Journal of Information Systems Engineering and Management
2025, 10(18s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Various Labelling for Double Star Graph

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ARTICLE INFO

Received: 18 Dec 2024

Revised: 28 Jan 2025

Accepted: 12 Feb 2025

ABSTRACT

In this work, we introduce labeling techniques for the double star graph, a basic graph theory structure. In particular, we concentrate on calculating two different kinds of labeling: fortunate labeling and correct labeling. Adhering to the restrictions of vertex labeling in graph theory, appropriate labeling guarantees that neighboring vertices receive unique labels. A more recent and interesting idea is the fortunate labelling, which gives vertices positive numbers so that the labels on neighboring vertices add up to a unique value for each edge. We calculate and examine various labeling methods using algorithmic methods, proving their usefulness and effectiveness for the double star graph. Our findings further enhance the theoretical and practical capabilities of various labeling systems by offering insightful information about how to optimize them for bipartite and tree-like graphs.

Keywords: Double star graph, Graph labelling, Proper labelling, Lucky labelling, Algorithm, Bipartite graph, Tree-like graphs, Vertex labelling, Graph theory, Combinatorics.

INTRODUCTION

The study of graphs, which depict scenarios involving points (vertices) and lines (edges), is the main subject of graph theory, an important field in mathematics and computer science. Its multidisciplinary uses span disciplines like biology, chemistry, physics, and the social sciences. Data management, social networking, computational processes, data structures, data mining, scheduling, and resolving challenging issues like the traveling salesman problem are all areas of computer science where graph theory is essential. Graphs are an essential tool in contemporary problem-solving because they simplify and quantify the dynamics of systems.

This paper focuses on a particular kind of graph called a double star graph, which is made up of two-star graphs connected by an edge. Because of its applications in coding theory, communication networks, and the study of dynamic systems, labeling graphs has long been a vital field of study. Specifically, we investigate the double star graph's appropriate and fortunate labeling. While lucky labeling necessitates that the total of the labels on neighboring vertices be distinct for each edge, proper labeling guarantees that adjacent vertices have distinct labels. Our goal is to increase knowledge of labeling methods and their effectiveness when used on bipartite graphs, especially the double star graph.

REVIEW OF LITERATURE

Because graph labeling is relevant to combinatorics, coding theory, and communication networks, it has become an important area of study within graph theory. In order to comprehend the structural characteristics of graphs and their applications in other fields, a number of labeling approaches have been created. By examining 1-homogeneous graphs, particularly cocktail party μ-graphs, and investigating symmetry and regularity properties—found in structured graphs like the double star graph—Jurišić and Koolen [1] established the framework. Their study paved the way for additional investigation into specialized graph architectures. The correct and fortunate labeling of quadrilateral snake graphs and generic graphs was investigated by Kumar and Meenakshi [2, 3]. They created algorithms that guarantee lucky labeling, which makes sure the total of labels on neighboring vertices is unique, and appropriate labeling, which assigns distinct labels to adjacent vertices. These techniques are especially important for tree-like and bipartite graphs, such as the double star.

Maria Irudhaya, Chitra, and Murugan [4] investigated lucky edge labelling in star-related graphs, providing strategies to attain different edge sums, thus broadening the use of labeling. Since double star graphs and star graphs have structural similarities, their work is very relevant to the study of double star graphs. Proper fortunate labelling, a hybrid of proper and lucky labelling, was first proposed by Meenakshi and Kumar [5]. It occurs when the labels on neighboring vertices add up to a unique value and both adjacent labels are distinct. In structured graphs like the double star, this thorough labeling method is essential. For bipartite and k-regular graphs, recent work by Bala and Pandey [6] extended generalized labeling techniques, such as edge-magic and vertex-magic labeling. These techniques provide more information about how difficult it is to classify graphs with star-like patterns.

Rosa [7] laid the groundwork for subsequent research in combinatorial labeling by first introducing the idea of elegant labeling, which is a forerunner to many contemporary labeling techniques. Double star graphs have structural issues that are ideally suited to graceful labeling, which gives vertices and edges unique labels so that every difference is unique. Gallian [8] offered a thorough analysis of graph labeling that covered a wide range of methods, including prime, harmonic, and magic labeling, and gave a general summary of advancements in the subject. For scholars investigating sophisticated labeling methods in particular graph classes, such as bipartite and tree-like structures, his work provides a point of reference. A strong basis for the accurate and fortunate labeling of double star graphs is provided by the algorithms and labeling techniques covered in these works. In order to improve the effectiveness and usability of graph labeling in structured and bipartite graphs, this study attempts to further refine these labeling techniques for use with double star graphs.

PRELIMINARIES

For example, the graph G has a finite number of edges and vertices. It is an undirected, simple graph, which means that there are no loops or multiple connections connecting any two vertices, and the edges have no orientation. Every edge joins two distinct vertices. Both the number of vertices and edges are constrained since the graph is finite. There are numerous applications for simple undirected graphs, such as the representation of links in networks with reciprocal relationships between points.

STAR GRAPH

The star graph has n edges and $1 + n$ vertices. It is represented by S_n. $V(S_n) = \{u, u_1, u_2, \ldots, u_n\} \cup \{v\}$ and $E(S_n) = \{vu_i\}$ where $n \in \mathbb{N}$ and $1 \leq i \leq n$ [5].

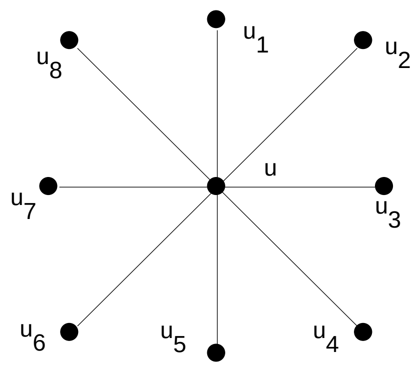

Figure 1: S_8

DOUBLE STAR GRAPH

The double star graph contains $2 + n + m$ vertices and $1 + n + m$ edges. It is represented by $S(U_n, V_m)$. 'n' represents number of pendent vertices in first star graph and 'm' represent number of pendent vertices in second star graph. $V\left(S(u_n, v_m)\right) = \{u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots., v_m\}$ and $E\left(S(u_n, v_m)\right) = \{uv\} \cup \{uu_i\} \cup \{vv_i\}$ [4].

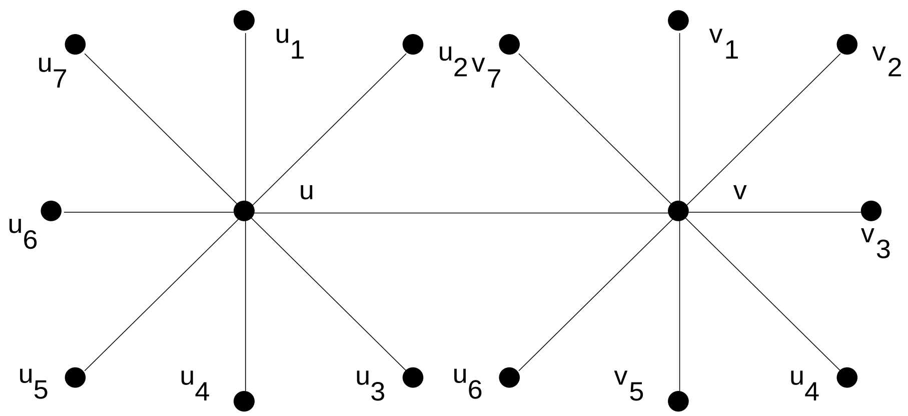

Figure 2: Double Star graph $S(u_7, v_7)$

PROPER LABELING

If adjacent labels are not equal, then it is said to be Proper [1]. It is denoted as $\varphi(G)$.

LUCKY LABELING

The Lucky is the graph which has sum of adjacent nodes are not equal [2]. It is denoted by $\eta(G)$.

PROPER LUCKY LABELING

If the graphs sum of adjacent nodes are not equal as well as its adjacent labels are not equal then it is said to be Proper lucky [3]. It is represented as $\eta_P(G)$. s and f denote sum and its labels respectively.

ALGORITHM

Step 1: Organize the graph's vertices in convenient order.

Step 2: Select the first vertex and Label it with the least positive integer.

Step 3: Select another vertex and label it with the next least labeling that has not yet been labelled on any adjacent vertices. If all of the surrounding vertices are labelled with this label, then choose next least positive integer. Repeat until all of the vertices are labelled.

MAIN RESULTS

Theorem 1: A double-star graph is proper then $\varphi(S(U_n, V_m)) = 2$.

Proof:

Considering the following iteration

i. If $n = 0, m = 0$ $then$ $\varphi(S(U_n, V_m)) = 2$.
ii. If $n = 1, m = 0$ $then$ $\varphi(S(U_n, V_m)) = 2$.
iii. If $n = 1, m = 1$ $then$ $\varphi(S(U_n, V_m)) = 2$.
iv. If $n = 2, m = 1$ $then$ $\varphi(S(U_n, V_m)) = 2$.
v. If $n = 2, m = 2$ $then$ $\varphi(S(U_n, V_m)) = 2$.

So, $f(S(U_n, V_m)) \to \{1, 2\}$ then $f(v) = 1, f(u) = 2, f(v_m) = 1, f(u_n) = 2$. Here adjacent are not same. So a double star graph is proper with $\varphi(S(U_n, V_m)) = 2$.

Theorem 2: A double star graph is lucky with $\eta\big(S(U_n, V_m)\big) = \begin{cases} 1 & for\ n \neq m \\ 2 & for\ n = m \end{cases}$

Proof:

Case (i) When $n \neq m$

Considering the following iteration

i. If $n = 0, m = 1$ *then* $\eta(S(U_n, V_m)) = 1$.
ii. If $n = 1, m = 4$ *then* $\eta(S(U_n, V_m)) = 1$.
iii. If $n = 2, m = 1$ *then* $\eta(S(U_n, V_m)) = 1$.
iv. If $n = 3, m = 1$ *then* $\eta(S(U_n, V_m)) = 1$.

So, $f(S(U_n, V_m)) = \{1\}$ for a double star graph be defined by $f(v) = 1, f(u) = 1, f(v_m) = 1, f(u_n) = 1$. The sum of the neighbor is $S(u) = 1 + n, S(v) = 1 + m, S(v_m) = 1, S(u_n) = 1$. Here sum of adjacent becomes not equal. So a double star graph is Lucky by $\eta(S(U_n, V_m)) = 1$ for $n \neq m$.

Case (i) When $n = m$

Considering the following iteration

i. If $n = 0, m = 0$ *then* $\eta(S(U_n, V_m)) = 2$.
ii. If $n = 1, m = 1$ *then* $\eta(S(U_n, V_m)) = 2$.
iii. If $n = 2, m = 2$ *then* $\eta(S(U_n, V_m)) = 2$.
iv. If $n = 3, m = 3$ *then* $\eta(S(U_n, V_m)) = 2$.

So, $f(S(U_n, V_m)) = \{1, 2\}$ for a double star graph be defined by $f(v) = 2, f(u) = 1, f(v_m) = 1, f(u_n) = 2$. The sum of the neighbor is $S(u) = 2(1 + n), S(v) = 2(1 + m), S(v_m) = 2, S(u_n) = 1$. Here sum of adjacent becomes not equal. So a double star graph is Lucky by $\eta(S(U_n, V_m)) = 1$ for $n = m$.

Theorem 3: A double star graph is properly lucky by $\eta_p(S(U_n, V_m)) = 2$ for $n \neq m$.

Proof:

Considering the following iteration

i. If $n = 2, m = 4$ then $\eta_p(S(U_n, V_m)) = 2$.
ii. If $n = 2, m = 5$ then $\eta_p(S(U_n, V_m)) = 2$
iii. If $n = 3, m = 4$ then $\eta_p(S(U_n, V_m)) = 2$
iv. If $n = 4, m = 6$ then $\eta_p(S(U_n, V_m)) = 2$

So, $f(S(U_n, V_m)) = \{1, 2\}$ for a double star graph be defined by $f(v) = 2, f(u) = 1, f(v_m) = 1, f(u_n) = 2$. The sum of the neighbor is $S(u) = 2(1 + n), S(v) = 1 + m, S(v_m) = 1, S(u_n) = 2$. Here sum of adjacent becomes not equal . So a double star graph is proper Lucky by $\eta_p(S(U_n, V_m)) = 2$ for $n \neq m$.

CONCLUSION

With an emphasis on appropriate labeling, fortunate labeling, and proper lucky labeling, we have investigated a number of labeling strategies for the double star graph in this study. To make sure that these labeling schemes satisfy the necessary requirements, algorithms were created and used. While fortunate labeling ensures that the total of the labels of adjacent vertices is unique, proper labeling ensures that neighboring vertices are assigned distinct labels. Proper fortunate labelling, which combines the two methods, offers an effective way to label double star graphs while guaranteeing that both requirements are met. Our results corroborate the applicability of all three labeling strategies in structured graphs, such as bipartite and star-related graphs, by showing that double star graphs admit them. These findings advance our knowledge of graph labeling and its optimization, which may find use in domains including coding theory, network architecture, and data organization. Future research can investigate the effectiveness of these algorithms in bigger, more complex graph classes by extending them to more sophisticated graph topologies.

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