TNT SYSTEM PARAMETERS IN AUTOMOBILE MANUFACTURING:

To guarantee that every component satisfies strict safety and performance requirements, quality control is essential in the automotive manufacturing sector. To reduce the possibility of flaws that could jeopardize the performance and safety of vehicles, careful inspection is necessary throughout the production of automotive elements such engine parts, suspension systems, and safety features.

Think about a situation where a manufacturing facility manufactures engine parts and must guarantee that the parts fulfill the necessary standards while lowering the risks to the manufacturer and the consumer. The components are inspected by the quality control team using the Tightened–Normal–Tightened (TNT) approach, which is based on statistical techniques.

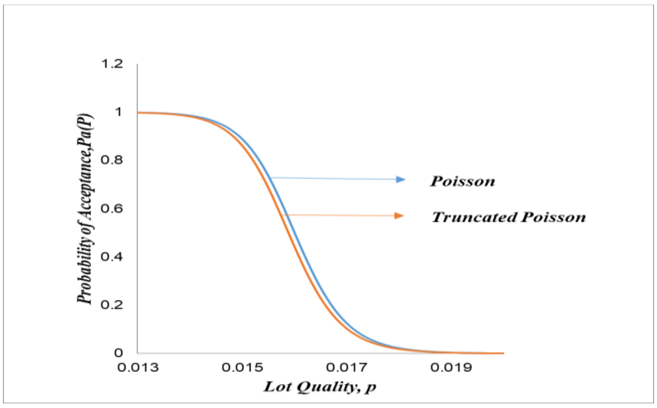
If one fixes $p_1 = 0.02$, $p_2 = 0.058$ with $\alpha = 0.02$ and $\beta = 0.10$ one gets the following plan using the table given in K.Govindaraju and K.Subramani (1992 b) for which the tabulated operating ratio 2.9 with $n = 500$, $c_T = 8$, $c_N = 20$, $t = 1$ and $s = 1$ with $\alpha = 0.002$ and $\beta = 0$, $\alpha+\beta = 0.002$. Under the same condition for TNT (n, c_T, c_N) system using Truncated Poisson distribution we get $n = 500$, $c_T = 9$, $c_N = 21$, $t = 1$ and $s = 1$ with $\alpha = 0$ and $\beta = 0$, $\alpha+\beta = 0$.

Making sure engine parts fulfill safety and performance requirements is crucial in the context of auto manufacturing. The comparison shows that the Truncated Poisson distribution gives a considerable advantage in terms of risk minimization, even if both the Poisson and Truncated Poisson distributions offer comparable sample sizes (500 parts) and inspection parameters. The Truncated Poisson method provides a more dependable quality control system for the production facility by lowering both the producer's and consumer's risk to zero.

Table-2 Comparison of TNT systems

Given Values				Poisson Distribution K.Govindaraju and K.Subramani (1992b)						Truncated Poisson distribution					
p_1	p_2	α	β	c_T	c_N	t	s	α	β	c_T	c_N	t	s	α	β
0.03	0.063	0.002	0.005	4	20	1	1	0.002	0.00	5	21	1	1	0.00	0.00
0.02	0.028	0.005	0.004	4	20	2	1	0.005	0.003	5	21	2	1	0.00	0.00
0.02	0.03	0.001	0.001	3	20	2	1	0.001	0.001	5	21	2	1	0.00	0.00
0.01	0.019	0.001	0.002	2	20	1	2	0.001	0.000	4	21	1	1	0.00	0.00

Figure 1 Operating Characteristic Curves of TNT System



CONCLUSION

This study illustrates how the Truncated Poisson distribution-optimized Tightened–Normal–Tightened (TNT) system effectively reduces risks in modern manufacturing processes. The suggested approach outperforms current selection methods in lowering producer and customer risks by choosing TNT systems according to specified Acceptable Quality Levels (AQL) and Limiting Quality Levels (LQL). The approach and table shown here provide useful tools for putting risk-minimized TNT systems into practice, demonstrating their enormous potential to improve quality control and guarantee more dependable production results in contemporary manufacturing settings.

Table – 1.1 Parameters of the TNT – (n; c_T, c_N) system for given p_2/p_1 and np_1

p_2/p_1	np_1									
	0.10	**0.15**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.75**	**1.00**	**1.50**
45.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.7	1 , 17 8 , 7 0 , 0.1	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 12 8 , 7 0 , 0	2 , 16 8 , 6 0 , 0
40.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.5	1 , 17 8 , 7 0 , 0.2	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 2 8 , 7 0 , 0	2, 16 8 , 7 0 , 0
35.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.6	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 2 8 , 7 0 , 0	2 , 16 8 , 7 0 , 0
30.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.5	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 2 8 , 7 0 , 0.4	2, 16 8 , 7 0 , 0
27.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.1	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 2 8 , 7 0 , 1	2, 16 8 , 7 0 , 0
25.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 16 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 2 8 , 7 0 , 3	2, 16 8 , 2 0 , 0
22.0		1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.1	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 3 8 , 7 0 , 7	2, 16 8 , 7 0 , 0.5
20.0		1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.2	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 3 8 , 7 0 , 0	2, 16 8 , 6 0 , 2
17.0		1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.7	1 , 17 8 , 7 0 , 0.6	1 , 17 8 , 7 0 , 0.1	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	2, 16 8 , 7 0 , 7
15.0			1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0.5	1 , 17 8 , 7 0 , 0.4	1 , 17 8 , 7 0 , 0	1 , 15 8 , 7 0 , 0	1 , 11 8 , 6 0 , 0
12.	Key c_T, c_N; t , s; α%, β%		1 , 11 8 , 7 0, 0.5	1 , 12 8 , 7 0 , 0.1	1 , 13 8 , 7 0, 0	1 , 14 8 , 7 0 , 0.2	1 , 14 8 , 7 0 , 1	1 , 16 8 , 7 0 , 0.1	2 , 14 8 , 6 0 , 0	1 , 11 8 , 6 0 , 0
10.0				1 , 12 8 , 7 0 , 0	1 , 13 8 , 7 0 , 0	1 , 14 8 , 7 0 , 0	1 , 14 8 , 7 0 , 0.3	1 , 16 8 , 7 0 , 0.4	2 , 14 8 , 6 0 , 0	1 , 13 8 , 6 0 , 0

Risk less than 0.1% is tabulated as 0%

Table 1 Continued

p_2/p_1	np_1									
	2.0	**2.5**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**
45.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0

40.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
35.0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
30.0	5 , 25 8 , 4 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
27.0	5 , 25 8 , 4 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
25.0	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	6 , 27 8 , 3 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
22.0	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	6 , 27 8 , 3 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
20.0	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	6 , 27 8 , 3 0 , 0	2 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
17.0	6 , 27 8 , 3 0 , 0	6 , 27 8 , 3 0 , 0	6 , 27 8 , 3 0 , 0	2 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
15.0	5 , 22 8 , 4 0 , 0	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
12.0	5 , 22 8 , 4 0 , 0.5	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
10.0	5 , 22 8 , 4 0 , 0.4	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 14 8 , 6 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0

Risk less than 0.1% is tabulated as 0%

Table 1 Continued

p_2/p_1	np_1									
	2.0	2.5	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
15.0	5 , 22 8 , 4 0 , 0	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0

12.0	5 , 22 8 , 4 0 , 0.5	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
10.0	5 , 22 8 , 4 0 , 0.4	5 , 25 8 , 4 0 , 0	6 , 27 8 , 3 0 , 0	2 , 14 8 , 6 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
9.0	5 , 25 8 , 4 0 , 1	5 , 25 8 , 4 0 , 0.1	6 , 27 8 , 3 0 , 0	1 , 17 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0
8.0	5 , 25 8 , 4 0 , 4	5 , 25 8 , 4 0 , 0.4	6 , 27 8 , 3 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 17 8 , 7 0 , 0
7.0	5 , 25 8 , 4 0 , 6	5 , 25 8 , 4 0 , 2	6 , 27 8 , 3 0 , 0.2	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0
6.0		5 , 25 8 , 4 0 , 6	6 , 27 8 , 3 0 , 1	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0
5.0		5 , 25 8 , 4 0 , 2	6 , 27 8 , 3 0 , 6	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0
4.0				1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	1 , 17 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0	2 , 18 8 , 7 0 , 0	2 , 18 8 , 7 0 , 0	6 , 14 8 , 2 0 , 0
2.90	Key c_T, c_N ; t , s; α%, β%			1 , 21 5 , 7 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 1 , 6 0 , 0	2 , 21 1 , 2 0 , 0	4 , 21 1 , 1 0 , 0	6 , 21 1 , 1 0 , 0	9 , 21 1 , 1 0 , 0
2.80				1 , 21 5 , 3 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 1 0 , 0	3 , 21 1 , 1 0 , 0	6 , 21 1 , 1 0 , 0	8 , 21 1 , 1 0 , 0
2.70				1 , 21 5 , 7 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 1 0 , 0	3 , 21 1 , 1 0 , 0	5 , 21 1 , 1 0 , 0	8 , 21 1 , 1 0 , 0

Risk less than 0.1% is tabulated as 0%

Table 1Continued

p_2/p_1	np_1										
	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
2.60	2 , 14 8 , 6 0 , 0	2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 21 8 , 7 0 , 0	1 , 21 8 , 7 0 , 0	1 , 20 4 , 5 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 1 0 , 0	3 , 21 1 , 1 0 , 0	5 , 21 1 , 1 0 , 0	7 , 21 1 , 1 0 , 0

2.50	2 , 14 8 , 6 0 , 0	2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 21 8 , 2 0 , 0	1 , 16 8 , 5 0 , 0	1 , 21 5 , 3 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 1 0 , 0	3 , 21 1 , 1 0 , 0	5 , 21 1 , 1 0 , 0	7 , 21 1 , 1 0 , 0
2.40	2 , 14 8 , 6 0 , 0	2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 19 8 , 7 0 , 0	1 , 21 8 , 6 0 , 0	2 , 21 1 , 1 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 2 0 , 0	2 , 21 1 , 1 0 , 0	4 , 21 1 , 1 0 , 0	7 , 21 1 , 1 0 , 0
2.30	2 , 14 8 , 6 0 , 0	2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 18 8 , 7 0 , 0	1 , 21 8 , 7 0 , 0	2 , 21 1 , 1 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 4 0 , 0	2 , 21 1 , 1 0 , 0	4 , 21 1 , 1 0 , 0	6 , 21 1 , 1 0 , 0
2.20		2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 18 8 , 7 0 , 0	1 , 21 8 , 2 0 , 0	2 , 21 1 , 1 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 1 , 7 0 , 0	2 , 21 1 , 1 0 , 0	4 , 21 1 , 1 0 , 0	6 , 21 1 , 1 0 , 0
2.10		2 , 14 8 , 6 0 , 0	4 , 12 8 , 4 0 , 0	1 , 17 8 , 7 0 , 0	1 , 21 8 , 5 0 , 0	2 , 21 1 , 3 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 2 , 1 0 , 0	2 , 21 1 , 3 0 , 0	3 , 21 1 , 1 0 , 0	5 , 21 1 , 1 0 , 0
2.00		1 , 10 8 , 3 2 ,2	1 , 15 8 , 6 0 , 0.8	1 , 17 8 , 7 0 , 0	1 , 20 8 , 6 0 , 0	1 , 21 5 , 7 0 , 0	1 , 21 3 , 2 0 , 0	1 , 21 2 , 1 0 , 0	2 , 21 2 , 1 0 , 0	1 , 21 1 , 1 0 , 0	3 , 21 1 , 1 0 , 0	5 , 21 1 , 1 0 , 0
1.90		1 , 10 8 , 7 2 ,2	1 , 13 8 , 2 0 , 1	1 , 17 8 , 7 0 , 0	1 , 19 8 , 2 0 , 0	1 , 21 5 , 1 0 , 0	1 , 21 3 , 6 0 , 0	1 , 21 2 , 3 0 , 0	2 , 21 2 , 1 0 , 0	1 , 21 1 , 1 0 , 0	3 , 21 1 , 2 0 , 0	4 , 21 1 , 1 0 , 0
1.80		1 , 10 8 , 7 2 ,3	1 , 13 8 , 7 0 , 1	1 , 16 8 , 4 0 , 0.5	1 , 19 8 , 5 0 , 0	1 , 21 5 , 1 0 , 0	1 , 21 3 , 7 0 , 0	1 , 21 2 , 7 0 , 0	2 , 21 2 , 1 0 , 0	3 , 21 2 , 1 0 , 0	2 , 21 1 , 1 0 , 0	4 , 21 1 , 1 0 , 0
1.70		1 , 9 8 , 2 2 ,4	1 , 12 8 , 1 1 , 1	1 , 16 8 , 7 0 , 1	1 , 19 8 , 5 0 ,0.6	1 , 21 6 , 2 0 , 0	1 , 21 4 , 1 0 , 0	1 , 21 2 , 4 0 , 0	2 , 21 2 , 1 0 , 0	3 , 21 2 , 1 0 , 0	2 , 21 1 , 2 0 , 0	3 , 21 1 , 1 0 , 0
1.60		1 , 9 8 , 6 2 ,5	1 , 12 8 , 5 1 , 2	1 , 15 8 , 4 0.6 , 1	1 , 18 8 , 3 0 , 1	1 , 21 6 , 4 0 , 0	1 , 21 4 , 1 0 , 0	1 , 21 3 , 1 0 , 0	2 , 21 2 , 1 0 , 0	3 , 21 2 , 2 0 , 0	4 , 21 2 , 1 0 , 0	3 , 21 1 , 1 0 , 0
1.50	Key c_T, c_N ; t , s; α%, β%		1 , 11 8 , 3 1 , 3	1 , 14 8 , 1 1 , 1	1 , 17 8 , 1 0 , 1	1 , 21 7 , 1 0 , 0	1 , 21 4 , 4 0 , 0	1 , 21 3 , 1 0 , 0	2 , 21 2 , 1 0 , 0	2 , 21 2 , 1 0 , 0	4 , 21 2 , 1 0 , 0	5 , 21 2 , 1 0 , 0
1.40			1 , 11 8 , 3 2 , 3	1 , 14 8 , 5 1 , 2	1 , 16 8 , 7 0 , 1	1 , 21 7 , 3 0 , 0	1 , 19 4 , 3 0 , 0	1 , 21 3 , 2 0 , 0	2 , 21 3 , 1 0 , 0	2 , 21 2 , 1 0 , 0	4 , 21 2 , 4 0 , 0	5 , 21 2 , 1 0 , 0

Risk less than 0.1% is tabulated as 0%

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