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Research Article

A New Method to Find the Balanced Solution to the Transportation Problem with Fuzzy Sets of Type-2 for Supply and Demand

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ABSTRACT

The Transportation Problem is a fundamental problem widely studied in the Operational Research domain. The main objective of this paper is to contribute to a deeper study of the topics of fuzzy transportation problems of linear programming when the supply and demand are represented as a fuzzy set of type-2. Here, we are using a newly proposed method to solve a transportation problem with fuzzy sets type-2 of supply and demand to find a balanced solution. The membership function of a fuzzy set of type-2, which is the set of its feasible solutions, is constructed. The properties of this set are investigated, and the problems of choosing balanced solutions are considered. The newly proposed method of the fuzzy transportation problem is discussed with the help of an illustrative numerical example.

Keywords: Fuzzy type-2, fuzzy transportation problem, multi-objective transportation problem, decision-making.

INTRODUCTION

Traditional Transportation Problems (TP) are particular Linear Programming Problems (LPPs) that arise in many critical applications. These problems usually describe the movement of some goods from points of departure (places of production) to points of destination (warehouses, shops). In a natural interpretation, it is considered the problem of the optimal transportation plan from suppliers to consumers with minimal costs. The main goal is to determine the volume of traffic from origins to destinations with the minimum cost of transportation, and this should take into account restrictions imposed on the volume of goods at the destination (supply constraints) and restrictions that take into account the need for goods at destinations (demand constraints). It is assumed in the TP that the cost of transporting cargo along a route is directly proportional to the volume of cargo transported along this route. Numerous generalizations of the TP are known, which are widely used in practice: a multicriteria TP, in which, along with cost minimization, other indicators (transportation time, reliability, delivery time, etc.) can be optimized (Senapati P.,2008); multi-level hierarchical TP (Raskin L.G.,1982); multi-product TP, in which there are several types of cargo, a fuzzy TP, in which both various problem parameters (unit costs, volumes of stocks and needs) and constraint (for example, in the form of fuzzy inequalities) can be fuzzily specified (Zinmenoz F.,1999); single objective TP with mixed constraints, such that method has been developed on the basis of combinatorial procedure to solve the particular type of TP and also discovered a simple algorithm for solving maximum flow problem in transportation network (Hitchcock, F.,2016); an efficient solution of TP is found by using a new method called harmonic mean method (Palanievel, M.,2018); formulation a new model of multi-objective capacitated TP with mixed constraints, to choose the optimal order of the product quantity which is to be shipped from origin to the target based on the capacitated constraint on each route (Gupta,S.,2018); many practical ideas were presented to deal with a restricted fixed charge solid TP in an uncertain environment involving fuzzy type-2 parameters (Das, A.,2019); analyze the multi-objective fixed-charge TP under rough programming with made a comparison of the obtained solution method (Midya, S.,2020); developing a method for solving a fully intuitionistic fuzzy multi-objective fractional TP, in which the problem is transformed into a linear one using transformations using the accuracy function for each objective. Then the linear model is decreased to an apparent multi-objective TP (El Sayed, M.A.,2021). Propose Z-fuzzy numbers approach in TP with fuzzy unit costs, where assume that demand and supply are deterministic numbers and the uncertainty associated with the transportation costs and is modelled by using Z-fuzzy numbers (Gładysz, B.,2022). Optimization time–cost trade-off decisions in an interval TP with multiple shipment options by proposing an efficient iterative algorithm for generating the Pareto frontier that solves a minimum cost flow problem at each iteration (Shalabh Singh,2023), using a fuzzy-based decision-making approach to select the warehouse site for the automotive industry. Well-located and well-designed warehouses can make reaching these aims for the automotive industry possible and more accessible. Hence, determining a location for a warehouse is a highly critical, tactical, and managerial resolution for the automotive industry, as there is a strong correlation between well-located warehouses and the well-structured logistics network in the automotive industry (Abhijit Saha,2023). In this paper, we will consider a generalization of the TP for the case of fuzzy sets type-2 of supply and demand.

OBJECTIVES

The main objective of this paper is to contribute to a deeper study of the topics of fuzzy transportation problems of linear programming when the supply and demand are represented as a fuzzy set of type-2.

METHODS

1. Formulation of TP with fuzzy sets of supply & demand

First, we present a traditional crisp of the TP. Assume that homogeneous goods are concentrated at $\{m\}$ supply in capacity $(a_1, a_2, \dots, a_m)$. Let us denote $\mathcal{M} = \{1,2,\dots,m\}$ be the universal set of supply. These goods must be delivered to $\{n\}$ demand in capacities $(\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_n)$. Assume that $\mathcal{N} = \{1,2,\dots,n\}$ be the universal set of demand. Also, we known $\left(c_{ij} > 0, i \in \mathcal{M}, j \in \mathcal{N}\right)$, be the unit cost of transportation goods from each $\{i^{th}\}$ suppliers to each $\{j^{th}\}$ demanders. It is required to draw up a transportation plan with minimization total cost $(MinTC)$. Denote that $x_{(ij)} \geq 0$, $\{i \in \mathcal{M}\}$, $\{j \in \mathcal{N}\}$ be the quantities transportation from the $\{i^{th}\}$ suppliers for each $\{j^{th}\}$ demanders. At that point, the objective function of the problem will take the form: $(MinTC) \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)}$. In the general case of unbalanced TP, the problem constraint system consists of two groups of inequalities. The first group of $\{m\}$ inequalities describe the condition that transportation quantities don't exceed the inventories of all $\{m\}$ supply and has the pattern: $\sum_{j\in\mathcal{N}} x_{(ij)} \leq a_i$, $\{i \in \mathcal{M}\}$. And, second group of $\{n\}$ inequalities express the requirement to satisfy the needs of all $\{n\}$ demand and has the mode: $\sum_{i\in\mathcal{M}} x_{(ij)} \geq \mathscr{b}_j$, $\{j \in \mathcal{N}\}$. Suppose that the Decision-Maker (DM) cannot certainly say which supply and demand are actually ready to work at the time of making the decision but can only set the Membership Functions (MFs) as follows:

- $\mu(i)$, $\{i \in \mathcal{M}\}$, a fuzzy set of indices $\widetilde{\mathcal{M}} \subseteq \mathcal{M}$ of supply who intend to ship goods;
- $\delta(j)$, $\{j \in \mathcal{N}\}$, fuzzy set of indices $\widetilde{\mathcal{N}} \subseteq \mathcal{N}$ of demand who are ready to receive shipment.

Thus, a TP appears with fuzzy sets of supply and demand in the following formulation:

$$(MinTC) \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)}; \tag{1}$$

$$\sum_{j\in\mathcal{N}} x_{(ij)} \leq a_i, \{i \in \mathcal{M}\}; x_{(ij)} \geq 0, \{i \in \mathcal{M}\}, \{j \in \mathcal{N}\}; \tag{2}$$

$$\sum_{j\in\mathcal{N}} x_{(ij)} > 0, \{i \in \widetilde{\mathcal{M}}\}; \tag{3}$$

$$\sum_{i\in\mathcal{M}} x_{(ij)} \geq \mathscr{b}_j, \{j \in \widetilde{\mathcal{N}}\}. \tag{4}$$

Where fuzzy set $\widetilde{\mathcal{M}}$ of constraints (3) corresponds to non-zero supply quantities of those suppliers who intend to release goods, and fuzzy set $\widetilde{\mathcal{N}}$ of constraints (4) meets the requirement to satisfy the needs of consumers who are ready to receive shipment. In more detail, the meaning of the model from (1) to (4). Indeed, if for some supply $i \in \widetilde{\mathcal{M}}$ the corresponding condition (3) is not satisfied (i.e., $\sum_{j\in\mathcal{N}} x_{(ij)} \leq 0$) with a membership degree of $(1 - \mu(i))$, then from (2), it follows that the transportation volumes: $x_{(ij)} = 0$, $\forall j \in \mathcal{N}$, with the same membership degree. Then, the quantities of goods transported from the supply $i \in \mathcal{M}$ to different demanders will be non-zero with the degree of membership $\mu(i)$ if and only if the inequality $\left(\sum_{j\in\mathcal{N}} x_{(ij)} > 0\right)$ is satisfied with the same degree of membership. Likewise, for the demanders. Suppose that the corresponding condition (3) $\left(\text{i.e.}, \sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j\right)$ with the membership degree $(1-\delta(j))$ is not satisfied for some demand $j \in \mathcal{N}$. Thus, since the coefficients of the objective function (1) are positive, it is clear that in the optimal solution x^* problem from (1) to (2) under the condition $\left(\sum_{i\in\mathcal{M}} x_{(ij)} < b_j\right)$ we get the values of transportation quantities $x^*_{(ij)} = 0$, $\forall i \in \mathcal{M}$; with the same degree of membership. Therefore, the quantities of transportation of goods to the demander $j \in \mathcal{N}$ from all suppliers will be non-zero with the degree of membership $\delta(j)$ if and only if the inequality $\left(\sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j\right)$ is satisfied with the same degree of membership. Let X be the set of feasible solutions for a system of inequalities (2), which we will further call the universal set of solutions to the TP from (1) to (4) with fuzzy sets of supply and demand; $f_i = \{x \in X | \sum_{j\in\mathcal{N}} x_{(ij)} > 0\}$ be the set of feasible solutions from the universal set X, that satisfy a constraint of the form (3) with index $i \in \mathcal{M}$, and $h_j = \{x \in X | \sum_{i\in\mathcal{M}} x_{(ij)} \leq b_j\}$ is a similar set for the constraint $j \in \mathcal{N}$ of the form (4). Then problems from (1) to (4) can be represented as: $(MinTC) \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)}$, s.to: $x \in \tilde{\mathcal{F}}$, where: $\tilde{\mathcal{F}} = \tilde{f} \cap \tilde{h}$. That is, $\tilde{f} = \bigcap_{i\in\widetilde{\mathcal{M}}} f_i$ represents the set of feasible solutions of systems (2) and (3), which is the intersection of the fuzzy set $\widetilde{\mathcal{M}}$ of crisp sets f_i, $i \in \mathcal{M}$. Moreover, $\tilde{h} = \bigcap_{j\in\widetilde{\mathcal{N}}} h_j$ is the set of feasible solutions to systems (2) and (4), which is the intersection of the fuzzy set $\widetilde{\mathcal{N}}$ of crisp sets, h_j, $j \in \mathcal{N}$; $\tilde{\mathcal{F}} = \tilde{f} \cap \tilde{h}$, be the set of feasible solutions to the system from (2) to (4). Let us define the concept of the intersection of a fuzzy set of crisp sets under the approach proposed in (Mashchenko S.O.,2013).

2. The intersection of a fuzzy set of crisp sets

Assume that $\mathcal{Q}_t$, $t \in \mathcal{T}$, is some finite collection of crisp sets that are subsets of some universal set $\mathcal{Q}$. Let $\tilde{\mathcal{T}}$ be some fuzzy subset of the index set $\mathcal{T}$ with MF: $\lambda(t)$, $t \in \mathcal{T}$. On the universal set of $\{\mathcal{T}, \forall t \in \mathcal{T}\}$, we define the MF of the crisp set $\mathcal{Q}_t$ as follows: $\phi_t(x) = \phi_t(x) = [0] \iff \{x \notin \mathcal{Q}_t\}$ and $[1] \iff \{x \in \mathcal{Q}_t\}$. Consider now the intersection $\tilde{\mathcal{Q}} = \bigcap_{t\in\tilde{\mathcal{T}}} \mathcal{Q}_t$ of a fuzzy set of $\tilde{\mathcal{T}}$ crisp sets $\mathcal{Q}_t$, $t \in \mathcal{T}$. The classical generalization of operation intersection leads to the fact that the set $\tilde{\mathcal{T}}$ will be given by the MF:

$$x \in \mathcal{Q},\ g(x) = \underset{t\in\tilde{\mathcal{T}}}{Min}\{\phi_t(x)\}. \qquad (5)$$

The value of MF: $g(x)$, for each fixed $x \in \mathcal{Q}$ will be determined as the value of the objective function of the Fuzzy Linear Programming Problem (FLPP):

$$g = \underset{t\in\tilde{\mathcal{T}}}{Min}\{\phi_t\}. \qquad (6)$$

According to (Orlovsky S.A.,1981), a solution to the problem (6) is a fuzzy set $\{\tilde{\mathcal{T}}^*\}$, whose vector is the set of Non-Dominated Optimal (NDO) alternatives (we denote it by $\mathcal{T}^{\{ndo\}}$) of a bi-objective discrete optimization problem:

$$Minimization(Min)\ \phi_t,\ Maximization\ (Max)\ \beta\ (t), t \in \mathcal{T}. \qquad (7)$$

The MF: $\tilde{\beta}$ of a fuzzy set $\{\tilde{\mathcal{T}}^*\}$ is the constraint of the MF: $\beta(t)$, $t \in \mathcal{T}$, from the universal set of indices $\mathcal{T}$ to the set $\mathcal{T}^{\{NDO\}} \subseteq \mathcal{T}$. In other words, this MF will look like this: $\tilde{\beta}(t) = [0] \iff \left\{t \notin \mathcal{T}^{\{ndo\}}\right\} \wedge \tilde{\beta}(t) = \beta(t) \iff \left\{t \in \mathcal{T}^{\{ndo\}}\right\}$. The set of solutions to a problem (6), which is the fuzzy set $\{\tilde{\mathcal{T}}^*\}$ with the MF: $\tilde{\beta}(t)$, $t \in \mathcal{T}$, according to (Orlovsky S.A.,1981), corresponds to the fuzzy set $\Omega \subseteq \{0,1\}$ of optimal values of the objective function of this problem with the MF: $\omega(g) = \underset{\phi_t=\{g\}}{Max}\left\{\tilde{\beta}(t)\right\}$, $\omega: \{0,1\} \to [0,1] \wedge g \in \{0,1\}$. It should be noted that the universal set of the fuzzy set Ω of optimal values of the objective function of problem (6) will be the set $\{0,1\}$, consisting of two elements: $g = [0] \wedge g = [1]$. Here is explained by the fact that the variable g can take values equal only to the values of $\phi_t(x)$, $t \in \mathcal{T}$, which in turn can be equal to either: $\{0\} \vee \{1\}$ for any fixed $x \in \mathcal{T}$. Thus, for each fixed $x \in \mathcal{T}$, the values $g(x)$ of the MF: (5) of the fuzzy set $\tilde{\mathcal{Q}} = \bigcap_{t\in\tilde{\mathcal{T}}} \mathcal{Q}_t$ also form a fuzzy subset Ω of the universal set $\mathcal{G} = \{0,1\}$. That implies the fuzzy set $\tilde{\mathcal{Q}}$ is the so-called fuzzy set type-2 (we denote it by

FST2) [13]. According to [13], we formalize the concept of the intersection $\tilde{Q} = \cap_{t\in\tilde{T}} Q_t$ of a fuzzy set $\tilde{T}$ of crisp sets Q_t, $t \in T$. For an arbitrary $x \in Q$, consider the dominance relation generated by the functions $\phi_t(x)$ and $\beta(t)$ on the index set T. We will say that the index $i \in T$ dominates the index $j \in T$ for the solution $x \in Q$ and denote it by $(i \overset{x}{\succ} j)$ if the following inequalities hold: $Min\, \phi_t$: $(\phi_i(x) \leq \phi_j(x)) \wedge Max\, \beta\,(t)$: $(\beta(i) \geq \beta(j))$, and at least one of them is strict. This concept allows us to define the set of NDO alternatives of the bi-objective problem (7), which will be the vector of the fuzzy set of solutions to the problem (6). For $x \in Q$, we denote this vector; $T^{\{ndo\}}(x) = \{t \in T | j \overset{x}{\nsucc} t, \forall j \in T\}$. For arbitrary $x \in X$, $t \in T$, we define the MF of the fuzzy set of solutions to the problem (7): $\tilde{\beta}(x,t) = [0] \Leftrightarrow t \notin T^{\{ndo\}}(x) \wedge \tilde{\beta}(x,t) = \beta(t) \Leftrightarrow t \in T^{\{ndo\}}(x)$. The intersection of a fuzzy set $\tilde{T}$ of crisp sets Q_t, $t \in T$, is called $\tilde{Q} = \cap_{t\in\tilde{T}} Q_t$ an FST2, which is given by triples relations $(x, \omega(x,g))$, such that: $\omega: X \times G \to [0,1]$ is a fuzzy mapping that plays the role of a fuzzy MF and is specified by: $\phi(x,g) = [0] \Leftrightarrow \phi_t(x) \neq g, \forall t \in T \wedge \phi(x,g) = \underset{t\in T}{Max}\{\tilde{\beta}(x,t) | \phi_t(x) = g\} \Leftrightarrow \exists t \in T: \phi_t(x) = g$. Such that: x represent the element of the universal set Q, and g: is an element of the universal set $G = \{0,1\}$ of values of the MF: $\phi(x,g)$ of the FST2 $\tilde{Q}$. The values of the MF: $\phi(x,g)$ for a fixed $\{x^0 \in Q\}$ form a fuzzy subset $\Omega_G(x^0)$ of the set $G = \{0,1\}$ with the MF: $\phi(x^0,g)$, $g \in \{0,1\}$. The value of $\phi(x^0,1)$ can be understood as the degree to which the solution $x^0 \in Q$ belongs to the set $\tilde{Q}$. Accordingly, the value of $\phi(x^0,0)$ means the degree of does not belong of $\{x^0 \in Q\}$ in the set $\tilde{Q}$. On the other hand, if we fix $g = \{1\}$ in the MF: $\phi(x,g)$, we obtain a fuzzy set of solutions $x \in Q$ belonging to the set $\tilde{Q}$ with the MF: $\phi(x,1)$. We denote this set by $\Omega_Q(1)$. Similarly, for a fixed value $g = \{0\}$, we obtain a fuzzy set of alternatives $x \in Q$ that does not belong to the set $\tilde{Q}$, with the MF: $\phi(x,0)$. Denote it by $\Omega_Q(0)$. Interestingly, in the general case, $\langle 1 - \Omega_Q(1) \neq \Omega_Q(0)\rangle$, and, accordingly, $\langle 1 - \omega(x,1) \neq \omega(x,0)\rangle$. Therefore, both a fuzzy set $\Omega_Q(0)$ and $\Omega_Q(1)$ are fuzzy sets of sections for $g = \{0\} \wedge g = \{1\}$ of the FST2 $\tilde{Q}$; and are its integral components. The following theorem makes it possible to simplify the construction of the MF $\omega(x,g)$.

Theorem [13]. Let Q_t, $t \in T$, be a crisp set that is defined on the universal set Q by the corresponding MFs: $\phi_t(x)$, $x \in Q$, $t \in T$; $\lambda(t)$, $t \in T$, of fuzzy set $\tilde{T}$. In order for the FST2 $\tilde{Q}$, which is given by the MF: $\omega(x,g)$, $x \in Q$, $g \in \{0,1\}$, to be the intersection of a fuzzy set $\tilde{T}$ to the crisp sets Q_t, $t \in T$, (i.e., $\tilde{Q} = \cap_{t\in\tilde{T}} Q_t$) it is essential and acceptable $\forall x \in Q$:

- $\omega(x,0) = \{0\} \Leftrightarrow \phi_t(x) = \{1\}\ \forall i \in T \wedge \omega(x,1) = \{0\}, \exists i \in Arg\underset{j\in T}{Max}\lambda(j) \Leftrightarrow \phi_t(x) = \{0\}$;
- $\omega(x,0) = \underset{\phi_t(x)=\{0\}}{Max} \lambda(t) \Leftrightarrow \exists t \in T\!: \phi_t(x) = \{0\} \wedge \omega(x,1) = \underset{t\in T}{Max}\lambda(t) \Leftrightarrow \phi_t(x) = \{1\}\ \forall t \in Arg\underset{j\in T}{Max}\lambda(j)$.

3. FST2 feasible solutions TP with fuzzy sets of supply & demand

It follows from the above theorem that the set $\tilde{\mathcal{F}} = \tilde{f} \cap \tilde{h}$ of feasible solutions to the system from (2) until (4) is the intersection of FST2 $\tilde{f} \wedge \tilde{h}$. The intersection $\tilde{f} = \cap_{i\in\tilde{M}} f_i$ is the set of feasible solutions to inequalities (3) that the MF gives $\omega_{\tilde{f}}(x,g), \therefore g \in \{0,1\}$, where:

$$\omega_{\tilde{f}}(x,0) = \begin{cases} \underset{i\in M}{Max}\{\mu(i) | \sum_{j\in N} x_{(ij)} \leq 0\}; & \exists i \in M\!: \sum_{j\in N} x_{(ij)} \leq 0, \\ (0); & \forall i \in M\!: \sum_{j\in N} x_{(ij)} > 0. \end{cases} \quad (8)$$

Such that $\omega_{\tilde{f}}(x,0)$ represents the reliability of the infeasible solution (x) for inequalities (3);

$$\omega_{\tilde{f}}(x,1) = \begin{cases} \underset{i\in M}{Max}\mu(i); & \forall i \in Arg\underset{i\in M}{Max}\mu(i)\!: \sum_{j\in N} x_{(ij)} > 0, \\ (0); & \exists i \in Arg\underset{i\in M}{Max}\mu(i)\!: \sum_{j\in N} x_{(ij)} \leq 0. \end{cases} \quad (9)$$

Where $\omega_{\tilde{f}}(x,1)$ represents the reliability of its feasibility.

Since inequalities (3) describe the set of goods transportation plans in which all suppliers participate non-zero, the value of $\omega_{\tilde{f}}(x,0)$ can be explained as the reliability of non-participation of all suppliers in the goods transportation plan (x). Furthermore, $\omega_{\tilde{f}}(x,1)$ can be understood as the reliability of their participation. The

FST2 $\tilde{h} = \bigcap_{j \in \tilde{N}} h_j$ is the set of feasible solutions to inequalities (4), which is given by the MF $\omega_{\tilde{h}}(x, g)$, $\therefore g \in \{0,1\}$, where:

$$\omega_{\tilde{h}}(x,0) = \begin{cases} \underset{j \in \mathcal{N}}{Max}\{\delta(j) \mid \sum_{i \in \mathcal{M}} x_{(ij)} < b_j\}; & \exists j \in \mathcal{N}: \sum_{i \in \mathcal{M}} x_{(ij)} < b_j, \\ (0); & \forall j \in \mathcal{N}: \sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j. \end{cases} \quad (10)$$

That is $\omega_{\tilde{h}}(x,0)$ represents the reliability of the infeasibility of the solution (x) for inequalities (4);

$$\omega_{\tilde{h}}(x,1) = \begin{cases} \underset{j \in \mathcal{N}}{Max}\delta(j); & \forall j \in Arg\underset{j \in \mathcal{N}}{Max}\delta(j): \sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j, \\ (0); & \exists j \in Arg\underset{j \in \mathcal{N}}{Max}\delta(j): \sum_{i \in \mathcal{M}} x_{(ij)} < b_j. \end{cases} \quad (11)$$

Where $\omega_{\tilde{h}}(x,1)$ represents the reliability of its feasibility.

Since inequalities (4) define a set of goods transportation plans in which all demanders take non-zero cooperation. Moreover, the value $\omega_{\tilde{h}}(x,0)$ can be interpreted as the reliability of non-cooperation of all demanders in the goods transportation plan (x), and $\omega_{\tilde{h}}(x,1)$ can be understood as the reliability of their cooperation. From (8) to (11), it follows that the set $\tilde{\mathcal{F}} = \tilde{f} \cap \tilde{h}$ of feasible solutions to a system of (2) until (4) is an FST2. Denote its MF: $\omega_{\tilde{\mathcal{F}}}(x, g)$, $g \in \{0,1\}$. Now, we can use the operation of the intersection of FST2 to obtain the following; $\omega_{\tilde{\mathcal{F}}}(x, g) = \underset{\substack{\kappa,\ell \in \{0,1\}, \\ min\{\kappa,\ell\}=(g)}}{Max} Min\{\omega_{\tilde{f}}(x,\kappa), \omega_{\tilde{h}}(x,\ell)\}$.

From here: $\omega_{\tilde{\mathcal{F}}}(x,0) = \underset{\substack{\kappa,\ell \in \{0,1\}, \\ min\{\kappa,\ell\}=(0)}}{Max} Min\{\omega_{\tilde{f}}(x,\kappa), \omega_{\tilde{h}}(x,\ell)\} = Max\{Min\{\omega_{\tilde{f}}(x,0), \omega_{\tilde{h}}(x,0)\}, Min\{\omega_{\tilde{f}}(x,0), \omega_{\tilde{h}}(x,1)\}, Min\{\omega_{\tilde{f}}(x,1), \omega_{\tilde{h}}(x,0)\}\}$; where $\omega_{\tilde{\mathcal{F}}}(x,0)$ represents the reliability of the infeasibility of the solution (x) for the system of (2) till (4), and $\omega_{\tilde{\mathcal{F}}}(x,1) = \underset{\substack{\kappa,\ell \in \{0,1\}, \\ min\{\kappa,\ell\}=(1)}}{Max} Min\{\omega_{\tilde{f}}(x,\kappa), \omega_{\tilde{h}}(x,\ell)\} = Min\{\omega_{\tilde{f}}(x,1), \omega_{\tilde{h}}(x,1)\}$; that is $\omega_{\tilde{\mathcal{F}}}(x,1)$ represents the reliability of its feasibility. Let us build these functions. To do this, consider the three possible options shown below:

1) Suppose that $\sum_{j \in \mathcal{N}} x_{(ij)} > 0 \ \forall i \in \mathcal{M}$, hence $\omega_{\tilde{f}}(x,0) = (0)$, $\omega_{\tilde{f}}(x,1) = \underset{i \in \mathcal{M}}{Max}\mu(i)$. Therefore $\omega_{\tilde{\mathcal{F}}}(x,0) = Max\{0,0, Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,0)\}\} = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,0)\}$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,1)\}$. The following cases are possible:

a) If $\sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j \ \forall j \in \mathcal{N}$, then $\omega_{\tilde{h}}(x,0) = (0)$, $\omega_{\tilde{h}}(x,1) = \underset{j \in \mathcal{N}}{Max}\delta(j)$, $\omega_{\tilde{\mathcal{F}}}(x,0) = Max\{0,0,0\} = (0)$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \underset{j \in \mathcal{N}}{Max}\delta(j)\}$; (12)

b) If $\sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j \ \forall j \in Arg\underset{j \in \mathcal{N}}{Max}\delta(j) \wedge \exists j \in \mathcal{N}: \sum_{i \in \mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j \in \mathcal{N}}{Max}\{\delta(j) \mid \sum_{i \in \mathcal{M}} x_{(ij)} < b_j\}$, $\omega_{\tilde{h}}(x,1) = \underset{j \in \mathcal{N}}{Max}\delta(j)$, so $\omega_{\tilde{\mathcal{F}}}(x,0) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \underset{j \in \mathcal{N}}{Max}\{\delta(j) \mid \sum_{i \in \mathcal{M}} x_{(ij)} < b_j\}\}$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \underset{j \in \mathcal{N}}{Max}\delta(j)\}$; (13)

c) If $\exists j \in Arg\underset{j \in \mathcal{N}}{Max}\delta(j) \ \sum_{i \in \mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j \in \mathcal{N}}{Max}\delta(j)$, $\omega_{\tilde{h}}(x,1) = 0$. Also, we get $\omega_{\tilde{\mathcal{F}}}(x,0) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \underset{j \in \mathcal{N}}{Max}\delta(j)\}$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), 0\} = 0$. (14)

2) Assume that $\sum_{j \in \mathcal{N}} x_{(ij)} > 0 \ \forall i \in Arg\underset{i \in \mathcal{M}}{Max}\mu(i) \ \wedge \ \exists i \in \mathcal{M} \ \sum_{j \in \mathcal{N}} x_{(ij)} \leq 0$. Either $\omega_{\tilde{f}}(x,0) = \underset{i \in \mathcal{M}}{Max}\{\mu(i) \mid \sum_{j \in \mathcal{N}} x_{(ij)} \leq 0\}$, $\omega_{\tilde{f}}(x,1) = \underset{i \in \mathcal{M}}{Max}\mu(i)$. That is why $\omega_{\tilde{\mathcal{F}}}(x,0) = Max\{Min\{\underset{i \in \mathcal{M}}{Max}\{\mu(i) \mid \sum_{j \in \mathcal{N}} x_{(ij)} \leq 0\}, \omega_{\tilde{h}}(x,0)\}, Min\{\underset{i \in \mathcal{M}}{Max}\{\mu(i) \mid \sum_{j \in \mathcal{N}} x_{(ij)} \leq 0\}, \ Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,0)\}\}$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,1)\}$. The following cases are possible:

a) If $\sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j \ \forall j \in \mathcal{N}$, then $\omega_{\tilde{h}}(x,0) = 0$, $\omega_{\tilde{h}}(x,1) = \underset{j \in \mathcal{N}}{Max}\delta(j)$, $\omega_{\tilde{\mathcal{F}}}(x,0) = Min\{\underset{i \in \mathcal{M}}{Max}\{\mu(i) \mid \sum_{j \in \mathcal{N}} x_{(ij)} \leq 0\}, \underset{j \in \mathcal{N}}{Max}\delta(j)\}$, $\omega_{\tilde{\mathcal{F}}}(x,1) = Min\{\underset{i \in \mathcal{M}}{Max}\mu(i), \underset{j \in \mathcal{N}}{Max}\delta(j)\}$. (15)

b) If $\sum_{i \in \mathcal{M}} x_{(ij)} \geq b_j \ \forall j \in Arg\underset{j \in \mathcal{N}}{Max}\delta(j) \ \wedge \ \exists j \in \mathcal{N}: \sum_{i \in \mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j \in \mathcal{N}}{Max}\{\delta(j) \mid \sum_{i \in \mathcal{M}} x_{(ij)} < b_j\}$, $\omega_{\tilde{h}}(x,1) = \underset{j \in \mathcal{N}}{Max}\delta(j)$. Therefore, we get $\omega_{\tilde{\mathcal{F}}}(x,0) = Max\{Min\{\underset{i \in \mathcal{M}}{Max}\{\mu(i) \mid \sum_{j \in \mathcal{N}} x_{(ij)} \leq$

$0\}, \underset{j\in\mathcal{N}}{Max}\{\delta(j)|\sum_{i\in\mathcal{M}} x_{(ij)} b_j\}\}, Min\{\underset{i\in\mathcal{M}}{Max}\{\mu(i)|\sum_{j\in\mathcal{N}} x_{(ij)} \leq$ $0\}, \underset{j\in\mathcal{N}}{Max}\delta(j)\}, Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\{\delta(j)|\sum_{i\in\mathcal{M}} x_{(ij)} b_j\}\}\}, \omega_{\tilde{F}}(x,1)\ Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\};$(16)

c) If $\exists j \in Arg\underset{j\in\mathcal{N}}{Max}\delta(j): \sum_{i\in\mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j\in\mathcal{N}}{Max}\delta(j)$, $\omega_{\tilde{h}}(x,1) = 0$, $\omega_{\tilde{F}}(x,0) = Max\{Min\{\underset{i\in\mathcal{M}}{Max}\{\mu(i)|\sum_{j\in\mathcal{N}} x_{(ij)} \leq 0\}, \underset{j\in\mathcal{N}}{Max}\delta(j)\}, 0, Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}\} = Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}, \omega_{\tilde{F}}(x,1) = Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), 0\} = 0.$ (17)

3) Let $\exists i \in Arg\underset{i\in\mathcal{M}}{Max}\mu(i): \sum_{j\in\mathcal{N}} x_{(ij)} \leq 0$. Then $\omega_{\tilde{f}}(x,0) = \underset{i\in\mathcal{M}}{Max}\mu(i)$, $\omega_{\tilde{f}}(x,1) = 0$. Therefore $\omega_{\tilde{F}}(x,0) = max\{min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,0)\}, Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \omega_{\tilde{h}}(x,1)\}, 0\}$; $\omega_{\tilde{F}}(x,1) = Min\{\omega_{\tilde{f}}(x,1), \omega_{\tilde{h}}(x,1)\} = 0$. (18) There are the following cases:

a) If $\sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j\ \forall j \in \mathcal{N}$, then $\omega_{\tilde{h}}(x,0) = 0$, $\omega_{\tilde{h}}(x,1) = \underset{j\in\mathcal{N}}{Max}\delta(j)$, that's why $\omega_{\tilde{F}}(x,0) = Max\{0, Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}, 0\} = Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}$; (19)

b) If $\sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j\ \forall j \in Arg\underset{j\in\mathcal{N}}{Max}\delta(j)$ and $\exists j \in \mathcal{N}: \sum_{i\in\mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j\in\mathcal{N}}{Max}\{\delta(j)|\sum_{i\in\mathcal{M}} x_{(ij)} < b_j\}$, $\omega_{\tilde{h}}(x,1) = \underset{j\in\mathcal{N}}{Max}\delta(j)$, so $\omega_{\tilde{F}}(x,0) = Max\{Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\{\delta(j)|\sum_{i\in\mathcal{M}} x_{(ij)} < b_j\}\}, Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}, 0\} = Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}.$ (20)

c) If $\exists j \in Arg\underset{j\in\mathcal{N}}{Max}\delta(j): \sum_{i\in\mathcal{M}} x_{(ij)} < b_j$, then $\omega_{\tilde{h}}(x,0) = \underset{j\in\mathcal{N}}{Max}\delta(j)$, $\omega_{\tilde{h}}(x,1) = 0$. Additionally we get $\omega_{\tilde{F}}(x,0) = Max\{Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}, 0, 0\} = Min\{\underset{i\in\mathcal{M}}{Max}\mu(i), \underset{j\in\mathcal{N}}{Max}\delta(j)\}.$ (21)

4. Find a balanced solution for TP with FST2 of supply & demand

When searching for a Balanced Solution (BS), the DM will try to minimize the objective function (1) as well as maximize the reliability of objective functions (2) and (3), respectively, of non-participation and participation of supply and demand in terms of goods transportation. In other words, the DM faces the following multi-objective programming problem:

$$Objective\ function\ (1): (MinTC) \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)};$$

$$Objective\ function\ (2): (Max\mathcal{R})\ \omega_{\tilde{F}}\ (x,0);$$

$$Objective\ function\ (3): (Max\mathcal{R})\ \omega_{\tilde{F}}\ (x,1);$$

$$s.to: x \in \mathcal{D}. \qquad (22)$$

Let $(WNDO)$ denote the set of Weakly Non-Dominated Optimal solutions to this problem. Recall that a solution $x^* \geq 0$ is called Slater's optimal solution for a problem of the form (22) if $\nexists x \geq 0$, for which the following inequalities hold:

- $\sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)} > \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x^*_{(ij)}$,
- $\omega_{\tilde{F}}(x,0) > \omega_{\tilde{F}}(x^*,0)$,
- $\omega_{\tilde{F}}(x,1) > \omega_{\tilde{F}}(x^*,1)$.

It is pretty clear that the definition of a BS to problem for (1) to (4) should include only solutions from the set of $WNDO$. These considerations lead to the following definition.

The general BS to the TP from (1) until (4), with fuzzy sets of supply and demand, will be an FST2 $\tilde{\mathcal{D}}$ with the MF: $\omega_{\tilde{\mathcal{D}}}(x,g) = \omega_{\tilde{\mathcal{D}}}(x,g), x \in WNDO: g \in \mathcal{G} = \{0,1\} \wedge \omega_{\tilde{\mathcal{D}}}(x,g) = 0, x \notin WNDO$.

When the DM is interested in a specific BS x^*, it can be selected from the set of $WNDO$ using one or another method of multi-objective optimization by solving a problem (22). Then we will call it a BS to the TP for (1) till (4) with certainties $\omega_{\tilde{f}}(x,0)$ and $\omega_{\tilde{f}}(x,1)$, respectively, of non-participation and participation of supply in the plan of goods transportation. Let us denote the functions:

$$\mathcal{R}^{\{demand\}}(x) = \begin{cases} \underset{j\in\mathcal{N}}{Max}\{\delta(j)|\sum_{i\in\mathcal{M}} x_{(ij)} \leq 0\}; & \exists j \in \mathcal{N}: \sum_{i\in\mathcal{M}} x_{(ij)} \leq 0, \\ (0); & \forall j \in \mathcal{N}: \sum_{i\in\mathcal{M}} x_{(ij)} > 0. \end{cases} \qquad (23)$$

$$\mathcal{R}^{\{supply\}}(x) = \begin{cases} \underset{i\in\mathcal{M}}{Max}\{\mu(i)|\sum_{j\in\mathcal{N}} x_{(ij)} < b_j\}; & \exists i \in \mathcal{M}\colon \sum_{j\in\mathcal{N}} x_{(ij)} < b_j, \\ (0); & \forall i \in \mathcal{M}\colon \sum_{j\in\mathcal{N}} x_{(ij)} \geq b_j. \end{cases} \quad (24)$$

We can allow denoting that $\{\mathcal{I}^*\} = Arg\underset{i\in\mathcal{M}}{Max}\mu(i) \wedge \{\mathcal{J}^*\} = Arg\underset{j\in\mathcal{N}}{Max}\delta(j)$. Multi-objective programming problem (22) can be simplified if and only if the MFs $\mu(i)$ of the fuzzy set of indices $\widetilde{\mathcal{M}} \subseteq \mathcal{M}$ of suppliers intending to release goods and $\delta(j)$ of the fuzzy set of indices $\widetilde{\mathcal{N}} \subseteq \mathcal{N}$ of consumers ready to receive goods, are standard, i.e., $\underset{i\in\mathcal{M}}{Max}\mu(i) = 1$ and $\underset{j\in\mathcal{N}}{Max}\delta(j) = 1$, then for each given value of the parameter $\mathcal{R} \in (0,1]$, at which the problem (25) and (26) has an optimal solution, this solution will be balanced for the TP of (1) until (4) with the reliability of the participation of supply in the plan of transportation of goods equal to (one), and the reliability of their non-participation is not less than $\mathcal{R}$, i.e.,

$(MinTC)\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} c_{ij}x_{(ij)}, (Max)\{\mathcal{R}^{\{demand\}}(x), \mathcal{R}^{\{supply\}}(x)\} \geq \mathcal{R}, x \in \mathcal{D}$, (25)

$\sum_{j\in\mathcal{N}} x_{(ij)} > 0 \;\; \forall i \in \mathcal{I}^*, \; \sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j \;\; \forall j \in \mathcal{J}^*.$ (26)

Let be now see how we can simplify the solution to the problem (25) and (26). Indicate that $\mathcal{M}^{\mathcal{R}} = \{i \in \mathcal{M}|\mu(i) \geq \mathcal{R}\}$ and $\mathcal{N}^{\mathcal{R}} = \{j \in \mathcal{N}|\delta(j) \geq \mathcal{R}\}$ sets of supply and demand indices, respectively, have degrees of membership in the corresponding fuzzy sets $\widetilde{\mathcal{M}}$ and $\widetilde{\mathcal{N}}$ of at least $\mathcal{R} \in (0,1]$. Thus, the problem (25) and (26) can be written as follows:

$$\underset{\{v\in\mathcal{M}^{\mathcal{R}}, w\in\mathcal{N}^{\mathcal{R}}\}}{Min}(MinTC)\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} c_{ij}x_{(ij)}; \; x \in \mathcal{D};$$

$$\sum_{j\in\mathcal{N}} x_{(vj)} \leq 0, \; \sum_{i\in\mathcal{M}} x_{(iw)} < b_j; \quad (27)$$

$$\sum_{j\in\mathcal{N}} x_{(vj)} > 0 \;\; \forall i \in \mathcal{I}^*, \; \sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j \;\; \forall j \in \mathcal{J}^*. \quad (28)$$

Since $\underset{i\in\mathcal{M}}{Max}\mu(i) = 1$ and $\underset{j\in\mathcal{N}}{Max}\delta(j) = 1$, it is evident that for $(\mathcal{R} = 1) \; \forall \; v \in \mathcal{I}^*, w \in \mathcal{J}^*$, constraints (27) and (28) will be a priori inconsistent. Therefore, we can obtain the following final result from the theorem above.

I. Signify by $\bar{\mathcal{M}}^{\mathcal{R}} = \{i \in \mathcal{M}|1 > \mu(i) \geq \mathcal{R}\}$ and $\bar{N}^{\mathcal{R}} = \{j \in \mathcal{N}|1 > \delta(j) \geq \mathcal{R}\}$ the sets of supply and demand indices, which have a certainty of membership degrees, not less than $\mathcal{R} \in (0,1)$, but not equal to one.

II. If the MFs of $\mu(i)$, the fuzzy set of indices $\widetilde{\mathcal{M}} \subseteq \mathcal{M}$ of supply who intend to release goods, and $\delta(j)$, the fuzzy set of indices $\widetilde{\mathcal{N}} \subseteq \mathcal{N}$ of demand ready to receive goods, are typical. Then for each given value of the parameter $\mathcal{R} \in (0,1)$, for which the problem has an optimal solution. It will be BS for TP from (1) to (4), with the reliability of the participation of supply in the plan for transportation goods equal to one. Furthermore, the reliability of their non-participation is at least $\mathcal{R}$.

$$\underset{\{v\in\bar{\mathcal{M}}^{\mathcal{R}}, w\in\bar{\mathcal{N}}^{\mathcal{R}}\}}{Min}(MinTC)\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} c_{ij}x_{(ij)}; x \in \mathcal{D};$$

$$\sum_{j\in\mathcal{N}} x_{(vj)} \leq 0; \sum_{j\in\mathcal{N}} x_{(ij)} > 0; \sum_{j\in\mathcal{N}} x_{(ij)} \leq a_i \; \forall i \in \mathcal{I}^*; \quad (29)$$

$$\sum_{i\in\mathcal{M}} x_{(iw)} < b_w; \sum_{i\in\mathcal{M}} x_{(ij)} \geq b_j \; \forall j \in \mathcal{J}^*; x_{(ij)} \geq 0, i \in \mathcal{M}, j \in \mathcal{N}. \quad (30)$$

Based on what was stated in the previous theorem and its subsequent properties. It is now possible to propose a new method consisting of five steps, to obtain the optimal BS of a TP with a fuzzy set of supply and demand indices that finally satisfies the DM, as displayed below:

1) Choose the number of $\mathcal{R}^{\{max\}} \in (0,1)$, which according to (I) and (II), characterizes the maximum reliability for the DM of the infeasibility of the goods transportation plan.

2) Compose a set of supply $\mathcal{M}^{\mathcal{R}} = \{(i) \in \mathcal{I}|\mu(i) \leq \mathcal{R}\}$, which have a membership degree of the fuzzy set of supply indices not greater than $\mathcal{R}^{\{max\}} \in (0,1)$.

3) For indices, $\mathcal{M}^{\mathcal{R}} = \{(j) \in \mathcal{J}|\delta(j) \leq \mathcal{R}\}$ constructs the set of supply with a degree of membership to the fuzzy set of supply indices not greater than $\mathcal{R}^{\{max\}} \in (0,1)$.

4) Minimizing the total cost of transportation $z(x) = \sum_{i\in\mathcal{M}^{\{supply\}}}\sum_{j\in\mathcal{N}^{\{demand\}}} c_{ij}x_{(ij)}$ for each supply and demand with index $(k,l) \in \mathcal{M}^{\mathcal{R}}$ on the set of feasible transportation plans $\Psi^{\{supply,demand\}}$ under additional

constraints: $\sum_{j\in\mathcal{N}_i} x_{(ij)} > 0 \ \forall i \in \{\mathcal{I}^*\backslash\mathcal{M}^{\mathcal{R}}\}$ and $\sum_{i\in\mathcal{M}_j} x_{(ij)} \geq b_j \ \forall j \in \{\mathcal{J}^*\backslash\mathcal{M}^{\mathcal{R}}\}$ on supply who release goods and on active demand respectively. Moreover, two additional constraints: $\sum_{j\in\mathcal{N}_k} x^l_{(kj)} \leq 0$ and $\sum_{i\in\mathcal{M}_k} x^l_{(ij)} < b^l_k$, which determines the zero goods of products from the supply and associated with an inactive demand respectively, with index (k,l), i.e., to solve the problem:

$$\left(\underset{\psi(supply,demand)}{Minimization}\right) z(x) = \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} c_{ij} x_{(ij)};$$

$$\sum_{j\in\mathcal{N}} x_{(ij)} \leq a_i,\ i \in \mathcal{M};\ \sum_{j\in\mathcal{N}_k} x^l_{(kj)} \leq 0;\ \sum_{j\in\mathcal{N}_i} x_{(ij)} > 0 \ \ \forall i \in \{\mathcal{I}^*\backslash\mathcal{M}^{\mathcal{R}}\}; \quad (31)$$

$$\sum_{i\in\mathcal{M}_k} x^l_{(ij)} < b^l_k;\ \sum_{i\in\mathcal{M}_j} x_{(ij)} \geq b_j \ \forall j \in \{\mathcal{J}^*\backslash\mathcal{M}^{\mathcal{R}}\};\ x_{(ij)} \geq 0,\ i \in \mathcal{M},\ j \in \mathcal{N}. \quad (32)$$

(We indicate its solution by $x^{(k,l)}$).

5) From the obtained solutions $x^{(k,l)}, \therefore (k,l) \in \mathcal{M}^{\mathcal{R}}$, choose $(\bar{x})$ the record one in terms of the value of the objective function, i.e., $\bar{x} = Arg \underset{\{(k,l)\in\mathcal{M}^{\mathcal{R}}\}}{\text{Min}} z\left(x^{(k,l)}\right)$.

5. Illustrative numerical example:

Consider a single-product TP as described below:

$$\left.\begin{array}{l}(MinTC)\, z\,(x_{11},\dots,x_{67}) = \\ \quad 5x_{(1,1)} + 8x_{(1,2)} + 7x_{(1,3)} + 3x_{(1,4)} + 3x_{(1,5)} + 9x_{(1,6)} + 5x_{(1,7)} + \\ +8x_{(2,1)} + 4x_{(2,2)} + 2x_{(2,3)} + 8x_{(2,4)} + 7x_{(2,5)} + 10x_{(2,6)} + 16x_{(2,7)} + \\ +10x_{(3,1)} + 2x_{(3,2)} + 6x_{(3,3)} + 3x_{(3,4)} + 9x_{(3,5)} + 10x_{(3,6)} + 18x_{(3,7)} + \\ +6x_{(4,1)} + 6x_{(4,2)} + 2x_{(4,3)} + 9x_{(4,4)} + 12x_{(4,5)} + 8x_{(4,6)} + 10x_{(4,7)} + \\ +3x_{(5,1)} + 6x_{(5,2)} + 5x_{(5,3)} + 7x_{(5,4)} + 4x_{(5,5)} + 10x_{(5,6)} + 8x_{(5,7)} + \\ +9x_{(6,1)} + 8x_{(6,2)} + 10x_{(6,3)} + 6x_{(6,4)} + 14x_{(6,5)} + 6x_{(6,6)} + 5x_{(6,7)}\end{array}\right\} (1)$$

Subject to constraints of supply:

$$\left.\begin{array}{l}x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 18000, \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \leq 10000, \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \leq 6000, \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} \leq 12000, \\ x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} \leq 8000, \\ x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} \leq 6000;\end{array}\right\} (2)$$

Subject to constraints of demand:

$$\left.\begin{array}{l}x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} \geq 11000, \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} \geq 12000, \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} \geq 8000, \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} \geq 10000, \\ x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} \geq 7000, \\ x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} \geq 15000, \\ x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} \geq 5000;\end{array}\right\} (3)$$

Non-negativity of variables: $x_{(ij)} \geq 0;\ (i = 1,2,\cdots,6);\ (j = 1,2,\cdots,7)$.

Let us denote: $\mathcal{I} = \{1,2,3,4,5,6\}$ is the set of supply, and $\mathcal{J} = \{1,2,3,4,5,6,7\}$ represents the demand set. Suppose that the DM cannot clearly say which supply will exactly release products but can only set a fuzzy set $\{\tilde{\mathcal{I}}\}$ with MFs: $\mu_{(1)} = 0.65$; $\mu_{(2)} = 0.90$; $\mu_{(3)} = 0.35$; $\mu_{(4)} = 1.0$; $\mu_{(5)} = 0.70$; $\mu_{(6)} = 0.40$. Furthermore, suppose that the DM cannot clearly say which demand will be precisely active (accept products in the stated amount) but can only set a fuzzy set $\{\tilde{\mathcal{J}}\}$ with MFs: $\delta_{(1)} = 0.60$; $\delta_{(2)} = 0.80$; $\delta_{(3)} = 0.75$; $\delta_{(4)} = 1.0$; $\delta_{(5)} = 0.90$; $\delta_{(6)} = 0.85$; $\delta_{(7)} = 1.0$. Let's now perform the procedure of choosing a BS.

1. Choosing the maximum reliability for the DM of the infeasibility of the transportation plan, for example: $\mathcal{R}^{\{max\}} = \left(\frac{1}{2}\right)$.

2. Then the set of indices of supply and demand, which have a degree of membership in the fuzzy set $\{\tilde{\mathcal{I}}\}$ and $\{\tilde{\mathcal{J}}\}$ not more than $\mathcal{R}^{\{max\}} = \left(\frac{1}{2}\right)$, will take the form $\mathcal{M}^{\mathcal{R}(ij)} = \{(i) \in \mathcal{I} \,\middle|\, \mu_{(i)} \leq \left(\frac{1}{2}\right)\} = \{1,3,5,6\} \wedge \{(j) \in \mathcal{J} \,\middle|\, \delta_{(j)} \leq \left(\frac{1}{2}\right)\} = \{1\}$.

a. For $\{(i) = 1; (j) = 1\}$ we solve the TP as follows:

$$\left.\begin{aligned}&(MinTC)\, z\,(x_{11},\dots,x_{67}) = 5x_{(1,1)} + 8x_{(1,2)} + 7x_{(1,3)} + 3x_{(1,4)} + 3x_{(1,5)} + 9x_{(1,6)} + 5x_{(1,7)} + \\ &\qquad + 8x_{(2,1)} + 4x_{(2,2)} + 2x_{(2,3)} + 8x_{(2,4)} + 7x_{(2,5)} + 10x_{(2,6)} + 16x_{(2,7)} + \\ &\qquad + 10x_{(3,1)} + 2x_{(3,2)} + 6x_{(3,3)} + 3x_{(3,4)} + 9x_{(3,5)} + 10x_{(3,6)} + 18x_{(3,7)} + \\ &+6x_{(4,1)} + 6x_{(4,2)} + 2x_{(4,3)} + 9x_{(4,4)} + 12x_{(4,5)} + 8x_{(4,6)} + 10x_{(4,7)} + \\ &+3x_{(5,1)} + 6x_{(5,2)} + 5x_{(5,3)} + 7x_{(5,4)} + 4x_{(5,5)} + 10x_{(5,6)} + 8x_{(5,7)} + \\ &+9x_{(6,1)} + 8x_{(6,2)} + 10x_{(6,3)} + 6x_{(6,4)} + 14x_{(6,5)} + 6x_{(6,6)} + 5x_{(6,7)}\end{aligned}\right\}$$

Subject to constraints of supply:

$$\left.\begin{aligned}&x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 18000,\\ &x_{(2,1)} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \leq 10000,\\ &x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \leq 6000,\\ &x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} \leq 12000,\\ &x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} \leq 8000,\\ &x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} \leq 6000,\\ &x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} > 0.000,\\ &x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} > 0.000,\\ &x_{(1,1)} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 0.000;\end{aligned}\right\} (4)$$

Subject to constraints of demand:

$$\left.\begin{aligned}&x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} \geq 12000,\\ &x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} \geq 8000,\\ &x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} \geq 10000,\\ &x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} \geq 7000,\\ &x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} \geq 15000,\\ &x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} \geq 5000,\\ &x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} < 11000;\end{aligned}\right\} (5)$$

Subject to Non-negativity of variables: $x_{(ij)} \geq 0;\ (i = 1,2,\cdots,6);\ (j = 1,2,\cdots,7)$.

The optimal solution of this problem: $x_{(2,2)}^{\{i=1,j=1\}} = 10000, x_{(3,4)}^{\{1,1\}} = 6000, x_{(4,2)}^{\{1,1\}} = 2000, x_{(4,6)}^{\{1,1\}} = 4000, x_{(5,4)}^{\{1,1\}} = 1000, x_{(5,5)}^{\{1,1\}} = 7000, x_{(4,6)}^{\{1,1\}} = 3000, x_{(6,6)}^{\{1,1\}} = 11000$, all other variables have the value of zero. The optimal value of the objective function: $z_{(MinTC)}x^{\{i=1,j=1\}} = 221000$.

b. For $\{(i) = 3; (j) = 1\}$ we solve the TP as follows:

$$\left.\begin{aligned}&(MinTC)\, z\,(x_{11},\dots,x_{67}) = 5x_{(1,1)} + 8x_{(1,2)} + 7x_{(1,3)} + 3x_{(1,4)} + 3x_{(1,5)} + 9x_{(1,6)} + 5x_{(1,7)} + \\ &\qquad + 8x_{(2,1)} + 4x_{(2,2)} + 2x_{(2,3)} + 8x_{(2,4)} + 7x_{(2,5)} + 10x_{(2,6)} + 16x_{(2,7)} + \\ &\qquad + 10x_{(3,1)} + 2x_{(3,2)} + 6x_{(3,3)} + 3x_{(3,4)} + 9x_{(3,5)} + 10x_{(3,6)} + 18x_{(3,7)} + \\ &+6x_{(4,1)} + 6x_{(4,2)} + 2x_{(4,3)} + 9x_{(4,4)} + 12x_{(4,5)} + 8x_{(4,6)} + 10x_{(4,7)} + \\ &+3x_{(5,1)} + 6x_{(5,2)} + 5x_{(5,3)} + 7x_{(5,4)} + 4x_{(5,5)} + 10x_{(5,6)} + 8x_{(5,7)} + \\ &+9x_{(6,1)} + 8x_{(6,2)} + 10x_{(6,3)} + 6x_{(6,4)} + 14x_{(6,5)} + 6x_{(6,6)} + 5x_{(6,7)}\end{aligned}\right\}$$

Subject to constraints of supply:

$$\left.\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17} \leq 18000, \\ x_{(2,1)}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27} \leq 10000, \\ x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37} \leq 6000, \\ x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47} \leq 12000, \\ x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57} \leq 8000, \\ x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{66}+x_{67} \leq 6000, \\ x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27} > 0.000, \\ x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47} > 0.000, \\ x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37} \leq 0.000; \end{array}\right\} (6)$$

Subject to constraints of demand:

$$\left.\begin{array}{l} x_{12}+x_{22}+x_{32}+x_{42}+x_{52}+x_{62} \geq 12000, \\ x_{13}+x_{23}+x_{33}+x_{43}+x_{53}+x_{63} \geq 8000, \\ x_{14}+x_{24}+x_{34}+x_{44}+x_{54}+x_{64} \geq 10000, \\ x_{15}+x_{25}+x_{35}+x_{45}+x_{55}+x_{65} \geq 7000, \\ x_{16}+x_{26}+x_{36}+x_{46}+x_{56}+x_{66} \geq 15000, \\ x_{17}+x_{27}+x_{37}+x_{47}+x_{57}+x_{67} \geq 5000, \\ x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61} < 11000; \end{array}\right\} (5)$$

Subject to Non-negativity of variables: $x_{(ij)} \geq 0;\ (i = 1,2,\cdots,6);\ (j = 1,2,\cdots,7)$.

The optimal solution to this problem: $x_{(1,4)}^{\{i=3,j=1\}} = 10000, x_{(1,5)}^{\{3,1\}} = 7000, x_{(2,2)}^{\{3,1\}} = 10000, x_{(4,2)}^{\{3,1\}} = 2000, x_{(4,6)}^{\{3,1\}} = 1000, x_{(6,6)}^{\{3,1\}} = 14000$, all other variables have a value of zero. The optimal value of the objective function: $z_{(MinTC)}x^{\{i=3,j=1\}} = 195000$.

c. For $\{(i) = 5; (j) = 1\}$ we solve the TP as follows:

$$\left.\begin{array}{l} (MinTC)\ z\,(x_{11},\dots,x_{67}) = 5x_{(1,1)} + 8x_{(1,2)} + 7x_{(1,3)} + 3x_{(1,4)} + 3x_{(1,5)} + 9x_{(1,6)} + 5x_{(1,7)} + \\ \quad + 8x_{(2,1)} + 4x_{(2,2)} + 2x_{(2,3)} + 8x_{(2,4)} + 7x_{(2,5)} + 10x_{(2,6)} + 16x_{(2,7)} + \\ \quad + 10x_{(3,1)} + 2x_{(3,2)} + 6x_{(3,3)} + 3x_{(3,4)} + 9x_{(3,5)} + 10x_{(3,6)} + 18x_{(3,7)} + \\ +6x_{(4,1)} + 6x_{(4,2)} + 2x_{(4,3)} + 9x_{(4,4)} + 12x_{(4,5)} + 8x_{(4,6)} + 10x_{(4,7)} + \\ +3x_{(5,1)} + 6x_{(5,2)} + 5x_{(5,3)} + 7x_{(5,4)} + 4x_{(5,5)} + 10x_{(5,6)} + 8x_{(5,7)} + \\ +9x_{(6,1)} + 8x_{(6,2)} + 10x_{(6,3)} + 6x_{(6,4)} + 14x_{(6,5)} + 6x_{(6,6)} + 5x_{(6,7)} \end{array}\right\}$$

Subject to constraints of supply:

$$\left.\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17} \leq 18000, \\ x_{(2,1)}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27} \leq 10000, \\ x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37} \leq 6000, \\ x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47} \leq 12000, \\ x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57} \leq 8000, \\ x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{66}+x_{67} \leq 6000, \\ x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27} > 0.000, \\ x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47} > 0.000, \\ x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57} \leq 0.000; \end{array}\right\} (7)$$

Subject to constraints of demand:

$$\left.\begin{array}{l} x_{12}+x_{22}+x_{32}+x_{42}+x_{52}+x_{62} \geq 12000, \\ x_{13}+x_{23}+x_{33}+x_{43}+x_{53}+x_{63} \geq 8000, \\ x_{14}+x_{24}+x_{34}+x_{44}+x_{54}+x_{64} \geq 10000, \\ x_{15}+x_{25}+x_{35}+x_{45}+x_{55}+x_{65} \geq 7000, \\ x_{16}+x_{26}+x_{36}+x_{46}+x_{56}+x_{66} \geq 15000, \\ x_{17}+x_{27}+x_{37}+x_{47}+x_{57}+x_{67} \geq 5000, \\ x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61} < 11000; \end{array}\right\} (5)$$

Subject to Non-negativity of variables: $x_{(ij)} \geq 0;\ (i = 1,2,\cdots,6);\ (j = 1,2,\cdots,7)$.

The optimal solution for this problem is: $x_{(1,4)}^{\{i=5,j=1\}} = 10000, x_{(1,5)}^{\{5,1\}} = 7000, x_{(2,2)}^{\{5,1\}} = 6000, x_{(3,2)}^{\{5,1\}} = 6000,\ x_{(4,6)}^{\{5,1\}} = 1000, x_{(6,6)}^{\{5,1\}} = 14000$, all other variables have zero value, and the optimal value of the objective function is: $z_{(MinTC)}x^{\{i=5,j=1\}} = 179000$.

d. For $\{(i) = 6; (j) = 1\}$ we solve the TP as follows:

$$\left.\begin{aligned}(MinTC)\ z\,(x_{11},\dots,x_{67}) = {}& 5x_{(1,1)} + 8x_{(1,2)} + 7x_{(1,3)} + 3x_{(1,4)} + 3x_{(1,5)} + 9x_{(1,6)} + 5x_{(1,7)} + \\ & + 8x_{(2,1)} + 4x_{(2,2)} + 2x_{(2,3)} + 8x_{(2,4)} + 7x_{(2,5)} + 10x_{(2,6)} + 16x_{(2,7)} + \\ & + 10x_{(3,1)} + 2x_{(3,2)} + 6x_{(3,3)} + 3x_{(3,4)} + 9x_{(3,5)} + 10x_{(3,6)} + 18x_{(3,7)} + \\ & +6x_{(4,1)} + 6x_{(4,2)} + 2x_{(4,3)} + 9x_{(4,4)} + 12x_{(4,5)} + 8x_{(4,6)} + 10x_{(4,7)} + \\ & +3x_{(5,1)} + 6x_{(5,2)} + 5x_{(5,3)} + 7x_{(5,4)} + 4x_{(5,5)} + 10x_{(5,6)} + 8x_{(5,7)} + \\ & +9x_{(6,1)} + 8x_{(6,2)} + 10x_{(6,3)} + 6x_{(6,4)} + 14x_{(6,5)} + 6x_{(6,6)} + 5x_{(6,7)}\end{aligned}\right\}$$

Subject to constraints of supply:

$$\left.\begin{array}{l} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \le 18000, \\ x_{(2,1)} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \le 10000, \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} \le 6000, \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} \le 12000, \\ x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} \le 8000, \\ x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} \le 6000, \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} > 0.000, \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} > 0.000, \\ x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} \le 0.000; \end{array}\right\} (8)$$

Subject to constraints of demand:

$$\left.\begin{array}{l} x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} \ge 12000, \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} \ge 8000, \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} \ge 10000, \\ x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} \ge 7000, \\ x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} \ge 15000, \\ x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} \ge 5000, \\ x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} < 11000; \end{array}\right\} (5)$$

Subject to Non-negativity of variables: $x_{(ij)} \ge 0;\ (i = 1,2,\cdots,6);\ (j = 1,2,\cdots,7)$.

RESULTS

The optimal solution of this problem: $x_{(1,4)}^{\{i=6,j=1\}} = 10000, x_{(1,5)}^{\{6,1\}} = 7000, x_{(1,6)}^{\{6,1\}} = 1000, x_{(2,2)}^{\{6,1\}} = 6000,\ x_{(2,6)}^{\{6,1\}} = 2000, x_{(3,2)}^{\{6,1\}} = 6000, x_{(4,6)}^{\{6,1\}} = 12000$, all other variables have a value of zero. The optimal value of the objective function is: $z_{(MinTC)}x^{\{i=6,j=1\}} = 212000$. Since $x^{\{i^*=6,j^*=1\}}$ has the smallest value of the objective function: $z^*_{(MinTC)}x^{\{i^*=6,j^*=1\}} = 212000$, it will be the solution to the problem with the reliability of the feasibility of the obtained solution equal to one, and the reliability of infeasibility is not more than $\left(\frac{1}{2}\right)$.

CONCLUSION

In conclusion, it should be noted that the proposed method extends the scope of fuzzy mathematical programming to the case of a transportation problem of linear programming with a fuzzy set of type-2 for supply and demand. The new method showed its effectiveness in optimal decision-making by obtaining a balanced solution with a fuzzy environment for supply and demand indexes for the transportation problem. Furthermore, it can provide a new approach to solving other optimization problem formulations under fuzzy information.

REFERENCES

[1] Abhijit Saha, Dragan Pamucar, Omer F. Gorcun, Arunodaya Raj Mishra, Warehouse site selection for the automotive industry using a fermatean fuzzy-based decision-making approach, Expert Systems with Applications, Volume 211, 2023, 118497, https://doi.org/10.1016/j.eswa.2022.118497.

[2] Das, A., Bera, U.K., Maiti, M.: A solid transportation problem in an uncertain environment involving a type-2 fuzzy variable. Neural Comput. Appl. 31(9), 4903–4927 (2019).

[3] El Sayed, M.A., Abo-Sinna, M.A.: A novel approach for fully intuitionistic fuzzy multi-objective fractional transportation problem. Alex. Eng. J. 60(1), 1447–1463 (2021).

[4] Gupta, S., Ali, I., Ahmed, A.: Multi-choice multi-objective capacitated transportation problem: a case study of uncertain demand and supply. J. Stat. Manag. Syst. 21(3), 467–491 (2018).

[5] Gładysz, B. (2022). Transportation Problem with Fuzzy Unit Costs. Z-fuzzy Numbers Approach. In: Nguyen, N.T., Kowalczyk, R., Mercik, J., Motylska-Kuźma, A. (eds) Transactions on Computational Collective Intelligence. Lecture Notes in Computer Science, vol 13750. Springer, Berlin, heidelberg. https://doi.org/10.1007/978-3-662-66597-8_6.

[6] Hitchcock, F.: The distribution of a product from several sources to numerous localities. Int. J. Pharm. Technol. 8(1), 3554–3570 (2016).

[7] Mashchenko S.O., Mathematical programming problem with a fuzzy set of constraint indices// Cybernetics and System Analysis. - 2013. - No. 1. - P. (73 – 81).

[8] Midya, S., Roy, S.K.: Multi-objective fixed-charge transportation problem using rough programming. Int. J. Oper. Res. 37(3), 377–395 (2020).

[9] Orlovsky S.A., Decision-making problems with fuzzy initial information. -M.: Science. Main edition of physical and mathematical literature, 1981.

[10] Palanievel, M., Suganya, M.: A new method to solve transportation problem-Harmonic Mean approach. Eng. Technol. Open Access J. 2, 1–3 (2018).

[11] Raskin L.G., Kirichenko I.O., Multi-index problems of linear programming. Moscow: Radio and communication, 1982.

[12] Senapati P., Kumar R.T., Multi-objective transportation model into Fuzzy parameters: Priority based Fuzzy Goal programming approach. Journal of Transportation systems Engineering and Information Technology. – 2008. – 8. – P. (40–48).

[13] Shalabh Singh (2023) Optimizing time–cost trade-off decisions in an interval transportation problem with multiple shipment options, Engineering Optimization, 55:1, 53-70, DOI: 10.1080/0305215X.2021.1982931 .

[14] Zinmenoz F., Vudegay J.L., Solving the fuzzy solid transportation problem by an evolutionary algorithm based parametric approach// European Journal of Operations Research. –1999. - 117. - P. (485-510).