

Designing of Increased Distance Modified Linear Block Code with its Decoding Algorithm using Syndrome Polynomial

Seema Talmale¹, B.K.Lande²

¹ Faculty K.J.Somaiya School of Engineering, Department of Electronics Engineering, SVU, Mumbai, India

² Department of Electronics Engineering, Datta Meghe College of Engineering, Navi Mumbai, India

ARTICLE INFO

Received: 04 Jan 2025

Revised: 26 Feb 2025

Accepted: 06 Mar 2025

ABSTRACT

Introduction: This article gives the designing of increased distance modified linear block code (MLBC). Designing of this MLBC is initially obtained from the basis vectors an Upper Triangular Matrix (UTM). Considered UTM is of 9th order. Using the basis vectors of a 9th order UTM, MLBC is established which has less minimum distance (d_{\min}) between the codewords. A new approach is developed to increase this d_{\min} with the help of distance increasing mapping (DIM). The developed code here for 9th order UTM is (18, 9, $d_{\min} = 5$) code. A decoding algorithm is designed to decode the errors in the transmission with the help of syndrome polynomials. The developed algorithm avoids the limitation of multiple solutions and Maximum Likelihood decoding associated with Linear Block Codes. Performance of this code has been established through simulation. Examples are explored here to decode the various bit positional errors if occurred during the transmission process with the help of developed decoding algorithm through simulation. These designed codes can be used in high-frequency radio environments for transmission and reception of signals.

Keywords: Encoding, Decoding, Code rate, Modulation, Demodulation, Parity Check Matrix

1. INTRODUCTION

One of the challenging problems, in coding theory is to construct the code with the best possible error correction capability. There are several linear codes existing in the literature having the fixed error correction capability. That is, there are single error correcting, double error correcting, and multiple error correcting codes existing in the literature. But they are having fixed length that is the (n, k) parameters value for such codes is fixed.

The two main algorithmic tasks associated with the design of efficient Error-correcting codes are the implementation the encoding function E and the decoding function D . When an encoded information is transmitted over a channel at the receiver it is observed that information gets accompanied with the noise. It becomes very difficult to recover the transmitted information at the receiver end. So, Coding theory is associated with the two challenges. One is to design the effective and efficient and simple codes and the foremost important to design the efficient, effective and simple decoding techniques at the receiver end for the designed codes. The limitation of decoding algorithms is the search for the transmitted codeword in the vicinity of the received codeword and the decision based on the maximum likelihood principle. This approach involves considerable computational complexity and does not give a definite solution.

Most of the work in coding theory treats codes in a graph theoretical way [1-4]. We have approached the coding problem using system theoretic properties of the codes.

This correspondence gives designing of (18, 9): (n, k) modified linear block code from the basis vectors of an upper triangular matrix, where “ n ” is the length of the codeword and “ k ” represent the total number of message bits. It is observed that the minimum distance established for such a developed code is 2. In order to make this code a code with good error correction capability, a distance increasing mapping methodology is proposed here which leads to transform this (18,9, $d_{\min} = 2$) code to (18,9, $d_{\min} = 5$). [5-6] discusses about increasing the minimum distance of codes by twisting. This is a two bit error correcting code. This article also proposes a decoding algorithm for the developed (18,9, $d_{\min} = 5$) with the help of syndrome polynomials. [7-15] gives various decoding techniques.

The paper is organised as follows: Section 2 discusses the designing of the modified linear block code from UTM along with the construction of distance increasing mapping technique and a decoding algorithm for the designed MLBC. In section 3, results are presented. Paper is concluded in section 4.

2. RESEARCH METHOD: DSIGNING OF THE MODIFIED LINEAR BLOCK CODE FROM AN UPPER TRIANGULAR

Consider a 9th order UTM as given below:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In [16-18], generation of UTM from completely controllable discrete time system is discussed. If we consider the rows of this matrix as basis vectors and obtain the generator matrix $G = [I: P]$, where P is above considered UTM and I is an identity matrix.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots (I)$$

Designing of such generator matrix is discussed in [19-24]. In the process of establishing this generator matrix, an identity matrix of size equivalent to that of matrix P is merged to P . The simulation result in MATLAB on this matrix gives that the established code is (18, 9) modified linear block code, where $n = 18$ and $k = 9$.

The minimum distance between the codewords the 2 from simulation results. As well it can also be theoretically notified from the generator matrix of the designed code that lowest weight codeword in it is with weight 2, so $d_{\min} = 2$. According to [25], for a linear code, the minimum distance d , which satisfies, $d = \min \{w(c)\} = w$, where the minimum is overall codewords except the all-zero codeword, $w(c)$ = weight of the codeword.

It is well known that error correction capacity of a linear block code is associated with its minimum distance property. [Costello] gives $t = \{(d_{\min} - 1)/2\}$, where t = possible number of error correction in the received codeword. In the next section of this paper, we develop the methodology to increase the distance of this (18, 9, $d_{\min} = 2$) code.

2.1. Distance Increasing Mapping for the designed MLBC:

Mapping algorithm "A" proposed here, is the technique to increase the minimum distance between the codewords of a above designed MLBC. The devised MLBC with this technique is termed as increased distance MLBC.

Consider the nineth order of upper triangular matrix:

Construction: Mapping Algorithm “A”:

Input : $UTM(C_1, C_2, \dots, C_n) \in Z_2^n$

Output : $(\Omega_1, \Omega_2, \dots, \Omega_n) \leftarrow AUTM(C_1, C_2, \dots, C_n)$

Begin : $AUTM(C_1) \leftarrow (\Omega_1, \Omega_2, \dots, \Omega_n) \leftarrow$ Add all the rows of nineth order UTM except the last and the sixth row of this UTM.

For $i = 0$ to n do

Rotate one bit right of every $AUTM(C_{1+i})$ to get $AUTM(C_{1+i+1})$

end.

According to above construction, the first row of the generator matrix for designing of increased distance MLBC from developed (18, 9) MLBC can be obtained by addition of all the rows of 9th order UTM except the last and the sixth row of this UTM. Other rows of the generator matrix of increased distance MLBC can be obtained through rotation as suggested in the above construction.

The transformed generator matrix is as given below:

$$G_{NEW} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \dots (2)$$

MATLAB simulation performed on this new generator matrix gives the minimum distance between the codewords as 5. Thus, the proposed mapping algorithm is a DIM and the designed code is now transformed to (18, 9, $d_{\min} = 5$) MLBC.

2.2. Designing decoding algorithm for (18, 9, $d_{\min} = 5$) MLBC

In this section of this article, we discuss the designing of decoding algorithm for an increased distance MLBC developed in the above section with the help of syndrome polynomials.

The developed increased distance MLBC is a systematic code and therefore message part and the parity part of the codeword can be separated out very easily.

It is observed from the analysis of syndrome polynomials for the received codeword that the errors in the message part of the codeword and errors in the parity part of the codeword can be decoded separately for the above constructed increased distance MLBC.

Syndrome polynomial (S) can be calculated using $S = r * H^T$, where r = received codeword, H = Parity check matrix and H^T is the transpose of H . H can be obtained as $[P^T: I]$.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \dots(3)$$

$$H^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \dots\dots\dots(4)$$

There will be always (n-k) syndromes. For this constructed increased distance (18, 9) MLBC, (n, k) code, there will be 9 syndromes and these are represented here as generalised syndrome polynomials (GSP). Consider the received codeword as (r₁, r₂, r₃,r₁₈) and then compute the GSP for designed increased distance MLBC. GSPs for designed increased distance (18,9) MLBC are as follows:

$$S_0 = r_1 + r_2 + r_4 + r_6 + r_7 + r_9 + r_{10} \dots\dots\dots(5)$$

$$S_1 = r_1 + r_2 + r_3 + r_5 + r_7 + r_8 + r_{11} \dots\dots\dots(6)$$

$$S_2 = r_2 + r_3 + r_4 + r_6 + r_8 + r_9 + r_{12} \dots\dots\dots(7)$$

$$S_3 = r_1 + r_3 + r_4 + r_5 + r_7 + r_9 + r_{13} \dots\dots\dots(8)$$

$$S_4 = r_1 + r_2 + r_4 + r_5 + r_6 + r_8 + r_{14} \dots\dots\dots(9)$$

$$S_5 = r_2 + r_3 + r_5 + r_6 + r_7 + r_9 + r_{15} \dots\dots\dots(10)$$

$$S_6 = r_1 + r_3 + r_4 + r_6 + r_7 + r_8 + r_{16} \dots\dots\dots(11)$$

$$S_7 = r_2 + r_4 + r_5 + r_7 + r_8 + r_9 + r_{17} \dots\dots\dots(12)$$

$$S_8 = r_1 + r_3 + r_5 + r_6 + r_8 + r_9 + r_{18} \dots\dots\dots(13)$$

It can be noted that there are (n - k) that is in this case (18 - 9) = 9 generalized syndrome polynomials from S₀ to S₈. Depending upon the actual calculated syndrome polynomial for the received codeword, we propose the following decoding algorithm for the designed increased distance (18,9) modified linear block code.

Decoding Algorithm

Input : (18,9)MLBC with code rate 1/2

*k = number of message bits, $n = 2 * k$*

GSP for given (n,k) MLBC

Received codeword (r)

Output : Bit number r which is in error = r_ψ

Corrected codeword r_c = flipping of r_ψ in r .

Begin :

Get the received codeword (r)

*Compute the syndrome $S = r * H^T$ and
the syndrome polynomials S_0, S_1, \dots, S_{k-1} .*

for $i = 0 : k-1$

if $(S_i, S_{i+1}, \dots, S_{k-1} = 0)$

then $r_c = r$;

else $r_c \neq r$

end

for $i = 0 : k-1$

if (any one of the computed syndromes

$S_i, S_{i+1}, \dots, S_{k-1} = 1)$

then get which of the computed syndrome

polynomial from $S_0 : S_{k-1} = 1 = S_A$

go to the corresponding S_A of the

available GSP of the considered (n,k) MLBC.

*r_ψ = the last bit number in the corresponding S_A
from GSP*

r_c = flip the r_ψ bit in the received codeword

end

for $i = 0 : k-1$

if (more than one of the computed

syndromes $S_i, S_{i+1}, \dots, S_{k-1} = 1)$

then get which of the computed syndrome polynomial

from $S_0 : S_{k-1} = 1 = \{S_\Omega\}$

common bit in the set forms $\{S_\Omega\} = r_\psi$

r_c = flip the r_ψ bit in the received codeword

end

for $i = 0 : k-1$

if (all the computed syndromes

$S_i, S_{i+1}, \dots, S_{k-2} = 1$ but the computed syndrome $S_{k-1} = 0)$

*then there is always error in the first r_0 bit of the
parity check bit part of the received codeword.*

r_c = flip the r_0 bit in the received codeword

end

2.3. Check on the performance of Decoding Algorithm for MLBC with examples

Example 1: For error in parity part

Consider the actual transmitted codeword as follows:

Actual codeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 1

Received codeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 0

It can be observed that there is an error in r_{18} bit of the received code. Calculated syndrome is

$S = 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1$

It can be noticed that one of calculated syndrome is

$S_8 = 1$

From the earlier calculated generalised syndrome polynomial as shown in equation (13), as per the designed decoding algorithm, the last bit in the syndrome polynomial S_8 , is r_{18} which is in error. So, flipping this detected bit in error in the received codeword gives the corrected codeword as $r_c = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1$

Example 2: For error in r_{17} bit of parity part of the codeword

(Actual codeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 1)

(Receivedcodeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 1 1)

It can be observed that there is an error in r_{17} bit of the received code. Calculated syndrome is

$S = 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0$

It can be noticed that one of calculated syndrome is

$S_7 = 1$

From the earlier calculated generalised syndrome polynomial as shown in equation (12), as per the designed decoding algorithm, the last bit in the syndrome polynomial S_7 , is r_{17} which is in error. So, flipping this detected bit in error in the received codeword gives the corrected codeword as $r_c = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0$

Example 3: For two bit error in parity part r_{17} and r_{18}

(Actual codeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 1)

(Receivedcodeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 1 0)

It can be observed that there is an error in r_{17} and r_{18} bit of the received code. Calculated syndrome is

$S = 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1$

It can be noticed that two bits in calculated syndrome are non-zero that is $S_7 = 1$ and $S_8 = 1$

From the earlier calculated generalized syndrome polynomial as shown in equation (12) and equation (13) that is in GSP S_7 and S_8 as per the designed decoding algorithm, the last bit in these syndrome polynomial that is r_{17} and r_{18} are in error. So, flipping these detected bits in error in the received codeword gives the corrected codeword as $r_c = 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1$

As this designed increased distance (18, 9) MLBC has $d_{\min} = 5$, it is a two bit an error correcting code. Thus, any two bit error can be decoded in the received codeword.

Example 4: For error in message part r_2

(Actual codeword: 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 1)

(Receivedcodeword: 1 1 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 1)

It can be observed that there is an error in r_2 bit of the received code. Calculated syndrome is

$S = 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0$

$S_0=1, S_1=1, S_2=1, S_4=1, S_5=1, S_7=1.$

Here it can be observed that more than three calculated syndrome polynomial are non-zero syndrome polynomial. From the developed decoding algorithm, common bit in the corresponding GSP of these calculated non-zero syndrome bits, is the bit in error. From equation 5, 6, 7, 9, 10 and 12 the common bit is r_2 . So, r_2 is in error. So, flipping these detected bits in error in the received codeword gives the corrected codeword as $r_c = 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1$

Example 5: For Error in message part r_1

(Actual codeword: $1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1$)

(Received codeword: $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1$)

It can be observed that there is an error in r_1 bit of the received code. Calculated syndrome is

$1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1$

$S_0=1, S_1=1, S_3=1, S_4=1, S_6=1, S_8=1$

Here it can be observed that more than three calculated syndrome polynomial are non-zero syndrome polynomial. From the developed decoding algorithm, common bit in the corresponding GSP of these calculated non-zero syndrome bits, is the bit in error. From equation 5, 6, 8, 9, 11 and 13, common bit is r_1 . So, r_1 is in error. So, flipping these detected bits in error in the received codeword gives the corrected codeword as $r_c = 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1$

3. RESULTS AND DISCUSSION

Designed (18,9) increased distance modified linear block code (IDMLBC) is simulated in MATLAB. Figure 1 gives BER performance of the designed code. Figure 2 gives the comparison of the designed (18,9) IDMLBC and (7,4) Hamming Code. In Figure 3, the simulation result on the designed higher order (28,24) IDMLBC is shown which is having minimum distance between the code words as 6.

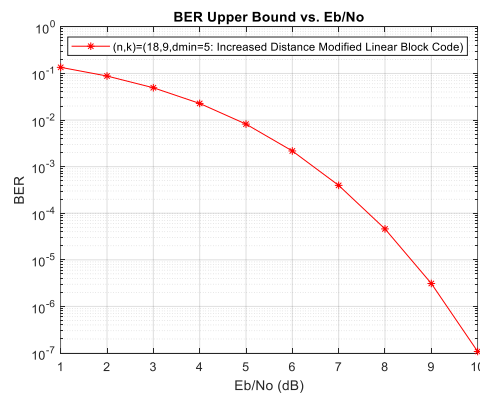


Figure 1: Eb/No verses BER

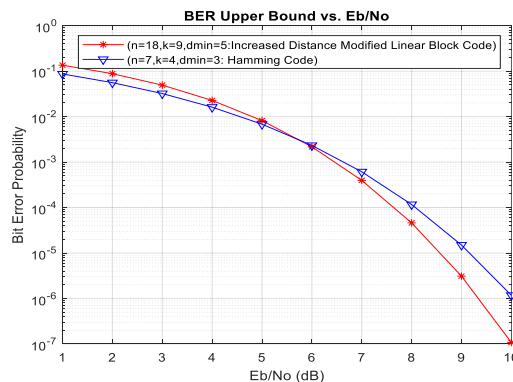


Figure 2: Eb/No verses BER

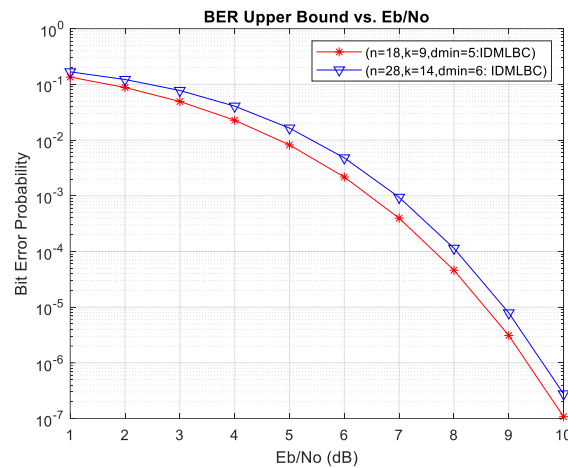


Figure 3: Eb/No verses BER

Decoding algorithms based on syndrome polynomials play a crucial role in error detection and correction within various communication systems. These algorithms are essential for maintaining data integrity when transmitting information across noisy channels. Space communication systems often experience high levels of noise and signal distortion. Syndrome polynomial-based decoding ensures the accuracy of transmitted data between satellites and ground stations, where retransmission is expensive or impossible. Deep space missions use error-correcting codes that rely on syndrome decoding to interpret signals from distant spacecraft. In cryptographic systems, decoding algorithms can be used to enhance data security by detecting and correcting intentional errors introduced by potential attackers. Syndrome decoding ensures that encrypted messages are correctly interpreted by authorized recipients. Algorithm developed in this research work can be used in the applications such as deep space missions, cryptographic systems etc.

4. RESULTS AND DISCUSSION

A modified linear block code is developed from the generator matrix established through an UTM. An increased distance modified linear block code is designed by using distance increasing mapping. The designed increased distance MLBC is (18, 9, dmin = 5) code. A decoding algorithm is designed for this increased distance MLBC using syndrome polynomials. The limitation of multiple solutions in syndrome decoding technique exist in literature is eliminated in the proposed decoding algorithm. The developed decoding algorithm is simple and based on the bit positions in the syndrome polynomial. We believe that this technique of designing increased distance MLBC can be further extended for higher order UTM. Such increased distance MLBC will have good error correction capability.

REFERENCES

- [1] J. Rosenthal, "Connection between Linear Systems and Convolutional Codes," Springer-Verlag New York Inc., 2001.
- [2] J. Rosenthal, "An optimal control theory for systems defined over finite rings," in *Open Problems in Mathematical Systems and Control Theory*, Springer Verlag, 1998, pp. 192–201.
- [3] J. Rosenthal, "Some Interesting Problems in Systems Theory which are of Fundamental Importance in Coding Theory," in *Proc. 36th Conf. Decision & Control*, San Diego, CA, USA, Dec. 1997.
- [4] R. E. Kalman, "On the general theory of Controls System," in *Proc. First IFAC Moscow Congress*, Butterworth Scientific Publications, 1960.
- [5] M. Akbari and N. I. Gillespie, "Increasing the minimum distance of codes by twisting," *Electron. J. Combinatorics*, vol. 25, no. 3, p. P3.36, 2018.
- [6] S. Talmale, S. Unnikrishnan, and B. K. Lande, "3 Bit Error Correcting Modified Linear Block Code," Indian Patent 415002, Dec. 2022.
- [7] X. Shao and W. Zhang, "Shortening the Turbo Codes Based on Unequal Error Protection," *Int. J. Multimedia Ubiquitous Eng.*, vol. 10, no. 8, pp. 73–82, 2015.
- [8] M. P. C. Fossorier and S. Lin, "Soft-Decision Decoding of Linear Block Codes Based on Ordered Statistics," *IEEE Trans. Inf. Theory*, vol. 41, no. 5, pp. 1379–1396, Sep. 1995.

- [9] P. A. Martin, D. P. Taylor, and M. P. C. Fossorier, "Soft-Input Soft-Output List-Based Decoding Algorithm," in *Proc. ISIT 2002*, Lausanne, Switzerland, Jun. 30–Jul. 5, 2002.
- [10] W. Godoy, E. C. G. Wille, and J. A. T. da Cunha, "Adaptive Decoding of Binary Linear Block Codes Using Information Sets and Erasures," in *Proc. 3rd Int. Conf. Commun. Theory, Reliability, and Quality of Service*, 2010.
- [11] W. Godoy and E. C. G. Wille, "A Simple Acceptance Criterion for Binary Block Codes Soft-Decision Algorithms," in *Proc. IEEE Int. Conf. Internet Web Appl. Services*, 2006.
- [12] G. G. de O. Brante, D. N. Muniz, and W. Godoy, "Information Set Based Soft-Decoding Algorithm for Block Codes," *IEEE Latin Am. Trans.*, vol. 9, no. 4, pp. 527–532, Jul. 2011.
- [13] Q. Guo, T. Johansson, E. Mårtensson, and P. Stankovski, "Information Set Decoding with Soft Information and some cryptographic applications," in *Proc. IEEE Int. Symp. Inf. Theory*, 2017.
- [14] J.-c. Chang, I.-t. Tsai, and H.-l. Wu, "Efficient decoding algorithm for constant composition codes," in *Proc. ISITA2010*, Taichung, Taiwan, Oct. 17–20, 2010.
- [15] T. G. Swart and H. C. Ferreira, "Decoding Distance-preserving Permutation Codes for Power-line Communications," in *Proc. IEEE Int. Symp. Power Line Commun. Appl.*, 2007.
- [16] S. D. Agashe and B. K. Lande, "A New Approach to State-transfer Problem," *J. Franklin Inst.*, vol. 333(B), no. 1, pp. 15–21, 1996.
- [17] B. K. Lande, "Some General Problems in Control Theory," Ph.D. dissertation, Indian Inst. Technol., Bombay, India, 1984.
- [18] V. Kotak, "Design of a Code: A Systematic Approach," Ph.D. dissertation, Veermata Jijabai Technol. Inst., Mumbai, India, 2015.
- [19] S. Talmale, S. Unnikrishnan, and B. K. Lande, "A Modified Block Code using Controllability of Linear System," in *Proc. ICAC3*, 2017.
- [20] S. Talmale, S. Unnikrishnan, and B. K. Lande, "Modified Linear Block Code with Code Rate $1/2$ and Less than $1/2$," *J. Discrete Math. Sci. Cryptogr.*, vol. 22, no. 2, pp. 139–150, Mar. 2019.
- [21] S. Talmale, S. Unnikrishnan, and B. K. Lande, "Distance Increasing Mapping for Variable Distance Block Code," *IET Commun.*, vol. 14, no. 9, pp. 1495–1501, Jun. 2020.
- [22] B. Honary, G. Markarian, "New simple encoder and trellis decoder for Golay codes", *Electronics Letters*, Vol. 29 No. 25, December, 1993.
- [23] B Pavithra , Parnasree Chakraborty, "Novel Polar Coded MIMO Power Domain NOMA Scheme for 5G New Radio (NR) ", *Indonesian J. of Electr. Eng. and Inform.*, Vol. 12, No. 3, pp. 700-710, September 2024.
- [24] Ojo, S. I., Abolade, R. O., Alagbe, O. L., Ojerinde, I. A., & Tooki, O. O., "Performance Enhancement of Decode and Forward Relaying Network in a Log-normal Fading Channel using Diversity Technique," *Indonesian J. of Electr. Eng. and Inform.*, vol. 12, no. 2, pp. 255–265, Jun. 2024.
- [25] S. Lin and D. Costello, *Error Control Coding: Fundamentals and Applications*, Prentice-Hall, Englewood Cliffs, NJ, 1983.