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Research Article

Application of Philos-Type Difference Equations in Electromechanical Devices

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ABSTRACT

The oscillatory criteria for solutions of fourth-order neutral difference equation of the form,

$$\Delta\left(a(n)\Delta^3\big(x(n)+p(n)x(\sigma(n))\big)\right)+q(n)x^\gamma\big(\tau(n)\big)=0$$

where $\{a(n)\}$ is positive, $0\le\{p(n)\}<1,\{q(n)\}>0$, α and γ are non-negative integers are studied. The conditions for oscillatory criteria of above equation are obtained. The Philos type oscillatory criteria are gained by Riccati transformation technique and results are applied in Linear-Time-Invariant (LTI) system to study the dynamics of electromechanical devices. There are examples for proving the results.

Keywords: Oscillation, Neutral Difference Equations, Linear-Time-Invariant (LTI) system, Electromechanical Devices.

INTRODUCTION

The paper aims at establishing some oscillatory and asymptotic criteria for the solutions of fourth order neutral difference equation given by,

$$\Delta\left(a(n)\Delta^3\big(x(n)+p(n)x(\sigma(n))\big)\right)+q(n)x^\gamma\big(\tau(n)\big)=0 \tag{1}$$

where $\{a(n)\}>0, 0\le\{p(n)\}<1,\{q(n)\}>0, n\ge n_0$, α and γ are non-negative integers and satisfies

$$\sum_{n=n_0}^{\infty}\left(\frac{1}{a(s)}\right)=\infty \tag{2}$$

where $n\in N$. A non-trivial solution of (1) is oscillating for every term of $\{x_n\}$ thus the sequence is neither eventually positive nor eventually negative; otherwise non-oscillatory. The difference equations have their applications in various fields and some includes, economics, biology, circuit system, control system, etc., for example see [1-14]. In recent times, the difference equations extend their growth in studying the dynamical system and respective behaviour for applications in real world problems. The difference equation studies the qualitative behaviour with analytical and numerical solutions of discrete dynamical system. Their application includes control systems, trade, signal processing, population dynamics, etc. The fourth order difference equations are important for modelling devices as they enhance the relations between mechanical and electrical components which include complex dynamics, thus providing complete illustration of dynamical behaviour. The electromechanical devices are incorporated in terms of Philos type neutral difference equations to enrich the performance and to hold a stable control by permitting predictive maintenance, adaptive tuning, and optimization of system parameters and fault detection. The reliability and accuracy of model can be improved using model updating algorithms, data fusion techniques, and sensor feedback, thus leading to efficient system operations and enhanced accuracy. Also, the methodology of approaching the Philos type difference equations improves the knowledge of designing the devices by utilising and understanding a systematic study about stability, oscillatory behaviour, and input responses. This facilitates the engineers over control strategy, and for optimizing the performance of system. The existing challenges for demonstrating the models via Philos type difference equations includes the uncertainties, handling nonlinearities, system disturbances and progressive control strategy. In practical applications these challenges can be minimised by adapting certain control measures, feedback mechanism and robust optimization. The Linear Time Invariant (LTI) systems play a vital role in dynamical systems. Here they are used to solve equations by converting the input of control signals and thus producing the output. In electromechanical device, the robots are considered and the result is applied in controlling the movements and gestures with respect to time. The paper aim at establishing new Philos-type conditions for all solutions of equation (1) and using LTI systems in electromechanical devices, say the robots under condition (2). This paper is arranged in multiple sections such as Methodology, Preliminaries and Definitions, Oscillatory criteria for Philos-type equations, Examples, the LTI system with its mathematical procedure in defining and solving the difference equations, and finally the limitations are conveyed. This is followed by the conclusion and future work. Examples are provided.

OBJECTIVES

The objective is to understand the concept of Philos type difference equations and their applications. To analyse the oscillatory and asymptotic behavior of neutral difference equations in electromechanical devices using Linear-Time-Invariant (LTI) system.

METHODS

The fourth order neutral difference equations are studied for solutions of oscillatory behaviour using Riccati technique, summation averaging technique, summation by parts, comparison and substitution methods. Using LTI system the electromechanical devices using robots are studied.

RESULTS

Dynamical Systems:

The dynamical system analyse the qualitative behaviour for solutions of both difference and differential equations. The difference equations involve the discrete dynamical systems where modelling techniques, analytical solutions, numerical solutions, modelling techniques, etc., are examined. In a state space, the dynamical systems consist of a state defined by a set of real numbers or a set of points in general. The dynamical systems exhibit both discrete and continuous dynamical behaviour where the system can flow and jump. The system that can flow is defined by the differential equations and the system that can jump is defined by automation or a state machine. There is a wide range of applications for the system which includes biology, engineering, medicine, economics, etc.

Also a Hybrid Dynamical Systems consist of both the continuous and discrete systems. The digital process interacts with the mechanical environments. They are used in mathematical modelling to construct and solve intelligent or reactive system with high complexity. In every time step a constant number is being multiplied to the system, hence the solutions for a linear discrete dynamical system becomes exponential and the constants can be determined easily during iteration process. If the dynamical systems are expressed in difference form then it is transformed to function iteration form and then the process of solving is illustrated.

Electromechanical Devices:

This term typically indicates a device that produces a motion when an electrical signal is passed in the device and reverse also holds true. The electromechanical devices are a combined process of electrical and mechanical system and they use properties of both the systems to operate in a device. They are equiped in day-to-day life and few examples of them includes vacuum cleaners and refrigerators for household work; electric motors in automobile industry, power companies, all fuel based generators; hydraulic press and CD & DVD players. Since used in uel based generators and power companies, the renewable energies are produced. A brief description on working system of slectromechanical devices are given as follows: Three systems are commonly operated: 1) Electric motors, 2) Solenoids and 3) Mechatronics.

1) The electric motors has its significance for altering a mechanical energy by magnetic fields. In terms of electrical systems, a large amount of direct and indirect sources are used such as, inverters, batteries, etc. whereas,

2) the solenoids has its applications in inductor replacing any electromagnet delay that changes in electric current. They are cylindrical object where current flows take place in wires generating a magnetic field and thus creating a linear motion. They are controlled by currents and so valves and swtiches uses them in devices such as car, doorbell, fan etc. Finally,

3) the Mechatronics, ehich is another vast study of telecommunications and electronics, robots and other engineering fields. They are a combined field of electrical, mechanical and computer system. Frequenrly seen as the future of automated manufacturing in industreis. Examples include anti lock brake, digital SLR cameras, consumer items, etc.

Robots as Electromechanical Device:

The Philos type difference equations of fourth order models many electromechanical devices like solenoids, robots, mechatronic systems, electric motors, etc. In the system of robotics, the microcontroller is the central control of directions and movements. The electromechanical end effectors are constructed by components called gripper, actuator with control systems. Using a gripper, the objects are griped; the component called actuator regulated the gripper in instructed direction. Finally they are operated in control system to produce gestures, signals, etc. Certain usual robot types includes Collaborative Robots, Cartesian, SCARA, Cylindrical type, Cartesian, Polar and Delta types. To implement electromechanical models of Philos type equations, software tools like simulation software, Python, MATLAB are extensively used in differential equations, difference equations and control systems. They are used for numerical analysis and in generating mathematical simulations.

Linear-Time-Invariant (LTI) Systems:

They are a class of system where many real world problems are studied and solved. Using a fixed rule, input signals $x(n)$ are converted to output signals $x(t)$ with respect to time t using any discrete signal. The working system of LTI includes 3 main perceptions: 1) the difference equation (1), the transfer function, and the block diagram. Each perspective has their own weakness and strength for choosing the necessary work factors to synthesis the process in systems. This is common feature in systems theory and signal processing.

Thus, 2) the transfer functions, which are used for tracking coefficients and delays of (1). It relates the input value $x(n)$ to output $y(n)$ value specifying one variable of a polynomial system. Finally, 3) the Block diagrams, that represent a graph for any LTI system. Main parts included in block diagrams are gain, delay and summation. With gain, the input signal $x(n)$ is multiplied with a constant, say R and output is produced as $Rx(n)$. The delay specifies about a decrease or reduction of one unit at time t followed by the output production. More than two signals are summed up to create the output signal in summation part. Each block is completely characterized by transfer function concerning inputs and outputs of the system. With validation technique, mathematical integration method, and appropriate discretization scheme the convergence and accuracy of solutions for the models are derived.

Oscillatory results using Philos type equations:

This section deals with a study on neutral difference equations and new oscillatory and asymptotic results for (1) are established. Some lemmas are given and Philos type theorems are proved using Riccati transformation. For each solution $\{x(n)\}$ define a corresponding sequence$\{y(n)\}$,

$$y(n) = x(n) + p(n)x\big(\sigma(n)\big) \qquad (3)$$

Consider the mapping $G: N \times N \to R$, $G(n,n) = 0$ for $n \geq n_0$, $H(n,t) > 0$ for $n > t \geq n_0$,

$$\Delta_2 G(n,t) \leq 0, \qquad -\Delta_2 G(n,t) = g(n,t)\sqrt{G(n,t)}$$

where $g(n,t) = G(n,t)\Delta z(t)$ and $\Delta_2 G(n,t) = G(n,t+1) - G(n,t)$. We consider the initial conditions, properties of equation and parameters for the system to establish the existence and uniqueness of solutions of fourth order difference equations and mathematical analysis is carried out.

Lemma 1: If $x(n)$ is eventually non-negative result for (1), then below inequality is true.

$$\Delta\Big(a(n)\Delta^3\big(y(n)\big)^{\alpha}\Big) \leq -q(n)(1-p)^{\gamma} y^{\gamma}\big(\tau(n)\big)$$

Proof: Here $x(n)$ is a positive result for (1) is the assumption and so $\exists\, n_1 \geq n_0 \ni x(n),\ x(\sigma(n)),\ x(\tau(n))$ are all positive for $n \geq n_1$. Because $\Delta a(n)$ is also positive then, $y(n),\ \Delta y(n),\ \Delta^3 y(n)$, are positive but $\Delta(a(n)\Delta^3(y(n)) \leq 0$ and from (3) we have, $x(n) \geq y(n) - px(\tau(n)) \geq y(n) - py(\tau(n)) \geq (1-p)y(n)$. Thus above inequality becomes $\Delta\left(a(n)\Delta^3(y(n))^{\alpha}\right) + q(n)(1-p)^{\gamma} y^{\gamma}(\tau(n)) \leq 0$ and the proof is completed.

Lemma 2: Let $x(n)$ be a non-negative solution for (1). If Lemma 1 holds then,

$$\psi(n) = \frac{a(n)\Delta^3(y(n))^{\alpha}}{y^{\gamma}(\tau(n))} \tag{4}$$

then $\Delta\psi(n) \leq 0$.

Proof: Here $x(n)$ is positive solution of (1) is our assumption and with (4), $\psi(n) > 0$ for $n \geq n_1$ then,

$$\Delta\psi(n) \leq \frac{\Delta\left(a(n)\Delta^3(y(n))^{\alpha}\right)}{y^{\gamma}(\tau(n))} - \gamma \frac{a(n)\Delta^3(y(n))^{\alpha}}{y^{\gamma+1}(\tau(n))}$$

By Lemma 1 we get, $\Delta\psi(n) \leq -q(n)(1-p)^{\gamma} y^{\alpha-\gamma}(\tau(n)) - \gamma \frac{a(n)\Delta^3(y(n))^{\alpha}}{y^{\gamma+1}(\tau(n))}$. From last inequality we get,

$$\Delta\psi(n) + q(n)(1-p)^{\gamma} y^{\alpha-\gamma}(\tau(n)) + \gamma \frac{a(n)\Delta^3(y(n))^{\alpha}}{y^{\gamma+1}(\tau(n))} \leq 0$$

The proof is completed.

Theorem 1: Assume G to be a function of $y(n)$ and if there exists another function $g: N_0 \times N_0$ such that,

$$\sum_{s=n_0}^{n} g^2(n,t) < \infty$$

for every fixed value $n \geq n_0$, then (1) is said to be oscillatory if the following condition is satisfied

$$\lim_{n\to\infty} sup \frac{1}{G(n,n_0)} \sum_{t=n_0}^{n} \left[H(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma \frac{1}{4} g^2(s,t)\right] = \infty$$

Proof: The contradiction method is used to prove the result and $x(n)$ is not an oscillatory solution for (1) then $\exists$ an integer $N \geq n_0 \ni x(n) > 0\ \forall\ N \leq n$. By Riccati transformation define a condition,

$$z(n) = \frac{a(n)\Delta^3(y(n))^{\alpha}}{x^{\gamma}(\tau(n))}$$

then we have, $q(n)x^{\gamma}(\tau(n)) = \gamma\Delta z(n) - \Delta^2(z(n)$. If $G(n,n) = 0$ then using summation-by-parts implies,

$$\sum_{t=N}^{n} G(n,t)(q(t)x^{\gamma}(\tau(t)) = \sum_{t=N}^{n-1} G(n,t)\gamma\Delta z(t) - \sum_{t=N}^{n-1} G(n,t)\Delta^2(z(t))$$

$$= \gamma G(n,N)z(N) + \gamma \sum_{t=N}^{n-1} g(n,t)\sqrt{G(n,t)}z(t) - \sum_{t=N}^{n-1} G(n,t)\Delta^2(z(t)) \tag{5}$$

$$\sum_{t=N}^{n} G(n,t)(q(t)x^{\gamma}(\tau(t)) = \gamma G(n,N)z(N) + \sum_{t=N}^{n-1}\left(\gamma g(n,t)\sqrt{G(n,t)}z(t) - G(n,t)\Delta^2(z(t)\right) \tag{6}$$

where $n \geq N$ and now using the perfect-square expression we get,

$$\gamma g(n,t)\sqrt{G(n,t)}z(t) - G(n,t)\Delta^2(z(t)) = \gamma \frac{1}{4} g^2(s,t) + \gamma\left(\frac{1}{2} g(n,t) - \sqrt{G(n,t)}z(t)\right)^2$$

$$g(n,t)\sqrt{G(n,t)}z(t) - G(n,t)\Delta^2(z(t)) \geq \frac{1}{4}g^2(s,t)$$

for every $(n,t) \in N_0$ and converges to a finite value eventually. Therefore the following inequality is obtained,

$$\sum_{t=N}^{n} \gamma g(n,t)\sqrt{G(n,t)}z(t) - G(n,t)\Delta^2(z(t) \leq \gamma\frac{1}{4}\sum_{t=N}^{n} g^2(s,t) < \infty$$

where $n \geq N$. When the above inequality is combined with (6) then,

$$\sum_{t=N}^{n}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma\frac{1}{4}g^2(s,t)\right) \leq \gamma G(n,N)z(N)$$

Therefore,

$$\sum_{t=n_0}^{n}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma\frac{1}{4}g^2(s,t)\right)$$

$$= \sum_{t=n_0}^{N}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma\frac{1}{4}g^2(s,t)\right) + \sum_{t=N}^{n}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma\frac{1}{4}g^2(s,t)\right)$$

$$\leq \sum_{t=n_0}^{n}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma G(n,N)z(N)\right)$$

Considering our assumption of $-\Delta_2 G(n,t)$ we know that $H(n,n_0) \geq H(n,t) > 0$. Thus,

$$\frac{1}{G(n,n_0)}\sum_{s=M}^{n-1}\left(G(n,t)(q(t)x^{\gamma}(\tau(t)) - \gamma\frac{1}{4}g^2(s,t)\right) \leq \sum_{t=n_0}^{N}\left(\frac{G(n,t)}{G(n,n_0)}(q(t)x^{\gamma}(\tau(t)) - \frac{G(n,N)}{G(n,n_0)}z(N)\right)$$

$$\leq \sum_{t=n_0}^{N}\left((q(t)x^{\gamma}(\tau(t)) - \gamma z(N)\right) < \infty$$

for every $n > n_0$, implies contradiction which completes the proof.

Theorem 2: Considering our previous assumptions, $-\Delta_2 G(n,s)$ for every $n > s \geq n_0$, then,

$$\lim_{n\to\infty} sup\frac{1}{G(n,N)}\sum_{s=N}^{n-1}\left[G(n,s)q(n) - \frac{g(n,s)z(n+1)}{G(n,s)q(s)}\right] = \infty$$

then all solutions of (1) become oscillatory or tends to 0 when n tend to ∞.

Proof: If the solution, $x(n)$ is not oscillatory for (1) then take $x(n)$ to be positive. Only this case is considered as the proof is similar for $x(n)$ to be a solution which is non-positive for (1). From $n_1 \in (n_0,\infty)$ there is $n_2 \in (n_1,\infty)$ such that, $\Delta^3 y(n) > 0$ and $a(n)\Delta^3(y(n)) < 0$. Define the function $\ni x(n)$ be the solution of (1) for $n = \eta$ then,

$$z(\eta) = \frac{a(\eta)\Delta^3(y(\eta))^{\alpha}}{x^{\gamma}(\tau(\eta))}$$

for $\alpha > 0, \gamma > 0$ then,

$$\Delta z(\eta) \leq \frac{q(\eta)x^{\gamma}(\tau(\eta))}{x^{\gamma}(\tau(\eta+1))} - \frac{z(\eta+1)}{x(\tau(\eta+1))} + \gamma x^{\gamma+1}(\tau(\eta))z(\eta)$$

From above inequality we have,

$$\sum_{s=N}^{\eta-1} G(\eta,s)\frac{q(s)x^{\gamma}(\tau(s))}{x^{\gamma}(\tau(s+1))} \leq -\sum_{s=N}^{\eta-1} G(\eta,s)\frac{z(s+1)}{x(\tau(s+1))} + \sum_{s=N}^{\eta-1} G(\eta,s)\gamma x^{\gamma+1}(\tau(s))z(s)$$

By applying summation by parts, we get

$$\sum_{s=N}^{\eta-1} G(\eta,s)\gamma x^{\gamma+1}(\tau(s))z(s) \leq x^{\gamma+1}(\tau(N))z(N) - \sum_{k=1}^{\eta} z(k)\left(G(\eta,s)x^{\gamma+1}(\tau(k+1)) - G(\eta,s)x^{\gamma+1}(\tau(k))\right)$$

By substitution we obtain,

$$\sum_{s=N}^{\eta-1} G(\eta,s)\frac{q(s)x^{\gamma}(\tau(s))}{x^{\gamma}(\tau(s+1))} \leq -\sum_{s=N}^{n-1} G(\eta,s)\frac{z(s+1)}{x(\tau(s+1))} + \gamma\left(x^{\gamma+1}(\tau(N))z(N) - \sum_{k=1}^{\eta}\left(z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)\right)\right) \quad (7)$$

Hence from (7) we have,

$$\sum_{s=N}^{\eta-1} G(\eta,s)\frac{q(s)x^{\gamma}(\tau(s))}{x^{\gamma}(\tau(s+1))} \leq -\sum_{s=N}^{n-1} G(\eta,s)\frac{z(s+1)}{x(\tau(s+1))} + \gamma x^{\gamma+1}(\tau(N))z(N) - \gamma\sum_{k=1}^{\eta} z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)$$

$$\leq \gamma x^{\gamma+1}(\tau(N))z(N) - \sum_{s=N}^{\eta-1} G(\eta,s)\frac{z(s+1)}{x(\tau(s+1))} - \gamma\sum_{k=1}^{\eta} z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)$$

From Theorem 1 we get,

$$\sum_{s=N}^{\eta-1} G(\eta,s)\frac{q(s)x^{\gamma}(\tau(s))}{x^{\gamma}(\tau(s+1))} \leq \gamma x^{\gamma+1}(\tau(N))z(N) + \sum_{s=N}^{\eta-1} g(\eta,s)\frac{z(s+1)}{x(\tau(s+1))} - \gamma\sum_{k=1}^{\eta} z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)$$

Now letting the assumptions such that, $u = G(\eta,s)q(s)$, $v = x(\tau(s))$, $A = \gamma x^{\gamma+1}(\tau(N))z(N)$, $B = g(\eta,s)z(n+1)$, $C = x(\tau(s+1))$, $D = \gamma z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)$. By the last inequality we have,

$$u\left(\frac{v}{C}\right)^{\gamma} - A + \frac{B}{C} + D \leq \frac{B}{u}\left(\frac{\Delta v}{C-v}\right)$$

Therefore,

$$\sum_{s=N}^{\eta-1} G(n,s)\frac{q(\eta)x^{\gamma}(\tau(\eta))}{x^{\gamma}(\tau(\eta+1))} - \gamma x^{\gamma+1}(\tau(N))z(N) - \sum_{s=N}^{\eta-1} g(\eta,s)\frac{z(\eta+1)}{x((\eta+1))} + \gamma\sum_{k=1}^{\eta} z(k)\left(\Delta G(\eta,s)x^{\gamma+1}(\tau(k))\right)$$
$$\leq \sum_{s=N}^{\eta-1}\frac{g(\eta,s)z(n+1)}{G(\eta,s)q(s)}$$

With (7) and since $G(\eta,\eta) = 0$ then,

$$\sum_{s=N}^{n-1}\left[G(\eta,s)q(n)x^{\gamma}(\tau(\eta)) - \frac{g(\eta,s)z(\eta+1)}{G(\eta,s)q(s)}\right] \leq \gamma x^{\gamma+1}(\tau(N))z(N)$$

For $\eta \geq N$,

$$\lim_{n\to\infty} sup\frac{1}{G(\eta,N)}\sum_{s=N}^{n-1}\left[G(n,s)q(n) - \frac{g(\eta,s)z(\eta+1)}{G(\eta,s)q(s)}\right] \leq \gamma x^{\gamma+1}(\tau(N))z(N)$$

This implies a contradiction to our assumption and the proof is complete.

Examples: To prove the main results, some examples are provided.

Example 1: Consider the difference equation given as,

$$\Delta(n+1)\Delta^3 x(n) + (16n+24)y(n+1) = 0 \qquad (8)$$

Here $\{x(n)\} = \{(-1)^n\}$ becomes the solution for (8). Hence every solution in (1) oscillates.

Example 2: The following difference equation is considered,

$$\Delta^4 x_n - \frac{128}{(2+3^n)^2}\left(\frac{3^n}{8} + \frac{(-1)^n}{2^{n+4}}\right) x_n^2 = \frac{(-1)^{n+1}}{2^{n-3}} \qquad (8.1)$$

for $n \geq 3$ that satisfies all the conditions of Theorem 2. Here the solution of (8.1) is $\{x_n\} = \{2+3^n\}$. Thus (7) has non-oscillatory solution that approaches a non-zero real number.

Linear Time Invariants (LTI) System in Robotics:

The Philos type fourth order neutral difference equation is formulated denoting the accuracy of dynamical behaviour for electromechanical devices such as generators and motors. This involves recognition of key components of system, interactions and its dynamics. The mechanical and electrical properties, external influences and control inputs are considered and integrated while formulating the equation. We know from section 2 that LTI forms a significant role for controllers. Here robots are taken as an example of electromechanical device. For a difference equation, the LTI system can be developed easily in a unique way. Thus the LTI system is used for moving the parts of robots such as movement of arms and several other complex joints with certain degrees of freedom. The key components and variables for the systems to influence the formulation and structure of Philos type fourth order neutral difference equations consist of capacitance, inductance, electrical resistance, control signals, damping coefficients, time delays, mechanical inertia, and external loads. They play a vital role to define the dynamics of the system and to determine respective terms and coefficients of the equation. The function of an integer for any discrete signal converts the input function $x(n)$ to an output function $y(n)$ with time t.

Now we consider (1) and the fourth order difference equation is, $\Delta(a(n)\Delta^3(x(n) + p(n)x(\sigma(n))^\alpha)$ such that by [14] the input function is defined and given as $\{x(n)\}$ then,

$$\big(a(n+1) - a(n)\big)\{x(n+4) - 4x(n+3) + 6x(n+2) - 4x(n+1) + x(n)\}$$

Then the respective output function $\{y(n)\}$ of (1) become,

$$\big(a(n+1) - a(n)\big)\{y(n+4) - 4y(n+3) + 6y(n+2) - 4y(n+1) + y(n)\}$$

The delay in philos difference equations indicates the time taken for the system to respond for inputs or variations or any stimuli. This influence oscillatory behavior, complete dynamics and stability of the system.

Limitations:

The neutral difference equations are appropriate to model dynamical systems as they possess the capability in integrating past and present time data which makes them compatible for systems having memory effects or delays. Also they are able to capture the interactions between various components and their relations over a period of time. However, in real time there are challenges while employing philos type fourth order neutral difference equations for electromechanical devices of dynamic systems which include stability issues, computational complexity, need for fast response times and parameter uncertainties. Overcoming such issues can be attained using multiple techniques such as model simplification techniques, advanced control algorithms, hardware-in-the-loop simulations and adaptive strategies which helps in validating the performance of the model. In practical applications deviations in environmental condition and material properties significantly affect the efficiency in accuracy for Philos type difference equations based models. The variations in factors like temperature, friction, external disturbances, etc leads to discrepancy in predicting the model compared to actual model predictions. Thus the sensitivity analysis and robust conditions are vital while the uncertainties are considered.

CONCLUSION

The paper shows that the Philos type fourth order neutral difference equations gives an understanding about the interaction between mechanical and electrical components. When comparing to other mathematical model the Philos type difference equations provide a better insight in dynamics with oscillatory behavior. We conclude from this paper that the Philos type oscillatory criteria of (1) are obtained by Riccati technique and summation averaging method. A study on electromechanical devices led to the application of fourth order difference equation in robots using LTI system.

FUTURE WORK

The future work include a study on models of Philos type difference equations with higher order and develop advanced strategies by investigating the disturbances and uncertainties and also to study on the technique of Hybrid modelling for establishing the behaviour of electromechanical systems with varying parameters. The work also includes a study on dynamics of hand-control robots using higher order difference equations.

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