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Research Article

A Study on Odd Prime Labeling of Octopus Graphs Families

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ABSTRACT

An odd prime labeling of a graph G (V,E), is defined as a bijective function f mapping the vertex set V to the set {1,3,5,...,2|V(G)|-1}, such that for every edge uv∈E. the greatest common divisor gcd(f(u),f(v))=1. A graph that permits such a labeling is referred to as an odd prime graph. In this study, we explore the odd prime labeling properties of various graph structures, including the octopus chain graph, octopus ladder graph, twisted octopus ladder graph, and hexa-octopus chain graph.

Keywords: Prime labeling, octopus chain graph, octopus ladder graph, twisted octopus ladder graph, and hexa-octopus chain graph.

INTRODUCTION:

In this paper, we consider finite, simple, undirected, non-trivial, and connected graphs. The definitions and notations in graph theory and number theory are adopted from [5,2] and [7], respectively. The greatest common divisor of two positive integers m and n is denoted by (m, n) . The concept of prime labeling was introduced by Entringer and later appeared in [8] through the work of Tout, Dabboucy, and Howalla in 1982. A graph G with vertex set $V(G)$ is said to have a prime labeling if there exists a bijective function $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ such that for every edge $xy \in E(G)$, the values $f(x)$ and $f(y)$ are relatively prime. A graph that admits such a labeling is called a prime graph. The prime labeling of various families of graphs has been extensively studied by numerous researchers. Entringer conjectured that all trees admit prime labeling, and Fu and Huang [9] later proved that all complete binary trees satisfy this property.

Definition 1. A graph G with vertex set $V(G)$ is said to have an odd prime labeling if there exists an injective function $f : V(G) \rightarrow \{1, 3, 5, \ldots, |V(G)|\}$ such that for every edge $xy \in E(G)$, $f(x)$ and $f(y)$ are relatively prime. A graph G that admits an odd prime labeling is called an odd prime graph.

Definition 2. [1] For a graph $G(V, E)$ with n vertices, a bijection $f: V \rightarrow [n]$ is called prime labeling if $gcd\ (f(u), f(v)) = 1$, for each $uv \in E$. The graph admitting prime labeling is called a prime graph.

Definition 3. [2] For a graph $G(V, E)$ with n vertices, a bijective function $f: V \rightarrow O_n$ is called odd prime labeling if for each $uv \in E$, $gcd\ (f(u), f(v)) = 1$. The graph admitting this labeling is called an odd prime graph.

Definition 4. [3] A function f is called an odd prime labeling, given that f is a bijection from V to $\{1, 3, 5, \ldots, 2|V|-1\}$, satisfying $gcd\ (f(u), f(v)) = 1$, for each $uv \in E$. A graph admitting this labeling is called an odd prime graph.

Definition 5. [4] The octopus chain graph, denoted by $O_2(n)$, is a graph constructed from n copies of O_2 and connecting one leg of the i^{th} copy to the $(i\ +\ 1)^{th}$ copy, for every $i\ =\ 1, 2, \ldots\ , n-1$ and $n \geq 1$. Graph $O_2(n)$ is given in Figure 1.

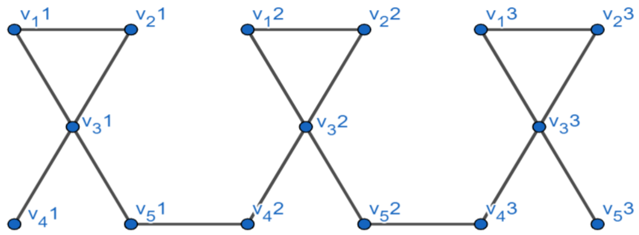

Figure 1. Graph $O_2(3)$

Definition 6. [4] The octopus ladder graph, denoted by $O'_2(n)$, is a graph constructed from $O_2(n)$ by connecting vertex v_5^i to a vertex v_4^{i+1} , for every $i\ =\ 1, 2, \ldots\ , n-\ 1$.

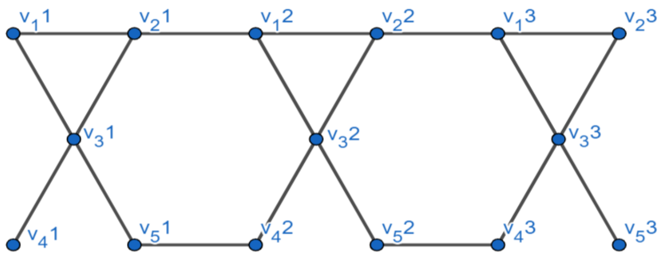

Figure 2. Graph $O'_2(3)$

The notation of the vertex v_j^i on the octopus chain graph $O_2(n)$ and the octopus chain graph $O_2(n)$ shows the vertex j on the i^{th} copy.

New Families Structural of octopus graph:

Definition 7: Twisted Octopus Ladder Graph $TO_2(n)$: Let $TO_2(n)$ be a connected graph with vertex set $V(TO_2(n)) = \{v_j^i | 1 \leq i \leq n, 1 \leq j \leq 5\}$, where each $TO_2(i)$ consists of five vertices. The edge set $E(TO_2(n))$ is defined as follows: Internal Edges within each $TO_2(i)$: $E_{int} = \{(v_1^i, v_2^i), (v_1^i, v_3^i), (v_2^i, v_1^i), (v_2^i, v_3^i), (v_3^i, v_4^i), (v_3^i, v_5^i)\}$ for $1 \leq i \leq n$, Ladder Connections: $E_{lad} = \{\ (v_5^i, v_4^{i+1})\ |\ 1 \leq i < n\}$ and Twisted Cross Connections: $E_{twist} = \{\ (v_1^i, v_3^{i+1}), (v_5^i, v_2^{i+1})\ |\ 1 \leq i < n\}$. The total number of vertices is $|V(TO_2(n))| = 5n$ and the total number of edges is $|E(TO_2(n))| = 8n - 3$.

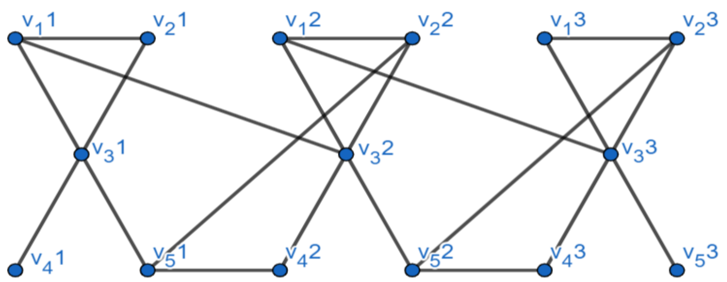

Figure 3. Graph $TO_2(3)$

Definition 8: The Hexa-Octopus Chain Graph (HOC_n) consists of n hexagonal units, each with 6 vertices. The total number of vertices is: $|V(HOC_n)| = 6n$. The edge set $E(HOC_n)$ is defined as: $E_{int} = \{(v_1^i, v_2^i), (v_2^i, v_3^i), (v_3^i, v_4^i), (v_4^i, v_5^i), (v_5^i, v_6^i), (v_6^i, v_1^i)\}$ and Ladder Connections: $E_{lad} = \{(v_6^i, v_2^{i+1}), (v_5^i, v_3^{i+1}) \mid 1 \le i < n\}$ hence the The total number of edges is: $|E(HOC_n)| = 8n - 2$.

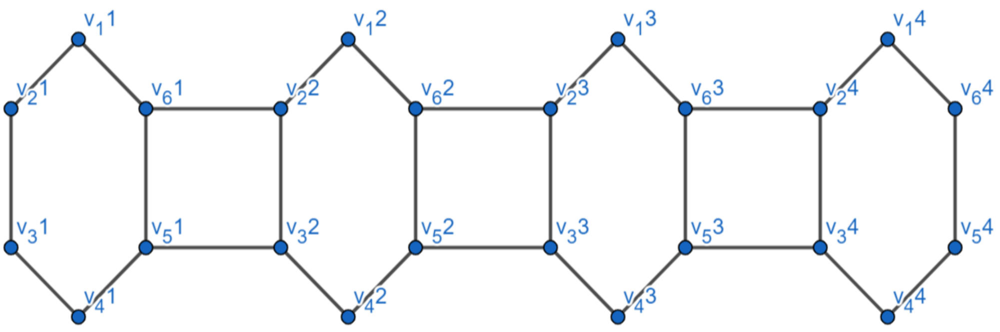

Figure 4. Graph HOC_4

Main results:

Theorem:1 The octopus chain graph $O_2(n)$ admits an odd prime labeling for any $n \ge 1$.

Proof: Let $V(O_2(n)) = \{v_j^i | 1 \le i \le n, 1 \le j \le 5\}$ be the vertex set of the graph, where each subgraph $O_2(i)$ consists of five vertices. The edge set $E(O_2(n))$ defines the connectivity of these vertices.

A function $f: V(O_2(n)) \to \{1,3,5, \dots, 2|V(O_2(n))| - 1\}$ is an odd prime labeling if it satisfies the following conditions:

1. f is bijective
2. $\gcd(f(u).f(v)) = 1, \forall (u,v) \in E(O_2(n))$.

Define the labeling function as:

$$f(v_1^i) = 2(5i-4) - 1, f(v_2^i) = 2(5i-3) - 1, f(v_3^i) = 2(5i-2) - 1$$

$$f(v_4^i) = 2(5i-1) - 1, f(v_5^i) = 2(5i) - 1.$$

Since all values are distinct odd numbers within the range $\{1,3,5, \dots, 2|V(O_2(n))| - 1\}$, f is bijective and the holds for every edge in $E(O_2(n))$. Hence $O_2(n)$ admits an odd prime labeling for any $n \ge 1$.

Result-2 : $O_2(3)$ admits an odd prime labeling for any $n \ge 1$.

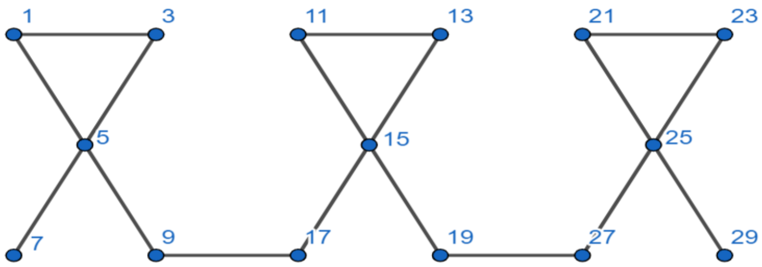

Figure 5. Graph $O_2(3)$

Theorem:3 Let $O_2'(n)$ be an octopus ladder graph with $5n$ vertices. Then, $O_2'(n)$ admits an odd prime labeling for any $n \ge 1$.

Proof: : Let $V(O_2'(n)) = \{v_j^i | 1 \le i \le n, 1 \le j \le 5\}$ be the vertex set of the graph, where each subgraph $O_2(i)$ consists of five vertices. The edge set $E(O_2'(n))$ defines the connectivity of these vertices.

A function $f: V(O_2'(n)) \to \{1,3,5,\dots,2|V(O_2'(n))| - 1\}$ is an odd prime labeling if it satisfies the following conditions:

1. f is bijective
2. $\gcd(f(u).f(v)) = 1, \forall (u,v) \in E(O_2'(n))$.

Define the labeling function as:

$$f(v_1^i) = 10i - 9, f(v_2^i) = 10i - 7, f(v_3^i) = 10i - 5$$
$$f(v_4^i) = 10i - 3, f(v_5^i) = 10i - 1.$$

Since all values are distinct odd numbers within the range $\{1,3,5,\dots,2|V(O_2'(n))| - 1\}$, f is bijective and the holds for every edge in $E(O_2'(n))$. Hence $O_2(n)$ admits an odd prime labeling for any $n \ge 1$.

Result-4: $O'_2(5)$ admits an odd prime labeling for any $n \ge 1$.

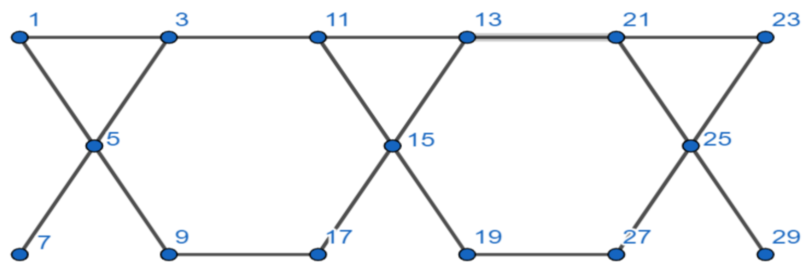

Figure 6. Graph $O'_2(3)$

Theorem:5 Let $TO_2(n)$ be a twisted octopus ladder graph with $5n$ vertices and $8n - 3$ edges. Then, $TO_2(n)$ admits an odd prime labeling for any $n \ge 1$.

Proof: : Let $V(TO_2(n)) = \{v_j^i | 1 \le i \le n, 1 \le j \le 5\}$ be the vertex set of the graph, where each subgraph $TO_2(i)$ consists of five vertices. The edge set $E(TO_2(n))$ is $8n - 3$.

A function $f: V(TO_2(n)) \to \{1,3,5,\dots,2|V(TO_2(n))| - 1\}$ is an odd prime labeling if it satisfies the following conditions:

1. f is bijective
2. $\gcd(f(u).f(v)) = 1, \forall (u,v) \in E(TO_2(n))$.

Define the labeling function as:

$$f(v_1^i) = 10i - 9, f(v_2^i) = 10i - 7, f(v_3^i) = 10i - 5$$
$$f(v_4^i) = 10i - 3, f(v_5^i) = 10i - 1.$$

Since all values are distinct odd numbers within the range $\{1,3,5,\dots,2|V(TO_2(n))| - 1\}$, f is bijective and the holds for every edge in $E(TO_2(n))$. Hence $TO_2(n)$ admits an odd prime labeling for any $n \ge 1$.

Result-6 : $TO_2(4)$ admits an odd prime labeling for any $n \ge 1$.

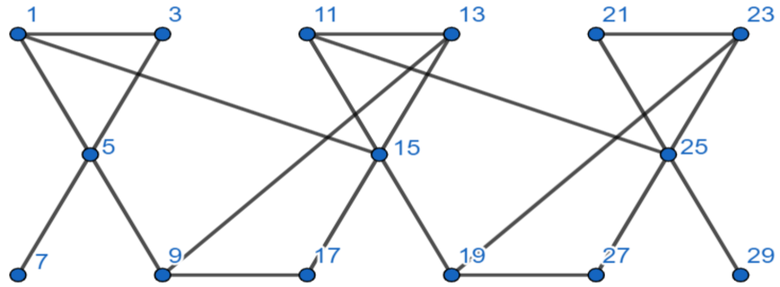

Figure 7. Graph $TO_2(3)$

Theorem:7 Let $\mathrm{HOC_n}$ be a hexagonal octopus chain graph with $6n$ vertices and $8n-2$ edges. Then for any $n \geq 1$, HOC_n admits an odd prime labeling .

Proof: Let $V(HOC_n) = \{v_j^i | 1 \leq i \leq n, 1 \leq j \leq 5\}$ be the vertex set of the graph, where each hexagonal unit has 6 vertices, and there are nnn such units in the chain. The edge set $E(HOC_n) = E_H \cup E_L$, E_H represents the internal hexagonal edges (each hexagon forms a cycle of 6 vertices). E_L represents the ladder edges connecting consecutive hexagons: (v_6^i, v_2^{i+1}) , (v_5^i, v_3^{i+1})

A function $f: V(HOC_n) \rightarrow \{1,3,5, \ldots, 2|V(HOC_n)| - 1\}$ is an odd prime labeling if it satisfies the following conditions:

1. f is bijective
2. $\gcd(f(u).f(v)) = 1, \forall (u,v) \in E(HOC_n)$.

Such that: $f(v_1^i) = 10(i-1)+1$, $f(v_2^i) = 10(i-1)+3$, $f(v_3^i) = 10(i-1)+5$

$$f(v_4^i) = 10(i-1)+7, f(v_5^i) = 10(i-1)+9, \ f(v_6^i) = 10(i-1)+11.$$

Since all values are distinct odd numbers within the range $\{1,3,5, \ldots, 2|V(HOC_n)| - 1\}$,f is bijective and the holds for every edge in $E(HOC_n)$. Hence HOC_n admits an odd prime labeling for any $n \geq 1$.

Result-8: HOC_4 admits an odd prime labeling for any $n \geq 1$.

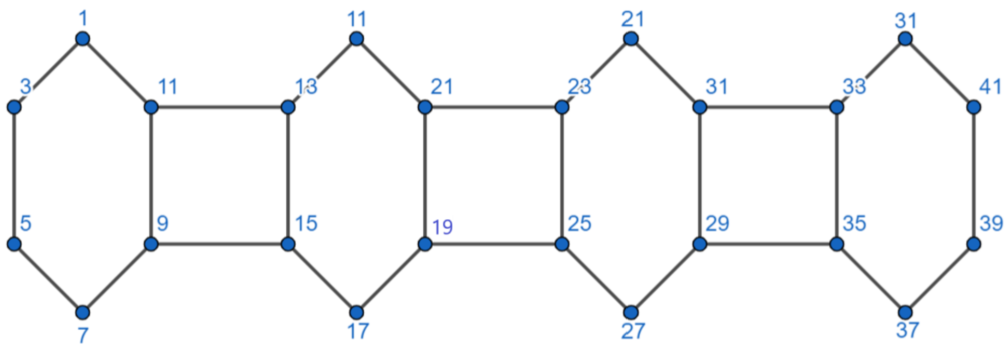

Figure 8. Graph HOC_4

CONCLUSION:

In this study, we explored the concept of odd prime labeling for a family of octopus-related graphs, including the octopus chain graph, octopus ladder graph, twisted octopus ladder graph, and hexa-octopus chain graph. We formulated and proved mathematical theorems establishing the conditions under which these graphs admit odd prime labeling. Through rigorous analysis, we demonstrated that each vertex labeling satisfies the bijective mapping condition and the greatest common divisor (GCD) property, ensuring that adjacent vertices have relatively prime labels.

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