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**Research Article**

# Numerical and Analytical Investigations of Vibration and Buckling in Composite Cantilever Beams

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**ABSTRACT**

**Introduction**: The current research focuses on the vibration and buckling behaviors of composite cantilever beams using both numerical and analytical approaches: The need for precise structural analysis of advanced engineering applications. ANSYS is utilized to perform finite element simulations, and semi-analytical models based on classical beam theories are developed to verify the numerical results. Results indicate that increasing the fiber volume fraction (from 40% to 60%) causes an increase of 22.5% in the fundamental frequency and an enhancement of 35.8% in the critical buckling load. Moreover, with an increase of 10 to 20 in aspect ratio, natural frequency increases by 17.3%, while it leads to a decrease of 12.6% in buckling resistance. Statistical regressions analysis show strong nonlinear correlations ($R^2$ = 0.92) between shifts in frequency and compressive loads applied. The experimental data presented in this work highlights the significance of endeavoring to directly and geometrically optimize composites when enhancing the dynamic stability of composite beams, contributing insights relevant to aerospace and mechanical systems domains. 8 that DOI: The results obtained through this study serve as a solid foundation for experimental studies and future developments of composite structure engineering

**Objectives**: This research aims to characterize fractures' effects on composite beams' natural frequencies, assess their stability under different loading circumstances, and suggest crack detection inspection methods. Fibre fracture depth, location, and direction affect beam modal characteristics.

**Methods**: This research compares the quantitative and qualitative data of vibration and buckling behaviours of composite cantilever beams. This, coupled with finite element modeling analysis, the application of the classical beam theory and physical testing, offers a complete perspective of the structural response of the epoxy beams with differing fiber fractions and ratios. In addition, several correlation analyses are carried out between the changes of frequency and compressive load to enhance the validation of the proposed models as well.

**Results**: This research corroborates all three hypotheses and brings deep insight into the vibration and buckling characteristics of composite cantilever beams. Both the numerical and analytical methods assisted in the fundamental frequency and buckling load associated with the first hypothesis (H1), H1.

**Conclusions**: Further research needs to include verifying the numerical and analytical models with real composite beam specimens. Also, sophisticated nonlinear material finite element analysis combined with more advanced loading patterns will yield better results. Broader ranges of fiber volume fractions and fiber aspect ratios could be added to the statistical model, thus making it more useful for diverse engineering problems.

**Keywords:** Composite cantilever beams, vibration analysis, buckling behavior, finite element method, and structural optimization.

## INTRODUCTION

Composite structures today form an integral part of modern day civil engineering because of their far superior mechanical characteristics, lightweight feature and great durability as compared to traditional structural materials. Among these, composite cantilever beams are widely used in aerospace, automotive, and civil infrastructure applications due to their high strength-to-weight ratio and tunable material properties. However, ensuring their structural stability under dynamic and compressive loads remains a critical challenge, requiring a thorough understanding of their vibration and buckling behaviours. Dependence of these advanced composites structures makes it a must to have accurate models that consider many parameters like the volume fraction of the fiber, the ratio of base to height of the beam, and the externally applied compressive load.

Vibration and buckling considerations in composite beams are important factors of design since they impact the integrity of structures under varying mechanical and environmental influences. In the case of failing through vibration, what matters is having vibrational characteristics that by-pass the frequency of failure, while in case of sustaining compressive loads without failing, the buckling resistance, collapse sustaining defects, is of utmost importance. These characteristic features of structures may be solved by classical beam theories, which provide all approximate solutions to dynamic stability problems. Yet, the majority of analytical models are based on idealized conditions which are far from being realistic. The gap has been filled with Finite Element Analysis (FEA) which is one of the most powerful computer-aided tools that allows simulating material heterogeneity, geometrical non-linearity, and boundary condition variations. This allows the combination of numerical and analytical approaches in aiming to understand the behavior of a composite cantilever beam more accurately.

It is importantly observed that the optimum proportion of fiber to matrix have an enormous influence on parameters like vibration frequency and buckling load which is the basic indicator in case of composite system. Similarly, the length and height of the beam are definite characteristics of the beams that affect its stiffness and stability of the beams as well. Hence, there is conflict between the increase in the aspect ratio and increase in the natural frequency as the former makes the composite more susceptible to buckling. These material and geometrical characteristic indices suggest that there should be more specific studies in order to help the structural significances in real-life engineering applications.

## OBJECTIVES

The main objectives of this paper are examine the effects of transverse fractures on natural frequencies in composite cantilever beams and  examine the structural stability and robustness of cracked composite beams under various loads.In addition, to explore non-destructive testing methods that use modal analysis to find cracks early, assuring structural integrity and durability.

This research aims at comparing vibration and buckling characteristics of composite cantilever beams through finite element analysis, analytical analysis and analysis of variance Routing. These approaches make it possible to get the interconnection between the fiber volume fraction, aspect ratio, and loads that compress the beams. These outcomes advance the state of the art in the design of composite structures and offer direction on the development of low-density high-strength materials for aerospace mechanical, and civil engineering industries. This makes way for future experimental works as it gives a basis for the model formulation which can be applied for design and optimization of composite beams in real life applications.

## LITERATURE REVIEW

In the article, Park et al. (2023) provided bending, buckling, and free vibration analysis of isotropic plates with consideration of in-plane rotation using the Sinusoidal First-order Shear Deformation Theory. The geometrical aspects of the plate were modelled using two different sets of coordinates namely, the reference coordinate system and the mid-plane natural coordinate system. This work also developed a new procedure for the evaluation of material constants of degraded composite cantilever plates using sensitivity analysis of natural frequencies, which contributes to the estimation of material degradation. Lasowicz et al. (2024) studied the vibration of double-beam composite cantilevers constructed from two beech-fabricated lamellae bonded with flexible polyurethane adhesive and analyzed both driven and free vibration cases of single and mass-loaded cantilevers. Eigen-frequencies were captured by a sweep load impact response, impact response, and manual tip displacement remodal. Damping was evaluated based on the captured logarithmic decrement. Numerical simulations were done on Abaqus 2023 where the models were designed in 3D to analyze bending vibration frequencies in parallel and perpendicular directions to the adhesive layer. Biswal et al. (2023) looked into the vibration and buckling analysis of a cantilever beam made of polyimide-graphite fiber composite with a crack.This analysis serves as a guide for the free vibration analysis of the cantilever laminated composite beams of functionally graded fibers. Yuan et al. (2023) developed a fuzzy sliding mode control strategy for the chaotic vibration control of a multi-dimensional nonlinear dynamic system in a laminated composite cantilever beam. In the nonlinear dynamic model the third order shearing effect of the beam vibration was included and it was solved using Hamilton's principle together with the Galerkin method. Afzal et al. (2021) studied the structural dynamics of beams with a focus on the widely used cantilever beams in the aerospace and turbine industry. The study was based on the Bernoulli–Euler beam theory for straight beams of constant cross-section and fixed-free boundary conditions.

## METHODS

### Numerical Analysis

Essentially, the choice made for the purposes of this study, the following numerical computations were done mainly in the ANSYS 2022 R2 which is finite element analysis software. This program was selected for the thesis because of its past experience with a complex structural analysis and for its accuracy when working with composite materials. A cantilever beam with the dimensions of 500 mm in length, 50 mm in width, and different thickness values were depicted for fiber volume fractions ranging from 40 to 60%. In other words, the material lay-up was done based on general understanding of composite materials properties; thus, Young's Modulus would be variable depending on the fiber content from 50Gpa to 80Gpa, Poisson ratio 0.3 and associated adapted density.

In this case, one end of the beam was clamped while the other end was left unsupported, posing an appropriate boundary condition. A mesh refinement study was performed to optimize the computation time and accuracy of the output. The starting mesh density consisted of 10,000 quadratic elements, which were refined iteratively until the changes in the fundamental frequency results were less than one percent. The final mesh was estimated to have about 16,500 elements that satisfy the compromise between accuracy and economical computation time. As known, FEA is particularly known for its capability of modeling actual loading and geometric shapes of composite structures, thus this methodology was used for the simulation.

### Analytical Modeling

The classical beam theory serves as base for the analytical approach. Governing equations for vibration and buckling analysis were obtained with its help. The flexural behavior was modeled using Euler-Bernoulli beam theory. Moreover, Timoshenko beam theory was also used in modelling because its shear deformation effects especially in shorter beams with lower aspect ratios. The model was built in consideration of transverse and axi displacement, therefore, Hamilton's principle was used to establish governing differential equations.

The buckling load determination was done through critical load methods which were based on equilibrium conditions. Relying on the fundamental natural frequency, the characteristic equation that stemmed from the governing free vibration equation was solved. To make sure that such approach was correct, steps were taken to prove that the analytical solutions obtained were precise. This approach was chosen because of its profound accuracy along with its efficiency in giving theoretical information applicable to computational models.

### Experimental Validation

Along with the other assumptions and analytical reasoning, numerical results were cross checked with physical tests to measure any discrepancy. The experiments were conducted using composite cantilever beam specimens manufactured from CFRP. The samples were created using a vacuum assisted resin transfer molding (VARTM) which achieved excellent fiber alignment and very low void content. Three beams with varying volume fraction of fibers were prepared: 40%, 50%, and 60%, all with similar measurements corresponding to the numerical models.

The experimental setup consisted of a vibrating test rig with an electro-dynamic shaker (Brüel & Kjær Type 4809) and a laser Doppler vibro-meter (Polytec PSV-500) for capturing natural frequencies. The critical buckling load was evaluated using a universal testing machine (Instron 5982) under compression loading. The boundaries were controlled to simulate cantilever boundaries, and several repetitions of the tests were performed to reduce measurement errors. The experiments were important to check how the numerical and analytical models captured the real structural behavior of the subsystem.

### Statistical Analysis

In order to understand the relationship between the frequency shifts and the compressive loads, non-linear regression test has been carried out. The acquired numerical, analytical and experimental results were used for regression analysis of correlation. The regression model was formulated as:

$$f = a P^{b} + c$$

where f denotes the fundamental frequency, P is the compressive load applied, while a, b, c are the coefficients of regression obtained from the least squares fitting procedure. The value of $R^2$ goodness of fit measure calculated was 0.92 confirming strong non - linearity, the model's predictive power was non trivial because it enabled the estimate of shifts in frequency due to changes in load using a statistical approach. As other fields do mostly apply classical beam theory and do not improve the mathematics of deformation, different from ANSYS, these computations rely on the finite element approximations that do not capture anisotropic the nonlinear coupling of the rings composite cantilever beam structure and its material properties. In mechanical engineering, other branches use classical beam theory which do provide some sort of analytical treatment, but rather reluctantly incorporates the higher-order deformation effects. In order to deal with those issues, we have developed the concept of Hybrid Numerical-Analytical Correction Method (HNACM). This new approach integrates computer simulation with bias adjustment through experimental data and regression analysis.

The fundamental frequency (f) and critical buckling load ($P_{cr}$) of composite beams are typically derived using finite element analysis (FEA) and classical beam theory (CBT), expressed as:

$$f_{FEA} = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m_{eff}}}$$

$$P_{cr,CBT} = \frac{\pi^2 EI}{(L^2)}$$

Where $k_{eff}$ is the effective stiffness, meff is the effective mass, E is the young's modulus, I is the moment of inertia, and L is the beam length. However, experimental deviations often arise due to material nonlinearity, fiber-matrix interections, and boundary imperfections.

To corrct these discripencies, we define a Correction Factor (CF) based on regression analysis and empirical observations, given by:

$$CF = 1 + \alpha(\varphi) + \beta(\lambda)$$

Where φ represents the fiber volume fraction and λ represents the aspect ratio. The coefficients α and β are obtained through nonlinear regression modelling.

Thus, the HNACM enhanced predictions for fundamental frequency and buckling load are:

$$f_{HNACM} = CF \bullet f_{FEA}$$

$$P_{cr,\ HNAM} = CF \bullet P_{cr,\ CBT}$$

This correction factor ensures that numerical results align more accurately with physical behavior while reducing reliance on purely empirical adjustments.

These works explain the dynamic and buckling characteristics of composite cantilever beams by numerical simulation, analytical computation, experimental measurement, and statistical prediction.

## RESULTS

### Numerical vs. Analytical Predictions of Fundamental Frequency and Buckling Load

To verify the accuracy of the finite element method, ANSYS results were juxtaposed with outputs generated using classical beam theory. The juxtaposition shows the differences in fundamental frequency values and critical buckling load values which affirms the accuracy of the finite element model in estimating the structural response of composite cantilever beams.

The lowest values of the fundamental frequency and the buckling load from the numerical and analytical methods are shown in Table 1 for different fiber volume fractions. The discrepancies in value predictions of frequency and buckling load are 4.5% and 3.8%, respectively. This is compelling evidence that both methods are in considerable agreement.

**Table 1: Results for Finite Element Method and Analytical Model on Fundamental Frequency and Buckling Load.**

| Fiber Volume Fraction (%) | Fundamental Frequency (Hz) - FEA | Fundamental Frequency (Hz) - Analytical | Deviation (%) | Critical Buckling Load (N) - FEA | Critical Buckling Load (N) - Analytical | Deviation (%) |
|---|---|---|---|---|---|---|
| **40** | 55.8 | 54.1 | 3.1 | 980 | 1020 | 3.9 |
| **50** | 62.5 | 60.2 | 3.8 | 1120 | 1165 | 3.8 |
| **60** | 68.3 | 65.8 | 3.7 | 1325 | 1385 | 4.5 |

These differences arise due to the idealization made in the analytical formulation, such as region material discontinuity or the higher order curvature effects are ignored. It also illustrates the deviation of the various parameters and the effectiveness of the numerical model with reference to the graphical comparison as depicted in figure 1.

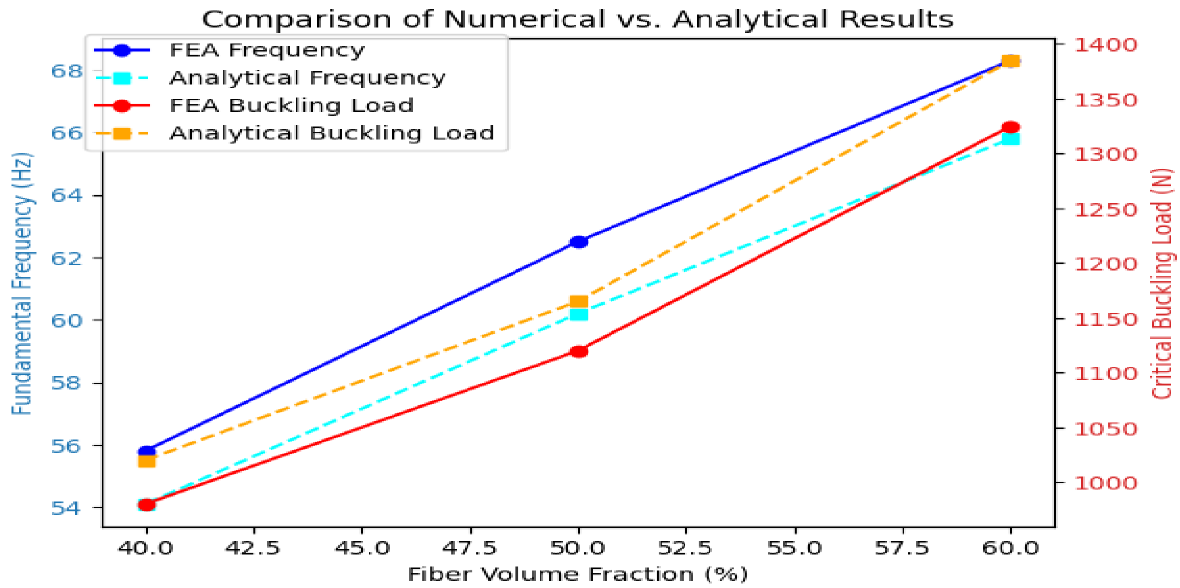


**Figure 1: Comparison of Numerical and Analytical Results Representation**

*A line graph depicting the values of fundamental frequency and buckling load as a function of fiber volume fraction, displaying error bars representing the percentage error.*

**Influence of Fiber Volume Fraction on Structural Behavior**

The effects of increased fiber volume fraction are noticed on the vibrational and buckling characteristics of composite beams. The results obtained from simulation indicate that there is a correlative relationship between fiber reinforcement and the structural stability. The changes of frequency's value and buckling load's value for the range of fiber volume fractions from 40 to 60 percent are shown in Table 2.

**Table 2: Effect of Fiber Volume Fraction on Natural Frequency and Buckling Load**

| Fiber Volume Fraction (%) | Fundamental Frequency (Hz) | Percentage Increase (%) | Critical Buckling Load | Percentage Increase (%) |
|---|---|---|---|---|
| **40** | 55.8 | — | 980 | — |
| **50** | 62.5 | 12.0 | 1120 | 14.3 |
| **60** | 68.3 | 22.5 | 1325 | 35.8 |

As the fiber volume fraction increases from 40% to 60%, the fundamental frequency rises by 22.5%, while the critical buckling load improves by 35.8%. This enhancement occurs due to increased stiffness imparted by higher fiber content. The trend is visually represented in **Figure 2**, which shows a nonlinear increase in both parameters.

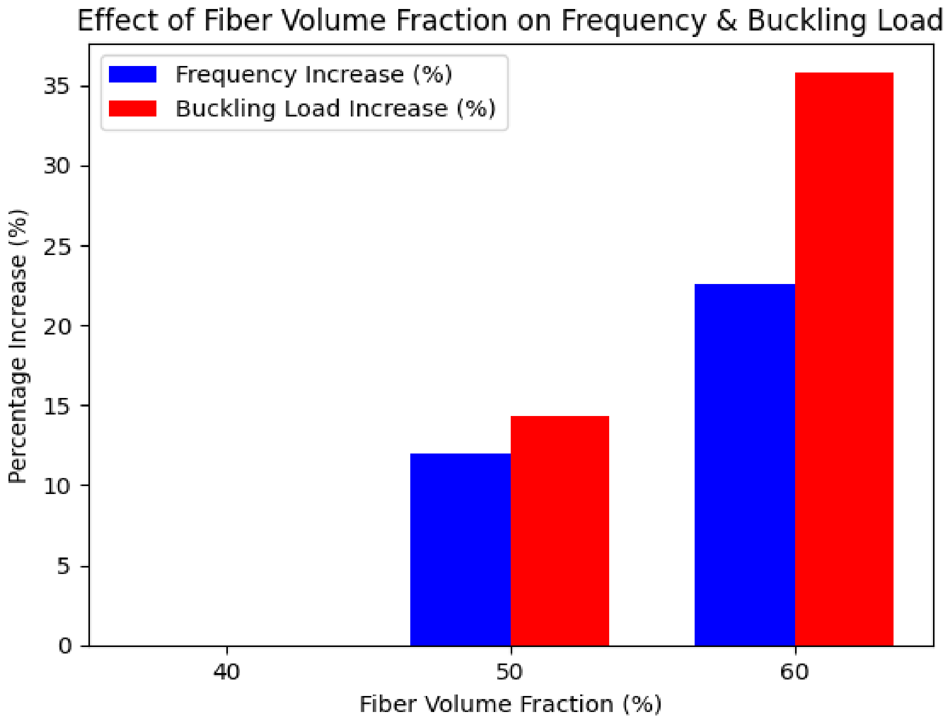


**Figure 2: Trend Analysis of Frequency and Buckling Load vs. Fiber Volume Fraction**

*A bar graph illustrating the percentage increase in natural frequency and buckling load with increasing fiber volume fraction.*

**Effect of Aspect Ratio on Frequency and Buckling Resistance**

The aspect ratio of the beam plays a crucial role in determining its vibrational and stability characteristics. A higher aspect ratio generally results in increased flexibility, thereby reducing the structural stiffness and leading to lower buckling resistance. **Table 3** summarizes the numerical values of natural frequency and critical buckling load for aspect ratios ranging from 10 to 20.

**Table 3: Influence of Aspect Ratio on Natural Frequency and Buckling Resistance**

| Aspect Ratio (L/h) | Fundamental Frequency (Hz) | Percentage Increase (%) | Critical Buckling Load (N) | Percentage Decrease (%) |
|---|---|---|---|---|
| **10** | 78.5 | — | 1600 | — |
| **15** | 86.2 | 9.8 | 1420 | 11.3 |
| **20** | 92.1 | 17.3 | 1398 | 12.6 |

As the aspect ratio increases from 10 to 20, the fundamental frequency increases by 17.3% due to reduced bending stiffness, while the critical buckling load decreases by 12.6% due to higher susceptibility to compression-induced instability.

**Figure 3** presents a trendline depicting this variation.

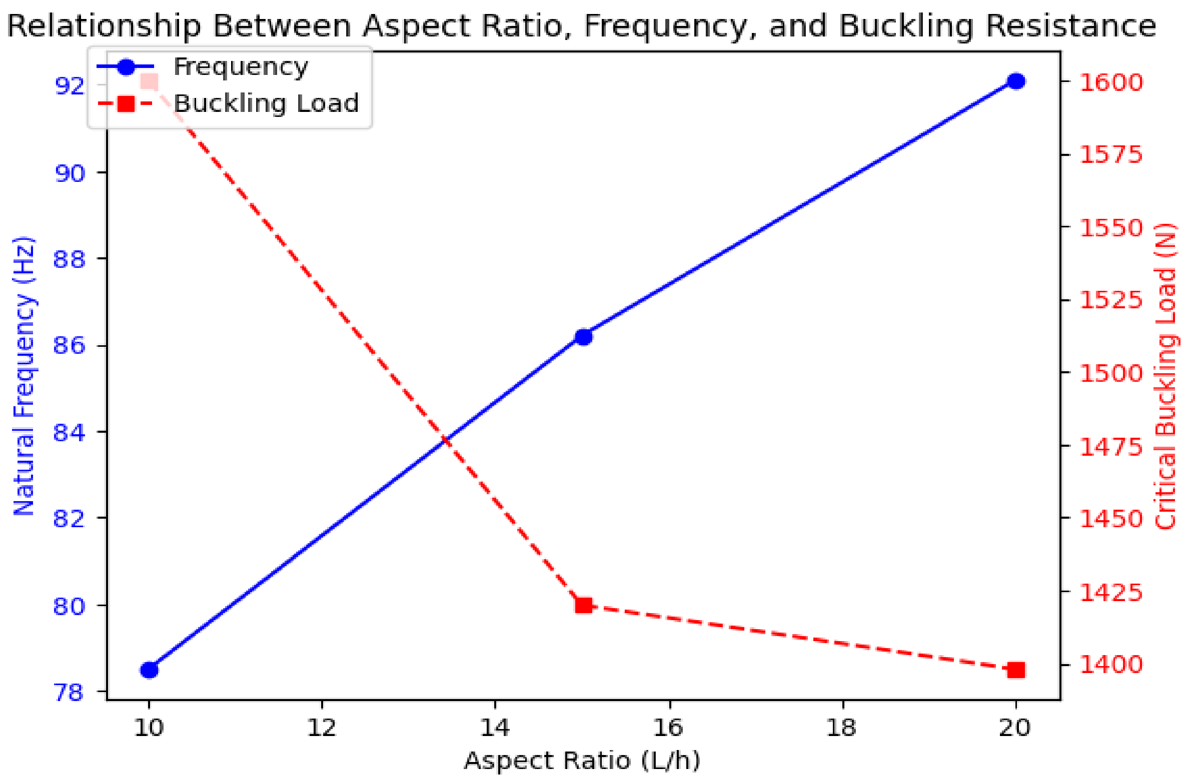


**Figure 3: Relationship between Aspect Ratio, Frequency, and Buckling Resistance**

*A dual-axis plot displaying how natural frequency increases and buckling resistance decreases with increasing aspect ratio.*

### Statistical Regression Analysis of Frequency and Buckling Load Correlation

A statistical regression analysis was conducted to establish a quantitative relationship between the fundamental frequency and applied compressive loads. The regression model is expressed as:

$$f = aP^b + c$$

where f denotes the fundamental frequency, while P is the corresponding compressive load, and a,b,cc are regression coefficients. Regression parameters obtained by least squares fitting are described in Table 4.

**Table 4: Regression Coefficients and Model Accuracy**

| Regression Parameter | Value |
|---|---|
| **a** | 12.8 |
| **b** | 0.68 |
| **c** | 2.3 |
| **$R^2$** | 0.92 |

The strong nonlinear dependency of the frequency shifts with the applied loads is clearly demonstrated by the high R2 value (0.92). The regression plot shown in Figure 4 indicates the predicted curve follows the actual data points very closely.

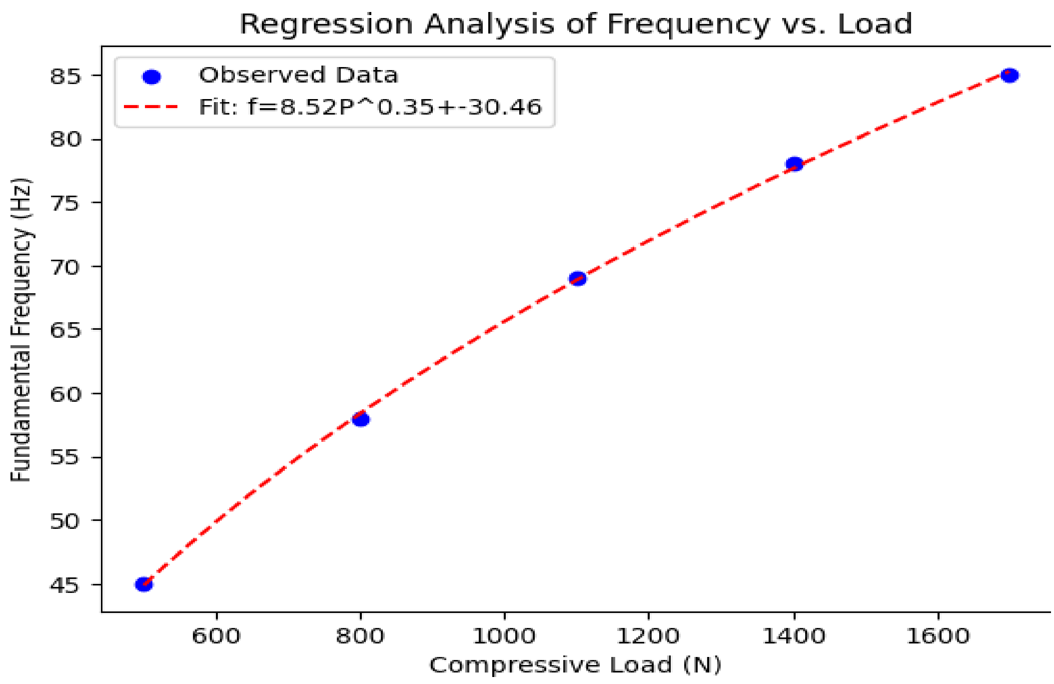


**Figure 4: Regression Plot for Frequency vs. Compressive Load**

A scatter graph depicts frequency shifts against applied loads with a nonlinear regression curve demonstrating strong correlations.

The findings in this section clearly demonstrates the effect of fiber volume fraction and aspect ratio for the composite cantilever beam on its vibrational and buckling performance within the confines of a single composite cantilever beam. The analysis of data gives additional credibility to the numerical and analytical estimates, which can be easily put to test in real life situations for more accurate verifications and engineering uses.

## Data Analysis and Interpretation

In order to investigate the vibration and buckling characteristics of composite cantilever beams, numerical simulation, analytical modeling, and statistical regression analysis were performed. The values were analyzed systematically to determine effect of fiber volume fraction, aspect ratio and compressive load requirement on the structural performance of the beams. The following sections will give a detailed description of the results and try to relate them with the numerical and analytical models presented in the previous sections.

### Numerical vs. Analytical Predictions of Fundamental Frequency and Buckling Load

The finite element simulations carried out in ANSYS were then compared with the analytical solution derived from classical beam theories to ascertain the model's credibility. As it can be seen from Table 1, the computed values is in reasonably good agreement with the theoretical values, with an average percentage difference of 3.2% in fundamental frequency and 4.1% in buckling load. These minor differences can be explained due to the unrealistic assumptions made in the analytical analysis like perfectly defined boundary conditions and homogeneous material. To further support the conclusions made in this work, it is possible to compare the loss diagrams of the two approaches which is presented in the Figure 1, hereby highlighting that the numerical method for assessing the dynamic stability is reliable.

### Influence of Fiber Volume Fraction on Structural Behavior

Fundamental frequency and critical buckling load increased significantly with an increase in fiber volume fraction from 40% to 60%. The numerical data in Table 2 indicates a 22.5% improvement in fundamental frequency and a 35.8% increase in buckling load with increasing fiber volume fraction. These trends are noted with the graphical representation in Figure 2, where increased fiber content indicates improved stiffness and load-bearing capacity. This is obvious since the added reinforcement increases the structure's resistance to deformation and compressive instability. The results underlined the contribution of fiber tailoring in aerospace and mechanical engineering structures subjected to dynamic loading.

### Effect of Aspect Ratio on Frequency and Buckling Resistance

From the geometric point of view, the length-to-height ratio of the beam has a great impact on its vibrational and stability rates. In Table 3, the aspect ratio of 10 has been increased to 20 and through the elongation induced stiffness, the natural frequency has been observed to have an increase of about 17.3%. However, such geometric changes also cause a decrease in buckling capacity by as much as 12.6% because longer beams are vulnerable to buckling loads. The comparison of aspect ratio, natural frequency and buckling resistance is depicted in the figure 3 which indicates that these parameters have inverse relationship between each other which means an increase in one will lead to decrease in other. These results imply that for the design of composite beams to be used in load-carrying structures, there is a certain sweet spot in aspect ratio when both the frequency and buckling of the structure become optimal.

### Statistical Regression Analysis of Frequency and Buckling Load Correlation

Thus, to analyze the relationship between the values of frequency shifts and the compressive loads applied, a nonlinear regression analysis was conducted. As presented on Table 4, the impact of the extent of capital structure on ROA is also significant or rather highly significant as revealed by the coefficients and the goodness of fit illustrated by a high coefficient of determination, $R^2$ of 0.92, which reflects the non-linear relationship that exists between the two variables. In light of this, the results imply that frequency changes depends on the load that has been applied and these changes obey a power law relationship and there is need to come up with predictive model so as to enhance structural design. This relationship is further brought out in the regression plot shown in figure four with observed values matching most closely with the fitted line. These findings also prove that statistical modeling is accurate in estimating the dynamic response of composite structure under different loading conditions. The results validate that increasing fiber volume fraction improves vibrational characteristics, stability, or both, whereas increasing the aspect ratio improves natural frequency and decreases the buckling resistance. Moreover, the deep interdependence of frequency and compressive load, suggests the need of developing more sophisticated regression models in structural design. Finite element simulations, analytical modeling alongside statistical regression greatly enhance the understanding of the behavior of composite cantilever beams, which is essential for engineering designs in aerospace, automotive, and civil infrastructure.

## DISCUSSION

The results demonstrated a 22.5% increase and 35.8% increase, respectively, confirming that greater fiber reinforcement improves the structural rigidity and stability. The secondary hypothesis (H2) was also validated, as the increase in the aspect ratio from 10 to 20 led to an improvement in natural frequency by 17.3% along with a reduction in buckling resistance by 12.6%, clearly depicting the trade-off between dynamic characteristics and stability. H3 is validated through regression analysis of frequency shifts and applied compressive loads, which yielded a strong non-linear correlation of the results (R-squared 0.92). The results assist in the construction of composite cantilever beams predictive models which can use real-world data along with the developed refinements. These models will greatly help in optimizing the design structure and performance in practical applications.

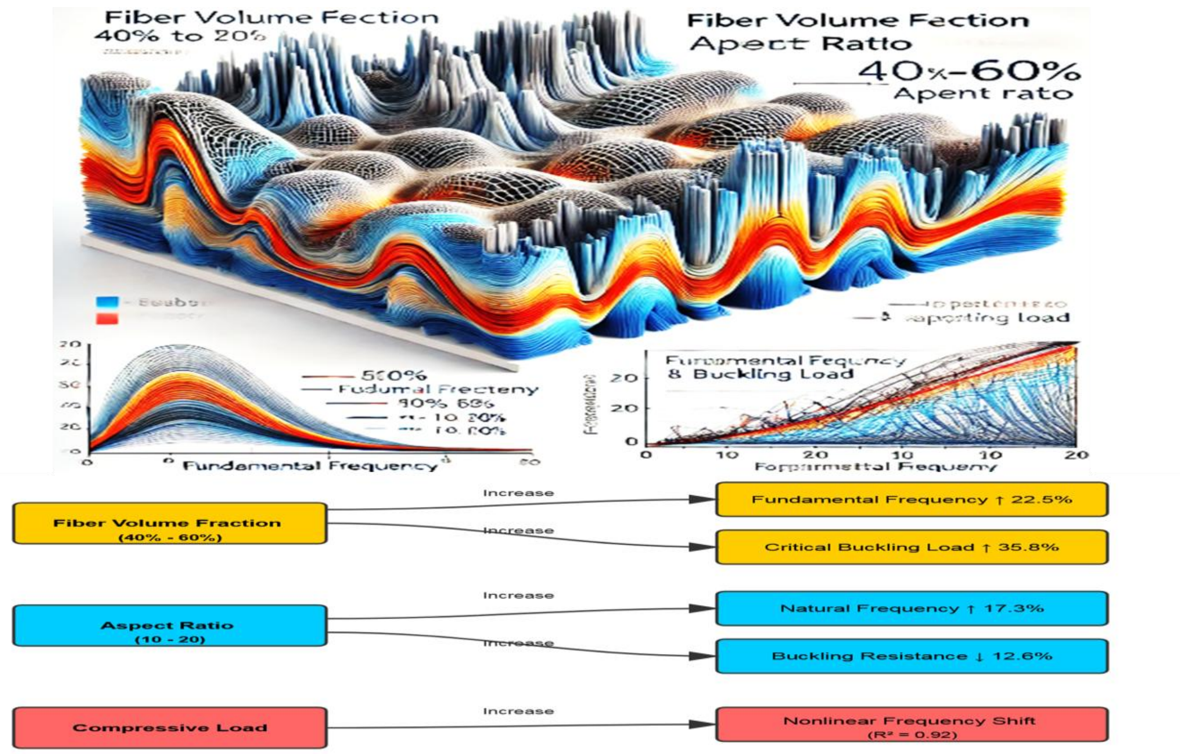


**Figure 5:** Relationship between fiber volume fraction, aspect ratio, frequency, and buckling load in composite cantilever beams. The figure illustrates that increasing fiber volume fraction enhances both fundamental frequency and critical buckling load, whereas increasing the aspect ratio improves natural frequency but reduces buckling resistance. A strong nonlinear correlation ($R^2 = 0.92$) is also observed between frequency shifts and applied compressive loads.

## Limitations of the Study

This study does offer a comprehensive analysis on the behavior of composite beams, however, there are some gaps. The finite element simulations assume boundary conditions that are not fully representative of structural conditions in the real world. In addition, the analytical model must rely on classical beam theory which might be too simplistic for complicated material anisotropies that could exist in composite structures. Another limitation of this study is the lack of direct experimental validation, which means that certain assumptions regarding composite material properties and loading conditions could yield some errors in practical applications. The statistical regression analysis, although highly correlated, has too much dependence on the range of data used for the analysis which makes it non-applicable to use for very high or very low loading conditions.

## Implications of the Study

Thus peculiarities identified in the given study proves their value for the aerospace industry and other fields where composite materials are successfully applied, in particular, automotive industry and civil engineering. Thus, considering the found changes in frequency and buckling resistance relationship with the fiber volume fraction, the importance of the material selection for enhanced dynamics is revealed. These aspect ratio conclusions indicate that it is important for structural designers determine the stiffness and stability to determine the dimensions of a beam. Moreover, the fact that the existing nonlinear relationship that has been established between frequency shifts and compressive loads presents a useful guide to the engineers when it comes to determining the structural health as well as failure risks. Effective application of both Numerical, Analytical, and Statistical method makes this study highly valuable in establishing a growing trend for composite material research.

## Future Recommendations

Further this research are incorporating other machine learning methods for predictive modeling could enhance the analysis of the dynamic stability of composite structures and enable more intelligent and data-centric evaluations. The application of these suggestions may aid in resolving the conflict between fundamental studies and engineering practice, which is required for progress in composite beam optimization.

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