Journal of Information Systems Engineering and Management
2025, 10(25s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Improving Healthcare Strategies with t-Fuzzy Graphs: A Decision Support Modelper

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ARTICLE INFO

Received: 28 Dec 2024

Revised: 16 Feb 2025

Accepted: 28 Feb 2025

ABSTRACT

Health management systems require intelligent decision-making to effectively analyze complex interactions between various medical, financial, and operational factors. This study explores the application of t-Fuzzy Graphs (t-FG) in modeling and managing intricate relationships within healthcare environments. By leveraging t-FG, this research highlights how these graphs can express uncertainty, capture multi-dimensional dependencies, and provide a structured representation of diverse health management variables. Fundamental t-FG operations, including homomorphism and isomorphism, are introduced to demonstrate their role in optimizing decision-making processes. Furthermore, the study discusses real-world applications of t-FG in healthcare, showcasing their potential in handling circular dimensions such as resource allocation, patient care strategies, and financial planning. The adaptability and efficiency of t-FG make them a valuable tool for policy development and strategic decision-making in health management, particularly in addressing complex social and numerical challenges within the healthcare sector.

Keywords: t-Fuzzy Graph, Health Management, Decision Support System, Homomorphism, Isomorphism, Uncertainty Modeling.

1. INTRODUCTION

The majority of conventional formal modelling and reasoning frameworks are clear-cut and deterministic. A statement in a crisp system has only two possible outcomes: true or false. In the same way, an element in classical set theory is either a member of a set or not. But instead of strict categories, Zadeh [1] established the idea of fuzzy sets, which provide a continuum of membership grades. The value of membership that gives all element a degree of membership in between 0 & 1 defines a fuzzy set. These days, fuzzy set theory is essential for simulating uncertainty in a variety of fields, including as business, society, and the environment. It is a potent mathematical tool that facilitates approximation thinking in decision-making processes.

Data and details about how components or objects interact may be conveniently represented using a graph [2, 3]. It makes sense to create a fuzzy graph when there is ambiguity in the depiction of the components or their interactions. A binary condtion of fuzzy subset is symmetric and also known as fuzzy graph [4]. The learning of fuzzy graphs based on fuzzy relations [6] was first introduced by Rosenfeld [5]. Bhattacharya [7] shed light on fuzzy graphs, while Sunitha and Vijayakumar [8] investigated their opposites. A number of fuzzy graph and fuzzy hypergraph traits were studied by Mordeson and Nair [9]. Concepts like regular, entirely regular, and total degree of vertices in certain fuzzy graphs were offered by Nagoor Gani and Radha [10, 11]. Chen [13] suggested a matrix model of graphs including fuzzy information, whereas Bhutani and Battou [12] studied M-strong fuzzy graphs.

Graph structures were first proposed by Sampathkumar [14] and have since shown great utility in a number of computer science and artificial intelligence domains. Hausdorff [15] was the first to study the lexicographic product, and Radha and Arumugam [16] extended it to fuzzy graphs. Ramakrishnan and Dinesh [17] studied generalised fuzzy graph structures, whereas Dinesh [18] studied fuzzy graph structures and related ideas. The semistrong min-product and maximum product of fuzzy network topologies were presented by Akram and Sitara [19-20], who also examined their characteristics. Furthermore, the residue product of fuzzy graph architectures were examined by Akram et al. [21].

It has been demonstrated that t -FGs, which provide a flexible approach to decision-making, may efficiently handle ambiguity and uncertainty. These models act as a link between symbolic expert systems and traditional numerical engineering techniques. The t-fuzzy hypothesis effectively communicates imprecision and unpredictability in complicated and uncertain circumstances. In order to solve real-world issues, we provide the idea of a ŧ -FG in this study using linear operators. The parameter "t" is essential for eliminating uncertainty in the decision-making process, allowing for exact control and customised solutions. This parameter offers a personalised approach to uncertainty management while improving flexibility. ŧ -FGs are an effective tool for deciphering intricate circumstances involving decision-making, providing a thorough approach to overcoming obstacles in decision-making. This method greatly improves decision-making accuracy while reducing the limitations of binary logic.

After providing a quick overview of ŧ -FGs, this work is organised as follows: To assist readers understand the originality of this study, basic terminology is included in the "Preliminaries" section. The knowledge of ŧ -FGs is examined, along with some of its most important characteristics, in the " -FG " section. Set-theoretical operations on ŧ -FGs are studied in "Operations on the ŧ -FG," along with graphical representations. "Isomorphism of ŧ -FGs" presents the ideas of homomorphism and isomorphism that are unique to ŧ-FGs. The "Complement of ŧ -FG " section examines the fundamental properties of the complement of ŧ -FGs and defines it. In "Application of ŧ -FG," this novel framework is used to create a health management decision support system. Lastly, the "Comparative Analysis" and "Conclusion" sections present a summary of the main findings and compare different elements of the study.

Symbols	Meaning
FS	Fuzzy Set
FG	Fuzzy Graph
ŧ-FS	ŧ-Fuzzy Set
ŧ-FG	ŧ-Fuzzy Graph
ŧ -FSG	ŧ- Fuzzy Subgraph
$\boldsymbol{h}_{\mathcal{G}_{ŧ}}$	Membership Function

Table 1 List of symbols used

MOTIVATION

- The capacity of ŧ –FGs to manage intricate situations with unclear information and erratic, reluctant interactions between parts is the main reason for their selection. These graphs, which use the "ŧ" parameter, offer an organised method for assessing and simulating different levels of relationship confidence and uncertainty.
- The above structure uses an approach that makes use of -norms and t-conorms to handle the integration and separation of uncertain information. This method is especially well-suited for making decisions in real-world situations where a variety of inputs and results need to be taken into account.
- This approach is applicable in a number of fields, such as decision analysis, risk assessment, and system optimisation. Its objective is to reveal hidden links in ambiguous data while preserving adaptability in various circumstances.
- By enabling multi-layered analysis using ŧ-fuzzy graphs, medical practitioners may assess various degrees of clinical data confidence, resulting in a more intelligent and flexible decision-support system.

NOVELTY

- The "ŧ" parameter plays a crucial role in establishing a structured and comprehensible representation of uncertain connections by acting as a threshold that reflects hesitation.
- By incorporating the "ŧ" parameter, relationship visualization is enhanced, allowing for the selective display of edges and nodes based on a predefined confidence level. This ensures that only the most relevant connections are highlighted.
- This approach enables a more precise distinction between strong and subtle associations, facilitating a systematic approach to managing ambiguity.

- A t-fuzzy framework links different paramete values of "t" to various graph layers, supporting multi-layered analysis. This allows for a detailed examination of relationships within the graph while accounting for varying confidence levels, ultimately leading to a deeper understanding of the underlying structure.

OBJECTIVES

By using t –FGs to handle ambiguity, reluctance, and differing confidence levels in medical data, the main objective of this research is to create a strong decision-support model that improves healthcare methods.

2. t-FUZZY GRAPH

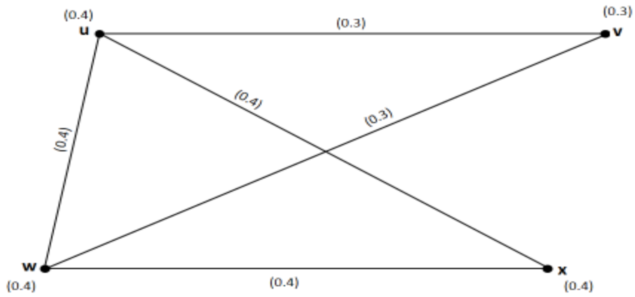
Definition.2.1 In a universal set Ų, Let G be the FS with t ∈ [0, 1]. A $FS_{\mathcal{G}_t}$ of Ų, also known as a t -FS, is defined as $h_{\mathcal{G}_t}(\upsilon_1) = \min\{h_{\mathcal{G}}(\upsilon_1), 1-t\}$, $\forall\, \upsilon_1 \in$ Ų . The form of t-FS is $\mathcal{G}_t = \{\upsilon_1, h_{\mathcal{G}}(\upsilon_1), \upsilon_1 \in$ Ų $\}$ where $h_{\mathcal{G}_t}$ a function that assign to each degree is. Moreover, the function $h_{\mathcal{G}_t}$, satisfy the condition $0 \le h_{\mathcal{G}}(\upsilon_1) \le 1$.

Definition. 2.2 Let $\mathcal{G} = (A, B)$ be a FG for a given simple graph H= (Ʋ, Ȩ). A t -FG is represented by the otation $\mathcal{G}_t = (A_t, B_t)$, where $A_t = \{(\upsilon_i, h_{\mathcal{G}}(\upsilon_i): \upsilon_i \in Ʋ\}$ is the t-FS on Ʋ & $B_t = \{\langle(\upsilon_i, \upsilon_j), h_{\mathcal{G}}(\upsilon_i, \upsilon_j)\rangle, (\upsilon_i, \upsilon_j) \in$ Ȩ$\}$ is the t-FS on Ȩ ⊆ Ʋ × Graph $h_{B_t}(\upsilon_i, \upsilon_j) \le 1$.

Example.2.3. Observe $H' = (Ʋ, Ȩ)$ where $Ʋ = \{u, v, w, x\}$ and $Ȩ = \{uv, uw, ux, vw, wx\}$. Given Ʋ, the node strengths of A *is* $\{(u, 0.5), (v, 0.3), (w, 0.7), (x, 0.4)\}$.

The Edge strength of is $\{(uv, 0.3), (uw, 0.5), (ux, 0.4), (vw, 0.3), (wx, 0.4)\}$. Applying the concept of t-FS to the two FS A and B that are provided, which correspond to the value t = 0.6, reveals that,

$A_{0.6} = \{(u, 0.4), (v, 0.3), (w, 0.4), (x, 0.4)\}$ and $B_{0.6} = \{(uv, 0.3), (uw, 0.4), (ux, 0.4), (vw, 0.3), (wx, 0.4)\}$.



Graph 1. 0.6-FG $\mathcal{G}_{0.6}$

t-FG, if t = 0.40; $\mathcal{G}_{0.40} = (A_{0.6}, B_{0.6})$

Definition. 2.4. Let $\mathcal{G}_t = (A_t, B_t)$ be an t-FG then $\mathcal{H}_t = (A'_t, B'_t)$ is considered a t-FSG if $A'_t \subseteq A_t$ and $B'_t \subseteq B_t$.

Definition. 2.5. The complete t-FG $\mathcal{G}_t$ is acknowlaged the following requirements listed below:

$$h_{B_t}(\upsilon_1, \upsilon_2) = \wedge\{h_{A_t}(\upsilon_1), h_{A_t}(\upsilon_2)\}, \forall\, (\upsilon_1, \upsilon_2) \in Ȩ.$$

Definition.2.6. In t -FG, the order is defined as follows

$$O(\mathcal{G}_t) = \left(\sum_{\upsilon_1 \in Ʋ} h_{A_t}(\upsilon_1)\right)$$

Example.2.8. The order of t-FG $\mathcal{G}_t$ is (1.5) from example 2.3.

Definition.2.9. The t-FG has a size defined by

$$S(\mathcal{G}_t) = \left(\sum_{(\upsilon_1, \upsilon_2) \in Ȩ} h_{B_t}(\upsilon_1, \upsilon_2)\right)$$

Definition. 2.10. t *-FG defines the degree of vertex* υ_1 in $\mathcal{G}_t$ *as follows:*

$$deg_{\mathcal{G}_t}(\upsilon_1) = \left(deg_{h_{\mathrm{B}_t}}(\upsilon_1)\right)$$

$$deg_{\mathcal{G}_t}(\upsilon_1) = \left(\sum_{(\upsilon_1,\upsilon_2)\in \mathrm{E}_t} h_{\mathrm{B}_t}(\upsilon_1,\upsilon_2)\right)$$

Example.2.11. Referring to example 2.3,

1. The degree of vertex in $\mathcal{G}_t$ are, $deg_{\mathcal{G}_t}(u) = (1.1)$; $deg_{\mathcal{G}_t}(v) = (0.6)$; $deg_{\mathcal{G}_t}(w) = (1.1)$; $deg_{\mathcal{G}_t}(x) = (0.8)$.
2. $\delta(\mathcal{G}_t)$ is minimum degree of t -FG , $\delta(\mathcal{G}_t) = (\delta_{h_{\mathrm{B}_t}}(\mathcal{G}_t), \delta(\mathcal{G}_t) = \left(\wedge\left\{deg_{h_{\mathrm{B}_t}}(\upsilon_1): \upsilon_1 \in \mathrm{U}\right\}\right)$.
3. $\Delta(\mathcal{G}_t)$ is maximum degree of t-FG , $\Delta(\mathcal{G}_t) = (\Delta_{h_{\mathrm{B}_t}}(\mathcal{G}_t)), \Delta(\mathcal{G}_t) = \left(\max\left\{deg_{h_{\mathrm{B}_t}}(\upsilon_1): \upsilon_1 \in \mathrm{U}\right\}\right)$.

From example 2.3: $\delta(\mathcal{G}_t)$ = (0.6); $\Delta(\mathcal{G}_t)$ = (1.1). In t-FG the following inequality holds, $\delta(\mathcal{G}_t) \leq \Delta(\mathcal{G}_t) \leq S(\mathcal{G}_t) \leq O(\mathcal{G}_t)$

Theorem.2.12 An t-FG is represented as $\mathcal{G}_t$= (A_t, B_t), then

$$\sum deg_{\mathcal{G}_t}(\upsilon_i) = \left(2\sum h_{\mathcal{G}_t}(\upsilon_i, w)\right)$$

Proof. Considering t-FG represented by $\mathcal{G}_t$= (A_t, B_t), let's investigate,

$$\sum deg_{\mathcal{G}_t}(\upsilon_i) = \left(\sum deg_{h_{\mathrm{B}_t}}(\upsilon_i)\right)$$

$$= \left(deg_{h_{\mathrm{B}_t}}(\upsilon_1) + \left(deg_{h_{\mathrm{B}_t}}(\upsilon_2)\right) \ldots + \quad (deg_{h_{\mathrm{B}_t}}(\upsilon_n)\right)$$

$$= \left(h_{\mathrm{B}_t}(\upsilon_1,\upsilon_2)) + (h_{\mathrm{B}_t}(\upsilon_1,\upsilon_3) + \ldots + (h_{\mathrm{B}_t}(\upsilon_1,\upsilon_n) + \cdots + (h_{\mathrm{B}_t}(\upsilon_n,\upsilon_1) + (h_{\mathrm{B}_t}(\upsilon_n,\upsilon_2) + \ldots + (h_{\mathrm{B}_t}(\upsilon_n,\upsilon_{n-1})\right)$$

$$= \left(2(h_{\mathrm{B}_t}(\upsilon_1,\upsilon_2) + 2(h_{\mathrm{B}_t}(\upsilon_1,\upsilon_3) + \ldots + 2(h_{\mathrm{B}_t}(\upsilon_1,\upsilon_n)\right)$$

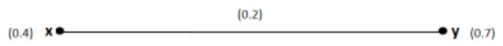
$$= \left(2\sum h_{\mathcal{G}_t}(\upsilon_i, w)\right).$$

4. OPERATION ON t-FUZZY GRAPH

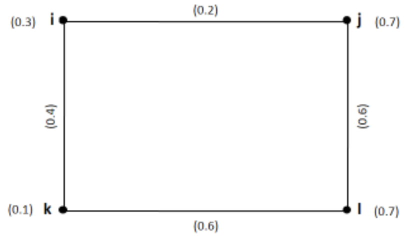
Definition.3.1. Consider two t-FG of G = (U, E) and G'=(U', E') correspond to $\mathcal{G}_t = (\mathrm{A}_t, \mathrm{B}_t)$ and $\mathcal{G}'_t = (\mathrm{A}'_t, \mathrm{B}'_t)$, respectively. $(\mathrm{A}_t \times \mathrm{A}'_t, \mathrm{B}_t \times \mathrm{B}'_t)$ defines the Cartesian product of two t -FG is denoted bt $\mathcal{G}_t \times \mathcal{G}'_t$. Where $\mathrm{A}_t \times \mathrm{A}'_t$ and $\mathrm{B}_t \times \mathrm{B}'_t$ are t -FS on $\mathrm{U} \times \mathrm{U}' = \{(\kappa_1, \varphi_1), (\varphi_2, \kappa_2): \varphi_1 \& \varphi_2 \in \mathrm{U}; \kappa_1 \& \kappa_2 \in \mathrm{U}'\}$ and $\mathrm{E} \times \mathrm{E}' = \{(\varphi_1, \kappa_1), (\varphi_2, \kappa_2): \varphi_1 = \varphi_2, \varphi_1 \& \varphi_2 \in \mathrm{U}, (\kappa_1, \kappa_2) \in \mathrm{E}'\} \cup \{(\varphi_1, \kappa_1), (\varphi_2, \kappa_2): \kappa_1 = \kappa_2, \kappa_1 \& \kappa_2 \in \mathrm{U}', (\varphi_1, \varphi_2) \in \mathrm{E}\} \cup \{(\varphi_1, \kappa_1), (\varphi_2, \kappa_2): \kappa_1 \neq \kappa_2, \varphi_1 \neq \varphi_2, (\kappa_1, \kappa_2) \in \mathrm{E}', (\varphi_1, \varphi_2) \in \mathrm{E}\}$ respectively, which fulfills the given requirement.

1. $\forall((\varphi_1, \kappa_1) \in \mathrm{U} \times \mathrm{U}', h_{\mathrm{A}_t \times \mathrm{A}'_t}(\varphi_1, \kappa_1) = \wedge\left\{h_{\mathrm{A}_t}(\varphi_1), h_{\mathrm{A}_{t'}}(\kappa_1)\right\}$
2. If $\varphi_1 = \varphi_2$ and $\forall (\kappa_1, \kappa_2) \in \mathrm{E}'$, $h_{\mathrm{B}_t \times \mathrm{B}'_t}((\varphi_1, \kappa_1), (\varphi_2, \kappa_2)) = \wedge\left\{h_{\mathrm{A}_t}(\varphi_1), h_{\mathrm{B}_{t'}}(\kappa_1, \kappa_2)\right\}$
3. If $\kappa_1 = \kappa_2$ and $\forall (\varphi_1, \varphi_2) \in \mathrm{E}$, $h_{\mathrm{B}_t \times \mathrm{B}'_t}((\varphi_1, \kappa_1), (\varphi_2, \kappa_2)) = \wedge\left\{h_{\mathrm{B}_t}(\varphi_1, \varphi_2), h_{\mathrm{A}_{t'}}(\kappa_1)\right\}$
4. If $\kappa_1 \neq \kappa_2$ *and* $\varphi_1 \neq \varphi_2$, $\forall (\kappa_1, \kappa_2) \in \mathrm{E}', (\varphi_1, \varphi_2) \in \mathrm{E}$, $h_{\mathrm{B}_t \times \mathrm{B}'_t}((\varphi_1, \kappa_1), (\varphi_2, \kappa_2)) = \wedge\left\{h_{\mathrm{B}_t}(\varphi_1, \varphi_2), h_{\mathrm{B}_{t'}}(\kappa_1, \kappa_2)\right\}$.

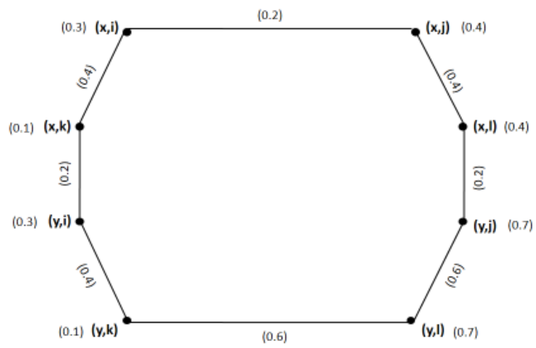
Example.3.2. The two 0.3-FG $\mathcal{G}_t$ and $\mathcal{G}'_t$, which are the elements to be taken into consideration, are shown in Graphs 2 and 3. Graph 4 displays the corresponding Cartesian product $\mathcal{G}_{0.3} \times \mathcal{G}'_{0.3}$.



Graph 2. 0.3-FG $\mathcal{G}_{0.3}$



Graph 3. 0.3-FG $\mathcal{G}'_{0.3}$



Graph 4. 0.3-FG $\mathcal{G}_{0.3} \times \mathcal{G}'_{0.3}$

Definition.3.3. $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(ϙ_1, ƙ_1) = \left(\deg\left\{h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big)\right\}\right)$

Where,

$$\deg\left\{h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big)\right\} = \sum_{ϙ_1 = ϙ_2, (ƙ_1, ƙ_2) \in Ȩ'} \wedge\left\{h_{Ą_t}(ϙ_1), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$$
$$+ \sum\nolimits_{ƙ_1 = ƙ_2, (ϙ_1, ϙ_2) \in Ȩ} \wedge\left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ą_{t'}}(ƙ_1)\right\}$$
$$+ \sum\nolimits_{ƙ_1 \neq ƙ_2, ϙ_1 \neq ϙ_2} \wedge\left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$$

Example.3.4. From example 3.2, the degree of vertex in $\mathcal{G}_{0.3} \times \mathcal{G}'_{0.3}$

$deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x, i) = (0.6)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x, j) = (0.6)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x, k) = (0.6)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x, l) = (0.6)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, i) = (0.6)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, j) = (0.8)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, k) = (1.0)$, $deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, l) = (1.0)$

Theorem.3.5. The Cartesian products of two t -FGs, result is a new t -FG

Proof: The requirement is clear for $Ą_t \times Ą'_t$. Considering that $(ƙ_1, ƙ_2) \in Ȩ'$ and $ϙ_1 \in Ʋ$,

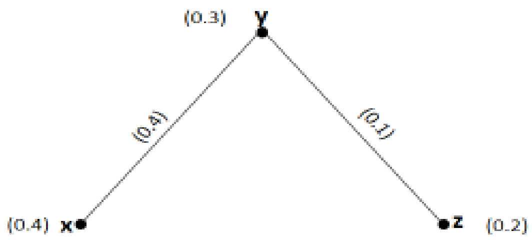
$$h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big) = \wedge\left\{h_{Ą_t}(ϙ_1), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$$
$$h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big) \leq \wedge\left\{h_{Ą_t}(ϙ_1), \wedge\{h_{Ą_{t'}}(ƙ_1), h_{Ą_{t'}}(ƙ_2)\}\right\}$$
$$h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big) \leq \wedge\left\{\wedge\{h_{Ą_t}(ϙ_1), h_{Ą_{t'}}(ƙ_1)\}, \wedge\{h_{Ą_t}(ϙ_1), h_{Ą_{t'}}(ƙ_2)\}\right\}$$
$$h_{Ɓ_t \times Ɓ'_t}\big((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)\big) = \wedge\left\{h_{Ą_t \times Ą'_t}(ϙ_1, ƙ_1), h_{Ą_t \times Ą'_t}(ƙ_1, ƙ_2)\right\}$$

Likewise we can demonstrate it for $ƙ_1 \in Ʋ', (ϙ_1, ϙ_2) \in Ȩ$.

Definition.3.6. Consider two t-FG of G = (Ʋ, E) and G'= (Ʋ',E') correspond to $\mathcal{G}_t = (Ą_t, Ɓ_t)$ and $\mathcal{G}'_t = (Ą'_t, Ɓ'_t)$, respectively. $(Ą_t \circ Ą'_t, Ɓ_t \circ Ɓ'_t)$ defines composition $\mathcal{G}_t \circ \mathcal{G}'_t$ of two t-FG. Where $Ą_t \circ Ą'_t$ and $Ɓ_t \circ Ɓ'_t$ are t-FS on $Ʋ \times Ʋ' = \{(ϙ_1, ƙ_1), (ϙ_2, ƙ_2): ϙ_1 \& ϙ_2 \in Ʋ; ƙ_1 \& ƙ_2 \in Ʋ'\}$ and $Ȩ \times Ȩ' = \{(ϙ_1, ƙ_1), (ϙ_2, ƙ_2): ϙ_1 = ϙ_2, ϙ_1 \& ϙ_2 \in Ʋ, (ƙ_1, ƙ_2) \in Ȩ'\} \cup \{(ϙ_1, ƙ_1), (ϙ_2, ƙ_2): ƙ_1 = ƙ_2, ƙ_1 \& ƙ_2 \in Ʋ', (ϙ_1, ϙ_2) \in Ȩ\} \cup \{(ϙ_1, ƙ_1), (ϙ_2, ƙ_2): ƙ_1 \neq ƙ_2, ϙ_1 \neq ϙ_2, (ƙ_1, ƙ_2) \in Ȩ', (ϙ_1, ϙ_2) \in Ȩ\}$ respectively, which fulfills the given requirement.

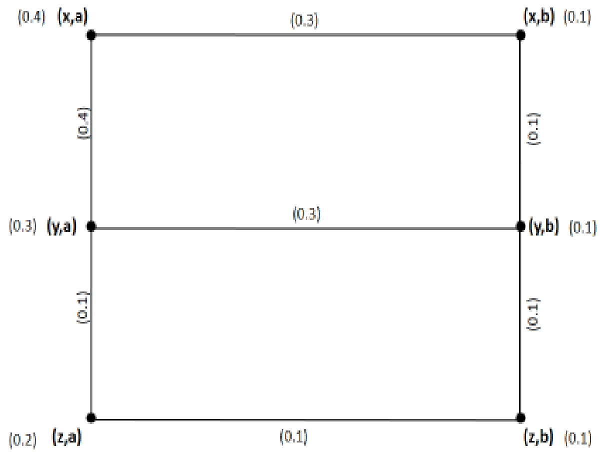
1. $\forall((ϙ_1, ƙ_1) \in Ʋ \circ Ʋ'$, $h_{Ą_t \circ Ą'_t}(ϙ_1, ƙ_1) = \wedge \left\{h_{Ą_t}(ϙ_1), h_{Ą_{t'}}(ƙ_1)\right\}$
2. If $ϙ_1 = ϙ_2$ and $\forall\ (ƙ_1, ƙ_2) \in Ȩ'$, $h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)) = \wedge\left\{h_{Ą_t}(ϙ_1), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$
3. If $ƙ_1 = ƙ_2$ and $\forall\ (ϙ_1, ϙ_2) \in Ȩ$, $h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)) = \wedge\left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ą_{t'}}(ƙ_1)\right\}$
4. If $ƙ_1 \neq ƙ_2$ and $\forall\ (ϙ_1, ϙ_2) \in Ȩ$, $h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)) = \wedge\left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ą_{t'}}(ƙ_1), h_{Ą_{t'}}(ƙ_2)\right\}$
5. If $ϙ_1 \neq ϙ_2$ and $\forall\ (ƙ_1, ƙ_2) \in Ȩ'$, $h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2)) = \wedge\left\{h_{Ą_t}(ϙ_1), h_{Ą_t}(ϙ_2), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$

Example.3.7**.** Contemplate two 0.6-FG $\mathcal{G}_t$ and $\mathcal{G}'_t$ illustrated in Graph 5 & 6.



Graph 5. 0.6-FG $\mathcal{G}_{0.6}$

Graph 6. 0.6-FG $\mathcal{G}'_{0.6}$



Graph 7. 0.6-FG $\mathcal{G}_{0.6} \circ \mathcal{G}'_{0.6}$

Graph 7. Shows their corresponding Compositon $\mathcal{G}_{0.6} \circ \mathcal{G}'_{0.6}$

Definition.3.8. From the composition of two t-FG, the degree of vertex is demonstrate as follows

$$(ϙ_1, ƙ_1) \in Ʋ \times Ʋ'; deg_{\mathcal{G}_t \circ \mathcal{G}'_t}(ϙ_1, ƙ_1)$$
$$= \left(\deg\left\{h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2))\right\}, \deg\left\{I_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2))\right\}, \deg\{F_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2))\}\right)$$

Where, $\deg\left\{h_{Ɓ_t \circ Ɓ'_t}((ϙ_1, ƙ_1), (ϙ_2, ƙ_2))\right\} = \sum_{ϙ_1 = ϙ_2, (ƙ_1, ƙ_2) \in Ȩ'} \wedge \left\{h_{Ą_t}(ϙ_1), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$

$$+ \sum_{ƙ_1 = ƙ_2, (ϙ_1, ϙ_2) \in Ȩ} \wedge \left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ą_{t'}}(ƙ_1)\right\}$$
$$+ \sum_{ƙ_1 \neq ƙ_2, (ϙ_1, ϙ_2) \in Ȩ} \wedge \left\{h_{Ɓ_t}(ϙ_1, ϙ_2), h_{Ą_{t'}}(ƙ_1), h_{Ą_{t'}}(ƙ_2)\right\}$$
$$+ \sum_{ϙ_1 \neq ϙ_2, (ƙ_1, ƙ_2) \in Ȩ'} \wedge \left\{h_{Ą_t}(ϙ_1), h_{Ą_t}(ϙ_2), h_{Ɓ_{t'}}(ƙ_1, ƙ_2)\right\}$$

Example.3.9. In Graph 7 shows that every vertex in $\mathcal{G}_t \circ \mathcal{G}'_t$ must have degrees,

$$deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x, a) = (0.7), deg_{\mathcal{G}_t \times \mathcal{G}'_t}(x.b) = (0.4), deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, a) = (0.8),$$
$$deg_{\mathcal{G}_t \times \mathcal{G}'_t}(y, b) = (0.5), deg_{\mathcal{G}_t \times \mathcal{G}'_t}(z, a) = (0.2), deg_{\mathcal{G}_t \times \mathcal{G}'_t}(z, b) = (0.2)$$

Definition.3.10. Consider two t -FG of G = (Ʋ,E) and G'=(Ʋ',E') correspond to $\mathcal{G}_t = (Ą_t, Ɓ_t)$ and $\mathcal{G}'_t = (Ą'_t, Ɓ'_t)$, respectively. $(Ą_t \cup Ą'_t, Ɓ_t \cup Ɓ'_t)$ defines the union $\mathcal{G}_t \cup \mathcal{G}'_t$ of two t -FG, $Ą_t \cup Ą'_t$ and $Ɓ_t \cup Ɓ'_t$, represent t -FS on $Ʋ \cup Ʋ'$ and $Ȩ \cup Ȩ'$, which fulfills the given requirement,

1) If $ϙ_1 \in Ʋ$ and $ϙ_1 \notin Ʋ'$. $h_{Ą_t \cup Ą'_t}(ϙ_1) = h_{Ą_t}(ϙ_1)$
2) If $ϙ_1 \notin Ʋ$ and $ϙ_1 \in Ʋ'$. $h_{Ą_t \cup Ą'_t}(ϙ_1) = h_{Ą'_t}(ϙ_1)$
3) If $ϙ_1 \in Ʋ \cap Ʋ'$. $h_{Ą_t \cup Ą'_t}(ϙ_1) = max\{h_{Ą_t}(ϙ_1), h_{Ą'_t}(ϙ_1)\}$
4) If $(ϙ_1, ƙ_1) \in Ȩ$ and $(ϙ_1, ƙ_1) \notin Ȩ'$. $h_{Ɓ_t \cup Ɓ'_t}(ϙ_1, ƙ_1) = h_{Ɓ_t}(ϙ_1, ƙ_1)$
5) If $(ϙ_1, ƙ_1) \notin Ȩ$ and $(ϙ_1, ƙ_1) \in Ȩ'$. $h_{Ɓ_t \cup Ɓ'_t}(ϙ_1, ƙ_1) = h_{Ɓ'_t}(ϙ_1, ƙ_1)$
6) If $(ϙ_1, ƙ_1) \in Ȩ \cap Ȩ'$. $h_{Ɓ_t \cup Ɓ'_t}(ϙ_1, ƙ_1) = max\{h_{Ɓ_t}(ϙ_1, ƙ_1), h_{Ɓ'_t}(ϙ_1, ƙ_1)\}$

Definition.3.11. Let $(ϙ_1, ƙ_1)$ is a degree of vertex in a t-FG for every $(ϙ_1, ƙ_1) \in Ʋ \cup Ʋ'$.

$$deg_{\mathcal{G}_t \cup \mathcal{G}'_t}(ϙ_1, ƙ_1) = \left(\deg\left\{h_{Ɓ_t \cup Ɓ'_t}(ϙ_1, ƙ_1)\right\}\right)$$

Where

$$deg\left\{h_{B_t\cup B_t'}(\varrho_1,\kappa_1)\right\}=\sum_{(\varrho_1,\kappa_1)\in E,(\varrho_1,\kappa_1)\notin E'} h_{B_t}(\varrho_1,\kappa_1)+\sum_{(\varrho_1,\kappa_1)\notin E,(\varrho_1,\kappa_1)\in E'} h_{B_t'}(\varrho_1,\kappa_1)$$
$$+\sum\nolimits_{(\varrho_1,\kappa_1)\in E\cap E'} max\{h_{B_t}(\varrho_1,\kappa_1),h_{B_t'}(\varrho_1,\kappa_1)\}.$$

Definition.3.12. let $\mathcal{G}_t=(A_t,B_t)$ and $\mathcal{G}_t'=(A_t',B_t')$ be a two t-FGs and the join operation of t–FG is $\mathcal{G}_t+\mathcal{G}_t'$ is demonstrated as (A_t+A_t',B_t+B_t'), A_t+A_t' yields a t-FG on $U\cup U'$ & B_t+B_t' forms a t-FG in $E\cup E'\cup E''$ has to give particular below conditions

1) If $\varrho_1\in U$ and $\varrho_1\notin U'$, $h_{A_t+A_t'}(\varrho_1)=h_{A_t}(\varrho_1)$
2) If $\varrho_1\notin U$ and $\varrho_1\in U'$, $h_{A_t+A_t'}(\varrho_1)=h_{A_t'}(\varrho_1)$
3) If $\varrho_1\in U\cap U'$, $h_{A_t+A_t'}(\varrho_1)=max\{h_{A_t}(\varrho_1),h_{A_t'}(\varrho_1)\}$
4) If $(\varrho_1,\kappa_1)\in E$ and $(\varrho_1,\kappa_1)\notin E'$, $h_{B_t+B_t'}(\varrho_1,\kappa_1)=h_{B_t}(\varrho_1,\kappa_1)$
5) If $(\varrho_1,\kappa_1)\notin E$ and $(\varrho_1,\kappa_1)\in E'$, $h_{B_t+B_t'}(\varrho_1,\kappa_1)=h_{B_t'}(\varrho_1,\kappa_1)$
6) If $(\varrho_1,\kappa_1)\in E\cap E'$, $h_{B_t+B_t'}(\varrho_1,\kappa_1)=max\{h_{B_t}(\varrho_1,\kappa_1),h_{B_t'}(\upsilon_1,w_1)\}$
7) If $(\varrho_1,\kappa_1)\in E''$, $h_{B_t+B_t'}(\varrho_1,\kappa_1)=max\{h_{B_t}(\varrho_1),h_{B_t'}(\kappa_1)\}$

Definition.3.13. Examine the subsequent pair of t -FGs, $\mathcal{G}_t$ and $\mathcal{G}_t'$. The t -FG has the degree for each vertex $\mathcal{G}_t+\mathcal{G}_t'$. If $(\varrho_1,\kappa_1)\in U+U'$, then $deg_{\mathcal{G}_t+\mathcal{G}_t'}(\varrho_1,\kappa_1)=\left(deg\left\{h_{B_t+B_t'}(\varrho_1,\kappa_1)\right\}\right)$

Where

$$deg\left\{h_{B_t\cup B_t'}(\varrho_1,\kappa_1)\right\}=\left(\sum_{(\varrho_1,\kappa_1)\in E,(\varrho_1,\kappa_1)\notin E'} h_{B_t}(\varrho_1,\kappa_1)+\sum_{(\varrho_1,\kappa_1)\notin E,(\varrho_1,\kappa_1)\in E'} h_{B_t'}(\varrho_1,\kappa_1)\right.$$
$$\left.+\sum_{(\varrho_1,\kappa_1)\in E\cap E'} max\{h_{B_t}(\varrho_1,\kappa_1),h_{B_t'}(\varrho_1,\kappa_1)\}+\sum_{(\varrho_1,\kappa_1)\in E''} max\{h_{B_t}(\varrho_1),h_{B_t'}(\kappa_1)\}\right)$$

Theorem. 3.14 For any two t-FGs, their union is a t-FG.

Proof. Let us assume a t-FG, $\mathcal{G}_t\cup\mathcal{G}_t'$. Let $(\varrho_1,\kappa_1)\in E$, $(\varrho_1,\kappa_1)\notin E'$ and $(\varrho_1,\kappa_1)\in U-U'$

Consider

$$h_{B_t}(\varrho_1,\kappa_1)=h_{B_t\cap B_t'}(\varrho_1,\kappa_1)$$
$$h_{B_t}(\varrho_1,\kappa_1)\le\wedge\{h_{A_t\cup A_t'}(\varrho_1),h_{A_t\cup A_t'}(\kappa_1)\}$$
$$h_{B_t}(\varrho_1,\kappa_1)=\wedge\{h_{A_t}(\varrho_1),h_{A_t}(\kappa_1)\}$$

Consequently $h_{B_t}(\varrho_1,\kappa_1)\le\wedge\{h_{A_t}(\varrho_1),h_{A_t}(\kappa_1)\}$.

Hence, $\mathcal{G}_t=(A_t,B_t)$ is established as a t -FG. Likewise, we deduce that $\mathcal{G}_t'=(A_t',B_t')$ is t-FG in G''. Given that $\mathcal{G}_t$ & $\mathcal{G}_t'$ are assumed, and since two t-FGs together produce a t-FG, we may deduce that $\mathcal{G}_t\cup\mathcal{G}_t'$.

4. ISOMORPHISM OF T-FUZZY GRAPH

Definition. 4.1. Let $\mathcal{G}_t$ & $\mathcal{G}_t'$ be any two t -FG. $\theta:\mathcal{G}_t\ to\ \mathcal{G}_t'$ is a homomorphism U *to* U', satisfies the ensuing requirements:

1. $h_{A_t}(\varrho_1)\le h_{A_t'}(\theta(\varrho_1))$; $\forall\varrho_1\in U$.
2. $h_{B_t}(\varrho_1,\kappa_1)\le h_{B_t'}(\theta(\varrho_1),\theta(\kappa_1)),\forall(\varrho_1,\kappa_1)\in E$.

Definition. 4.2. A weak isomorphism $\theta: Ʊ$ to $Ʊ'$, from t-FG $\mathcal{G}_t$ to $\mathcal{G}_t'$ has to meet the below.
$h_{A_t}(ϙ_1) = h_{A_t'}(\theta(ϙ_1))$, $\forall ϙ_1 \in Ʊ$.

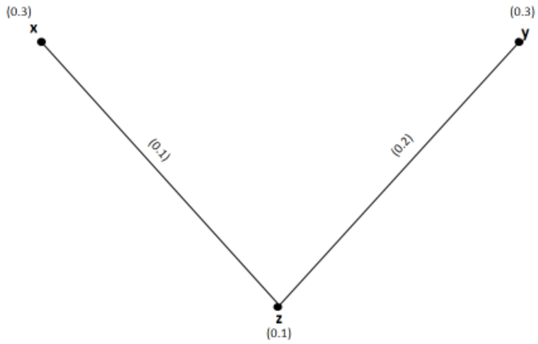
Definition. 4.3. A bijective mapping $\theta: Ʊ$ to $Ʊ'$ between two t-FGs, $\mathcal{G}_t = (A_t, B_t)$& $\mathcal{G}_t' = (A_t', B_t')$of G = (Ʊ, Ȩ) and G'= (Ʊ', Ȩ'), that satisfies the below criteria is called a strong co-isomorphism. $h_{A_t}(ϙ_1) \le h_{A_t'}(\theta(ϙ_1))$; $\forall ϙ_1 \in Ʊ$.

1. $h_{B_t}(ϙ_1, ķ_1) \le h_{B_t'}(\theta(ϙ_1), \theta(ķ_1))$,
2. $h_{B_t}(ϙ_1, ķ_1) = h_{B_t'}(\theta(ϙ_1), \theta(ķ_1))$, ; $\forall (ϙ_1, ķ_1) \in Ȩ$.

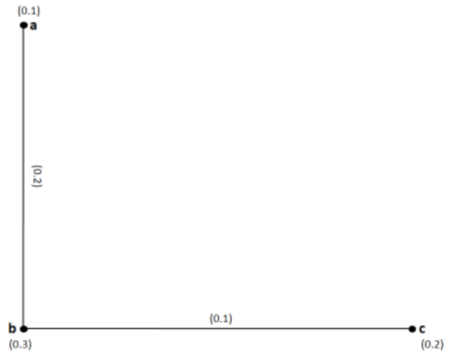
Definition. 4.4. An isomorphism of two t-FGs is a bijective homomorphism mapping $\theta: Ʊ$ *to* $Ʊ'$, which gives the below criteria

1. $h_{A_t}(ϙ_1) = h_{A_t'}(\theta(ϙ_1))$, $\forall ϙ_1 \in Ʊ$.
2. $h_{B_t}(ϙ_1, ķ_1) = h_{B_t'}(\theta(ϙ_1), \theta(ķ_1))$, ; $\forall (ϙ_1, ķ_1) \in Ȩ$.

Example. 4.5. From the below graphs, 0.7-$\mathcal{G}_t$ & $\mathcal{G}_t'$;



Graph 8. $0.7 - NG\ \mathcal{G}_{0.7}$



Graph 9. $0.7 - NG\ \mathcal{G}_{0.7}'$

Definition (4.4), the mapping ζ(x) = c, ζ(y) = b & ζ(z)=a gives us $\mathcal{G}_{0.7} \approx \mathcal{G}_{0.7}'$

Theorem. 4.6. The isomorphism between t-FGs satisfies the properties of an equivalence relation.

Proof: It is evident that there is symmetry and reflexivity. $\varphi: Ʊ$ to $Ʊ'$ and $\theta: Ʊ'$ to $Ʊ''$ indicate the isomorphisms of $\mathcal{G}_t$ onto $\mathcal{G}_t'$ & $\mathcal{G}_t'$ onto $\mathcal{G}_t''$. A bijective map from$Ʊ'$ to $Ʊ''$ is therefore $\theta \circ \varphi: Ʊ \to Ʊ''$, it is follows by

$$(\theta \circ \varphi)(ϙ_1) = \theta(\varphi(ϙ_1)), \forall ϙ_1 \in Ʊ$$

$\varphi: Ʊ$ to $Ʊ'$described by $\varphi(ϙ_1) = ķ_1, \forall ϙ_1 \in Ʊ$, it is an isomorphism. From def(4.4),

$$h_{A_t}(ϙ_1) = h_{A_t'}(\varphi(ϙ_1)) = h_{A_t'}(ķ_1), \forall ϙ_1 \in Ʊ$$

and

$$h_{B_t}(ϙ_1, ϙ_2) = h_{B_t'}(\varphi(ϙ_1), \varphi(ϙ_2)) = h_{B_t'}(ķ_1, ķ_2), \forall (ϙ_1, ϙ_2) \in Ȩ$$

Similarly, we have

$$h_{A_t'}(ķ_1) = h_{A_t''}(v_1), \forall ķ_1 \in Ʊ'$$

and

$$h_{B_t'}(ķ_1, ķ_2) = h_{B_t''}(v_1, v_2), \forall (ķ_1, ķ_2) \in Ȩ'$$

With the help of above relations, $\varphi(ϙ_1) = ķ_1$, $\forall ϙ_1 \in Ʊ$.

$$h_{A_t}(ϙ_1) = h_{A_t'}(\varphi(ϙ_1)) = h_{A_t'}(ķ_1) = h_{A_t''}(\theta(ķ_1)) = h_{A_t''}\left(\theta(\varphi(ϙ_1))\right)$$

and

$$h_{B_t}(ϙ_1, ϙ_2) = h_{B_t'}(ķ_1, ķ_2) = h_{B_t''}(\theta(ķ_1), \theta(ķ_2)) = h_{B_t''}\left(\theta(\varphi(ϙ_1)), \theta(\varphi(ϙ_2))\right)$$

Thus, $\mathcal{G}_t$ and $\mathcal{G}_t''$ are isomorphic using $\theta \circ \varphi$.

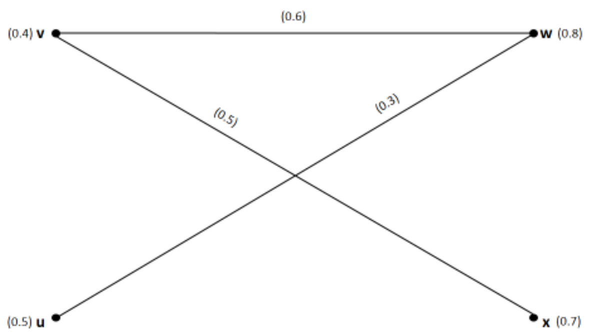
5. COMPLEMENT OF t-FUZZY GRAPH

Definition. 5.1. A t-FG of G = (Ʊ, Ȩ) is $\mathcal{G}_t = (A_t, B_t)$. A t-FG $\overline{\mathcal{G}_t}$ on $\overline{G} = (\overline{Ʊ}, \overline{Ȩ})$ is the complement of a t-FG $\mathcal{G}_t$ and is

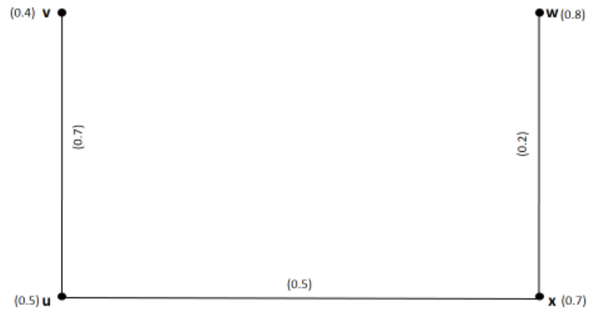
1. $\overline{Ʊ} = Ʊ$

2. $υ_1 \in Ʊ,\ h_{\overline{A_t}}(ϙ_1) = h_{A_t}(ϙ_1)$
3. If $h_{B_t}(ϙ_1, ϙ_2) \neq 0$
4. If $h_{B_t}(ϙ_1, ϙ_2) = 0$, then$h_{\overline{B_t}}(ϙ_1, ϙ_2) = \wedge\{h_{A_t}(ϙ_1), h_{A_t}(ϙ_2)\}$.

Example. 5.2. Consider a 0.2-FG $\mathcal{G}_t$ as shown in the below graph 10 . Then the complement $\overline{\mathcal{G}_t}$ of 0.2-FG $\overline{\mathcal{G}_t}$ is shown in graph 11



Graph 10. 0.2-FG$\mathcal{G}_{0.2}$



Graph 11. 0.2-FG$\overline{\mathcal{G}_{0.2}}$

Definition. 5.3. A t-FG$\mathcal{G}_t$ is said to be self-complementary t-FG if $\overline{\mathcal{G}_t} \approx \mathcal{G}_t$.

Preposition. 5.4. Let $\mathcal{G}_t = (A_t, B_t)$ be a self-complementary t-FG.

$$\sum_{ϙ_1 \neq ϙ_2} h_{B_t}(ϙ_1, ϙ_2) = \sum_{ϙ_1 \neq ϙ_2} min\{h_{A_t}(ϙ_1), h_{A_t}(ϙ_2)\}$$

Preposition. 5.5. Let $\mathcal{G}_t = (A_t, B_t)$, t -FG. If

$$\sum_{ϙ_1 \neq ϙ_2} h_{B_t}(ϙ_1, ϙ_2) = \sum_{ϙ_1 \neq ϙ_2} min\{h_{A_t}(ϙ_1), h_{A_t}(ϙ_2)\}, \forall\ ϙ_1, ϙ_2 \in Ʊ.$$

Then $\mathcal{G}_t$ is a self-complementary t-FG

Preposition. 5.6. For any two t-FG $\mathcal{G}_t$ and $\mathcal{G}_t'$. If $\mathcal{G}_t$ and $\mathcal{G}_t'$ have a strong homomorphism, then the strong isomorphism is $\overline{\mathcal{G}_t}$ and $\overline{\mathcal{G}_t'}$.

Proof. Let $\mathcal{G}_t$ and $\mathcal{G}_t'$ have a strong isomorphism, denoted by φ. The Given φ & φ^{-1} is ta bijective map, with $\varphi^{-1}(ς_1) = ϙ_1, \forall ς_1 \in Ʊ'$. Thus

$$h_{A_t}\left(\varphi^{-1}(ς_1)\right) = h_{A_t'}(ϙ_1), \forall ς_1 \in Ʊ'$$

Applying definition 23 makes it clear that:

$$h_{\overline{B_t}}(ϙ_1, ς_1) = \wedge\{h_{A_t}(ϙ_1), h_{A_t}(ς_1)\}$$

$$h_{\overline{B_t}}(ϙ_1, ς_1) \leq \wedge\left\{h_{A_t'}(\varphi(ϙ_2)), h_{A_t'}(\varphi(ς_2))\right\}$$

$$h_{\overline{B_t}}(ϙ_1, ς_1) \leq \wedge\left\{h_{A_t'}(ϙ_2), h_{A_t'}(ς_2)\right\}$$

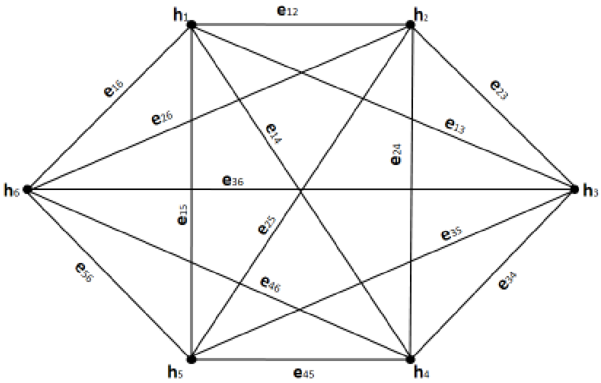
$$h_{\overline{B_t}}(ϙ_1, ς_1) = h_{\overline{B_t}}(ϙ_2, ς_2)$$

Thus $h_{\overline{B_t}}(ϙ_1, ς_1) \leq h_{\overline{B_t}}(ϙ_2, ς_2)$

It follows from this that $\overline{\mathcal{G}_t}$ and $\overline{\mathcal{G}_t'}$ are strongly isomorphic.

6. DECISION SUPPORT IN HEALTH MANAGEMENT USING t-FUZZY GRAPHS

Decision support in health management is becoming increasingly vital as healthcare systems move towards sustainable and efficient practices. Regenerative approaches, waste reduction, and optimized resource use are key objectives in reshaping health management. Let's consider a case where seven vertices represent significant factors in improving decision-making processes in health management. Resource Allocation Efficiency (h_1) focuses on maximizing the utilization of healthcare resources while minimizing waste. Healthcare Waste Management (h_2) involves effective methods for the disposal, recycling, and reduction of medical waste. Innovative Health Technologies (h_3) encourages the development and application of eco-friendly, health-oriented technologies. Sustainable Health Practices (h_4) promote responsible consumption of healthcare services and medications, fostering responsible healthcare behaviors in individuals and communities. Renewable Healthcare Energy (h_5) represents the transition to renewable and sustainable energy sources in healthcare infrastructure. Eco-friendly Healthcare Production (h_6) emphasizes the use of environmentally responsible production methods for medical supplies and equipment.



Graph 12.Fuzzy Graph

Let U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the set of factors contributing to decision support in health management. The edges represent the level of interaction between these factors. For instance, using a t-fuzzy model, decision-makers can assess the degree of correlation between the components. For example, if the Sustainable Health Practices (h_4) strongly impact Eco-friendly Healthcare Production (h_6), this would be reflected in a high membership degree between (h_4) and (h_6), suggesting a strong positive influence on sustainable practices in healthcare production. On the other hand, low membership degrees would indicate weaker connections or a lack of significant impact between factors.

In this model, decision-makers can examine fuzzy membership (connection). For instance, the edge e_{12} representing the connection between Resource Allocation Efficiency (h_1) and Healthcare Waste Management (h_2) might indicate a significant connection, with a membership value of 0.8, suggesting a moderately strong and uncertain correlation. By adjusting the parameter 't' in the model, decision-creators can modify the analysis permitting to their specific situation, expertise and risk preferences, offering a flexible and nuanced approach to decision support in health management.

Edges	FS	0.7-FS	Edges	FS	0.7-FS
$\boldsymbol{e_{12}}$=$(\boldsymbol{h_1}, \boldsymbol{h_2})$	(0.4)	(0.4)	e_{26}=(h_2, h_6)	(0.5)	(0.5)
$\boldsymbol{e_{13}}$=$(\boldsymbol{h_1}, \boldsymbol{h_3})$	(0.6)	(0.6)	e_{34}=$(h_3 h)$	(0.3)	(0.3)
$\boldsymbol{e_{14}}$=$(\boldsymbol{h_1}, \boldsymbol{h_4})$	(0.8)	(0.7)	e_{35}=(h_3, h_5)	(0.1)	(0.1)
$\boldsymbol{e_{15}}$=$(\boldsymbol{h_1}, \boldsymbol{h_5})$	(0.2)	(0.2)	e_{36}=(h_3, h_6)	(0.6)	(0.6)
$\boldsymbol{e_{16}}$=$(\boldsymbol{h_1}, \boldsymbol{h_6})$	(0.9)	(0.7)	e_{45}=(h_4, h_5)	(0.7)	(0.7)
$\boldsymbol{e_{23}}$=$(\boldsymbol{h_2}, \boldsymbol{h_3})$	(0.1)	(0.1)	e_{46}=(h_4, h_6)	(0.9)	(0.7)
$\boldsymbol{e_{24}}$=$(\boldsymbol{h_2}, \boldsymbol{h_4})$	(0.5)	(0.5)	e_{56}=(h_5, h_6)	(0.8)	(0.7)
$\boldsymbol{e_{25}}$=$(\boldsymbol{h_2}, \boldsymbol{h_5})$	(0.4)	(0.4)			

Table 2: Edges of FS and 0.7-NS

The table of fuzzy membership degree of each factor is given below;

Factors	Degree of each factor
$\boldsymbol{h_1}$	deg $(h_1) = (2.6)$
$\boldsymbol{h_2}$	deg $(h_2) = (1.9)$

$\boldsymbol{h_3}$	deg $(h_3) = (1.7)$
$\boldsymbol{h_4}$	deg $(h_4) = (2.9)$
$\boldsymbol{h_5}$	deg $(h_5) = (2.1)$
$\boldsymbol{h_6}$	deg $(h_6) = (3.2)$

Comparatively, deg (h_6) =3.2 is the greatest value. So, by our assumption h_6 factor has the high potential to Eco-friendly Healthcare Production.

6.1 COMPARATIVE ANALYSIS

The choice between t-fuzzy and fuzzy graphs depends on the specific requirements of the decision-making problem. Both models utilize the ' t ' parameter to adjust uncertainty levels; however, when a more precise, comprehensive, and detailed representation of uncertainty is required, t -fuzzy graphs are more effective. Their ability to independently assess connection makes them particularly useful in multifaceted decision-creation situations such as pattern recognition, medical identification, and advanced choice provision systems. Due to their exceptional flexibility and meticulous handling of uncertainty factors, t-fuzzy graphs are the preferred approach for achieving high precision and granularity in uncertainty modeling.

7. CONCLUSION

This study underscores the significance of t -Fuzzy Graphs (t-FG) in enhancing decision-making processes within health management systems. By effectively modeling uncertainty and multi-dimensional dependencies, t-FG provide a structured and adaptable framework for analyzing complex interactions among medical, financial, and operational factors. The exploration of fundamental t -FG operations, such as homomorphism and isomorphism, highlights their role in optimizing strategic planning. Additionally, real-world applications demonstrate their potential in critical areas like resource allocation, patient care strategies, and financial planning. Overall, t -FG emerge as a powerful tool for policy development and intelligent decision-making, offering a robust approach to addressing the intricate challenges of healthcare management.

Future work: Future research can investigate additional t -FG operations, such as edge contraction, decomposition, and clustering techniques, to enhance decision-making efficiency. The creation of advanced algorithms for t -FG-based analysis and optimization can improve computational performance and practical implementation in real-time healthcare systems.

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