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Research Article

Theoretical-Experimental Modelling Experience (TEME) as a Methodological Proposal for Learning Electrostatics: A Case Study of the Parallel Plate Capacitor

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ABSTRACT

This paper presents an alternative proposal within the framework of learning factual and formal sciences. It emphasizes the importance of focusing on student performance rather than the mechanization of operations. To achieve this, a Theoretical and Experimental Modeling Experience (TEME) is proposed, with the objective of contrasting theoretical arguments discussed and reinforced in the classroom. The primary topic selected for this approach is the study of the 2D Laplace equation within the field of potential theory, designed for an Advanced Mathematics and Physics course on Electricity and Magnetism at the Nueva Granada Military University and the Military Engineers School. The Theoretical Experimental Modeling Experience (TEME) is grounded in the need to establish dialogical relationships between different fields of knowledge, recognizing that science is inherently complex. Methodologically, this approach integrates theory, experimentation, and computational analysis. In this particular case, it refers to the theory of partial differential equations and potential theory, thus adopting an interdisciplinary stance.

Keywords: Science Learning, Electrostatic Phenomena, Theoretical Experimental Modeling Experience (TEME), Interdisciplinarity.

INTRODUCTION

Our work as teachers imposes great challenges on a pedagogical and didactic level to promote the understanding of fundamental concepts, both basic and advanced in the field of basic sciences. The purpose of this dissertation is to describe the benefits of the implementation of a Theoretical-Experimental Modeling Experience (TEME), promoting a convergence between the modeling of the phenomenon under study, the experimental practices designed with the aim of building causal relationships of the system and its simulation through the use of computational tools. The TEME is framed within the field corresponding to electromagnetic theory, particularly potential theory. It is intended that students interpret the related foundational aspects of a particular situation: solution of the Laplace's Equation.

In 1986 Ilya Prigogine wrote the second edition of his book, The New Alliance - Metamorphosis of Science, in which he problematizes the processes of intelligibility of science, by highlighting the need for a new relationship between human beings and nature, by deconstructing the relationships between the human sciences and the factual sciences and formal sciences (Prigogine & Stengers, 1986). His particular conjecture affirms that the intelligibility of the world can no longer be sustained from legality (existence of universal laws), determinism and reversibility, the new science (Contemporary Science) makes the world explicit through new categories; relativity, not – locality, indeterminacy, undecidability, incompleteness, uncertainty, bifurcation, in short, a new glimpse of intelligibility, science constitutes a bridge between culture and the theories, models and explanations reached and realized. The implementation of an TEME seeks to break with a construction of fragmented scientific knowledge, by taking uncertainty in the classroom as an element that generates discussion and criticism, by moving away from the dichotomous position of the traditional educational act: good or bad, in terms of Neil Postman with his book Teaching as a subversive activity, knowledge cannot be "transmitted", it does not emanate from a higher authority, and it needs to be questioned (Postman & Weingartner, 1969).

The relevance of Postman's reflections extends beyond the general educational realm and can be applied to the specific context of engineering education. In this regard, modeling and its causal relationship in the construction of Critical Scientific Thinking (CST) emerge as fundamental areas for the development of skills and competencies in future engineers, regardless of their specialization. These concepts involve the ability to understand and represent phenomena, starting from the simple, moving to the complicated, and finally to the complex, as well as the ability to use computational tools to analyze and solve problems specific to the field of engineering.

SCIENTIFIC THINKING

The epistemological foundation of our proposal is deeply influenced by the ideas of Jorge Wagensberg, a physicist and philosopher whose interdisciplinary approach and role as a science communicator were pivotal in the social appropriation of knowledge. At the University of Barcelona, where he taught Theory of Irreversible Processes, Wagensberg encouraged critical and reflective thinking among his students. He believed that education should transcend mere information transmission, fostering active dialogue and a profound appreciation for the natural world.

In (Velásquez González, 2008) Jorge Wagensberg regarding the question: how can learners construct knowledge? proposes three successive, though not disjointed, phases for the generation of new knowledge. The first phase concerns the stimulus: “in this phase, one decides what they want to know, being very aware that stimuli provoke changes in mood. It can generate a state of disinterest in knowing something or, on the contrary, create a sense of urgency to learn. Without this initial phase, the generation of cognitive processes would be nonexistent.” The second phase addresses conversation. According to (Wagensberg, El pensador intruso: el espíritu interdisciplinario en el mapa del conocimiento, 2014), there are three conversational categories, which interact dynamically with nonlinear logics, involving the entire cognitive process. Essentially, a conversation is an exchange of questions and answers. The first conversational category refers to conversation with reality (seeing, watching, observing, experimenting, simulating). The second refers to conversation with others (colleagues, teachers, learners, disciples), and the most important for us as learners and co-mediators of learning is the conversation with oneself (thinking, reflecting). “Conversation is the center of gravity for the construction of new knowledge and serves to confront reality with its possible interpretations and to decide between different alternatives.” By provoking conversational events, the path is constructed toward the moment before comprehension. Finally, we have intuition and understanding. For Wagensberg, making reality intelligible is achieving "the minimal expression of the maximum shared understanding.”

In (Wagensberg, On the Existence and Uniqueness of the Scientific Method, 2014) later revisits in the book Theory of Creativity (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017), he proposes three fundamental principles involved in the construction of Scientific Thinking: objectivity, intelligibility, and dialectical engagement with reality. He suggests that these principles are not absolute but can be applied to varying degrees depending on the complexity of the subject matter being studied. In other words, the understanding of reality, shaped by the relationship between the subject and the object of knowledge, is framed within three fundamental concepts: the first is reality, the second is the observation of reality, and the third is the understanding of the observation of that particular reality.

The foundational conjecture, which is indeed in line with the argumentative framework of this theoretical-methodological proposal, focuses on establishing the limits of scientific knowledge based on: 1) reality is observable, 2) observation is comprehensible, and 3) Understanding is falsifiable (which passes through a dialectical filter, in Thomas Kuhn's terms of 'falsifiability.”). In other words, understanding from a scientific standpoint must be delimited based on those three conjectures or hypotheses. We cannot speak of the intelligibility of a finite portion of reality if that reality in question cannot be observed or accessed directly or indirectly.

In the same argumentative logic, Wagensberg states, “it is also not possible to speak of science based on an unintelligible observation, even if reality is observable. Additionally, science cannot be made from a comprehensible observation if it turns out that it is not falsifiable, even if reality is observable and the observation is comprehensible.”

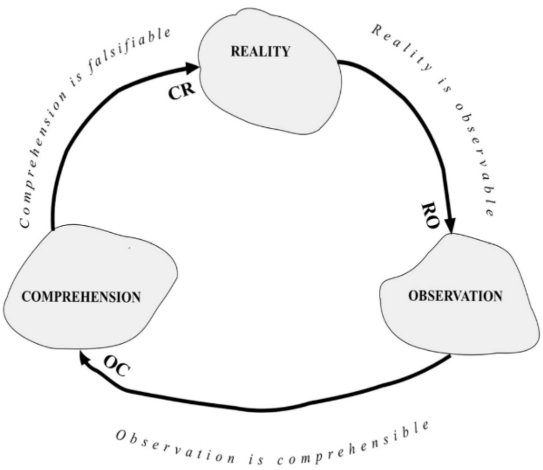


Figure 1: Connections between the three fundamental hypotheses: (1) reality is observable (RO); (2) observation is comprehensible (OC); and (3) comprehension is falsifiable (CR) in relation to the three initial concepts of reality (R), observation (O), and comprehension (C) (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017).

The Theoretical Experimental Modeling Experience (TEME), as a theoretical-methodological proposal, borrow from Wagensberg's thinking regarding the question: How do we know? This approach extrapolates his ideas to both formal and informal educational contexts by creating spaces for the concretization of the educational act. In our particular case, it understands criticality as the result of the looped relationships among the epistemological, ontological, and axiological dimensions intrinsic to learning.

The fundamental principles of objectivity, intelligibility, and the dialectical principle must be in accordance with an epistemological and ontological context.

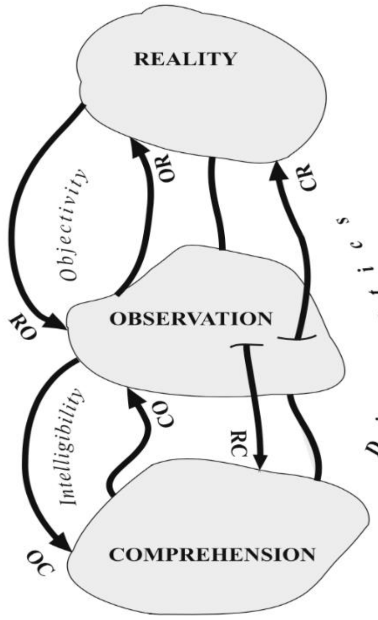


Figure 2: The three fundamental principles (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017).

FROM MODELING TO THE CONSTRUCTION OF CRITICAL SCIENTIFIC THINKING

In the early 1970s, following the maturation of Norbert Wiener's ideas in his book Cybernetics: Or Control and Communication in the Animal and the Machine (Wiener, 1948), the contributions of Mario Bunge with his biophilosophy and the concept of scientific model, as discussed in Intuition and Science (Bunge, 2016) , the legacy of Ludwig Von Bertalanffy reflected in his General Theory of Systems (Von Bertalanffy, 1976), and the contributions of Jay Forrester for the understanding and modeling of complex systems with Urban Dynamics (Forrester, 1970), allowed for the generation of approximate yet crucial responses by qualitatively and quantitatively making intelligible the global dynamics, both environmental and equitable among the countries and regions of the world, recognizing the complex nature and the impossibility of addressing them in isolation. By 1972, the first robust reflection emerged from the Club of Rome, a group composed of scientists, economists, biologists, politicians, among others, who, through the book The Limits to Growth (Meadows, Meadows, Randers, & Behrens III, 1972), demonstrated—via computational simulations—that the global patterns of economic, demographic, natural resource use, and environmental pollution growth would be unsustainable during the 21st century.

In other words, what the Club of Rome accomplished is nothing more than the structural synthesis upon which scientific thinking is built: striving to make reality intelligible. For this, the thinker constantly gravitates towards the idea of a model as a means of representing reality. Human beings naturally model all the time, even if they are unaware of this fact. When we speak or write, we construct models of what we think. The image of the world around us, based on what we perceive, is definitely simpler than the real world. If we focus on the image of the world of a blind person, their way of making reality intelligible will be significantly different from that of a person with all their senses. However, both perspectives represent a simplified version of the same reality. In simpler terms, a model is not a crude copy of reality; rather, it is an interpretation of it.

The meaning of "model" has been discussed by scientists, linguists, educators, and philosophers of science. However, many agree that a model is a representation of an idea, event, object, or objects, constructed and defined with a specific purpose. Without models, the development of scientific thinking would be unimaginable, as they allow us to simplify complex phenomena by accounting for abstract entities. They help us interpret causal relationships during experiments and enable decision-making based on the study's context, among many other things. It is clear that the success of a model hinges on achieving a balance between the realism and simplicity of the situation being studied. Since each model is a human construction designed for a specific purpose, its validity is defined within a particular field of knowledge, and thus, when we model, we do not "discover" an absolute truth, as no model is universally valid. Models allow for the formulation of answers, and this is where the observer $\boldsymbol{O}$ becomes relevant, as they are the ones who ask the pertinent questions: What problem needs solving? What question needs answering? From this stance, Aracil states: "An object $\boldsymbol{M}$ is a model of an object $\boldsymbol{S}$ for an observer $\boldsymbol{O}$, if the observer can use $\boldsymbol{M}$ to answer questions that concern them in relation to $\boldsymbol{S}$." As mediating instruments between reality and theory, models become research tools with the ability to be extrapolated to other fields of knowledge.

In the case at hand, learning in the construction and use of models can become an effective way to build scientific knowledge by creating representative structures. In the case of classical mechanics as a prototype of a physical theory, David Hestenes states: The great game of science is the modeling of the real world, where each scientific theory defines a system of rules for participating in that game. The object of the game is to build valid models of real objects and processes. These models form the core of scientific knowledge. To understand science, it is necessary to know how these models are built and validated. The main objective of science education should be to teach the game of modeling (Hestenes, 1993).

It is necessary to present the foundational concepts of Modeling from the perspective of the author of this dissertation in dialogue with its contemporary representatives.

A. Partition

We understand a partition as a portion of reality and, therefore, of intelligibility within the epistemological and ontological dimensions provided by the context. A partition, depending on the context—subpartitions can even be discussed—constitutes the first tool of science in the pursuit of a representation of reality. In other words, science corresponds to the study of finite nature partitions, serving as an epistemic and ontological bridge that brings the thinker closer to a reality of infinite nature. Understood this way, a particular partition equates to a particular form of scientific intelligibility. The history, sociology, and epistemology of science highlight the beginnings, development, and evolution of these forms of intelligibility.

According to authors such as Carlos Eduardo Maldonado (Maldonado Castañeda, 2022), Ilya Prigogine (Prigogine & Stengers, 1986), Ian Stewart (Stewart, 2008), and Jorge Wagensberg (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017), we can speak of two particular partitions, namely:

The Partition of Modern Logic:

This pertains to the understanding of realities that can be expressed in terms of linear causalities, through the use of mathematical objects, such as Differential Equations and/or Difference Equations, which seek to provide solutions to initial and/or boundary value problems inherent to the context of the reality under study. The search is thus reduced to the numerical or analytical formulation of integral trajectories, resulting from the linear combination of particular solutions with homogeneous solutions. In other words, it represents the theorem of existence and uniqueness at its maximum splendor. Concrete examples of this Modern Logic can be observed in Classical Mechanics in terms of Newton's fluxion notation, Analytical Mechanics in terms of Lagrangians and Hamiltonians, and most boundary problems typical of Thermodynamics, Electromagnetism, Fluid Mechanics, and Material Resistance, when expressed in terms of differences and/or finite elements (numerical solution), to mention just a few examples.

The Partition of Complex Logic:

This logic corresponds to partitions where unique properties arise from the interactions between the parts of the system under study. This particular type of attribute is known as emergent properties—Emergence—and was studied in the early 1970s and 1980s by authors such as Ilya Prigogine, Alan Turing with his Turing patterns, which currently constitutes an entire field of interdisciplinary knowledge called Morphogenesis. In the same argumentative line, the detection of nonlinear phenomena for climate prediction by Edward Lorenz and the contributions of Rolando García with his “Latin American” interpretative model, as well as the Club of Rome in the mid-1970s, are some examples of this shift in how reality is made intelligible. Unlike the Partitions of Modern Logic, the mathematical objects involved are expressed by nonlinear differential or difference equations, susceptible to minimal changes in initial and/or boundary conditions—a behavior known as the butterfly effect—where the principle of existence and uniqueness fails dramatically. We are talking about cascades of bifurcations, in contrast to integral trajectories. For the case of this paper, it falls within the study of Modern Logic partitions.

B. System

To avoid potential confusion between the concepts of system and partition, the partition refers to the epistemological and ontological lenses that allow for the intelligibility of the reality in question. Therefore, they establish the rules of modeling: linear or cyclical causalities, the existence of emergent attributes, and the system's adaptability or lack thereof.

Once the reality under study is delineated, the interactions between the parts and the environment will be described by the type of partition to which they belong; consequently, the language will change radically, transitioning to mathematical objects such as Differential Equations, Difference Equations, Stochastic Equations, Integral-Differential Equations, and Adaptive Genetic Algorithms, to mention a few. To account for the system, it is necessary to identify the variables, parameters, and constants specific to the reality being studied.

One of the foundational facts that encouraged the generation of epistemic bridges between Modern Logic and Complex Logic partitions revolves around the question: What happens with interactions? These are understood as the result of the exchange of "information": matter, energy, or information itself. We can speak of interactions among the parts of the system; a classic example would be the study of the cooling or heating of a liquid inside a specific container. When addressing the system under study, the parts would correspond to the liquid and the container. It is clear that the parts interact; for instance, the cooling time is not the same if we change the material of the container while keeping the same volume of liquid under identical initial conditions (initial temperature, initial volume, etc.).

From the results of these interactions, we can quantify parameters such as specific heat, the cooling or heating coefficient of the portion of liquid under study, and heat as a form of energy transfer. However, our thought experiment does not specify whether the system interacts with its surrounding environment, known in scientific language as the environment: that which exists outside the imaginary delimitation under study. This fact allows us to adopt a new form of understanding regarding interactions: the interactions between the system and its environment. Paying attention to these interactions was key to the development of fields such as biology, zoology, non-equilibrium thermodynamics, sociology, engineering, and education itself. In this way, terms like emergence (the formation of new properties arising from the interactions among the parts: the Whole), adaptability, and self-organization, among many others, enrich the idioms specific to Complex Logic partitions.

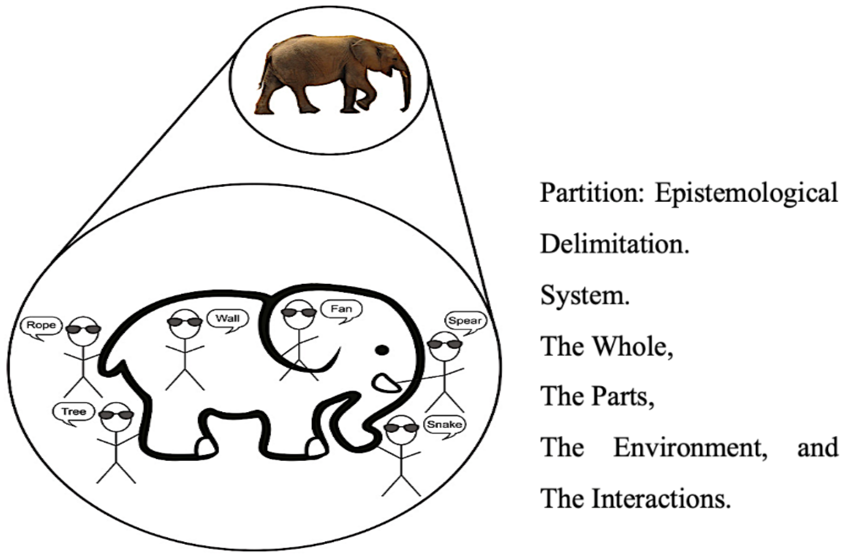


Figure 3: (Diaz Eaton, et al, 2019)

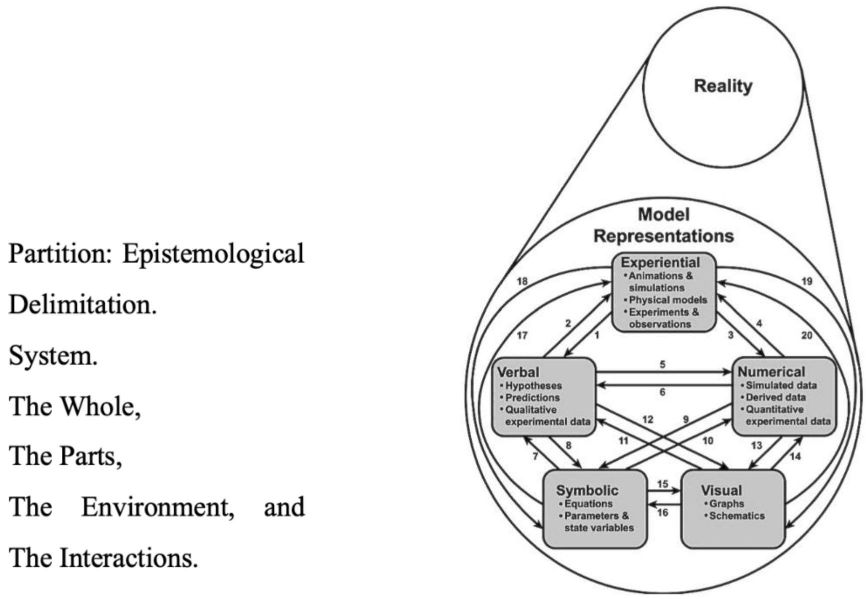


Figure 4: (Diaz Eaton, et al, 2019)

PEDAGOGICAL MEDIATION AND ITS ROLE IN THE CONSTRUCTION OF CRITICAL SCIENTIFIC THINKING

Before discussing pedagogical mediation, the foundational question must address: What is Pedagogy? We understand pedagogy as the genesis of the various possibilities for each human being to construct learning. In this way, pedagogy allows us to create spaces for reflection and understanding of the meaning of learning, as well as the need to form a critical pedagogical discourse grounded in knowledge and practice. Simply put, to exercise our inner freedom as co-mediators of learning, it is crucial to meaningfully appropriate a pedagogical discourse that demystifies the classroom as a symbol of the educational act, instead redefining it as the path one takes when constructing oneself as a human being.

A pedagogy of meaning has learning as its primary reference point. When, in an educational process, learning is frustrated, disoriented, or confused, problems with its very meaning arise. For the authors, with-meaning emerges as an interaction of complex nature, recognizing the meaning of the educational act not only in learning itself but also in the inherent uncertainties of those who are learning. What is remarkable is that this dimension, called with-meaning, is complex and requires more actors to fully comprehend it.

Theoretical, methodological, and practical proposals should aim to foster learning through genuine pedagogical environments, allowing Learners and Co-mediators of learning to fully leverage their capabilities. The educational task is not limited to merely transmitting knowledge or modifying behavior. A discourse acquires a pedagogical nature when it is aimed at promoting learning, thus establishing a meaningful connection between all the 'beings' involved in the educational act.

Now, if we take the dialogical relationship between the Learner and the Co-mediator of learning as the starting point—two equally important beings, without distinctions, and in a state of intellectual joy—the discourse undergoes a profound change, revealing one of the primary characteristics of the pedagogical task: interlocution. It is no longer merely about transmitting information, but about engaging with those beings to whom and for whom we express ourselves, whether orally or in writing. The pedagogical discourse is a public discourse, in which we interact with other beings; it is a discourse oriented toward the other.

Now, the second foundational question must address what Pedagogical Mediation is: As a conceptual-methodological stance, it emerged from two distance education projects at Rafael Landívar University and San Carlos University in Guatemala, conducted between 1987 and 1993. Its most influential proponents are educators Francisco Gutiérrez and Daniel Prieto Castillo, through their publications (Gutiérrez Pérez & Prieto Castillo, La Mediación Pedagógica: Apuntes para una Educación Alternativa a Distancia, 1991) and (Gutiérrez Pérez & Prieto Castillo, Mediación pedagógica para la educación popular, 1994). The term 'mediation' refers to the communicative treatment of content, forms of expression, and learning practices, whether in-person, virtual, or at a distance. In this way, the educational act, as a complex phenomenon, allows for the emergence of dimensions such as participation, creativity, expressiveness, and rationality.

The starting point for Pedagogical Mediation, as (Prieto Castillo & Van de Pol, 2006) puts it, is "a pedagogy of meaning, through which we assert that in the educational relationship, the fundamental meaning lies in the fact that beings are involved in the task of teaching and learning". However, the meaning of the educational act is not limited to learning alone, but also to the development of the beings who are learning. As educators (both individuals and institutions), we welcome beings who are in a process of personal construction, and who, at the same time, must learn. The meaning of the educational act also, and fundamentally, involves the possibility for someone to construct themselves as a human being. There are educational processes from which people emerge well-formed, others moderately formed, others poorly formed, and some even destroyed. From this perspective, the proposal by (Gutiérrez Pérez & Prieto Castillo, La Mediación Pedagógica: Apuntes para una Educación Alternativa a Distancia, 1991) can be summarized as follows: "Between an area of knowledge and human practice and those who are in a position to learn, society offers mediations. We refer to a mediation as pedagogical when it is capable of promoting and supporting learning".

In other words, we understand Pedagogical Mediation as the act of concretizing, in terms of promoting and supporting learning. The task of mediating culminates when the other has developed what is necessary to continue independently.

Pedagogical Mediation aims to educate in and for uncertainty, addressing the need for adaptability required by the changing contexts of our reality. As (Postman & Weingartner, 1969) and (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017) pointed out, contemporary educational systems often fail to adequately prepare individuals to face uncertainty, frequently associating it with error or failure. This can lead to a disconnection in learning practices, as is evident in issues such as climate change, social crises, and more recently, the Covid-19 pandemic, which has highlighted the urgency of incorporating uncertainty into educational frameworks.

Educating in uncertainty involves both a collective and individual construction of knowledge, encouraging respectful and constructive dissent. This process has an affective nature that must not be overlooked, despite the global tendency to prioritize the cognitive. In addition to acquiring knowledge, human beings must learn to live in society, which is why it is essential for education to provide flexible tools that enable learners to address their challenges without resorting to rigid learning practices.

Our worldview is based on a quest for intelligibility of a finite portion or object of study. In the words of (Wagensberg, Teoría de la creatividad: eclosión, gloria y miseria de las ideas, 2017), 'Science is a fiction of reality'. The issue lies in the type of fiction we conceive; it could be one grounded in linear and dichotomous causal relationships. However, despite this, as educators, we promote strategic planning and design educational programs for learners, offering them solutions to a reality of stochastic nature, especially when referring to the immersion process into the workforce. Nevertheless, uncertainty also presents opportunities to seek common ground, cooperate, and build collectively. We then speak of a Pedagogical Mediation centered on questions: questions, in (Freire, 1986) terms, demystify absolutist proposals, that is, the search for non-existent universal answers.

Pedagogical Mediation presents itself as a critical-reflective stance, centered on questions as generators of ideas. Ideas are understood as particles of knowledge that form part of the collective immersed in the educational act.

As (Gutiérrez Pérez & Prieto Castillo, La Mediación Pedagógica: Apuntes para una Educación Alternativa a Distancia, 1991) point out, 'Knowing, demystifying, and resignifying means being able to face different social texts in order to read them critically'. In this sense, Pedagogical Mediation, as a Theoretical-Methodological proposal, effectively addresses changing times. Each situation requires responses adapted to its context, and it is likely that these responses will not be applicable in other scenarios. Pedagogical Mediation focuses on discovery, creation, serious critique, and deep analysis. In addition to providing tools for individuals to act in the world and transform it, while simultaneously transforming themselves, it must also generate the necessary scenarios and resources for individuals to learn how to construct their own conceptual tools. This process is directed toward the construction of solid thinking, and in our case, we specifically refer to Critical Scientific Thinking.

In this discourse on the construction of the concept of pedagogical mediation, we will explore its application from the perspective of technology. The intrinsic value of a technology in supporting learning lies in its capacity to interact with users, particularly through the appropriation of its communication resources. It is clear that, beyond mere consumption of technologies, their true value resides in the ability to leverage them for integration into expressive resources at both individual and group levels. In this context, understanding the impact of technologies goes far beyond the mere concept of information and the supposed 'transmission' of content.

Therefore, in the educational act, Pedagogical Mediation in technological environments involves the creation of spaces that promote the search, processing, and application of information, giving it meaningful context. A technology acquires pedagogical value by utilizing its communication capacity to make the phenomenon under study intelligible. In this sense, the pedagogical value of technologies lies in their mediating function, which fosters and supports the learning process. This involves not only taking advantage of their communicative potential but also having an explicit purpose to mediate various materials, using these technologies reflectively and effectively in educational settings.

Thus, we can approach the use of technologies from four perspectives: their use, production, distribution, and application of information, allowing for varied interaction with other actors in the educational process. Furthermore, these technologies provide a space for intellectual enjoyment and the generation of new ideas, contributing to improved communication at both institutional and social levels.

The role of the teacher, hereafter referred to as the co-mediator of learning in the field of basic sciences and applied sciences, presents significant pedagogical challenges that have a profound impact on the development of critical scientific thinking. Learning must have intentionality, which we refer to as "meaningful learning," in contrast to "meaningless learning," characterized by the absence of context and, consequently, the inability to form emergent structures from peer-to-peer conversations, as well as dialogues between learners and the co-mediator of learning. This process ultimately culminates, without a specific order of occurrence, in an internal conversation that materializes at the moment knowledge is constructed, achieving what (Wagensberg, El gozo intelectual: teoría y práctica sobre la inteligibilidad y la belleza, 2007) calls "intellectual joy," a moment of revelation. From this argumentative cycle, it is clear that learning in basic sciences reveals a disconnection between the various academic spaces (islands of knowledge) that form the bulk of the curriculum in each specific degree, characterized by the absence of reflective dialogue between basic sciences, applied sciences, and humanities.

THEORETICAL EXPERIMENTAL MODELING EXPERIENCE (TEME)

This section offers an initial approach to theorizing the concept of Theoretical Experimental Modeling Experience (TEME), as understood by the authors of this dissertation. The epistemological and ontological foundation of this notion is derived from the discourse developed in the previous sections, which is synthesized into two fundamental questions: What do we understand by scientific thinking? and What is the role of pedagogical mediation in the construction of critical scientific thinking?

In this argumentative framework, the Theoretical-Experimental Modeling Experience (TEME) is presented as an epistemological bridge between the abstract and concrete dimensions, aimed at building Critical Scientific Thinking. This is achieved by materializing, through "experimental" practices, the fundamental laws that describe a specific phenomenon, correlating them with the mathematical objects employed, such as algebraic equations, differential equations, difference equations, integro-differential equations, and partial differential equations.

From this perspective, the Theoretical Experimental Modeling Experiences (TEME) seek to establish non-linear causal relationships between three forms of intelligibility represented by scientific thinking, mathematical thinking, and computational thinking. These interactions are explored both in school settings and non-school environments, with an emphasis on the “experimental” application to understand and solve real-world problems in physics and/or engineering, within the context of this article.

The Theoretical Experimental Modeling Experiences (TEME) represent an effort to materialize the scientific discourse in educational settings, emphasizing the necessity of establishing dialogical relationships for the study of partitions that aim to describe realities whether simple, complicated, or complex, depending on the study context. To achieve this, both the learner and the co-mediators of learning must have a clear understanding of the validity ranges of the models used, which is essential for robust description and the potential resolution of context-related problems. Furthermore, the Theoretical-Experimental Modeling Experiences (TEME) utilize experimentation as a dialectical filter in the pursuit of determining the validity ranges by the learner.

It is important to clarify that the concept of experimentation, as used in this dissertation, encompasses both measurement processes and model simulations. This includes:

1. Direct or indirect measurement: utilizing both analog and digital instruments to record data from the system under study. This process involves the acquisition of empirical data, which contributes to the understanding of the physical phenomenon under investigation.

2. Mathematical experimentation: refers to the simulation and analysis of mathematical models using algorithms implemented in programming languages or computational algebra systems.

For the case study on electrostatic phenomena presented in this article, four technological tools were employed: Excel as a spreadsheet, the Maxima CAS (Maxima, 2024), and two finite element modeling software packages: QuickField student edition (QuickField, 2024) and FEMM (Finite Element Method Magnetics, 2024) .The Theoretical Experimental Modeling Experiences (TEME) also aim to promote the integration of theoretical modeling, experimental practices, and computational simulation to offer a holistic approach to understanding phenomena. Given the inherent complexity of scientific knowledge, it is methodologically essential to adopt an integrative approach that combines theory, experimentation, and computational analysis.

In this context, the core thesis is that exposing students to the practices of scientific inquiry facilitates the process of knowledge construction, particularly in disciplines like Classical Electromagnetic Physics. This specific study focuses on potential theory, particularly the solution of the two-dimensional Laplace equation in steady-state conditions.

Roadmap for the Implementation of a Theoretical-Experimental Modeling Experience (TEME)

The implementation of a Theoretical Experimental Modeling Experience (TEME) is based on a series of stages that integrate academic, scientific, and technological contexts within an interdisciplinary framework. Below are the main steps to follow:

I. Identification of a Rich Context Experience (RCE): The first step is to select an experience with a sufficiently broad and deep context to encompass various mathematical objects, such as differential equations, difference equations, stochastic equations, integro-differential equations, and adaptive genetic algorithms, to name a few. Additionally, it should address the physical laws governing the phenomenon under study. In this case, a parallel plate capacitor, where an electrostatic field is confined, has been chosen as the "finite portion of reality." This system allows for epistemological connections between the study of physical phenomena, such as the formation of isotherms in confined systems (thermodynamics of equilibrium), as well as the understanding of velocity fields in simple hydrodynamic phenomena (introduction to fluid mechanics).

II. Establishing the Scientific-Technological Framework:

To successfully implement the Theoretical Experimental Modeling Experience (TEME), it is essential to define the scientific and technological foundations upon which the model will be built. Key guiding questions for this process include:

- What are electric field lines, and how do they enhance our understanding of a capacitor as a passive device, with widespread applications in everyday life?
- Why do we speak of equipotential profiles rather than equipotential lines? Understanding this distinction is crucial for grasping the behavior of electric fields in confined systems.
- What is a partial differential equation, and how is it applied to solve the two-dimensional Laplace equation? This step is critical in modeling the electric potential distribution in systems like parallel plate capacitors.
- What are Fourier series, and how do they correlate with equipotential profiles? The Fourier series play a significant role in breaking down complex periodic functions, making them essential for understanding the spatial distribution of electric potential.

III. Ease of Constructing a Low-Cost Experimental Setup:

A central aspect of implementing TEME (Theoretical Experimental Modeling Experiences) is ensuring that the experimental setup is accessible and feasible in a home environment. This has become particularly relevant with the shift to remote learning due to the COVID-19 pandemic. The ability to replicate the experiment at home facilitates active student participation and learning, while also minimizing economic and logistical barriers.

IV. Building the State of the Art:

Students, along with co-mediators of learning, should engage in the construction of the state of the art related to the RCE (Rich Contextual Experience). To this end, ten scientific articles were reviewed, five in Spanish and five in English, covering both the scientific framework and the experimental and mathematical approaches to the phenomenon under investigation. This critical analysis enriches the understanding of the phenomenon and the tools used; furthermore, the utilization of Chat PDF (Chat pdf, 2024) and Elicit (Elicit, 2024) as generative tools in constructing the state of the art enhances this process by facilitating deeper insights and streamlined research efforts.

V. Defining the Foundational Objectives of TEME:

Each working group must formulate a guiding research question. This question will serve as the foundation for analyzing, interpreting, and discussing the results obtained at each phase of the experience. It is important to note that responses to specific objectives should not be seen as definitive conclusions, as ERCs, by their interdisciplinary nature, do not lead to unique solutions. Rather, they aim to generate critical and in-depth reflections on the phenomena studied. This approach encourages collaboration between diverse disciplines, such as physics, mathematics, chemistry, and automation, thereby strengthening the integrative and educational nature of TEME.

VI. Final Defense:

The culminating activity corresponds to the final presentation in IEEE article format, with a duration of 20 minutes per working group. This defense constitutes the final "ritual" of the pedagogical process, in which it is assessed whether the students have achieved a significant conceptual change.

SYSTEMATIZATION OF TEME

Table 1. Systematization of TEME

ERC: portion actually of study: learning co-mediator learner	***Epistemic fields: Scientific construct - Mathematical construct learning co-mediator***	***Establish the validity ranges of the model and modeling - ERC***	***Taking the data***	***Validating data with reality***	***Experience filters: analyze and infer***	***Reflection from dialogue with:***	***Academic ritual***
Contrast exercises type - textbook	Dialogues between fields of knowledge - academic spaces	Ideal models to dissipative models	A. Low-cost assembly B. Portability (pandemic effect) independence from the physical existence of a laboratory C. Choosing computational tools	Identify and classify variables, parameters, constants	What if?	Peers, teacher, with himself	A. Orality
	State of the art: RAE-AP guided by the learning co-mediator and built by the learners			Dimensional analysis			B. Writing - IEEE

A Case Study of the Parallel Plate Capacitor.

The study of the parallel plate capacitor is fundamental in the field of electrostatics, as it integrates the concepts and applications of Coulomb's law for continuous charge distributions, considering in this case a constant surface charge density. This analysis also allows for the study of the formation and distribution of electric field lines, which are treated as a vector field from a mathematical standpoint. Complementarily, the equipotential profiles are examined, representing a scalar field orthogonal to the electric field lines.

Additionally, the interpretation of field flux through Gauss's law plays a key role in understanding potential functions. These functions, related to the gradient of a potential function, allow for the interpretation of the field of directions and its connection to the solution of partial and ordinary differential equations. If we add to this the scientific-technological application of the capacitor as a passive device for storing electrostatic charge when subjected to a direct current source, we can see it as an ideal case study within the context of this dissertation's proposal. This constitutes a Rich Experience in Context (ERC), which enables the consideration of this system as a Theoretical-Experimental Modeling Experience (ETEM) and its path towards building critical scientific thinking.

Two Parallel Infinite Non-Conducting Planes (Textbook Solution)

Consider two parallel infinite non-conducting planes lying in the xy-plane, subjected to a potential difference caused by a direct current source and separated by a distance d. Each plane is uniformly charged with equal but opposite surface charge densities σ. Find the electric field everywhere in space.

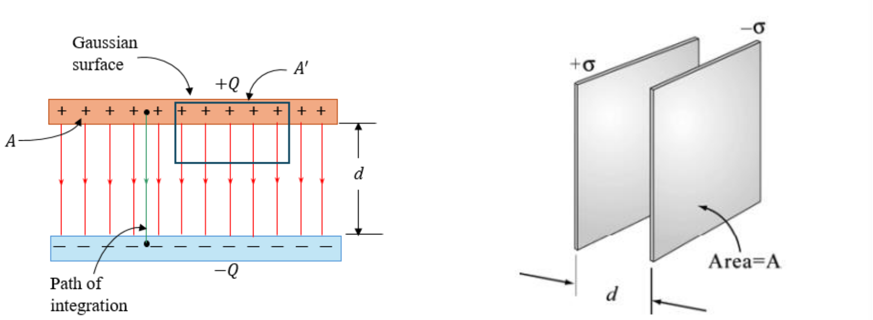


Figure 5. Positive and negative uniformly charged infinite planes b. Gaussian surface for calculating the electric field between the plates.

The solution is based on the application of Gauss's law for the electrostatic field inside the conductive plates. Subsequently, this result is correlated with the calculation of the electric potential, understood as the electric potential energy per unit charge. From this point, the concept of gradient is introduced to analyze how variations in the electric potential determine the magnitude and direction of the electric field in the studied region.

Determination of the Electric Field

To determine the electrostatic field between the plates with dispersor (+) and sink (-) polarities, a simplified initial model is assumed in which the change in the properties of space occurs in a vacuum. According to the principle of superposition, the electric field inside can be determined as follows:

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-}$$

Since the infinite parallel planes are homogeneous, meaning they are made of the same material, we assume that their charge densities are equal but of opposite polarity. Thus, the magnitude of the electrostatic field is determined as follows:

$$E_{+} = E_{-} = \frac{\sigma}{2\epsilon_0}$$

Therefore, when we add these fields together, we see that the field outside the parallel planes is zero, and the field between the planes has twice the magnitude of the field of either plane.

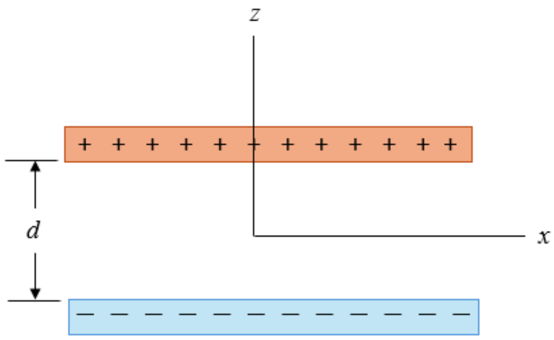


Figure 6. Electric field of two parallel planes

The electric field of the positive and the negative planes are given by

$$\vec{E}_+ = \begin{cases} +\dfrac{\sigma}{2\varepsilon_o}\hat{\boldsymbol{k}}, & z > d/2 \\ -\dfrac{\sigma}{2\varepsilon_o}\hat{\boldsymbol{k}} & z < d/2 \end{cases}, \qquad \vec{E}_- = \begin{cases} -\dfrac{\sigma}{2\varepsilon_o}\hat{\boldsymbol{k}}, & z > -d/2 \\ +\dfrac{\sigma}{2\varepsilon_o}\hat{\boldsymbol{k}} & z < -d/2 \end{cases}$$

Adding these two fields together then yields

$$\vec{E} = \begin{cases} 0\hat{\boldsymbol{k}}, & z > d/2 \\ -\dfrac{\sigma}{\varepsilon_o}\hat{\boldsymbol{k}}, & d/2 > z > -d/2 \\ 0\hat{\boldsymbol{k}}, & z < -d/2 \end{cases}$$

So the electric field between the plates is twice the magnitude of the field of a single plate

$$\vec{E} = -\frac{\sigma}{\varepsilon_o}\hat{\boldsymbol{k}}, \quad d/2 > z > -d/2$$

Determination of the Electric Potential

The potential difference between the plates is

$$\Delta V = V_- - V_+ = -\int_+^- \vec{E} \cdot d\vec{s} = -Ed$$

where we have taken the path of integration to be a straight line from the positive plate to the negative plate following the field lines. Since the electric field lines are always directed from higher potential to lower potential, $V_- < V_+$. However, in computing the capacitance C, the relevant quantity is the magnitude of the potential difference:

$$|\Delta V| = Ed$$

and its sign is immaterial.

Implementation of the Theoretical Experimental Modeling Experience TEME.

The experiment conducted involves an electrostatic tank where, through various geometries, equipotential profiles and electric field lines are drawn and constructed, immediately leveraging the physical-mathematical application of the gradient concept. Simultaneously, in many cases, the student becomes familiar with methods for solving partial differential equations. Initially, given the geometry of the problem, it is approached analytically using the method of separable variables and contrasted with a numerical approximation through finite difference methods (FDM) and finite element methods (FEM). The mediator of learning introduces a conceptual foundation for these numerical methods beforehand. Additionally, these numerical approximations were simulated using the QuickField student edition (QuickField, 2024) and FEMM (Finite Element Method Magnetics, 2024) software packages. The goal is for students to understand and describe the electrostatic phenomenon by applying physical-mathematical concepts, which are central to the cognitive development of future engineers, fostering their ability to interpret and solve given problems.

Since there are several possible configurations, we focus on the particular case of two parallel plates connected to a constant potential difference. In our specific case, these plates are represented by two wires of length L, made of the same conductive material.

Establishment of the State of the Art through IAG.

In this section, ten of the twenty articles developed to establish the state of the art will be considered. These articles were analyzed through the interaction of the tools Elicit and ChatPDF. Each group derived a specific state of the art from their analysis, which allowed them to determine the elements of the experimental setup. Some groups opted to use a direct current source from the laboratory, while others implemented the source using Arduino.

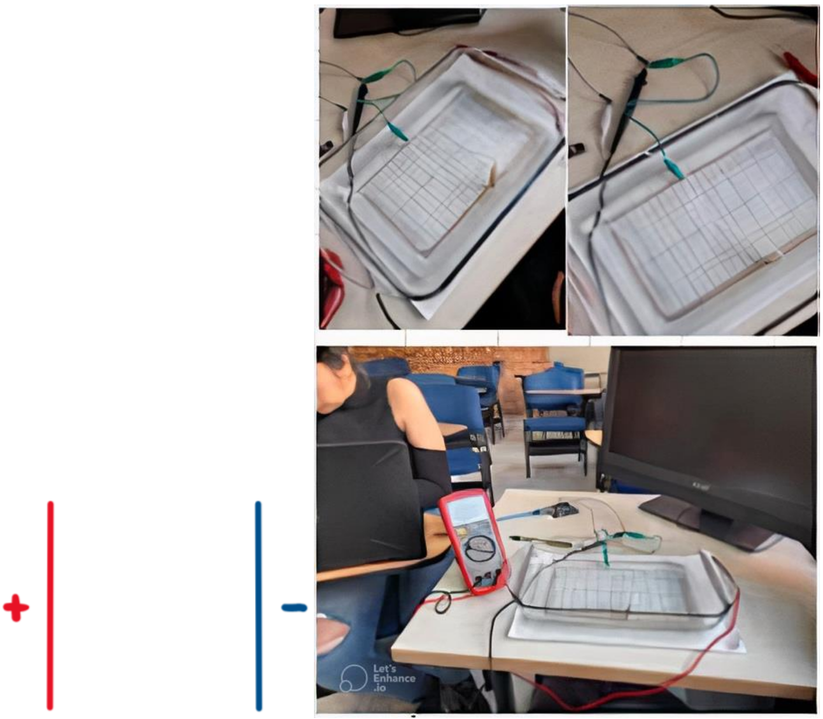
Furthermore, each group presented their proposal regarding the use of computational tools and the elements of the experimental setup. The formation of these groups, which interacted with each other, is framed within the TEME approach and is referred to as learning communities.

N°	Title	Summary	Relevance/Interpretation
1	Fourier transform terahertz near field imaging of one dimensional slit arrays	Presents 2D measurements of the THz electric field behind a sample consisting of multiple slits in a metal foil. The measurements reveal electric field lines, vortices, and saddle points. The magnetic field and Poynting vector are reconstructed, showing energy flow behind the sample.	Reveals how light flow can be studied near sub-wavelength plasmonic structures.
2	General Formulation of the Electromagnetic Field Distribution in Machines and Devices Using Fourier Analysis	Describes magnetic field distribution in electromagnetic machines using Fourier theory. Applicable to various 2D geometries, showing its utility in motors and actuators.	Provides a generalized approach for solving problems in electromagnetic geometries.
3	A method of calculating the propagation of electromagnetic fields both in waveguides and in free space	Evaluates field distribution in waveguides using FFT, applicable to guided and unguided propagation. Allows rapid decomposition into eigenmodes.	Provides an efficient method for analyzing electromagnetic fields in different configurations.
4	Advanced techniques for processing electromagnetic fields generated by lightning	Discusses techniques for reducing noise in electric and magnetic field recordings, enhancing time-frequency analysis.	Improves characterization of electromagnetic signals affected by noise, crucial for atmospheric studies.
5	Trivector Fourier transformation and electromagnetic field	Extends conventional Fourier transformation to analyze electromagnetic fields within Clifford algebra, providing deeper insights into their behavior and properties.	Facilitates more effective analysis of electromagnetic fields using advanced mathematical tools.
6	Mathematics III: Differential Equations, Fourier Series, and Applications	Covers differential equations and Fourier series, exploring their practical applications across scientific disciplines and engineering.	Provides essential mathematical foundations for understanding complex electromagnetic phenomena.
7	Fourier Series, Fourier Transforms, and Applications	Studies Fourier series and transforms, demonstrating their applications in mathematics and physics.	Reinforces the importance of Fourier analysis in solving physical and mathematical problems.
8	Fourier Transforms	Explains how the Fourier transform reveals important characteristics of sampled signals by transforming them into frequency domain analysis.	Essential for signal processing and analysis in various scientific applications.
9	The Fourier Transform	Comprehensive exploration of the Fourier transform as a fundamental mathematical tool used in engineering, physics, and signal processing, including properties and applications.	Key resource for students and professionals seeking to apply the Fourier transform in their research.
10	1D modeling of controlled source electromagnetic methods using a horizontal electric dipole as a source	Develops a CSEM method for marine exploration, facilitating the use of existing software for future academic work through modeling with marine electromagnetic data obtained using a horizontal electric dipole.	Contributes to advancements in controlled source electromagnetic methods applied to marine hydrocarbon exploration.

Table 2. State of the art development

Material / Resource	Cost (COP)	Cost (USD)
Multimeter	76000	19.0
DC Power Supply	300000	75.0
Alligator Clips	4000	1.0
Copper Wire	3500	0.88
Refractory	32000	8.0
Water	2000	0.5
Salt	500	0.13
Graph Paper	500	0.13
QuickField Software	N/A	N/A
FEMM Software	N/A	N/A
MAXIMA Software	N/A	N/A
Total	418500	104.63

Table 3. TEME: Experimental setup



Regarding the experimental setup, it is important to clarify that some students, depending on their field of engineering—in this case, telecommunications, mechatronics, and electronics—use a constant DC voltage source. Specifically, they utilize Arduino as a platform, particularly with the Arduino Uno or Arduino Leonardo. In addition to this, most students have their own multimeter and power supply, allowing for deductions within the values shown in the previous table.

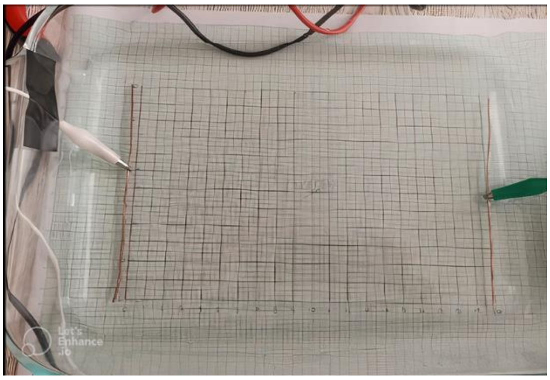


Figure 7. Visual Representation of the Experimental Setup

Using the data collected from the setups for each of the learning communities, which correspond to the measurements on the Y-axis and X-axis, as well as the voltage data at different points on the grid, data tables are created. These tables display the classified data using color formatting in Excel, allowing for the subsequent location and interpretation of the data for the generated graphs, as well as the calculation of error percentages considering the theoretical values from the software used as a reference.

Data Obtained Experimentally

For this theoretical-experimental TEME modeling experience, data collection was conducted in an experimental setup designed to construct equipotential profiles based on the mapping of 144 points. The resulting matrix from this collection was obtained through measurements taken with a multimeter. During this process, a potential difference of 1.5 volts was considered for one plate and 0 volts for the other. It is important to note that the generated matrix is not square; instead, it has dimensions of 16 × 9. This rectangular structure allows for a more effective representation of the collected data, facilitating the analysis and interpretation of the potential profiles within the context of the experiment.

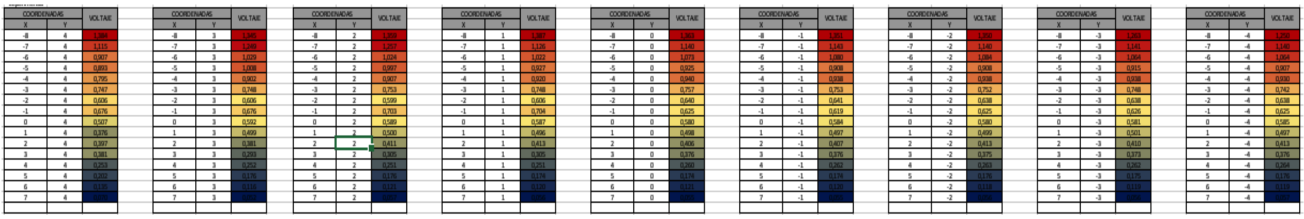


Figure 8. Equipotential Profiles

In this figure, the nine equipotential profiles obtained individually are displayed. Using Excel and its conditional formatting, the level curves are created based on the gradient of colors. In this case, the color red represents a higher potential, while the color blue indicates a lower potential.

4	1,384	1,115	0,907	0,893	0,795	0,747	0,606	0,676	0,507	0,376	0,397	0,381	0,253	0,202	0,135	0,070	0
3	1,345	1,249	1,029	1,008	0,902	0,748	0,606	0,676	0,592	0,499	0,381	0,293	0,252	0,176	0,116	0,052	0
2	1,359	1,257	1,024	0,997	0,907	0,753	0,599	0,703	0,589	0,500	0,411	0,305	0,251	0,176	0,121	0,057	0
1	1,387	1,126	1,022	0,927	0,920	0,748	0,606	0,704	0,587	0,496	0,413	0,305	0,251	0,174	0,120	0,056	0
0	1,363	1,140	1,073	0,925	0,940	0,757	0,640	0,625	0,580	0,498	0,406	0,376	0,260	0,174	0,121	0,055	0
-1	1,351	1,143	1,080	0,908	0,938	0,753	0,641	0,619	0,584	0,497	0,407	0,376	0,262	0,174	0,120	0,055	0
-2	1,350	1,140	1,084	0,908	0,938	0,752	0,638	0,625	0,580	0,499	0,413	0,375	0,263	0,176	0,118	0,056	0
-3	1,263	1,141	1,064	0,915	0,938	0,748	0,638	0,626	0,581	0,501	0,410	0,373	0,262	0,175	0,119	0,056	0
-4	1,250	1,140	1,064	0,907	0,930	0,742	0,638	0,625	0,585	0,497	0,413	0,376	0,264	0,176	0,119	0,057	0
	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8

Figure 9. Unified Mapping of Equipotential Profiles.

In this image, the complete mapping is displayed in terms of the positions on the X and Y axes, along with the corresponding electric potential for each coordinate of the experimental setup

Data Obtained through Simulation under the Same Experimental Conditions:

QuickField

The data analysis is conducted using QuickField, which allows us to observe the color map representing values approaching 0.0 volts in blue tones and 1.5 volts in red tones. The level curves are illustrated based on this color map, alongside the electric field lines, which are depicted by small black arrows. This visualization effectively demonstrates the direction of the electrostatic field in each of the configurations.

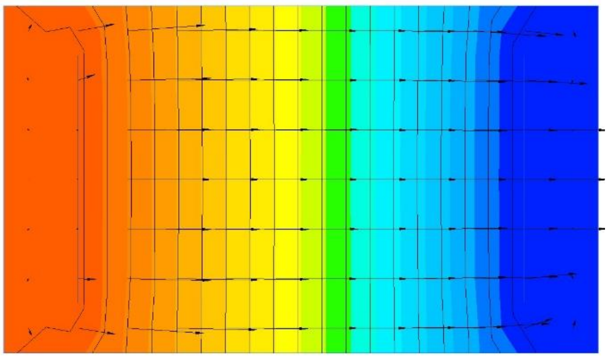


Figure 10. Parallel Plane Configuration in QuickField

FEMM

In the case of this computational tool, a graph is obtained along with its corresponding table, with the same voltage considerations and showing the level curves.

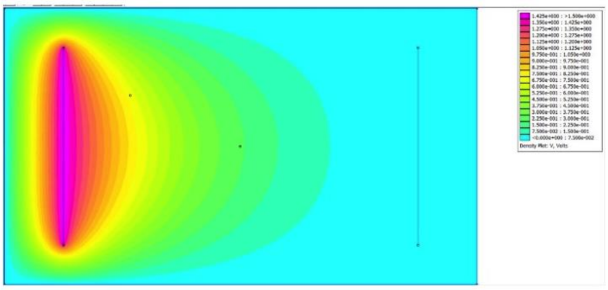


Figure 11. Parallel Plane Configuration in FEMM

To facilitate the understanding of data collection obtained through simulation, the information from QuickField was organized into an Excel spreadsheet. This approach simplified the comparison between the experimental data and the simulation results. It is important to note that the algorithm of this "black box" corresponds to the solution of the partial differential equation using the finite element method.

COORDENADAS		VOLTAJE
X	Y	
-8	4	1,409
-7	4	1,319
-6	4	1,231
-5	4	1,143
-4	4	1,056
-3	4	0,968
-2	4	0,881
-1	4	0,793
0	4	0,706
1	4	0,619
2	4	0,531
3	4	0,500
4	4	0,357
5	4	0,270
6	4	0,181
7	4	0,090

COORDENADAS		VOLTAJE
X	Y	
-8	3	1,410
-7	3	1,321
-6	3	1,232
-5	3	1,144
-4	3	1,056
-3	3	0,969
-2	3	0,881
-1	3	0,794
0	3	0,706
1	3	0,619
2	3	0,531
3	3	0,443
4	3	0,356
5	3	0,267
6	3	0,179
7	3	0,090

COORDENADAS		VOLTAJE
X	Y	
-8	2	1,411
-7	2	1,322
-6	2	1,233
-5	2	1,145
-4	2	1,057
-3	2	0,969
-2	2	0,881
-1	2	0,794
0	2	0,706
1	2	0,618
2	2	0,531
3	2	0,443
4	2	0,355
5	2	0,267
6	2	0,178
7	2	0,089

COORDENADAS		VOLTAJE
X	Y	
-8	1	1,411
-7	1	1,322
-6	1	1,234
-5	1	1,145
-4	1	1,057
-3	1	0,969
-2	1	0,881
-1	1	0,794
0	1	0,706
1	1	0,618
2	1	0,531
3	1	0,443
4	1	0,355
5	1	0,266
6	1	0,178
7	1	0,089

COORDENADAS		VOLTAJE
X	Y	
-8	0	1,411
-7	0	1,322
-6	0	1,234
-5	0	1,145
-4	0	1,057
-3	0	0,969
-2	0	0,881
-1	0	0,794
0	0	0,706
1	0	0,618
2	0	0,531
3	0	0,443
4	0	0,355
5	0	0,266
6	0	0,178
7	0	0,089

COORDENADAS		VOLTAJE
X	Y	
-8	-1	1,411
-7	-1	1,322
-6	-1	1,234
-5	-1	1,145
-4	-1	1,057
-3	-1	0,969
-2	-1	0,881
-1	-1	0,794
0	-1	0,706
1	-1	0,618
2	-1	0,531
3	-1	0,443
4	-1	0,355
5	-1	0,266
6	-1	0,178
7	-1	0,089

COORDENADAS		VOLTAJE
X	Y	
-8	-2	1,411
-7	-2	1,322
-6	-2	1,233
-5	-2	1,145
-4	-2	1,057
-3	-2	0,969
-2	-2	0,881
-1	-2	0,794
0	-2	0,706
1	-2	0,619
2	-2	0,531
3	-2	0,443
4	-2	0,355
5	-2	0,267
6	-2	0,178
7	-2	0,090

COORDENADAS		VOLTAJE
X	Y	
-8	-3	1,410
-7	-3	1,321
-6	-3	1,233
-5	-3	1,144
-4	-3	1,057
-3	-3	0,969
-2	-3	0,881
-1	-3	0,794
0	-3	0,706
1	-3	0,619
2	-3	0,531
3	-3	0,444
4	-3	0,356
5	-3	0,268
6	-3	0,179
7	-3	0,090

COORDENADAS		VOLTAJE
X	Y	
-8	-4	1,4101
-7	-4	1,32
-6	-4	1,2325
-5	-4	1,143
-4	-4	1,0556
-3	-4	0,96821
-2	-4	0,88086
-1	-4	0,79351
0	-4	0,70609
1	-4	0,61889
2	-4	0,53146
3	-4	0,44408
4	-4	0,3567
5	-4	0,26907
6	-4	0,18053
7	-4	0,090849

Figure 12. Equipotential Profiles with QuickField.

In the following image, the complete mapping is presented in terms of the positions on the X and Y axes, along with the corresponding electric potential for each coordinate resulting from the computational simulation using finite elements.

4	1,409	1,319	1,231	1,143	1,056	0,968	0,881	0,793	0,706	0,619	0,531	0,444	0,357	0,270	0,181	0,090	0,000
3	1,410	1,321	1,232	1,144	1,056	0,969	0,881	0,794	0,706	0,619	0,531	0,443	0,356	0,267	0,179	0,090	0,000
2	1,411	1,322	1,233	1,145	1,057	0,969	0,881	0,794	0,706	0,618	0,531	0,443	0,355	0,267	0,178	0,089	0,000
1	1,411	1,322	1,234	1,145	1,057	0,969	0,881	0,794	0,706	0,618	0,531	0,443	0,355	0,266	0,178	0,089	0,000
0	1,411	1,322	1,234	1,145	1,057	0,969	0,881	0,794	0,706	0,618	0,531	0,443	0,355	0,266	0,178	0,089	0,000
-1	1,411	1,322	1,234	1,145	1,057	0,969	0,881	0,794	0,706	0,618	0,531	0,443	0,355	0,266	0,178	0,089	0,000
-2	1,411	1,322	1,233	1,145	1,057	0,969	0,881	0,794	0,706	0,619	0,531	0,443	0,355	0,267	0,178	0,090	0,000
-3	1,410	1,321	1,233	1,144	1,057	0,969	0,881	0,794	0,706	0,619	0,531	0,444	0,356	0,268	0,179	0,090	0,000
-4	1,4101	1,32	1,2325	1,143	1,0556	0,97	0,88086	0,79351	0,70609	0,619	0,53146	0,44408	0,3567	0,2691	0,18053	0,090849	0
	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8

Figure 13. Unified Mapping of Equipotential Profiles Using QuickField

The fundamental reason for using two finite element software programs lies in the degree of reliability that a student must have when working with "black boxes," as mentioned earlier. This allows for contrasting the results with the portion of reality under study, in this case, the "electrostatic tank" fostering in each of the learning communities that critical thinking that we, as co-mediators of learning, seek in our learners.

For reasons of space in this publication, we have selected QuickField as the reference black box. Within the framework we refer to as the "dialectical filter," we contrasted the experimental values obtained by one of the randomly selected learning groups. From this, we calculated the percentage error at each of the 144 nodes that make up the data matrix, obtaining the following results:

4	1,73	15,45	26,34	21,87	24,66	22,82	31,17	14,84	28,22	39,32	25,34	23,83	29,01	25,07	25,16	22,56	
3	4,67	5,45	16,49	11,86	14,63	22,82	31,17	14,84	16,22	19,32	28,34	33,88	29,05	34,06	35,06	42,56	
2	3,67	4,89	16,92	12,91	14,21	22,31	31,98	11,43	16,61	19,19	22,55	31,15	29,28	34,06	32,04	36,51	
1	1,70	14,84	17,15	19,04	13,01	22,87	31,25	11,28	16,89	19,79	22,14	31,10	29,28	34,72	32,51	36,62	
0	3,409	13,810	12,990	19,200	11,060	21,910	27,350	21,270	17,820	19,440	23,560	15,150	26,590	34,810	32,100	38,410	
-1	3,41	13,81	12,99	19,20	11,06	21,91	27,35	21,27	17,82	19,44	23,56	15,15	26,59	34,81	32,10	38,41	
-2	4,29	13,79	12,09	20,68	11,27	22,36	27,63	21,21	17,86	19,25	22,14	15,25	25,93	34,08	33,73	37,91	
-3	10,43	13,63	13,66	20,08	11,22	22,78	27,54	21,18	17,71	19,06	22,74	15,89	26,31	34,84	33,73	38,02	
-4	11,34	13,63	13,66	20,62	11,87	23,33	27,52	21,18	17,09	19,63	22,25	15,43	26,01	34,53	33,87	37,55	
	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	

Figure 14. Percentage Error Obtained from the Comparison between Real Data and Simulation using QuickField.

The percentage error was calculated by comparing the experimental data with the results from the simulation performed using QuickField software. This error reflects the deviation between both data sets, allowing for an evaluation of the simulation's accuracy in relation to the real measurements. By conducting this comparison, it becomes possible to assess how closely the simulated behavior matches the actual physical phenomenon, which is essential for validating the models used and adjusting system parameters as necessary.

Physical-Mathematical Development by the Learners.

First, the variables of interest are identified along with their corresponding equations, based on the underlying mathematical and physical principles. The main variables to consider in this case are the electric field and the electric potential. Additionally, secondary parameters such as temperature, volume, and the solubility factor of the saline solution are included. In our study, this solution acts as the conductive medium between the electrodes, represented by brine. To coherently compare the experimentally obtained results with those from the QuickField simulation, the case was approached through the analytical solution of the equation that describes the behavior in an electrostatic confinement. This partial differential equation, known as the Laplace equation, elegantly models this phenomenon.

The following presents an abbreviated development carried out by the students, who were organized into learning communities to address the physical-mathematical treatment. The co-mediator of learning introduces the necessary theoretical framework, providing a detailed explanation of the physical and mathematical significance of the 2D Laplace equation. The equation is then developed for the specific case of a parallel plane configuration, allowing students to apply and understand this model in a practical context.

The Laplace equation, in our specific case in 2D and Cartesian coordinates, represents a fundamental physical-mathematical construct within the realm of potential theory. This equation allows for an understanding of the behavior of the electrostatic field in a specific region, based on the measurement or calculation of electric potential. Its application is essential for analyzing and predicting how the field is distributed in situations where no charges are present, which is foundational for the design of electrical devices and the understanding of electromagnetic phenomena.

$$\nabla^2 \cdot V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \quad [1]$$

$$V(x,y) = X(x) \cdot Y(y) \quad [2]$$

[2] en [1]

$$\frac{\partial}{\partial x}\left[\frac{\partial (X(x)\cdot Y(y))}{\partial x}\right] = \frac{\partial}{\partial x}\left[Y(y)\cdot X'(x)\right] = Y(y)\cdot X''(x)$$

$$\frac{\partial}{\partial y}\left[\frac{\partial (X(x)\cdot Y(y))}{\partial y}\right] = \frac{\partial}{\partial y}\left[X(x)\cdot Y'(y)\right] = X(x)\cdot Y''(y)$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = Y(y)\cdot X''(x) + X(x)\cdot Y''(y) = 0$$

$$\frac{Y(y)\cdot X''(x)}{X(x)\cdot Y(y)} + \frac{X(x)\cdot Y''(y)}{X(x)\cdot Y(y)} = \frac{0}{\underbrace{X(x)\cdot Y(y)}_{\text{Factor integrante}}}$$

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} = 0$$

$$\frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} \rightarrow \lambda$$

$$\lambda = \frac{X''(x)}{X(x)} \qquad \lambda = -\frac{Y''(y)}{Y(y)}$$

$$[a] \; X''(x) - \lambda \cdot X(x)$$

$$[b] \; Y''(y) + \lambda \cdot Y(y)$$

Figure 15. Handwritten Mathematical Developments.

The possible cases are established with the aim of concluding that the only valid and applicable case for the solution of the modeling configurations in question is Case 3, as the trivial solutions of Cases 1 and 2 are not acceptable.

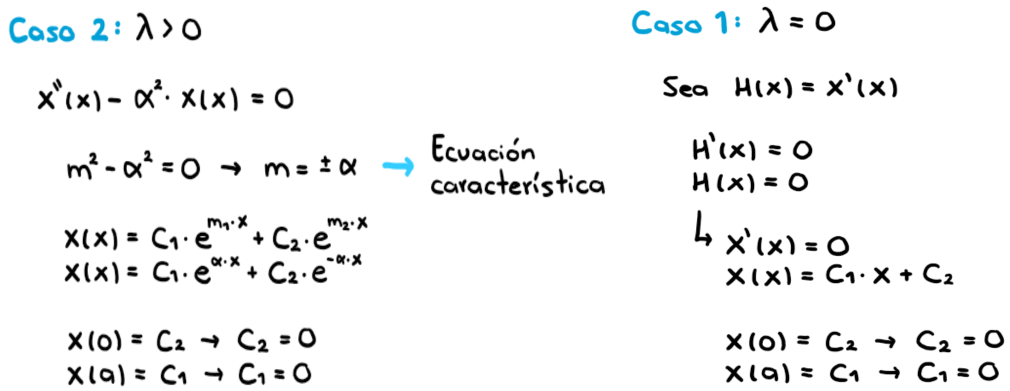


Figure 16. Handwritten Mathematical Developments.

For Case 3, where $\lambda < 0$, it is established that $\lambda = -\beta^2$. The corresponding analysis is conducted considering the transform, as well as the range and domain of the hyperbolic sine (*sinh*) and hyperbolic cosine (*cosh*) functions. This approach is relevant since any function over a closed interval can be represented in terms of sine and cosine, allowing for an adequate representation of the solutions in this context.

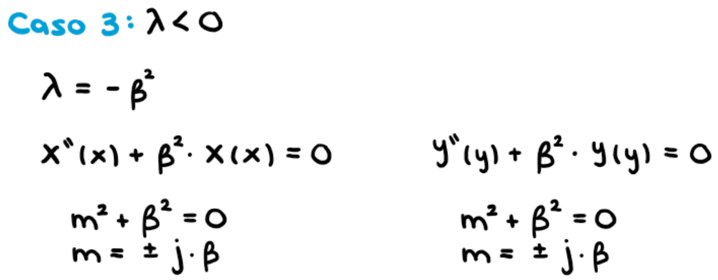


Figure 17. Handwritten Mathematical Developments.

Regarding the development of Fourier series, it is important to consider that each configuration presents specific criteria. However, certain equations and standard formulas remain consistent across these variations. This ensures a common foundation for the analysis and representation of functions using Fourier series, facilitating their application in different contexts. In the context of our "parallel plane" modeling configuration, the following analytical solution is obtained with boundary conditions.

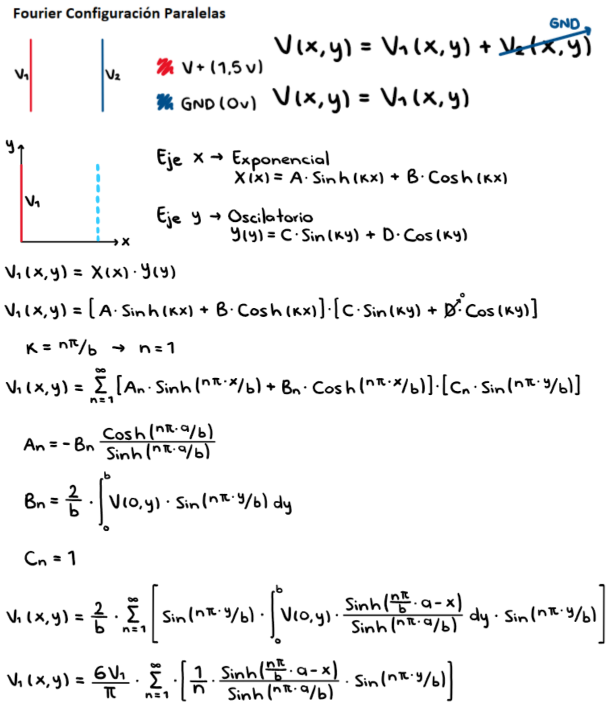


Figure 18. Handwritten Mathematical Developments.

$$V(x,y) = \frac{6V_1}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{n} \cdot \frac{\sinh\left(\frac{n\pi}{b} \cdot a - x\right)}{\sinh\left(n\pi \cdot a/b\right)} \cdot \sin\left(n\pi \cdot y/b\right) \right]$$

Figure 19. Handwritten Mathematical Developments- Solution by Fourier series

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 0\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 1{,}425\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 0{,}25\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 1{,}306\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 0{,}5\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 1{,}169\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 0{,}75\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 1{,}096\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 1\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 1{,}005\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 1{,}25\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}921\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 1{,}5\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}844\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 1{,}75\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}773\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 2\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}709\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 2{,}5\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}595\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 3\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}500\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 3{,}5\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}419\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 4\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}352\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 5\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}248\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 6\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}175\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 8\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0{,}086\ V$$

$$V(x,y) = \frac{6 \cdot 1{,}5}{\pi} \cdot \sum_{n=1}^{\infty} \cdot \left[\frac{1}{1} \cdot \frac{\sinh\left(\frac{1\pi}{9} \cdot 16 - 16\right)}{\sinh\left(1\pi \cdot 16/8\right)} \cdot \sin\left(1\pi \cdot 4/8\right) \right] = 0\ V$$

To verify whether the development of Fourier series aligns with reality, a dialectical filter is applied, using the results obtained from experimental measurements as a reference, alongside the results obtained from computational simulations.

Figure 20. Values obtained through the solution of the Fourier series, taking central values along the vertical axis of the setup in question.

Final Reflections

The TEME (Theoretical-Experimental Modeling Experience) framework. It emphasizes a critical approach to scientific knowledge construction, combining theoretical modeling and real-world data collection, followed by comparisons with simulations to establish a coherent and dialectical process of knowledge building.

As a first non-linear organization for the reader, regarding the proposal of this dissertation and in line with the systematization (TEME) proposed in previous pages, we have:

Table 4.

1. Introduction to TEME Approach	-Brief explanation of TEME (Theoretical-Experimental Modeling Experience) - Emphasis on the role of modeling as a toll to make finite portions of reality intelligible.
2. Critical Knowledge Construction	- Explanation of how TEME encourages a critical stance on the way scientific knowledge is built - Mention of the dialectical process between theoretical modeling and real-world data collection
3. Mathematical Objects and Modeling	- Description of the mathematical objects involved in the modeling process. - How these mathematical representations help to conceptualize the problem.
4. Real Data Collection and Simulation	- Explanation of the data collection process (tangible, real data). - Contrasting these real-world data points with simulations to create a coherent knowledge structure.
5. Dialectical Filters	- Detail the process of using dialectical filters to compare and integrate results from the model and experimental data. - Discuss the implications for understanding the scientific phenomenon.

The theoretical experimental modeling experiences (TEME) seek to involve both the learner and the co-learning mediator in the scientific endeavor, in their quest to understand reality. This process aims to generate critical thinking and awareness of their own reality, as well as to stimulate autonomy in learning and their role as thinking beings. Furthermore, it aims to make epistemological and ontological resistances visible, promoting respect for other forms of understanding.

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