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Research Article

Hospital Resource Management Using Complex t-Fuzzy Graph

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ABSTRACT

This paper addresses an concept of Complex ŧ-fuzzy Graphs (CTFGs) effectively utilized for realization and visualizing intricate collaborations thus may be challenging for grasp. This highlights how CTFGs can typically make complex relationships that involve multiple apparatuses or scopes under a certain context. The paper covers a fundamental set operations of CTFGs, while exploring concepts of homomorphism & isomorphism under this outline. In addition, this study presents a real-time solicitation of CTFGs, illustrating the utility of hospital resource management by accounting for a multiplicity of appropriate dynamics. Through this application, this apporoach demonstrates the tractability & efficiency of CTFGs by way like a decision-support apparatus for visualizing and prioritizing actions aimed at optimizing hospital resource management.

Keywords: Complex Fuzzy Graph, complex ŧ -Fuzzy Graph, Operations, Product Complex ŧ - Fuzzy Graph, optimizing hospital resource management

1. INTRODUCTION

1.1. DECISION-MAKING

Decision-making plays a crucial role in both personal and professional settings, as it directly influences the accomplishment and flaws of association. Analysts engage in decision-making throughout every stage that administrate the process and effectiveness of results determines overall managerial success. Without strong decision-making abilities, managers struggle to perform essential tasks like forecasting, consolidating, directing, adapting, and recruiting. This process must be in both progressive & collaborative, fostering structural growth. In uncertain scenarios, management solutions become essential. To address ambiguity, fuzzy decision-making methodologies leverage fuzzy set theory, which effectively handles situations where data is imprecise or uncertain. This theory assigns membership grade toward elements within a traditional set, accounting for the inherent vagueness present in decision-making.

1.2. FUZZY SET

Fuzzy set theory, first introduced by Zadeh [1], establishes a robust mathematical approach for managing uncertainty, imprecision, and ambiguity in computational processes. Since its inception, this theory has been effectively utilized in diverse scientific and technical fields, containing shopper micro electronics, automated mechanism organisations, pattern recognition, robotics, machine learning, and manufacturing computerisation. What's more, it has demonstrated its utility in various operational investigation zones, for instance project scheduling, decision support, logistics optimization, queuing theory, and quality assurance. Several scholars, including Kandel [2], Klir and Yuan [3], as well as Mendel [4] and Zimmermann [5], have authored foundational texts that elaborate on the key principles and methodologies of fuzzy set theory, providing readers with an in-depth comprehension and practical insights into its applications.

1.3. FUZZY GRAPH

Graphs are a handy way to depict connections among substances, with vertices (nodes) and edges (connection). The philosophy of graphs remains an effective utility for analysing & reducing complex networks. Molecular descriptors are significant in mathematical chemistry because they allow researchers to investigate the model of particles with sourece of mathematical methods. Chemical graph theory investigates the interplay between chemistry, graph theory, and mathematics by utilizing molecular graphs to illustrate atoms, chemical linkages, and their relationships within molecular frameworks. When dealing with uncertain or ambiguous relationships among elements, a Fuzzy Graph Model becomes essential. Fuzzy graphs serve as powerful mathematical tools for analyzing structures with unknown or imprecise components. Rosenfeld [17] was the first to do study on fuzzy graph theory, after that Morde-son and Chang-Shyh's [18] description of fuzzy graph operations and Bhattacharya's [19] verification of graph abstract discoveries. Bhutani [20] explored the automorphisms of fuzzy networks. FG model is applied across various scientific and technological domains, including telecommunications, manufacturing, social network analysis, machine intelligence, information processing, and neural networks.

The conceptual framework of complex fuzzy graphs (CFGs) serves as a powerful tool for addressing and clarifying complex and confusing issues encountered in real-life situations. This effectiveness is due to their ability to clearly capture the inherent sorts of changeability, intricacy, fuzziness, & ambiguity present within elements of sets. Yet, to resolve applied issues related to membership, these methods must be redefined by clear-cut arithmatical standards. Noticing these challenge, CTFG’s concepts was introduced, incorporating linear ŧ -norm and ŧ -co norm operators. By adopting the CTFGs rises the need for structured and flexible approach to effectively handle uncertainty and facilitate decision-making founded in established principles.

From the scenario, an argument ' ŧ ' streamlines a method specifying the basis for calculating membership degrees. In many real-world scenarios, decision-making must take into consideration varying degree of certianty. Adding the parameter ' ŧ ' to CTFGs aims to overcome normal FG restrictions by offering specific access over rigidity, enhancing customisation, permits to separate decision inceptions, boosting elasticity & reducing obscurity. These characteristics make CTFGs a active method to modeling insecurity and aiding well-versed decision-making on circumstances that require a customized & skilful method to managing indecision.

CTFGs offer a significant advantage in understanding and navigating intricate decision-making scenarios where conventional fuzzy graphs fall short. These graphical models equip decision-makers with powerful tools to explore and evaluate various alternatives by thoroughly representing the intricate relationships between inputs and outcomes. By incorporating complex fuzzy interconnections, decision-makers can systematically assess multiple criteria and their interdependencies, fostering a more comprehensive and well-rounded approach to tackling challenging decision-making problems. A complicated method in CTFGs significantly improves resolution-creation, particularly in circumstances described by membership & parameter ' ŧ '. This signifies a shift away from the limitations of binary logic, paving the path for enhanced precision in decision-making procedures.

1.3 MOTIVATION

Grasping intricate relationships in various perspectives, like biodiversity preservation, essential to building well-informed judgments. Nevertheless, conventional methods might struggle for fully capture the complexity kind of vast occurrences. Consequently, there is a demand for an advanced framework capable of precisely representing and analyzing these complex interactions.

1.4 NOVELTY

This research is distinguished by its use of CTFGs as an innovative approach to describing and evaluating intricate relationships. While various graph-based models exist, the incorporation of complex ŧ-fuzzy sets (CTFS) introduces a new perspective to the problem. This integration enhances the ability to represent uncertainty in sophisticated systems, making the proposed approach highly effective for modeling real-world phenomena with multiple networking components. Additionally, the study explores fundamental set operations, homomorphism, and isomorphism within the back ground of CTFGs, more contributing the inventiveness.

1.5 GOAL

- To familiarize CTFGs and their characteristics.
- The new CTFG framework aims to preserve biodiversity.
- Establish conceptual for CTFGs.
- Explain how CTFGs may be utilised for biodiversity conservation in real-world applications.
- Explore how isomorphism and homomorphism in CTNGs might improve biodiversity preservation decisions.

1.6 OBJECTIVE

- ŧ -fuzzy graphs are mathematical tools used to describe obscure, and vague information under the structured graph. To broaden this concept of conventional graphs, ŧ -fuzzy sets, a generalisation of FS and traditional sets, are introduced.
- Broaden the utilization of modern graph theory concepts and methodologies within the framework of complex ŧ-fuzzy graphs, facilitating a more comprehensive examination and enhanced problem-solving capabilities in evolving and interlinked systems.
- Use complicated ŧ -fuzzy graphs for decision-making with several incompatible conditions or ambiguous facts.
- Use complex ŧ -fuzzy graphs to tackle actual problems in disciplines such as biology, transportation, social networks, and communication.

1.7 KEY CONTRIBUTION

The primary impact of this research lies in the improvement of CTFGs, a powerful methodology for visualizing and comprehending intricate relationships, particularly within the domain of biodiversity preservation. This study illustrates how CTFGs effectively represent complex interdependencies among various conservation elements, facilitating more well-informed decision-making processes. Additionally, it highlights the adaptability of CTFGs in tackling challenges related to hospital resource allocation through practical implementations, accounting for multiple influential parameters. The incorporation of membership functions within CTFGs strengthens decision-makers' ability to analyze conservation strategies, thereby optimizing the prioritization of key initiatives. Moreover, the inclusion of the "ŧ" parameter in CTFGs introduces flexibility in responding to diverse sensitivities and conditions, equipping decision-makers with the means to address risk and uncertainty in multifaceted decision-making contexts. Collectively, this research establishes a comprehensive framework and analytical instruments that enhance decision-making in hospital resource administration, ultimately fostering improved societal well-being.

1.8 STRUCTURE OF PAPER

The following is how the paper moves forward: To emphasise the uniqueness of the provided work, the "Basics of CTFGs" section explains important terminology. The next section, Symmetric operations on CTFGs examines a number of set-theoretical procedures using graphical representations. After that, homomorphisms and isomophisms inside CTFGs are defined in the section titled "Isomorphism CTFGs." The newly created approach is then used to support ecosystem preservation in the section titled "Application of CTFG inbiodiversity conservation." Sensitivity analysis, comparison analysis, and conclusive closes succinct the results are the last steps in the research process.

2 ŧ-FUZZY GRAPH

Definition 1. Given a universal set U, let G be the fuzzy set (FS) and ŧ ∈ [0,1]. Well-known as a ŧ-fuzzy set (ŧ-FS), the $FS_{\mathcal{G}_ŧ}$ of U is well-defined as $\mu_{\mathcal{G}_ŧ}(n_1) = \wedge\{\mu_{\mathcal{G}_ŧ}(n_1), ŧ\}$, $\forall\, n_1 \in U$. The form of ŧ-FS is $\mathcal{G}_ŧ = \{n_1, \mu_{\mathcal{G}_ŧ}(n_1) : n_1 \in U\}$ where $\mu_{\mathcal{G}_ŧ}$, function that give a degree of membership are. Moreover, the function $\mu_{\mathcal{G}_ŧ}$ satisfy the condition $0 \le \mu_{\mathcal{G}}(n_1) \le 1$.

Definition 2. A CFS A, defined on a universe of discourse U is an objective of the form $Ꝑ = \{ꞑ_1, ɥ_{\mathcal{G}_ŧ}(ꞑ_1)e^{iɥꞙ_{\mathcal{G}ŧ}(ꞑ_1)}\}$, where $i = \sqrt{-1}$, $(ɥ_{\mathcal{G}_ŧ}(ꞑ_1) \in [0,1], 0 \leq ɥꞙ_{\mathcal{G}_ŧ}(ꞑ_1) \leq 2\pi$.

Definition 3. For a specific simple graph $G = (Ꝃ, Ŋ)$, consider $\mathcal{G}_ŧ = (Ꝑ, Ꞧ)$ is a FG. The form of annotation $\mathcal{G}_ŧ = (Ꝑ_ŧ, Ꞧ_ŧ)$ indicates CTFG, everywhere $Ꝑ_ŧ = \{(ꞑ_i, ɥς_{\mathcal{G}ŧ}(ꞑ_i)e^{iɥꞙ_{\mathcal{G}ŧ}(ꞑ_i)}): ꞑ_i \in Ꝃ\}$ it is a CTFS on Ꝃ and $Ꞧ_ŧ = \{((ꞑ_i, ꞑ_j), ɥς_{\mathcal{G}ŧ}(ꞑ_i, ꞑ_j)e^{iɥꞙ_{\mathcal{G}ŧ}(ꞑ_i, ꞑ_j)}) : (ꞑ_i, ꞑ_j) \in Ŋ\}$ is a CTFG on $Ŋ \subseteq Ꝃ \times Ꝃ$, As to ensure $(ꞑ_i, ꞑ_j) \in Ŋ$. *Satisfy the condition,* $0 \leq ɥς_{Ꝑ_ŧ}(ꞑ_i) \leq 1$ and $0 \leq ɥς_{Ꞧ_ŧ}(ꞑ_i, ꞑ_j) \leq 1$.

$$ɥς_{Ꞧ_ŧ}(ꞑ_i, ꞑ_j)e^{iɥꞙ_{Ꞧ_ŧ}(ꞑ_i, ꞑ_j)} \leq \wedge\{ɥς_{Ꝑ_ŧ}(ꞑ_i), ɥς_{Ꝑ_ŧ}(ꞑ_j)\}e^{i\wedge\{ɥꞙ_{Ꝑ_ŧ}(ꞑ_i), ɥꞙ_{Ꝑ_ŧ}(ꞑ_j)\}}$$

Example 1. Explore $G' = (Ꝃ, Ŋ)$ where $Ꝃ = \{ꞑ_1, ꞑ_2, ꞑ_3\}$ and $Ŋ = \{ꞑ_1ꞑ_2, ꞑ_1ꞑ_3, ꞑ_1ꞑ_4, ꞑ_2ꞑ_3, ꞑ_3ꞑ_4\}$. Let Ꝑ be a CTFSs of Ꝃ and Ŋ be a CTFG of $Ŋ \subseteq Ꝃ \times Ꝃ$, such as specified at $ŧ = 0.60e^{i0.9\pi}$ in fig 1.

$$Ꝑ_{0.60e^{i0.6\pi}} = \begin{Bmatrix} (ꞑ_1, 0.3e^{i0.4\pi}), \\ (ꞑ_2, 0.4e^{i0.3\pi}), \\ (ꞑ_3, 0.2e^{i0.5\pi}) \\ (ꞑ_4, 0.5e^{i0.1\pi}) \end{Bmatrix} \ and \ Ꞧ_{0.60e^{i0.6\pi}} = \begin{Bmatrix} (ꞑ_1ꞑ_2, 0.3e^{i0.3\pi}) \\ (ꞑ_1ꞑ_3, 0.2e^{i0.4\pi}) \\ (ꞑ_1ꞑ_4, 0.3e^{i0.1\pi}) \\ (ꞑ_2ꞑ_3, 0.2e^{i0.3\pi}) \\ (ꞑ_3ꞑ_4, 0.2e^{i0.1\pi}) \end{Bmatrix}$$

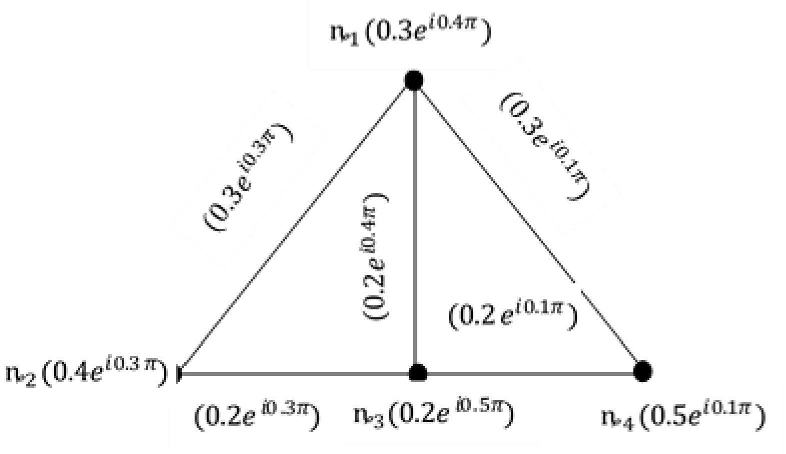


Figure 1. CTFG.

Definition 4. Let $\mathcal{G}_ŧ = (Ꝑ_ŧ, Ꞧ_ŧ)$ be a CTFG and then, $\mathcal{H}_ŧ = (Ꝑ'_ŧ, Ꞧ'_ŧ)$ is regarded as a CTFSG if $Ꝑ'_ŧ \subseteq Ꝑ_ŧ$ and $Ꞧ'_ŧ \subseteq Ꞧ_ŧ$.

Definition 5. A CTFG $\mathcal{G}_ŧ = (Ꝑ_ŧ, Ꞧ_ŧ)$ is termed a complete CTFG if it fulfils the subsequent requirement:

$$ɥς_{Ꞧ_ŧ}(ꞑ_1, ꞑ_2)e^{iɥꞙ_{Ꞧ_ŧ}(ꞑ_1, ꞑ_2)} = \wedge\{ɥς_{Ꝑ_ŧ}(ꞑ_1), ɥς_{Ꝑ_ŧ}(ꞑ_2)\} e^{i\wedge\{ɥꞙ_{Ꝑ_ŧ}(ꞑ_1), ɥꞙ_{Ꝑ_ŧ}(ꞑ_2)\}}, \forall (ꞑ_1, ꞑ_2) \in Ŋ.$$

Definition 6. The order is specified below in the CTFG

$$O(\mathcal{G}_ŧ) = \left(\sum_{ꞑ_1 \in Ꝃ} ɥς_{Ꝑ_ŧ}(ꞑ_1) \ e^{i\sum_{ꞑ_1 \in Ꝃ} ɥꞙ_{Ꝑ_ŧ}(ꞑ_1)} \right)$$

Example 2. The derived $(1.4e^{i1.3\pi})$ of CTFG $\mathcal{G}_ŧ$ since Ex 1.

Definition 7. The CTFG's size is determined in

$$S(\mathcal{G}_ŧ) = \left(\sum_{(ꞑ_1, ꞑ_2) \in Ŋ} ɥς_{Ꞧ_ŧ}(ꞑ_1, ꞑ_2)e^{i\sum_{(ꞑ_1, ꞑ_2) \in Ŋ} ɥꞙ_{Ꞧ_ŧ}(ꞑ_1, ꞑ_2)} \right)$$

Definition 8. The degree of vertex $ꞑ_1$ *in* $\mathcal{G}_ŧ$ is well-demarcated such as shadows in the CTFG.

1. $deg_{\mathcal{G}_t}(n_1) = \left(deg_{\mu R_t}(n_1)\right)$, $\quad deg_{\mathcal{G}_t}(n_1) = \left(\sum_{(n_1,n_2)\in N} \mu\varsigma_{R_t}(n_1, n_2) e^{i\, \mu f_{R_t}(n_1,n_2)}\right)$
2. The CTFG's minimal degree $\Delta(\mathcal{G}_t)$ is $\Delta(\mathcal{G}_t) = \left(\delta_{\mu\varsigma_{R_t}}(\mathcal{G}_t) e^{i(\delta_{\mu f_{R_t}}(\mathcal{G}_t))}\right)$

$$\delta(\mathcal{G}_t) = \left(\wedge\left\{deg_{\mu\varsigma_{R_t}}(n_1)\right\} e^{i\, \wedge\{deg_{\mu f_{R_t}}(n_1)\}}\right) n_1 \in K.$$

3. The the CTFG's maximal degree $\Delta(\mathcal{G}_t)$ is $\Delta(\mathcal{G}_\mu) = \left(\Delta_{\mu\varsigma B_\mu}(\mathcal{G}_\mu) e^{i(\Delta_{\mu f B_\mu}(\mathcal{G}_\mu))}\right)$

Example 3. According to Ex 1, the vertex's degree of $\mathcal{G}_t$

$deg_{\mathcal{G}_t}(n_1) = (0.8e^{i0.8\pi})$; $deg_{\mathcal{G}_t}(n_2) = (0.5e^{i0.6\pi})$

$deg_{\mathcal{G}_t}(n_3) = \quad (0.6e^{i0.8\pi})$; $deg_{\mathcal{G}_t}(n_4) = (0.5e^{i0.2\pi})$.

3. OPERATION ON CTFG

3.1. CARTESIAN PRODUCT OF CTFG

Definition 9. Let $\mathcal{G}_t = (P_t, R_t)$ and $\mathcal{G}'_t = (P'_t, R'_t)$ stand every twofold CTFGs in $G = (K, N)$ & $G' = (K', N')$, correspondingly. $\mathcal{G}_t \times \mathcal{G}'_t$ is Cartision product of twofold CTFGs, $\mathcal{G}_t$ and $\mathcal{G}'_t$ stand demarcated by $(P_t \times P'_t, R_t \times R'_t)$ anywhere $P_t \times P'_t$ and $R_t \times R'_t$ are CTNSs on $K \times K' = \{(n_1, \omega_1), (n_2, \omega_2): n_1 \& n_2 \in K;\ \omega_1 \ \&\ \omega_2 \in K'\}$ and $N \times N' = \{(n_1, \omega_1), (n_2, \omega_2): n_1 = n_2, n_1 \ \&\ n_2 \in K, (\omega_1, \omega_2) \in N'\}\ U\ \{(n_1, \omega_1), (n_2, \omega_2): \omega_1 = \omega_2, \omega_1 \& \omega_2 \in K', (n_1, n_2) \in N\}\ U\ \{(n_1, \omega_1), (n_2, \omega_2): \omega_1 \neq \omega_2, n_1 \neq n_2, (\omega_1, \omega_2) \in N', (n_1, n_2) \in N\}$, which satisfies the following condition:

1. $\forall\ (n_1, \omega_1) \in K \times K'$

$$\mu\varsigma_{P_t \times P'_t}(n_1, \omega_1)\, e^{i\mu f_{P_t \times P'_t}(n_1, \omega_1)} = \wedge\left\{\mu\varsigma_{P_t}(n_1), \mu\varsigma_{P_{t'}}(\omega_1)\right\} e^{i\, \wedge\left\{\mu f_{P_t}(n_1), \mu f_{P_{t'}}(\omega_1)\right\}}$$

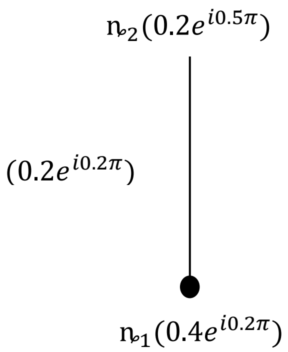
2. If $n_1 = n_2$ and $\forall\ (\omega_1, \omega_2) \in N'$

$$\mu\varsigma_{R_t \times R'_t}((n_1, \omega_1), (n_2, \omega_2)) e^{i\mu f_{R_t \times R'_t}((n_1, \omega_1),(n_2, \omega_2))} = \wedge\left\{\mu\varsigma_{P_t}(n_1), \mu\varsigma_{R_{t'}}(\omega_1, \omega_2)\right\} e^{i\, \wedge\left\{\mu f_{P_t}(n_1), \mu f_{R_{t'}}(\omega_1, \omega_2)\right\}}$$

3. If $\omega_1 = \omega_2$ and $\forall\ (n_1, n_2) \in N$

$$\mu\varsigma_{R_t \times R'_t}((n_1, \omega_1), (n_2, \omega_2)) e^{i\, \mu f_{R_t \times R'_t}((n_1, \omega_1),(n_2, \omega_2))} = \wedge\left\{\mu\varsigma_{R_t}(n_1, n_2), \mu\varsigma_{P_{t'}}(\omega_1)\right\} e^{i\, \wedge\left\{\mu f_{R_t}(n_1, n_2), \mu f_{P_{t'}}(\omega_1)\right\}}$$

Example 4. Figures 3(a) and 3(b) illustrate two 0.5$e^{i0.6\pi}$-FG $\mathcal{G}_t$ & $\mathcal{G}'_t$, Each of the variables that must be considered. *The Cartesian product $\mathcal{G}$ 0.5$e^{i0.6\pi}$×G'0.5$e^{i0.6\pi}$, this is shown in Figure 4 and relates to them.*



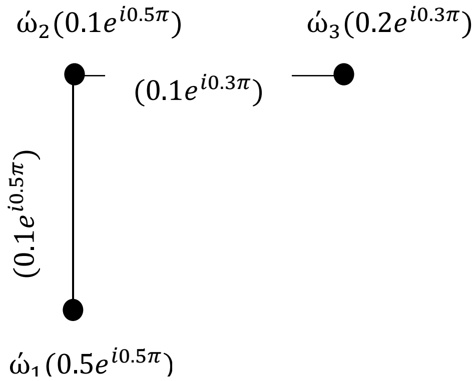


Figure 3(a). $\mathcal{G}_{0.5e^{i0.6\pi}}$.

Figure 3(b). $\mathcal{G}'_{0.5e^{i0.6\pi}}$.

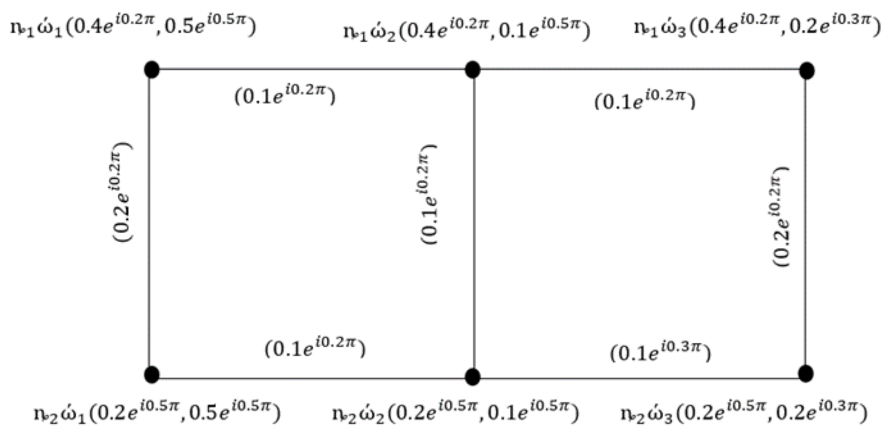


Figure 4. The corresponding Cartesian product $\mathcal{G}_{0.5e^{i0.6\pi}} \times \mathcal{G}'_{0.5e^{i0.6\pi}}$.

Definition 10. The vertex of degree $\mathcal{G}_t \times \mathcal{G}'_t$ is well-defined, for any $(ɳ_1, ɯ_1) \in Ꝁ \times Ꝁ'$.

$$deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_1, ɯ_1) = \left(deg\left\{ɰς_{R_t\times R'_t}\left((ɳ_1, ɯ_1),(ɳ_2, ɯ_2)\right)\right\} e^{iɰf_{R_t\times R'_t}((ɳ_1, ɯ_1),(ɳ_2, ɯ_2))}\right)$$

where

$$deg\left\{ɰς_{R_t\times R'_t}\left((ɳ_1, ɯ_1),(ɳ_2, ɯ_2)\right)\right\} e^{iɰf_{R_t\times R'_t}((ɳ_1, ɯ_1),(ɳ_2, ɯ_2))}$$

$$= \sum_{ɳ_1=ɳ_2,(ɯ_1, ɯ_2)\in Ŋ'} \wedge\left\{ɰς_{P_t}(ɳ_1), ɰς_{R_t'}(ɯ_1, ɯ_2)\right\} e^{i\sum_{ɳ_1=ɳ_2,(ɯ_1,ɯ_2)\in Ŋ'} \wedge\left\{ɰf_{P_t}(ɳ_1), ɰf_{R_t'}(ɯ_1,ɯ_2)\right\}}$$

$$+\sum_{ɯ_1=ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge\left\{ɰς_{R_t}(ɳ_1,ɳ_2), ɰς_{P_t'}(ɯ_1)\right\} e^{i\sum_{ɯ_1=ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge\left\{ɰf_{R_t}(ɳ_1,ɳ_2), ɰf_{P_t'}(ɯ_1)\right\}}$$

$$+\sum_{ɯ_1\neq ɯ_2, ɳ_1\neq ɳ_2} \wedge\left\{ɰς_{R_t}(ɳ_1,ɳ_2), ɰς_{R_t'}(ɯ_1, ɯ_2)\right\} e^{i\sum_{ɯ_1\neq ɯ_2,ɳ_1\neq ɳ_2} \wedge\left\{ɰf_{R_t}(ɳ_1,ɳ_2), ɰf_{R_t'}(ɯ_1,ɯ_2)\right\}},$$

Example 5. Every single vertex in $\mathcal{G}_t \times \mathcal{G}'_t$ from example 4 consumes the degree shown below

$deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_1\ ɯ_1) = (0.3e^{i0.3\pi})$, $deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_1\ ɯ_2) = (0.3e^{i0.6\pi})$, $deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_1\ ɯ_3) = (0.3e^{i0.4\pi})$,

$deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_2\ ɯ_2) = (0.3e^{i0.4\pi})$ $deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_2\ ɯ_2) = (0.3e^{i0.7\pi})$, $deg_{\mathcal{G}_t\times\mathcal{G}'_t}(ɳ_2\ ɯ_3) = (0.3e^{i0.5\pi})$,,

Theorem 1. A Cartesian product of two CTFGs yields another CTFG.

Proof: For $A_t \times A'_t$, It is clear that the situation. Considering that $u_1 \in Ꝁ$ and $(ɯ_1, ɯ_2) \in Ŋ'$,

$$ɰς_{B_t\times B'_t}\left((ɳ_1, ɯ_1),(ɳ_1, ɯ_2)\right)e^{i\,ɰf_{B_t\times B'_t}((ɳ_1, ɯ_1),(ɳ_1, ɯ_2))} = \wedge\left\{ɰς_{A_t}(ɳ_1), ɰς_{B_t'}(ɯ_1, ɯ_2)\right\} e^{i\wedge\left\{ɰf_{A_t}(u_1), ɰf_{B_t'}(ɯ_1, ɯ_2)\right\}}$$

$$\leq \wedge\left\{ɰς_{A_t}(u_1), \wedge\{ɰς_{A_t'}(ɯ_1), ɰς_{A_t'}(ɯ_2)\}\right\} e^{i\wedge\left\{ɰf_{A_t}(ɳ_1),\ \wedge\{ɰf_{A_t'}(ɯ_1), ɰ_{A_t'}(ɯ_2)\}\right\}}$$

$$\leq \wedge\left\{\wedge\left\{ɰς_{A_t}(ɳ_1), ɰς_{A_t'}(ɯ_1)\right\}, \wedge\left\{ɰς_{A_t}(ɳ_1), ɰς_{A_t'}(ɯ_2)\right\}\right\} e^{i\wedge\left\{\wedge\left\{ɰf_{A_t}(ɳ_1),\ ɰf_{A_t'}(ɯ_1)\right\},\ \wedge\{ɰf_{A_t}(ɳ_1), ɰf_{A_t'}(ɯ_2)\}\right\}}$$

$$ɰς_{B_t\times B'_t}\left((ɳ_1, ɯ_1),(ɳ_2, ɯ_2)\right)e^{i\,ɰf_{B_t\times B'_t}((ɳ_1, ɯ_1),(ɳ_2, ɯ_2))}$$

$$= \wedge\left\{ɰς_{A_t\times A'_t}(ɳ_1, ɯ_1), ɰς_{A_t\times A'_t}(ɯ_1, ɯ_2)\right\} e^{i\wedge\left\{ɰf_{A_t\times A'_t}(ɳ_1, ɯ_1),\ ɰf_{A_t\times A'_t}(ɯ_1, ɯ_2)\right\}}$$

Consequently,

$$ɰς_{B_t\times B'_t}\left((ɳ_1, ɯ_1),(ɳ_2, ɯ_2)\right)e^{i\,ɰf_{B_t\times B'_t}((ɳ_1, ɯ_1),(ɳ_2, ɯ_2))}$$

$$\leq \wedge\left\{ɰς_{A_t\times A'_t}(ɳ_1, ɯ_1), ɰς_{A_t\times A'_t}(ɯ_1, ɯ_2)\right\} e^{i\wedge\left\{ɰf_{A_t\times A'_t}(ɳ_1, ɯ_1),\ ɰf_{A_t\times A'_t}(ɯ_1, ɯ_2)\right\}}$$

3.2. CTFG IN COMPOSISTION

Definition 11. $\mathcal{G}_ŧ \circ \mathcal{G}_ŧ'$ is composition of two CTFGs, $\mathcal{G}_ŧ$ & $\mathcal{G}_ŧ'$ is a CTFG and demarcated as a pair$(Ᵽ_ŧ \circ Ᵽ_ŧ',\ Ʀ_ŧ \circ Ʀ_ŧ')$ everywhere $(Ᵽ_ŧ \circ Ᵽ_ŧ')$ and $(Ʀ_ŧ \circ Ʀ_ŧ')$ are CTFSs on $Ķ \times Ķ' = \{(ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2): ƞ_1 \& ƞ_2 \in Ķ;\ ɯ_1 \&\ ɯ_2 \in Ķ'\}$and $Ŋ \times Ŋ' = \{(ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2): ƞ_1 = ƞ_2, ƞ_1 \& ƞ_2 \in Ķ, (ɯ_1,\ ɯ_2) \in Ŋ'\}\ U\ \{(ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2):\ ɯ_1 =\ ɯ_2,\ ɯ_1 \&\ ɯ_2 \in Ķ', (ƞ_1, ƞ_2) \in Ŋ\}\ U\ \{(ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2):\ ɯ_1 \neq\ ɯ_2, ƞ_1 \neq ƞ_2, (ɯ_1,\ ɯ_2) \in Ŋ', (ƞ_1, ƞ_2) \in Ŋ\}$, respectively, which satisfies the following condition:

1. $\forall((ƞ_1,\ ɯ_1) \in\ Ķ \circ Ķ'$,

$$ɱς_{Ᵽ_ŧ \circ Ᵽ_ŧ'}(ƞ_1, ɯ_1)e^{i\, ɱϝ_{Ᵽ_ŧ \circ Ᵽ_ŧ'}(ƞ_1, ɯ_1)} = \wedge\left\{ɱς_{Ᵽ_ŧ}(ƞ_1), ɱς_{Ᵽ_{ŧ}'}(ɯ_1)\right\}e^{i \wedge\left\{ɱϝ_{Ᵽ_ŧ}(ƞ_1),\ ɱϝ_{Ᵽ_{ŧ}'}(ɯ_1)\right\}}$$

2. If $ƞ_1 = ƞ_2$ and $\forall\ (ɯ_1,\ ɯ_2) \in Ŋ'$,

$$ɱς_{\,Ʀ_ŧ \circ Ʀ_ŧ'}\big((ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2)\big)e^{i\, ɱϝ_{\,Ʀ_ŧ \circ Ʀ_ŧ'}((ƞ_1,\ ɯ_1),(ƞ_2,\ ɯ_2))} = \wedge\left\{ɱς_{Ᵽ_ŧ}(ƞ_1), ɱς_{\,Ʀ_{ŧ}'}(ɯ_1,\ ɯ_2)\right\}e^{i\wedge\left\{ɱϝ_{Ᵽ_ŧ}(ƞ_1),\ \ ɱϝ_{\,Ʀ_{ŧ}'}(ɯ_1, ɯ_2)\right\}}$$

If $ɯ_1 =\ ɯ_2$ and $\forall\ (ƞ_1, ƞ_2) \in Ŋ$, $ɱς_{\,Ʀ_ŧ \circ Ʀ_ŧ'}\big((ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2)\big)e^{i \wedge \{ɱς_{\,Ʀ_ŧ \circ Ʀ_ŧ'}((ƞ_1, ɯ_1),(ƞ_2, ɯ_2))\}}$

3. If $ɯ_1 \neq\ ɯ_2$ and $\forall\ (ƞ_1, ƞ_2) \in Ŋ$

$$ɱς_{\,Ʀ_ŧ \circ Ʀ_ŧ'}\big((ƞ_1,\ ɯ_1), (ƞ_2,\ ɯ_2)\big)e^{i\, ɱϝ_{\,Ʀ_ŧ \circ Ʀ_ŧ'}((ƞ_1,\ ɯ_1),(ƞ_2,\ ɯ_2))}$$

$$= \wedge\left\{ɱς_{\,Ʀ_ŧ}(ƞ_1, ƞ_2), ɱς_{Ᵽ_{ŧ}'}(ɯ_1), ɱς_{Ᵽ_{ŧ}'}(ɯ_2)\right\}e^{i\wedge\left\{ɱϝ_{\,Ʀ_ŧ}(ƞ_1, ƞ_2),\ ɱϝ_{Ᵽ_{ŧ}'}(ɯ_1), ɱϝ_{Ᵽ_{ŧ}'}(ɯ_2)\right\}}$$

Example 7. The composition complex membership grade of fig 6 is $0.6e^{i0.5\pi}$.

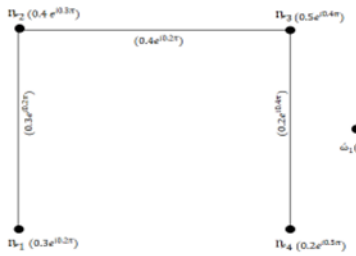


Figure 5(a). $0.5e^{i0.6\pi}$-$\mathcal{G}_ŧ$.

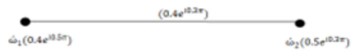


Figure 5(b). $0.5e^{i0.6\pi}$- $\mathcal{G}_ŧ'$.

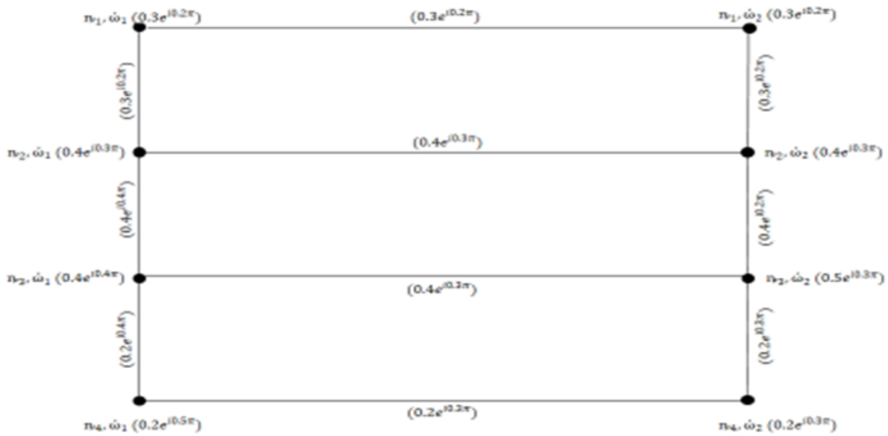


Figure 6. $\mathcal{G}_{0.5e^{i0.6\pi}} \circ \mathcal{G}_{0.5e^{i0.6\pi}}'$.

Definition 12. The degree of composition $\mathcal{G}_t$ & $\mathcal{G}_t'$ is define follows

$$(ɳ_1,\ ɯ_1) \in Ꝁ \times Ꝁ';\ deg_{\mathcal{G}_t \circ \mathcal{G}_t'}(ɳ_1,\ ɯ_1) = \left(deg\left\{ ɱς_{\,Ꞧ_t \circ Ꞧ_t'}\big((ɳ_1,\ ɯ_1),(ɳ_2,\ ɯ_2)\big)\right\} e^{i\left\{ɱϝ_{\,Ꞧ_t \circ Ꞧ_t'}((ɳ_1,\ ɯ_1),(ɳ_2,\ ɯ_2))\right\}} \right)$$

where

$$deg\left\{ ɱς_{\,Ꞧ_t \circ Ꞧ_t'}\big((ɳ_1,\ ɯ_1),(ɳ_2,\ ɯ_2)\big)\right\} e^{i\left\{ɱϝ_{\,Ꞧ_t \circ Ꞧ_t'}((ɳ_1,\ ɯ_1),(ɳ_2,\ ɯ_2))\right\}}$$

$$= \sum_{ɳ_1=ɳ_2,(ɯ_1,\ ɯ_2)\in Ŋ'} \wedge \left\{ ɱς_{Ꝓ_t}(ɳ_1), ɱς_{\,Ꞧ_{t}'}(ɯ_1,\ ɯ_2)\right\} e^{\,i\sum_{ɳ_1=ɳ_2,(ɯ_1,\ ɯ_2)\in Ŋ'} \wedge\left\{ɱϝ_{Ꝓ_t}(ɳ_1), ɱϝ_{\,Ꞧ_{t}'}(ɯ_1,\ ɯ_2)\right\}} +$$

$$\sum_{ɯ_1=ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge \left\{ ɱς_{\,Ꞧ_t}(ɳ_1, ɳ_2), ɱς_{Ꝓ_{t}'}(ɯ_1)\right\} e^{\,i\sum_{ɯ_1=ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge\left\{ɱϝ_{\,Ꞧ_t}(ɳ_1,ɳ_2), ɱϝ_{Ꝓ_{t}'}(ɯ_1)\right\}} +$$

$$\sum_{ɯ_1\neq ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge \left\{ ɱς_{\,Ꞧ_t}(ɳ_1, ɳ_2), ɱς_{Ꝓ_{t}'}(ɯ_1), ɱς_{Ꝓ_{t}'}(ɯ_2)\right\} e^{\,i\sum_{ɯ_1\neq ɯ_2,(ɳ_1,ɳ_2)\in Ŋ} \wedge\left\{ɱϝ_{\,Ꞧ_t}(ɳ_1,ɳ_2), ɱϝ_{Ꝓ_{t}'}(ɯ_1), ɱϝ_{Ꝓ_{t}'}(ɯ_2)\right\}}$$

Example 8. By using Definiton 7, and fig 6, the degree of composition is

$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_1,\ ɯ_1) = (0.6e^{i0.4\pi})$, $deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_1,\ ɯ_2) = (0.6e^{i0.4\pi})$

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_2,\ ɯ_1) = (1.1e^{i0.9\pi}),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_2,\ ɯ_2) = (1.1e^{i0.7\pi})$$

$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_3,\ ɯ_1) = (1.0e^{i1.1\pi}),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_3,\ ɯ_2) = (1.0e^{i0.8\pi})$$

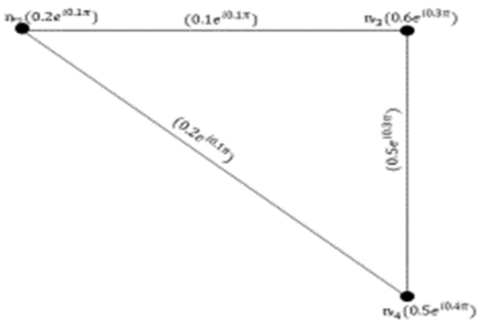
$$deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_4,\ ɯ_1) = (0.4e^{i0.7\pi}),\ deg_{\mathcal{G}_t\circ\mathcal{G}_t'}(ɳ_4,\ ɯ_2) = (0.4e^{i0.6\pi})$$

3.3. UNION OF CTFG

Definition 13. Supose $G = (Ꝁ, Ŋ)$ & $G' = (Ꝁ', Ŋ')$ stand every twofold CTFGs, such that $\mathcal{G}_t = (Ꝓ_t,\ Ꞧ_t)$, and $\mathcal{G}_t' = (Ꝓ_t',\ Ꞧ_t')$. The union $\mathcal{G}_t \cup \mathcal{G}_t'$ of these twofold CTFGs stands demarcated, in certain norms, as $(Ꝓ_t \cup Ꝓ_t',\ Ꞧ_t \cup Ꞧ_t')$, anywhere $Ꝓ_t \cup Ꝓ_t'$ and $Ꞧ_t \cup Ꞧ_t'$, correspondingly, characterize CTFGs on $Ꝁ \cup Ꝁ'$ and $Ŋ \cup Ŋ'$, this meets the subsequent criteria:

- If $ɳ_1 \in Ꝁ$ and $ɳ_1 \notin Ꝁ'$, $ɱς_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)e^{i\,ɱϝ_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)} = ɱς_{Ꝓ_t}(ɳ_1)e^{i\,ɱϝ_{Ꝓ_t}(ɳ_1)}$
- If $ɳ_1 \notin Ꝁ$ and $ɳ_1 \in Ꝁ'$, $ɱς_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)e^{i\,ɱϝ_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)} = ɱς_{Ꝓ_t'}(ɳ_1)e^{i\,ɱϝ_{Ꝓ_t'}(ɳ_1)}$
- If $ɳ_1 \in Ꝁ \cap Ꝁ'$, $ɱς_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)e^{i\,ɱϝ_{Ꝓ_t\cup Ꝓ_t'}(ɳ_1)} = max\{ɱς_{Ꝓ_t}(ɳ_1), ɱς_{Ꝓ_t'}(ɳ_1)\}e^{i\,max\{ɱϝ_{Ꝓ_t}(ɳ_1), ɱϝ_{Ꝓ_t'}(ɳ_1)\}}$
- If $(ɳ_1,\ ɯ_1) \in Ŋ$ and $(ɳ_1,\ ɯ_1) \notin Ŋ'$, $ɱς_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)e^{i\,ɱϝ_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)} = ɱς_{\,Ꞧ_t}(ɳ_1,\ ɯ_1)e^{i\,ɱϝ_{\,Ꞧ_t}(ɳ_1,\ ɯ_1)}$
- If $(ɳ_1,\ ɯ_1) \notin Ŋ$ and $(ɳ_1,\ ɯ_1) \in Ŋ'$, $ɱς_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)e^{i\,ɱϝ_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)} = ɱς_{\,Ꞧ_t'}(ɳ_1,\ ɯ_1)e^{i\,ɱϝ_{\,Ꞧ_t'}(ɳ_1,\ ɯ_1)}$
- If $(ɳ_1,\ ɯ_1) \in Ŋ \cap Ŋ'$, $ɱς_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)e^{i\,ɱϝ_{\,Ꞧ_t\cup Ꞧ_t'}(ɳ_1,\ ɯ_1)} = max\{ɱς_{\,Ꞧ_t}(ɳ_1,\ ɯ_1), ɱς_{\,Ꞧ_t'}(ɳ_1,\ ɯ_1)\}e^{max\{ɱϝ_{\,Ꞧ_t}(ɳ_1,\ ɯ_1), ɱϝ_{\,Ꞧ_t'}(ɳ_1,\ ɯ_1)\}}$

Example 9. The $0.7e^{i0.6\pi}$-CTFGs of $\mathcal{G}_t$ & , presented now Fig 7(a) & 7(b). The unon of CTFGs is show in Figu 8.



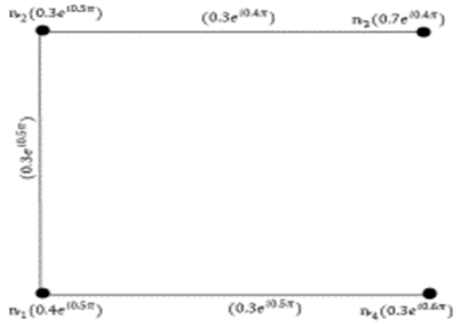


Figure 7(a). $0.7e^{i0.6\pi} - NG\mathcal{G}_{0.7e^{i0.6\pi}}$. Figure 7(b). $0.7e^{i0.6\pi} - NG\mathcal{G}'_{0.7e^{i0.6\pi}}$

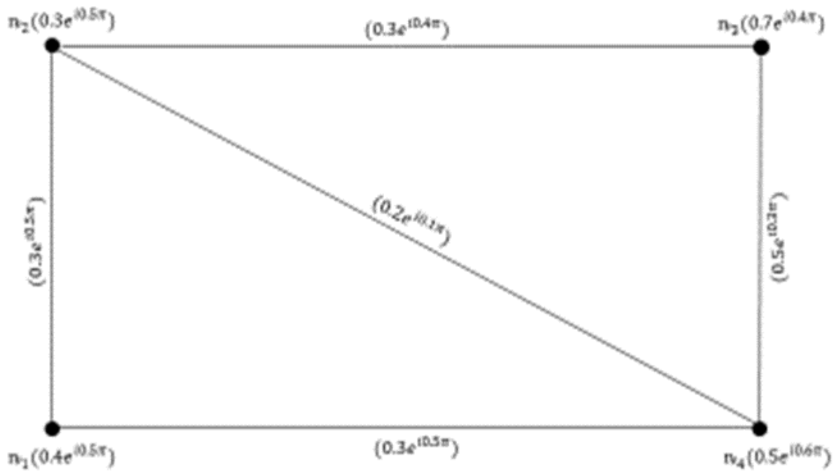


Figure 8. $\mathcal{G}_{0.7} \cup \mathcal{G}'_{0.7}$.

Definition 14. The degree of vertex (n_1, w_1) at a CTFG for every $(n_1, w_1) \in \mathcal{K} \times \mathcal{K}'$

$$deg_{\mathcal{G}_t \cup \mathcal{G}'_t}(n_1, w_1) = \left(deg\left\{\mu\varsigma_{R_t \cup R'_t}(n_1, w_1)\right\} e^{i\left\{\mu f_{R_t \cup R'_t}(n_1, w_1)\right\}}\right) \text{ where}$$

$$deg\left\{\mu\varsigma_{R_t \cup R'_t}(n_1, w_1) e^{i\,\mu f_{R_t \cup R'_t}(n_1, w_1)}\right\} = \sum_{(n_1, w_1) \in \mathcal{N}, (n_1, w_1) \notin \mathcal{N}'} \mu\varsigma_{R_t}(n_1, w_1)\, e^{i \sum_{(n_1, w_1) \in \mathcal{N}, (n_1, w_1) \notin \mathcal{N}'} \mu f_{R_t}(n_1, w_1)}$$

$$+ \sum_{(n_1, w_1) \notin \mathcal{N}, (n_1, w_1) \in \mathcal{N}'} \mu\varsigma_{R'_t}(n_1, w_1) e^{i \sum_{(n_1, w_1) \notin \mathcal{N}, (n_1, w_1) \in \mathcal{N}'} \mu f_{R'_t}(n_1, w_1)}$$

$$+ \sum_{(n_1, w_1) \in \mathcal{N} \cap \mathcal{N}'} max\{\mu\varsigma_{R_t}(n_1, w_1), \mu\varsigma_{R'_t}(n_1, w_1)\} e^{\sum_{(n_1, w_1) \in \mathcal{N} \cap \mathcal{N}'} max\{\mu f_{R_t}(n_1, w_1), \mu f_{R'_t}(n_1, w_1)\}}$$

3.4. JOINIFG OF CTFGS

Definition 15. Suppose two CTFGs $\mathcal{G}_t = (P_t, R_t)$ & $\mathcal{G}'_t = (P'_t, R'_t)$. These CTFGs' joining procedure $\mathcal{G}_t + \mathcal{G}'_t$ be presentas $(P_t + P'_t, R_t + R'_t)$, everywhere $P_t + P'_t$ yields a CTFG on $\mathcal{K} \cup \mathcal{K}'$ and $R_t + R'_t$ formulae a CTFG on $\mathcal{N} \cup \mathcal{N}' \cup \mathcal{N}''$

If $n_1 \in \mathcal{K}$ *and* $n_1 \notin \mathcal{K}'$, $\mu\varsigma_{P_t + P'_t}(n_1) e^{i\mu f_{P_t + P'_t}(n_1)} = \mu\varsigma_{P_t}(n_1)\, e^{i\mu f_{P_t}(n_1)}$

i. If $n_1 \notin \mathcal{K}$ and $n_1 \in \mathcal{K}'$, $\mu\varsigma_{P_t + P'_t}(n_1) e^{i\mu f_{P_t + P'_t}(n_1)} = \mu\varsigma_{P'_t}(n_1) e^{i\mu f_{P'_t}(n_1)}$

ii. If $n_1 \in \mathcal{K} \cap \mathcal{K}'$, $\mu\varsigma_{P_t + P'_t}(n_1)\, e^{i\mu f_{P_t + P'_t}(n_1)} = max\{\mu\varsigma_{P_t}(n_1), \mu\varsigma_{P'_t}(n_1)\} e^{i max\{\mu f_{P_t}(n_1), \mu f_{P'_t}(n_1)\}}$

iii. If $(n_1, w_1) \in \text{Ŋ}$ and $(n_1, w_1) \notin \text{Ŋ}'$, $\mu\varsigma_{R_t+R_t'}(n_1, w_1)e^{i\,\mu f_{R_t+R_t'}(n_1, w_1)} = \mu\varsigma_{R_t}(n_1, w_1)e^{i\,\mu f_{R_t}(n_1, w_1)}$

iv. If $(n_1, w_1) \notin \text{Ŋ}$ and $(n_1, w_1) \in \text{Ŋ}'$, $\mu\varsigma_{R_t+R_t'}(n_1, w_1)e^{i\,\mu f_{R_t+R_t'}(n_1, w_1)} = \mu\varsigma_{R_t'}(n_1, w_1)e^{i\,\mu f_{R_t'}(n_1, w_1)}$

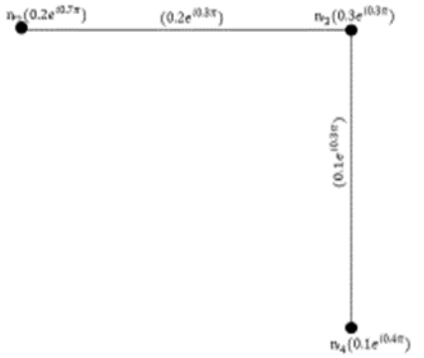
v. If $(n_1, w_1) \in \text{Ŋ} \cap \text{Ŋ}'$

$$\mu\varsigma_{R_t+R_t'}(n_1, w_1)e^{i\,\mu f_{R_t+R_t'}(n_1, w_1)} = max\{\mu\varsigma_{R_t}(n_1, w_1), \mu\varsigma_{R_t'}(n_1, w_1)\}e^{i\,max\{\mu f_{R_t}(n_1, w_1), \mu f_{R_t'}(n_1, w_1)\}}$$

vi. If $(n_1, w_1) \in \text{Ŋ}''$

$$\mu\varsigma_{R_t+R_t'}(n_1, w_1)\, e^{i\mu f_{R_t+R_t'}(n_1, w_1)} = max\{\mu\varsigma_{R_t}(n_1), \mu\varsigma_{R_t'}(w_1)\}e^{i\,max\{\mu f_{R_t}(n_1), \mu f_{R_t'}(w_1)\}}$$

Example 10. The join $\mathcal{G}_t + \mathcal{G}_t'$ *is* $0.4e^{i0.7\pi}$ is presented now Fig 12.



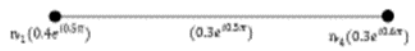


Figure 9(a). $0.4e^{i0.8\pi} - \mathcal{G}_{0.4e^{i0.8\pi}}$. Figure 9(b). $0.4e^{i0.8\pi} - \mathcal{G}'_{0.4e^{i0.8\pi}}$

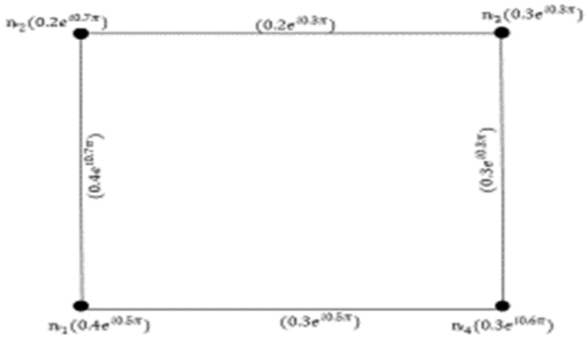


Figure 9. $\mathcal{G}_{0.4e^{i0.7\pi}} + \mathcal{G}'_{0.4e^{i0.7\pi}}$.

Definition 16. The two $\mathcal{G}_t$ & $\mathcal{G}_t'$ be CTFGs and the degree of CTFG $\mathcal{G}_t + \mathcal{G}_t'$ is termed below. For every $(n_1, w_1) \in \text{Ķ} \times \text{Ķ}'$. $deg_{\mathcal{G}_t+\mathcal{G}_t'}(n_1, w_1) = \left(deg\{\mu\varsigma_{R_t+R_t'}(n_1, w_1)\}e^{i\{\mu f_{R_t+R_t'}(n_1, w_1)\}}\right)$, wherever

$$deg\left\{\mu_{R_t+R_t'}(n_1, w_1)e^{i\mu f_{R_t+R_t'}(n_1, w_1)}\right\} =$$

$$\Big(\sum_{(n_1, w_1)\in \text{Ŋ},(n_1, w_1)\notin \text{Ŋ}'} \mu\varsigma_{R_t}(n_1, w_1)\, e^{i\sum_{(n_1, w_1)\in \text{Ŋ},(n_1, w_1)\notin \text{Ŋ}'} \mu f_{R_t}(n_1, w_1)} +$$

$$\sum_{(n_1, w_1)\notin \text{Ŋ},(n_1, w_1)\in \text{Ŋ}'} \mu\varsigma_{R_t'}(n_1, w_1)\, e^{i\sum_{(n_1, w_1)\notin \text{Ŋ},(n_1, w_1)\in \text{Ŋ}'} \mu f_{R_t'}(n_1, w_1)} +$$

$$\sum_{(n_1, w_1)\in \text{Ŋ}+\text{Ŋ}'} max\{\mu\varsigma_{R_t}(n_1, w_1), \mu\varsigma_{R_t'}(n_1, w_1)\}\, e^{i\sum_{(n_1, w_1)\in \text{Ŋ}\cap \text{Ŋ}'} max\{\mu f_{R_t}(n_1, w_1),\ \mu f_{R_t'}(n_1, w_1}$$

$$\sum_{(n_1, w_1)\in \text{Ŋ}''} max\{\mu\varsigma_{R_t}(n_1), \mu\varsigma_{R_t'}(w_1)\}\, e^{i\sum_{(n_1, w_1)\in \text{Ŋ}''} max\{\mu f_{R_t}(n_1), \mu f_{R_t'}(w_1)\}}\Big),$$

Theorem 2. The CTFGs $\mathcal{G}_{ŧ} = (\text{Ƥ}_{ŧ}, \text{Ɽ}_{ŧ})$ & $\mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ}')$ of $G = (\text{Ꝁ}, \text{Ŋ})$ and $G' = (\text{Ꝁ}', \text{Ŋ}')$, correspondingly, everywhere $\text{Ꝁ} \cap \text{Ꝁ}' \neq \emptyset$, $\mathcal{G}_{ŧ} \cup \mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ} \cup \text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ} \cup \text{Ɽ}_{ŧ}')$ is a CTFG of $G = G \cup G'$ if $\mathcal{G}_{ŧ}$ and $\mathcal{G}_{ŧ}'$ are the CTFGs of G & G', correspondingly.

Proof. Let $(\text{ƞ}_1, \text{ɯ}_1) \in \text{Ŋ}, (\text{ƞ}_1, \text{ɯ}_1) \notin \text{Ŋ}'$, & $(\text{ƞ}_1, \text{ɯ}_1) \in \text{Ꝁ} - \text{Ꝁ}'$.

Consider

$$\text{ɱ}\varsigma_{B_{\text{ɱ}}}(\text{ƞ}_1, \text{ɯ}_1)e^{i\,\text{ɱ}F_{B_{\text{ɱ}}}(\text{ƞ}_1, \text{ɯ}_1)} = \text{ɱ}\varsigma_{B_{\text{ɱ}} \cup B_{\text{ɱ}}'}(\text{ƞ}_1, \text{ɯ}_1)\, e^{i\,\text{ɱ}F_{B_{\text{ɱ}} \cup B_{\text{ɱ}}'}(\text{ƞ}_1, \text{ɯ}_1)}$$

$$\leq \wedge\{\text{ɱ}\varsigma_{A_{\text{ɱ}} \cup A_{\text{ɱ}}'}(\text{ƞ}_1), \text{ɱ}\varsigma_{A_{\text{ɱ}} \cup A_{\text{ɱ}}'}(\text{ɯ}_1)\}e^{i \wedge\{\text{ɱ}F_{A_{\text{ɱ}} \cup A_{\text{ɱ}}'}(\text{ƞ}_1),\ \text{ɱ}F_{A_{\text{ɱ}} \cup A_{\text{ɱ}}'}(\text{ɯ}_1)\}}$$

$$= \wedge\{\text{ɱ}\varsigma_{A_{\text{ɱ}}}(\text{ƞ}_1), \text{ɱ}\varsigma_{A_{\text{ɱ}}}(\text{ɯ}_1)\}e^{i \wedge\{\text{ɱ}F_{A_{\text{ɱ}}}(\text{ƞ}_1), \text{ɱ}F_{A_{\text{ɱ}}}(\text{ɯ}_1)\}}$$

Correspondingly, we determine $\mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ}')$ as a CTFG of G''. Undertake FG $\mathcal{G}_{ŧ}$ & $\mathcal{G}_{ŧ}'$ and considerate that the of two CTFGs creates a CTFG it tracks that $\mathcal{G}_{ŧ} \cup \mathcal{G}_{ŧ}'$. □

4. ISOMORPHISM OF CTFGS

Definition 17. Suppose $\mathcal{G}_{ŧ} = (\text{Ƥ}_{ŧ}, \text{Ɽ}_{ŧ})$ & $\mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ}')$ stand every twofold CTFGs of $G = (\text{Ꝁ}, \text{Ŋ})$ & $G' = (\text{Ꝁ}', \text{Ŋ}')$ correspondingly. A homomorphism θ from CTFG $\mathcal{G}_{ŧ}$ & $\mathcal{G}_{ŧ}'$ is a mapping $\theta: \text{Ꝁ} \to \text{Ꝁ}'$, adequate below

$$\text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)e^{i\,\text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)} \leq \text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))e^{i\,\text{ɱ}F_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))}, \forall \text{ƞ}_1 \in \text{Ꝁ}.$$

1. $\text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)} \leq \text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))\, e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))}, \forall (\text{ƞ}_1, \text{ɯ}_1) \in \text{Ŋ}.$

Definition 18. A weak isomorphism $\theta: \text{Ꝁ} \to \text{Ꝁ}'$, from CTFG $\mathcal{G}_{ŧ}$ ɱo $\mathcal{G}_{ŧ}'$, necessity below circumstances

$$\text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)e^{I\,\text{ɱ}F_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)} \leq \text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))e^{I\,\text{ɱ}F_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))}, \forall \text{ƞ}_1 \in \text{Ꝁ}$$

Definition 19. A strong co-isomorphism is demarcated such as a bijective mapping $\theta: \text{Ꝁ} \to \text{Ꝁ}'$ between every twofold CTFGs, $\mathcal{G}_{ŧ} = (\text{Ƥ}_{ŧ}, \text{Ɽ}_{ŧ})$ & $\mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ}')$ of $G = (\text{Ꝁ},E)$ & $G' = (\text{Ꝁ}',E')$, correspondingly, adequate below

1. $\text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)e^{i\,\text{ɱ}F_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)} \leq \text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))e^{\text{ɱ}F_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))}, \forall \text{ƞ}_1 \in \text{Ꝁ}$

2. $\text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)} \leq \text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))\, e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))}, \forall (\text{ƞ}_1, \text{ɯ}_1) \in \text{Ŋ}.$

3. $\text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)} = \text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))}, \forall (\text{ƞ}_1, \text{ɯ}_1) \in \text{Ŋ}.$

Definition 20. An isomorphism among two CTFGs $\mathcal{G}_{ŧ} = (\text{Ƥ}_{ŧ}, \text{Ɽ}_{ŧ})$ & $\mathcal{G}_{ŧ}' = (\text{Ƥ}_{ŧ}', \text{Ɽ}_{ŧ}')$ stands a bijective homomorphism mapping $\theta: \text{Ꝁ} \to \text{Ꝁ}'$ adequate below:

$$\text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)e^{i\,\text{ɱ}F_{\text{Ƥ}_{ŧ}}(\text{ƞ}_1)} \leq \text{ɱ}\varsigma_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))e^{\text{ɱ}F_{\text{Ƥ}_{ŧ}'}(\theta(\text{ƞ}_1))}, \forall \text{ƞ}_1 \in \text{Ꝁ}.$$

$$\text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}}(\text{ƞ}_1, \text{ɯ}_1)} = \text{ɱ}\varsigma_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))e^{i\,\text{ɱ}F_{\text{Ɽ}_{ŧ}'}(\theta(\text{ƞ}_1), \theta(\text{ɯ}_1))}, \forall (\text{ƞ}_1, \text{ɯ}_1) \in \text{Ŋ}.$$

Example 11. According to the following figures, take the Two $0.8e^{i0.7\pi}$-$\mathcal{G}_{ŧ}$ and $\mathcal{G}_{ŧ}'$ as shown in Figures 10(a) and 10(b).

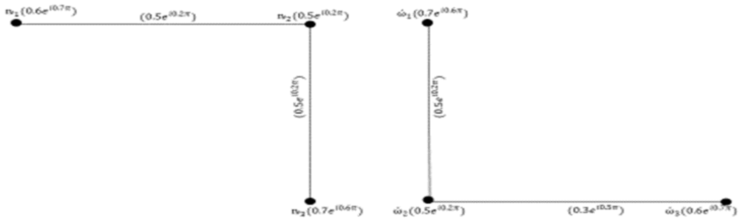


Figure 10(a) . $0.8e^{i0.7\pi} - NG\ \mathcal{G}_{0.8e^{i0.7\pi}}$. Figure 10(b). $0.8e^{i0.7\pi} - NG\ \mathcal{G}'_{0.8e^{i0.7\pi}}$.

Giving to Definition (20), the mapping ζ(a)=g, ζ(b)=f, and ζ(c) = e provides

$$\mathcal{G}_{0.8e^{i0.7\pi}} \approx \mathcal{G}'_{0.8e^{i0.7\pi}}$$

Theorem 3. By connecting an isomorphism between CTFGs, the properties of a relation of equivalents are met.

Proof. Symmetry as well as reflexivity are both evident. The notations $\varphi: Ꝁ \to Ꝁ'$ and $\theta: Ꝁ' \to Ꝁ''$ represent the isomorphism of $\mathcal{G}_{ŧ}$ onto $\mathcal{G}'_{ŧ}$ and $\mathcal{G}'_{ŧ}$ onto $\mathcal{G}''_{ŧ}$, respectively. it is defined as follows, and $\theta \circ \varphi: Ꝁ \to Ꝁ''$ is a bijective map from $Ꝁ'$ to $Ꝁ''$.

$$(\theta \circ \varphi)(ɳ_1) = \theta(\varphi(ɳ_1)), \forall ɳ_1 \in Ꝁ$$

For a map $\varphi: Ꝁ \to Ꝁ'$demarcated by $\varphi(ɳ_1) = \omega_1, \forall ɳ_1 \in Ꝁ$, iɥ is an isomorphism. In view of def (20)

$$ɥ\varsigma_{Ᵽ_ŧ}(ɳ_1)e^{iɥf_{Ᵽ_ŧ}(ɳ_1)} = ɥ\varsigma_{Ᵽ'_ŧ}(\varphi(ɳ_1))e^{iɥf_{Ᵽ'_ŧ}(\varphi(ɳ_1))} = ɥ\varsigma_{Ᵽ'_ŧ}(\omega_1)e^{iɥf_{Ᵽ'_ŧ}(\varphi(ɳ_1))}, \forall ɳ_1 \in Ꝁ \quad (1)$$

and

$$\begin{aligned} ɥ\varsigma_{R_ŧ}(ɳ_1, ɳ_2)e^{iɥf_{R_ŧ}(ɳ_1,ɳ_2)} &= ɥ\varsigma_{R'_ŧ}(\varphi(ɳ_1), \varphi(ɳ_2))e^{iɥf_{R'_ŧ}(\varphi(ɳ_1),\varphi(ɳ_2))} \\ &= ɥ\varsigma_{R'_ŧ}(\omega_1, \omega_2)e^{iɥf_{R'_ŧ}(\omega_1,\omega_2)}, \forall(ɳ_1, ɳ_2) \in Ŋ \end{aligned} \quad (2)$$

Similarly, we derive that

$$ɥ\varsigma_{Ᵽ'_ŧ}(\omega_1)e^{iɥf_{Ᵽ'_ŧ}(\omega_1)} = ɥ\varsigma_{Ᵽ''_ŧ}(v_1)e^{iɥf_{Ᵽ''_ŧ}(v_1)}, \forall\ \omega_1 \in Ꝁ' \quad (3)$$

and

$$ɥ\varsigma_{R'_ŧ}(\omega_1, \omega_2)e^{iɥf_{R'_ŧ}(\omega_1,\omega_2)} = ɥ\varsigma_{R''_ŧ}(v_1, v_2)e^{iɥf_{R''_ŧ}(v_1,v_2)}, \forall(\omega_1, \omega_2) \in Ŋ' \quad (4)$$

By using the relations (1) and (7) and $\varphi(ɳ_1) = \omega_1, \forall\, ɳ_1 \in Ꝁ$, we have

$$ɥ\varsigma_{Ᵽ_ŧ}(ɳ_1)e^{ɥf_{Ᵽ_ŧ}(ɳ_1)} = ɥ\varsigma_{Ᵽ'_ŧ}(\varphi(ɳ_1))e^{iɥf_{Ᵽ'_ŧ}(\varphi(ɳ_1))}$$

$$= ɥ\varsigma_{Ᵽ'_ŧ}(\omega_1)e^{iɥf_{Ᵽ'_ŧ}(\omega_1)}$$

$$= ɥ\varsigma_{Ᵽ''_ŧ}(\theta(\omega_1))e^{iɥf_{Ᵽ''_ŧ}(\theta(\omega_1))}$$

$$= ɥ\varsigma_{Ᵽ''_ŧ}\left(\theta(\varphi(ɳ_1))\right)e^{i\,ɥf_{Ᵽ''_ŧ}\left(\theta(\varphi(ɳ_1))\right)}$$

When using the relations (4) and (10), the outcome is

$$\begin{aligned} ɥ\varsigma_{R_ŧ}(ɳ_1, ɳ_2)e^{iɥf_{R_ŧ}(ɳ_1,ɳ_2)} &= ɥ\varsigma_{R'_ŧ}(\omega_1, \omega_2)e^{iɥf_{R'_ŧ}(\omega_1,\ \omega_2)} \\ &= ɥ\varsigma_{R''_ŧ}(\theta(\omega_1), \theta(\omega_2))e^{ɥf_{R''_ŧ}(\theta(\omega_1),\ \theta(\omega_2))} \\ &= ɥ\varsigma_{R''_ŧ}\left(\theta(\varphi(ɳ_1)), \theta(\varphi(ɳ_2))\right)e^{iɥf_{R''_ŧ}\left(\theta(\varphi(ɳ_1)),\ \theta(\varphi(ɳ_2))\right)} \end{aligned}$$

When using the relations (6) and (12), the outcome is

$$\begin{aligned} F\varsigma_{R_ŧ}(ɳ_1, ɳ_2)e^{iF\rho_{R_ŧ}(ɳ_1,ɳ_2)} &= F\varsigma_{R'_ŧ}(\omega_1, \omega_2)e^{iF\rho_{R'_ŧ}(\omega_1,\ \omega_2)} \\ &= F\varsigma_{R''_ŧ}(\theta(\omega_1), \theta(\omega_2))e^{F\rho_{R''_ŧ}(\theta(\omega_1),\ \theta(\omega_2))} \\ &= F\varsigma_{R''_ŧ}\left(\theta(\varphi(ɳ_1)), \theta(\varphi(ɳ_2))\right)e^{iF\rho_{R''_ŧ}\left(\theta(\varphi(ɳ_1)),\ \theta(\varphi(ɳ_2))\right)} \end{aligned}$$

Hence, $\mathcal{G}_{ŧ}$ and $\mathcal{G}''_{ŧ}$ are isomorphic to every other via $\theta \circ \varphi$.

5. REAL-WORLD APPLICATIONS IN HOSPITAL RESOURCE MANAGEMENT

Hospital resource management solutions based on CTFGs and TFGs are examined in this section.

5.1 DESCRIPTION OF THE EXPERIMENT

The study leveraged CTFGs to explore and identify challenges in hospital resource management. By utilizing CTFGs, healthcare administrators could examine uncertain data and evaluate multiple dimensions of resource distribution, including bed occupancy, workforce scheduling, medical equipment allocation, patient flow coordination, emergency response planning, and financial oversight. The CTFG-based model incorporated membership functions to depict the interdependencies among resource management variables, capturing the extent of conformity or divergence from optimal hospital resource utilization.

The results of the study are as follows:

- CTFGs proved instrumental in pinpointing key determinants impacting hospital resource administration, equipping decision-makers with a deeper understanding of the intricate connections between various allocation factors.
- Leveraging CTFGs to examine interdependencies among resource variables enabled more effective prioritization, thereby refining strategic decision-making in hospital management.
- The adjustable parameter 'ŧ' within CTFGs empowered administrators to customize the graphical models based on their specific expertise in healthcare operations and domain-specific challenges, leading to improved efficiency in resource distribution and management.
- Graphical depictions of CTFGs illuminated complex interrelationships among hospital resources, assisting in the formulation of robust strategies for streamlined hospital workflows and enhanced patient care.

5.2. APPLICATION OF CTFGS IN HOSPITAL RESOURCE MANAGEMENT

Efficient management of hospital resources is crucial for maintaining high standards of patient care and maximizing operational effectiveness. Various elements, including workforce availability, medical asset allocation, bed utilizetion, emergency preparedness, budgetary limitations, and shifts in patient demand, significantly impact hospital functionality. A comprehensive strategy that accounts for the interconnections among these factors is necessary for effective resource administration. By employing CTFGs, healthcare decision makers can analyze ambiguous data and evaluate key resource parameters. This approach facilitates the identification of critical determinants and strengthens strategic planning in hospital management.

Key aspects contributing to hospital resource management include:

- Bed availability (H1)
- Staff scheduling (H2)
- Medical equipment distribution (H3)
- Patient flow optimization (H4)
- Emergency preparedness (H5)
- Cost management (H6)

Let M = {H1, H2, H3, H4, H5, H6} represent a set of key resource factors that substantially influence hospital administration. The connections between these elements are quantified using t-fuzzy values, indicating the degree of interdependence among them. Within the CTFG framework, membership functions characterize the relationships among hospital resources, reflecting the extent to which each factor conforms to or deviates from optimal resource allocation. The incorporation of these functions offers a comprehensive understanding of resource interactions, enabling healthcare administrators to assess operational effectiveness. The parameter 't' serves as a customizable element that allows administrators to tailor CTFGs based on their domain knowledge and hospital-specific challenges, facilitating precise interventions and enhanced resource coordination. Additionally, fluctuations in the t parameter encapsulate diverse perspectives on risk tolerance and uncertainty in resource allocation. By modifying the t value, decision-makers can explore various operational scenarios, striking a balance between over-allocation and resource scarcity. This flexibility is essential for adapting to sudden increases in patient volume, financial constraints, and emergency conditions, ensuring resilient and efficient hospital management.

Furthermore, different values of the parameter ‘t’ reflect diverse viewpoints on risk assessment and sensitivity in resource distribution. Decision-makers can prioritize or mitigate specific resource constraints depending on their strategic objectives and tolerance for operational uncertainty. The ‘t’ parameter enables healthcare institutions to customize CTFG models to suit various hospital environments and fluctuations in demand, supporting resilient and data-driven resource management. Additionally, adjustments in the ‘t’ parameter represent distinct approaches to handling uncertainty and risk. By modifying edge values for membership and non-membership functions, administrators gain the flexibility to emphasize or downplay specific resource factors, ensuring alignment by their desired levels of acceptance and rejection in decision-making.

The parameter ‘ŧ’ enables the adaptation of CTFGs to various contexts and sensitivity levels. By modifying the ‘ŧ’ value, decision-makers can explore multiple scenarios, adjusting the equilibrium between supportive and opposing perspectives. This adaptability is especially vital when making decisions in dynamic or uncertain environments. Additionally, the ‘ŧ’ parameter permits decision-makers to tailor CTFGs to align with their domain expertise and specific problem requirements. Altered ideals of ‘ŧ’ correspond to varying approaches toward danger and ambiguity. By influencing verge values for membership, decision-makers can highlight or downplay specific resource factors based on their intended level of membership association. This ‘ŧ’ parameter serves as a crucial tool for fine-tuning CTFGs, ensuring they align with diverse operational settings and risk sensitivities. Adjusting ‘ŧ’ grants decision-makers the ability to assess a broad spectrum of alternatives by recalibrating the balance between affirmative and contrasting viewpoints. This flexibility is essential for navigating complex decision-making processes in unpredictable conditions or during continuous alteration. Table 2 presents the CFSs and 0.7-CFS, as demarcated by the vertices, illustrating their structural composition.

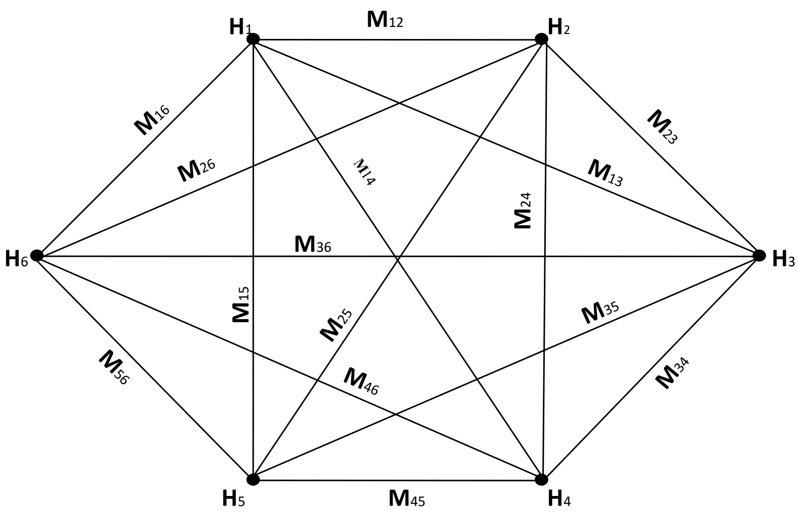


Figure 11. Graphical representation of Hospital Resource Management

Table 2. Vertices of FS and $0.6e^{0.7\pi}$-FS.

Vertices	CFS	Complex 0.7-FS
H1	$(0.7e^{i0.8\pi})$	$(0.6e^{i0.6\pi})$
H2	$(0.5e^{i0.4\pi})$	$(0.5e^{i0.4\pi})$
H3	$(0.4e^{i0.6\pi})$	$(0.4e^{i0.3\pi})$
H4	$(0.7e^{i0.1\pi})$	$(0.6e^{i0.1\pi})$

H5	$(0.8e^{i0.2\pi})$	$(0.6e^{i0.2\pi})$
H6	$(0.9e^{i0.4\pi})$	$(0.6e^{i0.4\pi})$

Table 5 presents the results derived from the scoring function formula outlined in Table 4. Meanwhile, Fig 12 provides a graphical representation of the score function corresponding to the parameters detailed in Table 5. Among the evaluated factors, H1 (Bed Availability) and H6 (Cost Management) exhibit the highest score of 2.992, indicating their predominant influence on hospital resource administration with respect to the parameter 'ŧ'. The data in Table 5 clearly highlights that Bed Availability (H1) and Cost Management (H6) play a critical role in shaping hospital resource management strategies, emphasizing their significance in operational decision-making.Table 3. Edges of CFS and complex 0.6$e^{0.7\pi}$-FS.

Edges	Complex 0.6-FS
M12	$(0.5e^{i0.4\pi})$
M13	$(0.4e^{i0.3\pi})$
M14	$(0.6e^{i0.1\pi})$
M15	$(0.6e^{i0.2\pi})$
M16	$(0.6e^{i0.4\pi})$
M23	$(0.4e^{i0.3\pi})$
M24	$(0.5e^{i0.1\pi})$
M25	$(0.5e^{i0.2\pi})$
M_{26}	$(0.5e^{i0.4\pi})$
M_{34}	$(0.4e^{i0.1\pi})$
M_{35}	$(0.4e^{i0.3\pi})$
M_{36}	$(0.4e^{i0.2\pi})$
M_{45}	$(0.6e^{i0.1\pi})$
M_{46}	$(0.6e^{i0.1\pi})$
M_{56}	$(0.6e^{i0.2\pi})$

Table 4. Table of membership degree of each factor.

Factor	Degree of Each Factor
H1	$\deg(H_1) = (2.7e^{i\,1.4\pi})$
H2	$\deg(H_2) = (2.4e^{i\,1.5\pi})$
H3	$\deg(H_3) = (2.0e^{i\,1.9\pi})$
H4	$\deg(H_4) = (2.7e^{i\,0.5\pi})$
H5	$\deg(H_5) = (2.7e^{i\,1.0\pi})$
H6	$\deg(H_6) = (2.7e^{i\,1.4\pi})$

Table 5. Score value of CTFGs.

Factor	Score Value of CTFG
Bed availability (H1)	2.992
Staff scheduling (H2)	2.638
Medical equipment distribution (H3)	2.3023
Patient flow optimization (H4)	2.779
Emergency preparedness (H5)	2.85
Cost management (H6)	2.992

5.3. PERFORMANCE COMPARATIVE ANALYSIS

CTFGs enhance conventional hospital resource management frameworks by incorporating the parameter 'ŧ', which refines the representation of uncertainty based on specific operational requirements. Modifying 'ŧ' enables decision-makers to more precisely simulate various resource allocation scenarios, subsequent in a additional nuanced description of uncertainty and fluctuations in demand. Given its applicability across multiple hospital functions, the adaptable nature of 'ŧ' allows for tailored adjustments to address varying concerns regarding resource availability, ensuring seamless hospital operations under diverse conditions. This flexibility is particularly vital in real-world hospital administration, where decision-makers must simultaneously navigate fluctuating patient admissions, workforce shortages, and medical equipment distribution challenges. The uniform structure of CTFGs implies that all resource elements contribute equally to hospital functions, regardless of their membership values. As well, allocating a membership rate of 0.6 to 'ŧ' signifies a strong interconnection among resources, granting administrators the ability to configure CTFGs according to their strategic priorities. The capacity to fine-tune 'ŧ'' ensures that CTFGs align with the distinct requirements of healthcare institutions, ultimately facilitating more accurate resource allocation and enhanced patient care delivery.

5.4. SENSITIVITY ANALYSIS

This study's sensitivity investigation centers on the parameter 'ŧ' in CTFGs, underscoring its precarious role in the decision-making process for hospital resource management. The 'ŧ parameter enables administrators to adjust the balance between resource allocation and demand variability, reflecting various approaches to risk and uncertainty. By altering the 'ŧ' value, hospital decision-makers can assess numerous scenarios, prioritizing particular resources in line with their confidence levels and operational goals. This flexibility proves invaluable when navigating unpredictable hospital environments, where factors like patient influx, resource shortages, and budget limitations demand adaptive allocation strategies. Additionally, the ability to modify 'ŧ' ensures that hospital management practices align with differing viewpoints on risk, uncertainty, and resource optimization. This customization provides decision-makers with the tools to fine-tune hospital strategies, guaranteeing that healthcare institutions maintain smooth operations while addressing the complexities inherent in real-world hospital resource management.

6. CONCLUSIONS

In this study, the CTFGs idea was maintained, and various fundamental apects of this concept were explored. Numerous studies and demonstrations have been conducted on graphical depictions for multiple set-abstract of CTFGs. Additionally, a classification and an analysis of some of the essential components of the complement of CTFGs be presented. The study also introduced the conceptions of CTFG homomorphisms and isomorphisms. Furthermore, the practical utility of the newly developed framework was demonstrated through applications in hospital resource management. In hospital resource management, CTFGs provided a structured approach to optimizing critical resources such as staff scheduling, bed availability, and emergency preparedness. By leveraging the flexibility of the 'ŧ' parameter, administrators could adjust their decision-making models to reflect varying degrees of confidence in resource allocation strategies. This capability enhanced the hospital's efficiency in managing uncertainties, ensuring improved patient care and operational effectiveness. The parameter 'ŧ' in CTFGs represents the degree of confidence or hesitation in decision-making processes. The two immoderations of this metric illustrate either a strong overtone or a insignificant effect on the preferred outcomes. By effectively tuning this parameter, choice-creators can refine ambiguity representation, ultimately leading to a more nuanced and adjustable basis aimed at addressing complex encounters across diverse domains, including biodiversity conservation and healthcare resource management.

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