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**Research Article**

# Mathematical Models for Calculating Structures Under a Combination of Loads

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**ABSTRACT**

The chosen direction of work, the loads acting on the structure, including some long-term ones, are of a random temporary nature. An unfavorable combination of several random loads over time has a low probability. The work of combinations of snow loads is devoted to solving the problem of combining loads. The combination coefficients largely determine the magnitude of the loads on the structure, and therefore its parameters, material consumption and cost on the one hand; on the other hand, the correct assignment of these coefficients ensures the reliability of the structures. The insufficiency of statistical data on loads and the imperfection of theoretical load models forces designers to create structures with a certain margin to compensate for these shortcomings, which leads to waste of materials. In the design standards in force in our country, when calculating structures and foundations for the action of two or more temporary loads, combination coefficients (...) 1 are introduced, which multiply the design values of each of the existing loads.

**Keywords:** Beam, support moments, load, moment of inertia, recurrence relation, conditional probability.

## INTRODUCTION

At present, when Uzbekistan is tasked with creating cost-effective structures of optimal reliability, the theoretical justification of building codes and regulations is acquiring important national economic significance.

The chosen direction of work seems relevant both for engineering practice and for the development of modern statistical methods of structural mechanics.

Using the methods of the theory of extreme order statistics, models of snow loads have been developed. The theoretically based choice of models made it possible to obtain models in a form quite convenient for subsequent use and with their help to solve the problem of combining 2, 3, 4 and (theoretically) more loads. Problems in such a formulation are considered for the first time, which determines the novelty of the main content of the work. The significance of the work lies in the fact that the proposed probabilistic calculation method makes it possible to take into account the random nature of the loads acting on the structure, snow loads, based on the principle of equireliability.

Standard snow load values are obtained based on the weight of the snow cover on 1 horizontal surface of the earth, taking into account the cover profile and the snow load pattern on it. The weight of the snow cover is determined as a result of snow surveys systematically carried out at weather stations.

During the survey, among a number of characteristics of the snow cover, the height and density of the snow are determined, which, in turn, serve as the basis for calculating the water reserve in the snow. The water reserve in snow, expressed in mm, corresponds to the weight of the snow cover in kg/or snow load [1].

The results of observations of snow cover, including the amount of water in the snow on various dates, are published in the monthly meteorological departments of the hydrometeorological service.

Calculations of probabilistic values of snow cover weight were carried out using a statistical series consisting of annual maximums of water reserves in snow with a repetition period of 2, 5, 10 and 20 years. Based on these calculations, meteorologists carried out zoning of the territory of the USSR according to the weight of snow cover, possible once every 5 years. [1,2].

## MATERIALS AND METHODS

Research into the patterns of snow removal from pavements under the influence of wind, carried out at TsNIISK by V.A. Otstavnov and L.S. Rozenberg [3,4,5,6,7], made it possible to take into account in the standards the reduction in the load from snow on a significant part of the types of pavements.

Snow load is a temporary load on structures, the nature of which varies from year to year according to a random law. Observations of snow load received during the winter period with a time interval τ are shown on the graph (Fig. 1).

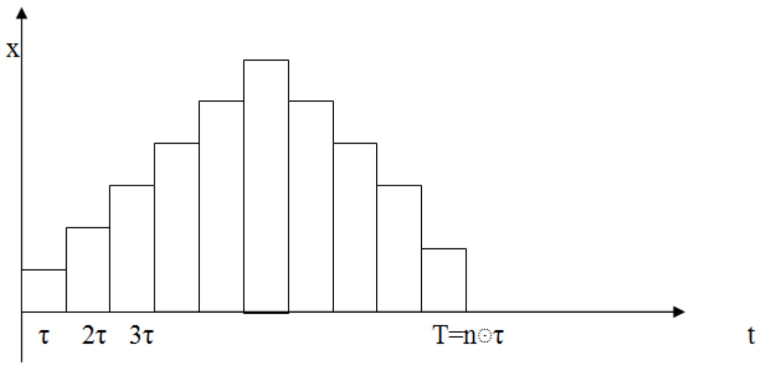


**FIGURE 1.**

If the snow load is the only load acting on the structure, then we are only interested in the distribution law for the maximum value of the snow load during the winter. This distribution is usually assumed to be double exponential.

$$F_0(x) = H_{3,0}\left(\frac{x-a}{b}\right) = \exp\left(-\exp\left(-\frac{x-a}{b}\right)\right) \qquad (1)$$

The notation is adopted in (22). $H_{3,0}(x) = \exp(-e^{-x})$ The numerical values of parameters $a$ and $b$ for Moscow are given in [4]. $a = 931\,\frac{н}{м^2}$, $b = 365\,\frac{н}{м^2}$

If the structure is subject to loads other than snow, then we need to create a model of the snow load as a function of time to calculate the structure for all existing loads. In the snow load models known in the literature, the latter is represented as a non-stationary random process. Such a representation greatly complicates the subsequent probabilistic calculation of structures under the influence of several random loads. The thesis proposes a model in the form of a set of independent identically distributed random variables. In this case, the actually observed snow load values are represented by the order statistics of this population [8,9,10,11,12,13].

Let us show the possibility of representing the snow load for the winter period in the form of a sample volume $X_1, X_2, \ldots, X_n$ $n$ where $X_i$ independent random variables with the same distribution law. The time interval τ corresponding to each random variable will be clarified later. The random process of snow load X(t) is represented as a step function:

$$X(t) = X_i\,, \quad t \in [(i-1)\tau : i\tau)], \quad i = 1, 2, \quad n \qquad (2)$$

Next, we make the following assumptions, which do not contradict common sense:

1. There are $n$ independent and identically distributed random variables $X_{n:n}$ with a distribution law $F(x)$ such that their order statistics $X_{1:n}, X_{2:n}, \dots, X_{n:n}$ are observable snow accumulation processes.

2. Random values of load increment $U_2 = X_{2:n} - X_{1:n}$, ... $U_n = X_{n:n} - X_{n-1:n}$ independent.

From assumption 2 it follows that the law of distribution of random variables $X_1, X_2, \dots, X_n$ is exponential, i.e. $F(x) = 1 - e^{-\lambda x}$. Evidence of this fact was given by J. Galambos.

Based on this theorem, it is easy to prove that for a sequence of independent random variables $X_1, X_2, \dots, X_n$ with a distribution function of $F(x) = 1 - e^{-x}$. limit distribution law is double exponential, i.e.:

$$\lim_{n\to\infty} P\{Z_n < a_n + b_n y\} = \exp(-e^{-y}), \qquad (3)$$

$a_n$, $b_n$ - normalizing constants, equal

$$. \ a_n = \ln n \ u \ b_n = 1, \qquad (4)$$

If

$$F(x) = 1 - e^{-\lambda x}, \text{ то}$$

$$a_n = \frac{\ln n}{\lambda}, \ b_n = \frac{1}{\lambda} \qquad (1.15)$$

Next we will do this. Size

$b = b_n$ we consider known $b = 365\frac{H}{M^2}$. The value $\lambda$ is obtained from the relation $\lambda = \frac{1}{b} = 0{,}00274$.

Let's adjust the value $a = a_n$ a little compared to (3) so that the number $n = \exp(\frac{a}{b})$ is equal to an integer.

We get $n = 13$, $a = 936{,}113\frac{H}{M^2}$.

For the obtained value, it is quite plausible to assign a value equal to 10 days to the time interval, and a value equal to 130 days to the duration of winter.

To substantiate our model, we can give the following reasoning. Let $X_1, X_2, \dots, X_n$ be an independently identical distribution of a quantity with a distribution function $F(x)$.

Let $X_{1:n}, X_{2:n}, \dots, X_{n:n}$ be the order statistics corresponding to the random variables $X_j$ $(1 \le j \le n)$. Let's put $U_1 = X_{1:n} U_2 = X_{2:n} - X_{1:n}$, ... $U_n = X_{j:n} - X_{j-1:n}$, $j \ge 2$.

For an exponential distribution with a distribution function of, it is easily shown that the random variables are independent and have an exponential distribution. It turns out that the converse of Theorem 2 holds: Let there be independent variables with a common continuous distribution function of. Let us assume that the random variables are independent. Then:

$$F(x)=1-e^{-\lambda x};\quad X_1, X_2, \dots, X_n;\quad F(x);\quad U_j\ (j\geq 1);\quad F(x)=1-e^{-\lambda x}.$$

Now a model is proposed that necessarily follows if we accept the assumption of independence of snow load increments over a time interval of $\tau = 10$ days.

To compare the value of the probability of the snow load not exceeding level x for k = 5, 10 and 20 years according to formula (1) and formula

$$\left[\left[1-e^{-\lambda x}\right]^{kn}\right]$$

A random implementation of the snow load process according to the proposed model is shown in Fig. 2.

Next, we will compare the developed model with the Poisson process model proposed by E.I. Fedorov, as well as with its possible generalization in the form of a process with random increments.

**TABLE 1.**

| | X | $\left[\left[1-e^{-\lambda x}\right]^{kn}\right]$ | $\exp\left[-k\exp\left(-\frac{x-a}{b}\right)\right]$ |
|---|---|---|---|
| | 2000 | 0,7625513 | 0,7621770 |
| | 2500 | 0,9334255 | 0,9334140 |
| | 3000 | 0,9826434 | 0,9826478 |
| | 3500 | 0,9955599 | 0,9955574 |
| | 2100 | 0,5814844 | 0,5809137 |
| | 2500 | 0,8712831 | 0,8712616 |
| 0 | 3000 | 0,9665862 | 0,9655967 |
| | 3500 | 0,9911400 | 0,9911345 |
| | 2000 | 0,3381242 | 0,3374608 |
| | 2500 | 0,7591343 | 0,7590967 |
| 0 | 3000 | 0,9323607 | 0,9323770 |
| | 3300 | 0,9823585 | 0,9823476 |

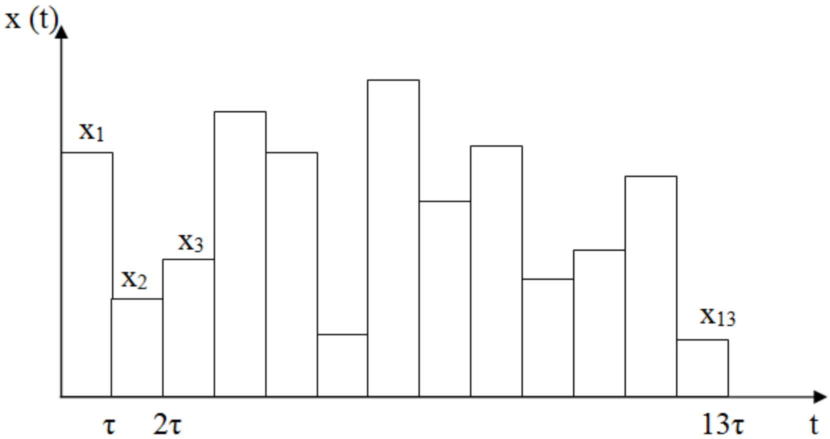


**FIGURE 2.** Sequence n.o.r. random variables.

In (..) the Poisson process was adopted as a snow load model. Let $Y(t)$ be the snow load. At $t=0$, $Y(0)=0$. On the segment [0, $\tau$ ] where T is the duration of winter, at $v_n$ time points process $Y(t)$ makes a jump by a certain constant value c. Discrete random variable $n$ distributed according to Poisson's law,

$$P_k(T) = e^{-\bar{n}} \cdot \frac{\bar{n}^k}{k!} , \qquad (6)$$

Where

$\bar{n}$ - mathematical expectation of value $n$ on a segment [0, $\tau$ ].

Moments in time at which process jumps occur - $T_1, T_2, \dots, T_n$

As is known, random variables

$T_1, T_2, \dots, T_n$

Distributed as a set of order statistics of volume n taken from a uniform distribution on the segment [0, $\tau$ ].

Random variables $\tau_1 = T_1$ , $\tau_2 = T_2 - T_1$ , ... $\tau_n = T_n - T_{n-1}$ are distributed uniformly in the region

$$\tau_i \geq 0 , \ i = 1, 2 , \dots , n , \ \sum_{i=1}^{n} \tau_i \leq T , \qquad (7)$$

Parameters $\bar{n}$ and c were selected based on the following considerations. Random value of maximum snow load

$$Z_1 = \max_{t \in [0,T]} E(\tau)$$

Has a mathematical expectation and variance equal to:

$$E(Z_i) = c \cdot \bar{n} , \quad D(Z_i) = c^2 \cdot \bar{n} , \qquad (8)$$

As is known [5], as the law of distribution of the random variable of the maximum winter snow load, we can take the double exponential

$$F_0(x) = \exp\left(-\exp\left(-\frac{x-a}{b}\right)\right) \qquad (9)$$

Numerical parameters a and b for Moscow are given in (62):

$$a = 931 \frac{\text{н}}{\text{м}^2} , \quad b = 365 \frac{\text{н}}{\text{м}^2} \qquad (10)$$

The mathematical expectation and variance of a random variable with distribution function (9) are equal to:

$$E(Z_2) = a + \gamma b , \quad D(Z_2) = \frac{b^2 \cdot \pi^2}{6} , \qquad (11)$$

Where: $-\gamma = \Gamma'(1) = -0,57721$ is the value of the first derivative of the gamma function $F(x)$ at the point

есть значение первой производной гамма-функция $x = 1$ .

If we equate

$$E[Z_1] = E(Z_2], \quad D[Z_1] = D(Z_2]$$

then you can find the parameters with and $\bar{n}$.

$$c = \frac{b^2 \cdot \pi^2}{6(a + b\gamma)}, \qquad (12)$$

$$\bar{n} = \frac{6(a + b\gamma)^2 b^2 \cdot \pi^2}{b^2 \cdot \pi^2}$$

For a and b, in accordance with (10), we obtain

$$c = 191{,}95 \ , \quad \bar{n} = 5{,}95 \approx 6 \ , \qquad (13)$$

Figure 3 shows the trajectory of the Poisson process. The moments in time are random.

$T_1, T_2, \ldots, T_n$

We emphasize that any load model requires that the distribution of the random variable of the maximum load on a certain time interval [0,T] coincide with the distribution of the maximum obtained as a result of processing the observed sample of maximum values. Fulfillment of this requirement is necessary, if only because at small values of all other loads, the reliability of the structure is determined by the extreme values of the load in question.

## RESULTS AND DISCUSSION

The disadvantage of the proposed model is the constancy of the snow load increment and, as a consequence, the discreteness of the random value of the maximum snow load.

A natural generalization of the process model is a model with random increments of snow load at the moment of the jump.

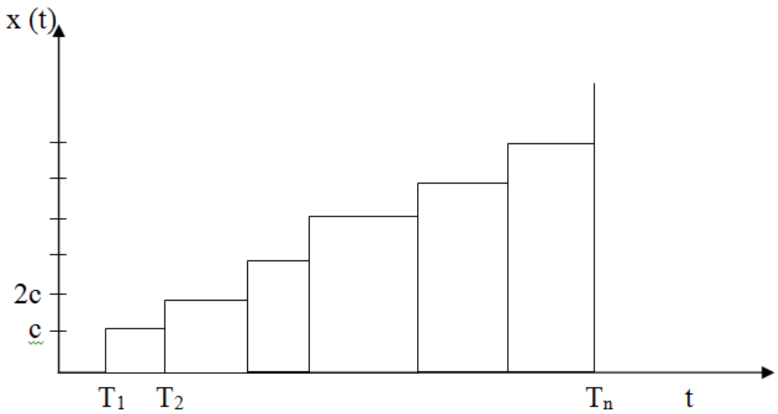


**FIGURE 3.** Poisson process model.

We will assume that at *n* time points the snow load increases by the value $U_i$ ($i$=1, 2, ... ,n). The number of jumps is a random variable with mathematical expectation. As mentioned above, from the assumption of independence of random variables $U_i$. It follows that their distribution law is exponential.

The trajectory of the random process for this model is shown in Fig. 4. In contrast to the Poisson process model, for the latter model we can consider the option of a non-random variable and a constant time interval

$$n = \bar{n}; \quad \tau = T/(n+1)$$

The law of distribution of the maximum snow load during the winter

$$F(x) = e^{-\bar{m}} \sum_{i=0}^{\infty} \frac{\bar{n}^{i}}{i!} \left[1 - e^{-\lambda_1 x}\right]^{i}, \qquad (15)$$

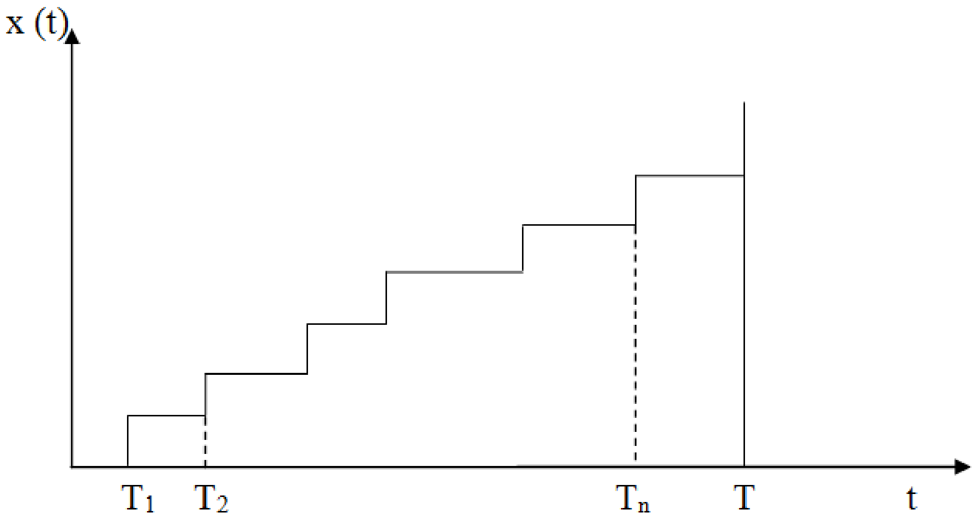


**FIGURE 4.** Process model with independent increments.

It is easy to show that the distribution function $F_1(x)$ и $F_2(x)$ in expressions (14) and (15) are comparable Parameters $\lambda_1$ и $\bar{n}$ can be selected from the condition that functions $F_1(x)$ и $F_2(x)$ and distribution function $F_0(x)$ from (9) are comparable.

To do this, the equalities must be satisfied:

$$\lambda_1 = \frac{1}{b}, \quad \bar{n} = \exp\left(\frac{a}{b}\right), \qquad (16)$$

$$\lambda_1 = 0{,}00274, \quad \bar{n} = 12{,}81 \approx 13$$

Thus, we have two snow load models. One model with jumps at random times, for which the maximum distribution law is (14) and the second model with jumps at a constant period of time and the maximum distribution law (15).

$$\bar{T} = T/(n+1)$$

Note that in cases of probabilistic calculation of a structure under the influence of one snow load, the use of these two models or the use of a random value of the maximum load with distribution law $F_0(x)$ gives almost identical results, since function $F_0^m(x)$, $F_1^m(x)$ и $F_2^m(x)$ . Where m is the number of years, very quickly $(m \geq 4)$ becomes almost indistinguishable. This difference can only affect the problem of probabilistic combination of loads.

Next, we will try to compare a model with random time intervals $\tau_1, \tau_2, \ldots \tau_{n+1}$ , where $\tau_1 + \tau_2 + \ldots + \tau_{n+1} = T$ and *n* are a random variable distributed according to Poisson's law, and a model with a constant number of $\bar{n}+1$ deterministic time intervals, equal to $\bar{T} = T/(\bar{n}+1)$ in the load combination problem.

Let P(*t*,*x*) be the conditional probability of failure-free operation of the structure over a period of time *t*, provided that the snow load is equal to *x*.

Then for the first model the probability of failure-free operation during time T is equal to

$$P(t) = e^{-\bar{n}} \sum_{k=1}^{\infty} \frac{\bar{n}^n}{n!} \int_{\Omega} \left[ \prod_{i=1}^{n+1} \int_0^{\infty} \lambda_1 e^{-\lambda_1 x} P(\tau_i, x) dx \right] d\tau_1 d\tau_2 ... d\tau_n \ , \qquad (17)$$

where $\tau_{n+1} = T - \tau_1 - \tau_2 - ... - \tau_n$

This expression requires complex calculations. A simpler expression for this probability is obtained for the second model

$$P_0(t) = \left[ \lambda_1 \int_0^{\infty} \lambda_1 e^{-\lambda_1 x} P(\bar{T}, x) dx \right]^{\bar{n}+1} , \qquad (18)$$

If we assume that the intervals $\tau_1 = \bar{T}$ and *n* are random, we obtain the following approximation for $P(t)$:

$$P_1(t) = e^{-\bar{n}} \sum_{n=1}^{\infty} \frac{\bar{n}^n}{n!} \left[ \lambda_1 \int_0^{\infty} e^{-\lambda_1 x} P(\bar{T}, x) dx \right]^{n+1} , \qquad (19)$$

Or in another form:

$$P_1(t) = \int_0^{\infty} \lambda_1 e^{-\lambda_1 x} P(\bar{T}, x) dx \cdot \exp\left[ -\bar{n} \left( 1 - \int_0^{\infty} \lambda_1 e^{-\lambda_1 x} P(\bar{T}, x) dx \right) \right] , \qquad (20)$$

Let us introduce the notation:

$$\varphi_n(\tau_1, \tau_2, ...\tau_n) = \prod_{i=1}^{n+1} \int_0^{\infty} \lambda_1 e^{-\lambda_1 x} P(\tau_i, x) dx \ , \qquad (21)$$

Then the mathematical expectation of the function $\varphi_n(\tau_1, \tau_2, ...\tau_n)$

$$\overline{\varphi_n} = \int_{\Omega} \varphi_n(\tau_1, \tau_2, ...\tau_n) d\tau_1 d\tau_2 ... d\tau_n \ , \qquad (22)$$

And formula (13) takes the form:

$$P(t) = e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \overline{\varphi_n} \ , \qquad (23)$$

Applying linearization to function $\varphi_n(\tau_1, \tau_2, ...\tau_n)$ in the vicinity of mathematical expectations, we obtain in a first approximation

$$\overline{\varphi_n} \approx \varphi_n(\overline{\tau_1}, \overline{\tau_2}, ...\overline{\tau_n}) \ , \qquad (24)$$

Next, formula (15), since $\overline{\tau_i = T/(n+1)}$ .

We get a more accurate formula in the form:

$$\overline{\varphi_n} \approx \varphi_n(\overline{\tau_1}, \overline{\tau_2}, ...\overline{\tau_n}) + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2 \overline{\varphi_n}}{\partial \tau_i^2} D[\tau_i] + \sum_{i<j} \frac{\partial^2 \overline{\varphi_n}}{\partial \tau_i \partial \tau_j} k_{i,j} \ , \qquad (25)$$

As known (4)

$$D[T_i] = T^2 \frac{i(n-i+1)}{(n+1)^2}\,,\quad E[(T_i-\overline{T})(T_j-\overline{T})] = T^2 \frac{i(n-j+1)}{(n+1)^2(n+2)}\,, \qquad (26)$$

After making some simple transformations, we arrive at the formula for

$P_2(\overline{T})$:

$$P_2(\overline{T}) = P_1(\overline{T})\left\{1 - e^{-\overline{n}} \sum_{n=1}^{\infty} \frac{\overline{n}^2}{n!} \frac{n(n^2-2)}{(n+1)^2(n+2)} \left[ \lambda_1^2 \int_0^{\infty} e^{-\lambda_1 x} P_{tt}''(\overline{T},x)dx \cdot \int_0^{\infty} e^{-\lambda_1 x} P(\overline{T},x)dx - \left( \lambda_1 \int_0^{\infty} e^{-\lambda_1 x} P_t'(\overline{T},x)dx \right)^2 \right] \right\}, (27)$$

Where can you get an estimate of the error by replacing $P_2(T)$ with $P_1(T)$.

Thus, models of a random process with independent increments for random times of load surges and non-random times of these surges can be considered little different from each other. A model based on the application of the theory of order statistics is actually identical to a process model with random increments at non-random points in time.

The good agreement of all these models is an argument in favor of their adequacy.

In the future, a model based on the theory of order statistics will be used, as it is the simplest for use in the task of combining loads.

## CONCLUSION

Thus, models of a random process with independent increments for random times of load surges and non-random times of these surges can be considered little different from each other [2].

A model based on the application of the theory of order statistics is actually identical to a process model with random increments at non-random points in time.

The good agreement of all these models is an argument in favor of their adequacy.

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