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Research Article

What the Monty Hall Problem can tell us (which it hasn't Already)

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ABSTRACT

The Monty Hall Problem is a well-known probabilistic brainteaser that gave rise to a huge amount of academic debate. This paper presents methodologies to discuss two lesser-known versions of the problem, displaying unexpected results. In the first variant, knowledge of the participants, and knowledge about knowledge of other participants impacts on the assessment of probability. In the second variant, different players receive the same information and reach different results, in another scenario they receive different information reaching the same result. In those variants it is unclear what factors probability depends on and what are its determinants. If probability's determinants are unclear, ontic ex-post probability itself can be deemed scarcely reliable. The findings of this paper follow the footprint of Bruno de Finetti (1906-1985) who, quite provocatively, stated that "probability does not exist". This anomaly is likely to invest other fields of human knowledge, especially those that more heavily rely on probability calculus. The dismaying conclusion of this paper is that probability, in its ex-post version, may not be an adequate tool for interpreting the world we live in and for understanding the complexity of reality and the provisional nature of our knowledge.

Keywords: Epistemic probability, ontic ex-post probability, decision-making, information, knowledge.

INTRODUCTION

The Monty Hall Problem (hereinafter MHP or 'the Problem') finds its origins in the popular TV show *Let's Make A Deal!* broadcasted in the US since the 1960's for nearly 30 years, although with some interruptions. Even though the format of the TV game could be changed at some extent by the host (Monty Hall) to the purpose of increasing suspense and the interest of spectators, the standard version is well-known. A player is shown three doors; one of them conceals a valuable prize, the other two doors hide no prize, kind of. The player chooses one of the doors and wins whatever is hidden behind it. However, after the player's initial choice, the host, who knows where the prize is, opens another door, which is empty, and offers the player a chance to stick to her initial choice or switch. Such a simple problem attracted attention of the scientific community. A lot of researchers published about the MHP on academic journals specializing in fields as varied as statistics, mathematics, physics, economics, psychology, education science and philosophy. A Google Scholar search of 'Monty Hall Problem' carried out in 2024 retrieved more than 35,000 academic papers, scientific articles and books, some of them cited hundreds of times. The fierce debate that revolves around the MHP demonstrates the importance and depth of topics it gives rise to. Indeed, the MHP is so rich in nuances and details that research, which started in 1975 with two letters to The American Statistician, is not over and there is still room for new and original reasoning about it.

The MHP displays features different from, say, tossing a coin in as much as the former deals with assessing probability of an event that has already occurred (hiding the prize behind a door) whereas the latter is about something which still has to occur. As shown in the section about Random Epistemic Sequence, this makes all the difference.

Probability has an impact on almost all sciences (Hájek, 2023), the most affected being quantum mechanics, economics, and financial trading, without forgetting that all sciences where hypothesis testing applies, are also heavily influenced by probability.

In the following, section 2 presents a review of the existing literature, section 3 describes some of the Problem displaying interesting features that will be analyzed in details in section 4. Section 5 discusses the results and section 6 concludes.

LITERATURE REVIEW

Academic interest in the Problem took off in 1975, when Steve Selvin sent a couple of letters to The American Statistician (Selvin, 1975a; Selvin, 1975b), presenting a solution to the statistical puzzle known as the Monty Hall Problem. Since then, the academic world started a debate around the MHP and its many variants, which has not terminated yet. Although the solution of the standard problem is pretty well-known, still it raises doubts and fifteen years after publication of Selvin's letters, when Marilyn vos Savant stated the correct answer in her column on the Parade Magazine (vos Savant, 2013), she received thousands of irate responses, some sexist comments about women's understanding of stats and even insults. A justification of those mistaken responses (of course, not of the sexist and insulting ones, that deserve no justification) is that the Problem is deeply counter-intuitive and based on numerous tacit assumptions. In a newspaper interview, Stanford University's statistician Diaconis said "Our brains are just not wired to do probability problems very well, so I'm not surprised there were mistakes" (Tierney, 1991). This is indirectly confirmed by the many papers published on psychology, economics, education and philosophy journals (Granberg and Dorr, 1998; Bradley and Fitelson, 2003; Slembeck and Tyran, 2004; Baumann, 2005; Saenen et al., 2015; Dupont and Durham, 2018; to mention just a few). When, in the final stage of the game, there are just two closed doors left and the prize is behind one of them, the most natural response seems to be that each door carries equal probabilities to conceal the prize. However, the untold assumption is 'lacking further information', a statement that completely overturns the problem. Despite a later attempt to unify epistemic and ontic probability (Nakajima, 2019), the two concepts are regarded as irreconcilable to each other by most authors. At that point, epistemic probability comes to the foreground and takes the dominant position in the face of statistical (or ontic) probability, valid until then. Indeed, the standard formulation of MHP has been criticized mainly because either some details are missing (VerBruggen, 2015) or for other reasons. One alternative criticism assumes Host is lazy or tired (Rosenthal, 2008) and tries to minimize his walking effort by choosing, whenever he got the freedom to do so, to open the lowest-numbered door. This assumption changes epistemic probability but is not usually specified in the description of the Problem. Indeed, some versions state that Host 'randomly opens an empty door' whereas in others he simply 'opens an empty door'. Verbalizing the question may make all the difference. As Morgan et al. (1991, p.284) point out, "this apparently innocuous little problem can be erroneously 'solved' in a variety of ways" and "the nature of these errors can be quite subtle". That is the power of not spelled-out assumptions that can influence the outcome. Among the many studies on the Problem, a prominent position must be assigned to Rosenhouse (2009), an entire book dedicated to the MHP that analyzes in depth 10 of its variations, producing a treatise on statistics suitable for undergrad and postgrad courses alike. The author digs into several interesting considerations related to the variants he presents. Conditional probability and Bayes' theorem are used to explain the Random Monty scenario whereas Fernandez and Piron (1999) make reference to the Nash equilibrium to suggest optimal strategies (Bierman and Fernandez, 1999). Ensslin and Westerkamp (2019) assume Host is trying to minimize Player's chances of success and propose a counter-strategy she can use to defend herself against what they call 'evil showmaster'. Baumann (2005) took off from the MHP to discuss, from a philosophical viewpoint, whether the single-case probability can equally sit within the statistical debate where results are subject to the Law of Large Numbers and are dubious outside it. A great deal of arguments has been generated by the interpretation of probability, with an apparently irreconcilable confrontation on whether qualification of probability is objective or subjective. This argument, which goes much beyond the Monty Hall Problem, gave rise to intense disagreements, reaching the most original conclusions, like 'subjective probability does not exist' (Zaman, 2019), 'ontic probability does not exist' (Slattery, 2015), 'imprecise probability does not exist' (Vicig and Seidenfeld 2012) to finish off with the definitive 'PROBABILITY DOES NOT EXIST' (capital in the text), an apparently paradoxical thesis stated by the authoritative statistician Bruno de Finetti (de Finetti, 1990), later supported by Nau (2001).

MATERIALS AND METHODS

The purpose of this research is assessing the epistemic characteristics of probability by analyzing two variations of the standard Monty Hall problem and applying novel methodologies. One of the most interesting features of the Problem is that any minimal change in its assumptions may modify the outcome because of the impact on its epistemic environment. Probability depends on the information available; when new information arrives, so changing the ex-ante environment into an ex-post environment, evaluation of probability changes accordingly. For this reason, the section on epistemic and ontic probability first takes into account two well-known scenarios, Random Monty and Monty Fall. Indeed, probability is closely related to uncertainty, on its turn depending on our knowledge. "Probability […] if regarded as something endowed with some kind of objective existence, is […] a misleading misconception, an illusory attempt to exteriorize or materialize our true probabilistic beliefs" (de Finetti, 1990, p.x). As shown in this paper, minimal variations of actors' knowledge do affect probabilistic evaluation; shifts in the epistemic sequence of events do affect probabilistic evaluation; apparently uninfluential changes do affect probabilistic evaluation, leading to wondering whether (ontic) probability exists at all. If different subjective evaluations materialize in different objective outcomes (and all at the same time), de Finetti's paradoxical statement about non-existence of probability no longer seems so paradoxical.

Random Epistemic Sequence

Methodology used: modification of the sequence of events.

The basic operations performed in the Monty Hall problem are:

A. Organizer conceals the prize behind a door

B. Player makes her initial guess about the location of prize

C. Host opens an empty door

D. Player makes her final decision about which door to open.

That is pretty standard stuff. However, from an epistemic viewpoint it is interesting to analyze what happens when the four events A, B, C and D do not occur in that sequence. For example, Organizer may have the chance to hide the prize after Player's initial selection (sequence B, A, C, D). In this case, knowledge of Player's choice takes a major role, as Organizer can be assumed to always be trying to minimize the cost to the show and therefore exploiting its knowledge to deceive Player. At the contrary, its ignorance of Player's choice may lead to different results. The analysis of all possible combinations of sequence and knowledge presents epistemic scenarios that completely overturn the solution patterns. A limitation in this analysis is the number of mutual knowledge levels that are considered. Organizer may, or may not, know Player's choice (if the latter occurs first) and Player may, or may not, know whether Organizer is aware of her choice. The complexity of the example goes rapidly up if we take into account schemes like 'I know that you know that I know, …' (as per common knowledge, Aumann, 1976). In this study, no more than two levels of mutual knowledge are investigated as the purpose is discovering the impact of epistemic probability on the decision-making process, not reaching the highest probability of winning the prize or suggesting an optimal strategy. Once the relationship between knowledge and probability is established, the goal of this section will be considered achieved.

Two Players

Methodology used: modification of epistemic statuses.

The scenario with two players is interesting because of their potentially different epistemic degrees. Two players are shown the three closed doors at the same time and they select one of the doors. They may (i) both select the winning door, (ii) one winning and one empty door or both empty doors. Moreover, in the last case, they may (iii) both choose the same empty door or (iv) different ones. Whereas scenarios (i) and (iii) do not provide particular food for thought, as both players share the same information and reach the same conclusion, scenarios (ii) and (iv) lead to interesting outcomes.

The two methodologies used in this study have wide application in many other real-life events.

ONTIC AND EPISTEMIC PROBABILITY

Before digging into the scenarios discussed in this paper, two scenarios are worth a brief presentation for reference and comparison. In the following, we will assume Organizer being an impersonal entity, Host being a gentleman (as in the TV show), and Player being a lady, with all pronouns declined accordingly, with an eye to inclusive language and the other to confusion avoidance.

Random Monty

Host is assumed to ignore the location of prize. He can open any door except the one selected by Player. In case Host opens the door concealing the prize, the game stops immediately. At that point, there are two possible ways to continue: the game is terminated and perhaps it continues with another participant or the game is repeated with the same participant. This scenario relaxes epistemic characteristics of Host, that is, of an Agent not directly involved in the decision-making process. Nevertheless, his behavior carries information and therefore influences Player's epistemic status. As explained in detail by Rosenhouse (2009), the winning probabilities of sticking or switching are now 50% each.

Monty Fall

A slightly different scenario is the case in which Host is supposed to slip on a banana´s peel and accidentally open any door. This scenario increases the percentage of null games, because either Host may reveal the winning door or may open the door selected by Player. In both cases the game terminates and again either of the two possible continuations described above could occur. This scenario modifies the epistemic environment even more profoundly than the Random Monty but conceptually (and mathematically) the outcome is similar: an information is revealed that should not. As described in Rosenthal (2008), winning probabilities are, again, 50% on either door.

Random Epistemic Sequence

The basic operations performed in the Monty Hall problem are:

Organizer hides prize

Player selects a door

Host opens an empty door

Player decides whether to stick or switch

Now we are going to investigate how Player's winning probabilities vary as events occur in a non-standard sequence and as agents' knowledge changes. Simple permutation of the four steps yields 4! = 24 possible sequences, as shown in Table 1. Yet, Table 1 can be, at a large extent, simplified. Events B, C, and D must always occur in this sequence, otherwise it would no longer be an instance of Monty Hall game. Only event A may change its position in the sequence. Therefore, several sequences in Table 1 can be canceled out; these are indicated by the grey rows.

Table 1: Possible permutations of MHP events' sequence. Grey rows indicate non-MHP scenarios

	Steps of the game			
Permutation	**I**	**II**	**III**	**IV**
1	A	B	C	D
2	A	B	D	C
3	A	C	B	D
4	A	C	D	B
5	A	D	B	C
6	A	D	C	B
7	B	A	C	D
8	B	A	D	C
9	B	C	A	D
10	B	C	D	A

Permutation	Steps of the game			
	I	II	III	IV
11	B	D	A	C
12	B	D	C	A
13	C	A	B	D
14	C	A	D	B
15	C	B	A	D
16	C	B	D	A
17	C	D	A	B
18	C	D	B	A
19	D	A	B	C
20	D	A	C	B
21	D	B	A	C
22	D	B	C	A
23	D	C	A	B
24	D	C	B	A

Only four scenarios need to be analyzed further, namely 1 (A, B, C, D), 7 (B, A, C, D), 9 (B, C, A, D), and 10 (B, C, D, A). In these, and many other examples in probability, information assumes great importance. This may sound a trivial statement, especially in the light of conditional probability and Bayes' theorem. Yet, the case under study is different from commonplace 'importance of information' as apparently irrelevant information has an impact on the outcome and the resulting decision. As seen in the Random Monty scenario, it is not immaterial whether Host knows, or ignores, the location of prize, but it is also important whether Organizer knows, or ignores, Player's choice before hiding the prize behind a door, and whether Player knows, or ignores, about the other two agents' knowledge. For the purpose of reasoning, the scheme will be limited to the following scenarios:

- Organizer may know (k), or not know (n), Player's choice
- Host may know (k), or not know (n), the location of prize but he never opens the door chosen by Player (i.e., Random Monty is allowed but Monty Fall is not)
- Player may know (k), or not know (n), about Organizer's knowledge of her choice
- Player may know (k), or not know (n), about Host knowledge of prize location.

Based on knowledge available to each Agent (a collective name comprising Organizer, Host and Player), the above scenarios present different epistemic probabilities, as shown in Table 2.

Table 2: Probabilities of events A, B, C and D under different scenarios (k=knows; n=ignores).

Scenario	I	I%	II	II%	III	III%	IV	IV%	Choice	Stick%	Switch%	Rational%	Mult
1nk==	A	1/3	B	1/3	C	1; 1/2	D	2/3	W	1/3	2/3	2/3	4
1nn=k	A	1/3	B	1/3	C	1/2*	D	1/2	=	1/2	1/2	1/2	2
1nn=n	A	1/3	B	1/3	C	1/2*	D	2/3	W	1/2	1/2	1/2	2
7kkk=	B	1/3	A	1	C	1/2	D	1	T	1	0	1	2
7kkn=	B	1/3	A	1	C	1/2	D	2/3	W	1	0	0	2
7knkk	B	1/3	A	1	C	1/2*	D	1	T	1	0	1	1
7knkn	B	1/3	A	1	C	1/2*	D	1	T	1	0	1	1
7knnk	B	1/3	A	1	C	1/2*	D	1/2	=	1	0	1/2	1
7knnn	B	1/3	A	1	C	1/2*	D	2/3	W	1	0	0	1

Scenario	I	I%	II	II%	III	III%	IV	IV%	Choice	Stick%	Switch%	Rational%	Mult
7nk==	B	1/3	A	1/3	C	1; 1/2	D	2/3	W	1/3	2/3	2/3	4
7nn=k	B	1/3	A	1/3	C	1/2*	D	1/2	=	1/2	1/2	1/2	2
7nn=n	B	1/3	A	1/3	C	1/2*	D	2/3	W	1/2	1/2	1/2	2
9knk=	B	1/3	C	1/2	A	1	D	1	T	1	0	1	2
9knnk	B	1/3	C	1/2	A	1	D	1/2	=	1	0	1/2	1
9knnn	B	1/3	C	1/2	A	1	D	2/3	W	1	0	0	1
9nn=k	B	1/3	C	1/2	A	1/2	D	1/2	=	1/2	1/2	1/2	2
9nn=n	B	1/3	C	1/2	A	1/2	D	2/3	W	1/2	1/2	1/2	2
10knk=	B	1/3	C	1/2	D	1	A	1	T	0	0	0	2
10knnk	B	1/3	C	1/2	D	1/2	A	1	=	0	0	0	1
10knnn	B	1/3	C	1/2	D	2/3	A	1	W	0	0	0	1
10nn=k	B	1/3	C	1/2	D	1/2	A	1/2	=	1/2	1/2	1/2	2
10nn=n	B	1/3	C	1/2	D	2/3	A	1/2	W	1/2	1/2	1/2	2
									Average	**56.67%**	**33.33%**	**50.83%**	

In the first column, an initial digit represents the scenario from Table 2, and subsequent letters ('k' or 'n') indicate the epistemic status of each Agent: the first letter refers to Organizer, the second to Host, third and fourth refer to Player's knowledge of Organizer's and Host's knowledge, respectively. An equal sign ('=') indicates non-relevance. In the 'Choice' column, 'T' means sTick and 'W' indicates sWitch. Asterisks ('*') indicate scenarios in which the randomly opened door may reveal the location of prize. All values displayed in Table 2 are discussed in detail in the Appendix.

Scenario 1 contains only 3 rows because it cannot happen that Organizer knows about Player's choice as in that scenario event A always occurs first; therefore, the sequence 1k** is impossible because Organizer cannot know about Player's choice. The purpose of Table 2 is computing statistics of wins under the Stick, Switch and Rational decision, which may not always be switching. The column 'Mult' represents the multiplying factor needed for working out averages: the default value 1 gets multiplied by 2 for every '=' sign in the scenario, so taking into account the fact that the equal sign stands for both 'k' and 'n'. The scenarios in which sticking wins are more numerous than those in which a win is given by switching, against an expected 33% versus 67%, as in the Classical Monty. The Stick+Switch percentages do not add up to 1 since in some cases under scenario 10, Player will lose no matter whether she sticks or switches. Knowledge definitely has an impact on outcome of the Monty Hall Problem. This variation of the MHP suggests that determinants of probability are the doors but also the information known by Player, Host's knowledge about prize location, Organizer's knowledge, and decision-making process, or any combination of the above. Moreover, as discussed in the Appendix, knowledge does display hierarchy. In particular, when Player is aware that Organizer knows her choice, this piece of information takes priority over knowledge (or ignorance) of Host's knowledge about prize location, which was instead crucial in other scenarios. The search for the determinants of probability is getting more and more complicated.

Two Players Scenario

Let us assume that two players (Mrs. A and Mr. B) participate in the same game at the same time. However, for obvious reasons, they do not know about presence of the other participant and each player is being shown the content of the open door privately, that is, Host may open a different door to each player. The four possible scenarios are those described in section 3.2. Without loss of generality the prize is assumed to be behind door X. Black doors are closed, white doors are open.

Scenario 1 (figure 1)

i) Pr(Prize behind door X | Prize behind door X or Y or Z) = 1/3

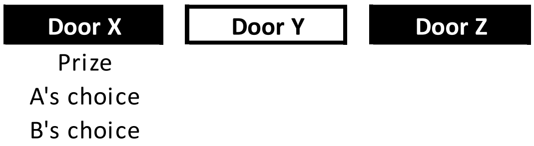

Figure 1. Both players choose door X, Host opens door Y

ii) Pr(Prize behind door X | Host opens door Y) = 1/3

iii) Pr(Prize behind door Z | Host opens door Y) = 2/3

In this case, following Bayes' theorem, both players assign ⅓ probability to win if they stick and ⅔ if they switch to door Z. This is the standard scenario.

Scenario 2 (figure 2)

i) Pr(Prize behind door X or prize behind door Y | Prize behind door X or Y or Z) = 1/3

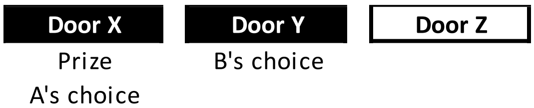

Figure 2. Player A chooses door X, Player B chooses door Y, Host opens door Z

ii-A) Pr(Prize behind door X | Host opened door Z) = 1/3

ii-B) Pr(Prize behind door Y | Host opened door Z) = 1/3

iii-A) Pr(Prize behind door Y | Host opened door Z) = 2/3

iii-B) Pr(Prize behind door X | Host opened door Z) = 2/3

According to Player A, door X carries ⅓ probability and door Y carries ⅔. The opposite is true for Player B. They share the same information but reach different conclusions.

Scenario 3 (figure 3)

i) Pr(Prize behind door Y | Prize behind door X or Y or Z) = 1/3

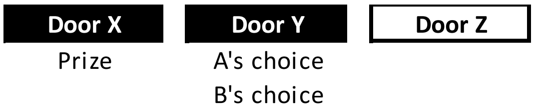

Figure 3. Both players choose door Y, Host opens door Z

ii) Pr(Prize behind door Y | Host opened door Z) = 1/3

iii) Pr(Prize behind door X | Host opened door Z) = 2/3

Again, Player A and Player B assign ⅓ probability to the selected door, and ⅔ to the other door.

Scenario 4 (figure 4)

i) Pr(Prize behind door Y or prize behind door Z | Prize behind door X or Y or Z) = 1/3

Figure 4. Player A chooses door Y, Player B chooses door Z. Door Y (black-white) is closed for Player A but open for Player B, door Z (white-black) is the other way around

ii-A) Pr(Prize behind door Y | Host opened door Z) = 1/3

ii-B) Pr(Prize behind door Z | Host opened door Y) = 1/3 iii-A) Pr(Prize behind door X |Host opened door Z) = 2/3

iii-B) Pr(Prize behind door X | Host opened door Y) = 2/3

In this scenario Host cannot open and show the same empty door to both players, as there is no such a door available. Door X cannot be opened because it conceals the prize, door Y has been selected by Player A and door Z by Player B. The only possible way ahead is opening door Z privately to Player A and door Y, privately as well, to Player B. The players do not know that the content of the other door has been shown to the other participant, otherwise it would be too easy to guess the prize location. The players receive different information. Player A assigns ⅓ probability to door Y, ⅔ to door X, and 0 to door Z whereas Player B assigns ⅓ probability to door Z, ⅔ to door X and 0 to door Y.

Scenarios 1 and scenario 3 are not particularly interesting. Yet, we have a paradoxical situation for which in scenario 2 both players share the same common knowledge but diverge in assigning probabilities, depending on their respective initial choice, whereas in scenario 4 they have different information about the open door but both agree in assigning ⅔ probability to door X (switch). This raises with even more strength the question: what are the determinants of probability? The door or the information players possess and their initial decision? Differently from the classical MHP case, and according to Bayes' theorem, conditional probability depends on information each player subjectively possesses. Objective probability does not seem to play any role.

DISCUSSION

Meaning of Probability

Non-standard Monty Hall scenarios display unexpected results. The Two-Player version makes it rather clear that not only information but also opinions (guess, in this case) impact on probability. Indeed, when the two players select different doors, one of which is right (figure 2 in section 4.4), they both assign ⅓ probability to the selected door and ⅔ to the other one – and they are both right! This results in an unclear overall evaluation since the same door carries ⅓ probability for one player and ⅔ for the other – and the opposite applies to the other door. Computer simulation confirms that both players win according to their expected probabilities, even though their probability assessment are different. At this point it is difficult to state what is the ontic probability associated to each door as it depends on the initial choice of the player, which is unrelated to the real location of the prize. This is compliant with the Bayesian interpretation of probability assignments as the representation of our degree of belief in a given proposition. However, despite the fact that the assessments are different, they are both rational and both turn out to be right. De Finetti (2008, p.3) summarizes it as follows: "it is senseless to speak of the probability of an event unless we do so in relation to the body of knowledge possessed by a given person". Yet, this example shows that probability does not only depend on 'body of knowledge' but on individual opinions, too. So far, one can conclude that each player assigns probability according to his/her level of knowledge and therefore that probability depends on his/her mind rather than on the door. The matter gets even more complicated than it already looks when we analyze Table 2. In this case, probabilities follow the random sequence of events and are heavily impacted by each agent's knowledge about other agents' knowledge. This consideration moves probability determinants from information owned by Player to Host's and Organizer's, or at least involves their knowledge, as it seems confirmed by discussion of the Random Epistemic Sequence. Moreover, in the Random Monty scenario, probability seems to depend on Host's knowledge and in the Monty Fall scenario, its determinants are in Fate's hands (the banana´s peel). In this case, regardless of whether Host got the information about the winning door or otherwise, Player should make her choice based on what happened on stage. One could even speculate on the probability of Host slipping on the banana peel or on the unlikely probability that there is one on the floor. Indeed, according to the examples above, probability seems to depend on a multiplicity of factors, not all of them directly related to the prize location – and some even totally unrelated to it. At this point one may be drawn to agree with de Finetti (1990, p.x): "My thesis, paradoxically, and a little provocatively, but nonetheless genuinely, is simply this: PROBABILITY DOES NOT EXIST". Less paradoxical but equally strong is Carnap (1945, p.8) who introduces two different concepts of 'probability', that he calls probability1 and probability2:

"When we look at the formulations which the authors themselves offer in order to make clear which meanings of 'probability' they intend to take as their explicanda, we find phrases as different as 'degree of belief,' 'degree of reasonable expectation,' 'degree of possibility,' 'degree of proximity to certainty,' 'degree of partial truth,' 'relative frequency,' and many others".

This leads to differentiate between the probability2 ('relative frequency in the long run', as the coin I am about to toss – ex-ante probability) as the one referred to events that have not happened yet versus probability1 ('degree of confirmation') to those events that have already occurred but whose outcome in still unknown (the door that hides the prize – ex-post probability). It is difficult to disagree with Carnap (1945, p.13), according to whom "[t]his is primarily due to the unfortunate fact that both concepts are designated by the same familiar, but ambiguous word 'probability'". The matter is taken up again by Hacking (2006, p.13):

"Poisson and Cournot said we should use the ready-made French words chance and probabilité to mark the same distinction. Before that Condorcet suggested facilité for the aleatory concept and motif de croire for the epistemic one […]. Bertrand Russell uses 'credibility' for the latter […]. There have been many other words. We have had Zuverlässigkeit, 'propensity', 'proclivity', as well as a host of adjectival modifiers of the word 'probability', all used to indicate different kinds of probability".

Confusion is great under the probability sky!

The issue of identification of probability determinants seems a hard nut to crack. However, we believe that the difficulty comes from the mistake (highlighted by the authors cited above) that the same word is being used to indicate two deeply different concepts, only apparently akin to each other. When I toss a coin, I must distinguish between 'before' and 'after'. Before tossing it, it is pretty unanimously agreed upon 50% chance to get head as well as tail. But let's suppose that after tossing the coin, I cover it with my hand and then assess the probability of head or tail. This is a question that gives rise to much more controversial answers. Some may insist that chances are 50-50 whereas others may state that at that point it is no longer a matter of probability. The single-case probability has been discussed by Baumann (2005), criticized by Levy (2007), counter-responded by Baumann (2008), with Sprenger (2010) temporarily settling the matter. For some authors, whatever the guess, the answer is either right or wrong: once the coin has landed, probability can only be 1 or 0, no more ½. Even if we agree on the different probabilistic result in the aftermath of an uncertain event, and on the different meaning of probability (in this case picking up Carnap's probability1), the question remains unanswered: what are the determinants of probability1? So far in the MHP we have identified five possible dependencies: the doors, Player's mind, Organizer's mind, Host's mind or Fate. Other probabilistic problems may show dependence on different factors. Nevertheless, this is not the end of the story. As section 4.3 points out, probability (whatever meaning is being assigned to the word) might be an entity unevenly shared between all those actors. If it depends on a complex relationship between the knowledge of Player, Host and Organizer – and dynamically changing as each one of them acquires more information – the concept of probability assumes some ethereal borders, a concept to which assigning mathematical formulas seems something more akin to wishful thinking than logical reasoning. The Random Epistemic Sequence is different from assessing the probability before tossing a fair coin, where all knowledge of the world would not change the fact that outcome probability is (as the number of tosses tends to infinity) 50% head and 50% tail. Therefore, it seems clear that the two kinds of probability are not the same and it is also deeply doubtful (agreeing with Carnap) whether the same name should be used to designate two such wide apart phenomena.

Probability pops up anywhere in our daily life, from crossing a road to investing money, to undergoing a surgical treatment, letting alone quantum mechanics that has probability at its core. Every decision we make is a decision about the future and, since nobody owns the crystal ball, we all make much heavier use, albeit informally, of probability than we usually think.

Interpretations of Probability

In the Stanford Encyclopedia of Philosophy, Alan Hájek (2023) explains in details the various and different interpretations of probability. However, whatever kind of interpretation one decides to adopt, it falls short of explicative power as far as ex-post probability is concerned. According to the author, "an interpretation should be precise, unambiguous, non-circular, and use well-understood primitives" (ibid, p.5). Section 4.3. about Random Epistemic Sequence, demonstrates that it is difficult, to say the least, to precisely and unambiguously identify probability determinants, something that clashes with those basic criteria, especially if some sort of similarity can be drawn between 'primitives' and 'determinants' in this context. Discussing the Applicability to the Frequencies criterion, the author states that "an interpretation should render perspicuous the relationship between probabilities and (long-run) frequencies" (ibid. p.6). This is fundamental to any interpretation of probability – and indeed, Two-Player scenario 2 complies with this basic requirement: despite each player having different information from the other, they are both right and long-run frequencies confirm their respective views. Also, the criteria Applicability to Rational Beliefs ("knowing that one event is more probable than another, a rational agent will be more confident about the occurrence of the former event", ibid, p.6) and Applicability to Rational Decision ("an interpretation should make clear how probabilities figure in rational decision-making", ibid, p.6) fail to apply to scenario 2. Although each player would choose the door that carries more probabilities, they will make different decisions (as they own different information) but still they will prove both right on long-run. Hájek's disheartening conclusion is "It should be clear from the foregoing that there is still much work to be done regarding the interpretations of probability" (ibid, p.34). If there is much work to do about ex-ante probability, the judgement on ex-post probability seems even more ethereal. A definitive conclusion suggests Jaynes (1968), according to whom, if two scenarios share the same epistemic condition, they should be assigned the same probability, which is not the case depicted by scenario 2. Ramsey (1926) admits that placing a bet changes the environment and therefore impacts on bettor's opinion, something that justifies a shift in probability assessment. But, again, the long-run trial, the definitive reality check, confirms that, whatever the bet and whatever the opinion, both bettors are always right. This sounds like a logical contradiction, and even more so if, as described by scenario 4, when two bettors have different epistemic states, they may share the same opinion. If there is a border between creative confusion and illogical conclusion, that border has definitely been trespassed. Human beings have a natural tendency to disagree with each other but, as pointed out by Frances (2018), if two people, that share the same epistemic condition, disagree, there is evidence that one of them made a mistake. Again, this conflicts with the results of this study, confirming, once more, that ex-post probability does not obey the fundamental rules of probability, leading to a logical contradiction.

CONCLUSION

The conclusion of the previous reasoning, "paradoxically, and a little provocatively, but nonetheless genuinely, is simply this: PROBABILITY DOES NOT EXIST". Indeed, "only subjective probabilities exist – i.e., the degree of belief in the occurrence of an event attributed by a given person at a given instant and with a given set of information. (De Finetti, 1990, p.3–4)", to which we may add "and other (unspecified) factors".

According to our study, probability seems to depend on very different factors, some directly related to the problem at hand (the doors and Player's information), others only indirectly related to it (Host's and Organizer's knowledge as well as Player's knowledge about Host's and Organizer's knowledge), and some totally unrelated to the problem at hand. Indeed, in our opinion this is the most striking finding of this paper. It may even depend on some completely unrelated factors, or others that we have not discovered, or hypothesized, yet. Given these premises, de Finetti's statement comes as the most, and perhaps only, logical conclusion. If probability depends on such a wide range of different factors, it can be concluded that, according to "[t]he radical part of de Finetti's claim [, ...] *there is no 'objective' semantics for probability*" (Jeffrey, 1984:85), at least in its ex-post, or Carnap's probability2, meaning. A mathematical concept that lacks an objective semantics does not exist as such, although, pragmatically, "in favourable circumstances frequency can be a good tool for evaluating probabilities" (Galavotti, 1989:246). Whereas on the one side probability is undeniably useful to help solving innumerable practical problems, on the other side it is still unclear whether it leads to 'real' knowledge. Our analysis of spurious variants of the Monty Hall problems found that probability may provide the right answer using the wrong procedure and that we can mistake the wrong solution for the right one despite making use of mathematically correct calculations. This is particularly worrying with respect to our confidence to be achieving probabilistic knowledge. If, as shown in the Two Players versions of the Problem, statistical results confirm that Player's ambiguous, or even plainly wrong, interpretation of the reality leads to right results, we cannot avoid the question to invade other fields of science, starting from, but not necessarily limiting to, those that more than others rely on probability, as quantum physics, that is, the very essence of our being. Research on the deepest meaning of probability goes on – and rightly so – together with other branches of science. Yet, according to scientific method, researchers must always be ready to work as hard to the goal of falsifying their own findings. Including supporters of ontic probability and, of course, the authors of this paper.

Appendix – EPISTEMIC TABLE OF EVENT SEQUENCES

In the following, the meaning of all lines contained in Table 2 are explained in details.

1nk==

I: (A=1/3) Organizer randomly hides the prize behind any of the three doors

II: (B=1/3) No matter whether Player knows, or ignores, that Organizer does not know her choice, she will pick a door at random

III: (C=1 or 1/2) If Player selected the right door, then Host can open any of the two empty doors, whereas in the opposite case, only one door can be opened

IV: (D=2/3) According to Bayes' theorem, switching door carries 2/3 probability against 1/3 sticking

Choice: Switch

Winning chances: 2/3

1nn=k

I: (A=1/3) As 1nk== I

II: (B=1/3) As 1nk== II

III: (C=1/2*) If Hosts does not know the location of prize, he can open any door except the one selected by Player. If he opens the door concealing the prize, the game immediately stops and may terminate or repeat. In any case (as demonstrated in section 4.2 Random Monty) the probability of opening an empty door is 1/2.

IV: (D=1/2) Player knows that Host ignores the location of prize and therefore, according to Random Monty, probability is 1/2 on each door

Choice: Not relevant

Winning chances: ½

1nn=n

I: (A=1/3) As in 1nk== I

II: (B=1/3) As 1nk== II

III: (C=1/2*) As in 1nn=k

IV: (D=2/3) Player ignores that Host does not know the location of prize and therefore she follows the Bayes' theorem, switching door as it carries 2/3 probabilities

Choice: Switch

Winning chances: ½

7kkk=

I: (B=1/3) Player selects any of the three closed doors

II: (A=1) Organizer knows Player's selection and therefore assumes that, based on Bayes' theorem, she will switch. In order to minimize costs for the show, it applies the most rational decision, that is, hiding the prize behind the door Player selected

III: (C=1/2) Host knows the location of prize and the selection of Player, which coincide. Thus, he opens any one of the two remaining doors with 1/2 probabilities

IV: (D=1) Player is aware that Organizer knows her selection, therefore she counters Organizer's strategy by sticking

Choice: Stick

Winning chances: 1

7kkn=

I: (B=1/3) As in 7kkk= (I)

II: (A=1) As in 7kkk= (II)

III: (C=1/2) As in 7kkk= (III)

IV: (D=2/3) Player is not aware that Organizer knows her selection, therefore she believes to be following the standard Bayes' theorem and that switching grants her 2/3 wining chances

Choice: Switch

Winning chances: 0

7knkk

I: (B=1/3) As in 7kkk= (I)

II: (A=1) As in 7kkk= /II)

III: (C=1/2*) As 1nn=k (III)

IV: (D=1) As in 7kkk= (IV)

Choice: Stick

Winning chances: 1

7knkn

I: (B=1/3) As in 7kkk= (I)

II: (A=1) As in 7kkk= (II)

III: (C=1/2*) As 1nn=k (III)

IV: As in 7kkk= IV. Despite not knowing that Host ignores the location of prize, Player's second choice is driven only by her awareness about Organizer knowledge of her initial selection. This scenario clearly shows that some information (awareness of Organizer's knowledge) carries priority over other information (awareness of Host's knowledge of prize location). When assessing probability, information is subject to hierarchy. For this reasons, scenarios 7knkk and 7knkn yield the same result

Choice: Stick

Winning chances: 1

7knnk

I: (B=1/3) As in 7kkk= (I)

II: (A=1) As in 7kkk= (II)

III: (C=1/2*) As 1nn=k (III)

IV: (D=1/2) If Player ignores that Organizer knows her initial selection but is aware that Host ignores the location of prize, she will follow the Random Monty scheme

Choice: Not relevant

Winning chances: ½

7knnn

I: (B=1/3) As in 7kkk= (I)

II: (A=1) As in 7kkk= (II)

III: (C=1/2*) As 1nn=k (III)

IV: (D=2/3) Whenever Player ignores whether Host knows the location of prize, she will follow Bayes' theorem

Choice: Switch

Winning chances: 0

7nk==

I: (B=1/3) As in 7kkk= (I)

II: (A=1/3) If Organizer does not know Player's selection, it will conceal the prize at random behind any of the three doors

III: (C=1 or 1/2) As in 1nk== (III)

IV: (D=2/3) No matter whether Player is aware or ignores that Host knows the location of prize, she will follow Bayes' theorem

Choice: Switch

Winning chances: 2/3

7nn=k

I: (B=1/3) As in 7kkk= (I)

II: (A=1/3) As in 7nk== (II)

III: (C=1/2*) As 1nn=k (III)

IV: (D=1/2) No matter whether Player knows, or otherwise, that Organizer ignores her initial selection but is aware that Host ignores the location of prize, she will follow the Random Monty scheme

Choice: Not relevant

Winning chances: ½

7nn=n

I: (B=1/3) As in 7kkk= (I)

II: (A=1/3) As in 7nk== (II)

III: (C=1/2*) As 1nn=k (III)

IV: (D=2/3) As in 1nn=n (IV)

Choice: Switch

Winning chances: ½

9kkk, 9kkn= and 9nk==

These scenarios do not exist as in all scenarios 9 Host opens a door before Organizer conceals the prize and therefore, he cannot be aware of location

9knk=

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) Host can open any of the two doors not selected by Player

III: (A=1) As in 7kkk= (II)

IV: (D=1) As in 7kkk= (IV)

The same reasoning of scenarios 7knkk and 7knkn about information priority applies

Choice: Stick

Winning chances: 1

9knnk

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (A=1) As in 7kkk= (II)

IV: (D=1/2) By being aware that Host ignores prize location, Player follows the Random Monty scheme

Choice: Not relevant

Winning chances: ½

9knnn

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (A=1) As in 7kkk= (II)

IV: (D=2/3) By ignoring that Host does not know prize location, Player follows Bayes' theorem

Choice: Switch

Winning chances: 0

9nn=k

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (A=1/2) Organizer does not know Player's choice and therefore it randomly conceals the prize behind any of the two remaining closed doors

IV: (D=1/2) Player knows about Organizer's random selection and therefore cannot but follow a random choice

Choice: Not relevant

Winning chances: ½

9nn=n

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (A=1/2) As in 9nn=k

IV: (D=2/3) As in 9knnn

Choice: Switch

Winning chances: ½

10kkk, 10kkn= and 10nk==

These scenarios do not exist as in all scenarios 10 Host opens a door before Organizer conceals the prize and therefore, he cannot be aware of location

10knk=

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (D=1) Player is aware that Organizer knows her selection and therefore she believes that it will try to minimize cost for the show by hiding the prize behind her initial selection, confident that she will switch. In order to counter Organizer's strategy, Player decides to stick, sure to be winning

IV: (A=1) Organizer conceals the prize after Player's second selection and easily does so nullifying her winning chances

The same reasoning of scenarios 7knkk and 7knkn about information priority applies

Choice: Stick

Winning chances: 0

10knnk

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (D=1/2) Whenever Player ignores that Organizer knows her selection but is aware that Host does not know the location of prize, she is left with random choice about the winning door

IV: (A=1) As in 10knk=

Choice: Not relevant

Winning chances: 0

10knnn

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (D=2/3) If Player does not know that Host ignores the prize location, she believes to be in a standard Bayes' theorem scenario, believing that switching will increase her winning chances

IV: (A=1) As in 10knk= (IV)

Choice: Switch

Winning chances: 0

10nn=k

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (D=1/2) As in 10knnk (III)

IV: (A=1/2) If Organizer ignores Player's selection, it will randomly conceal the prize

Choice: Not relevant

Winning chances: ½

10nn=n

I: (B=1/3) As in 7kkk= (I)

II: (C=1/2) As in 9knk= (II)

III: (D=2/3) As in 10knnn (III)

IV: (A=1/2) As in 10nn=k (IV)

Choice: Switch

Winning chances: ½

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