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Research Article

Analysis of Queuing Model having Multiple Servers with Environment Effects and with Retention of Impatient Consumers

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ABSTRACT

Through this paper, an innumerable definite latent alike queuing model with clients' restiveness and the action of absorbing of annoyed clients in two different environments is analyzed. The findings of the model in stable position are attained by Matrix Approach.

Keywords: Steady state solution, Environment, Retention, Reneging.

INTRODUCTION

1.1 Background of Research A queuing model is a mathematical model that describes the behavior of queues or waiting lines. Queues are common in various real-world scenarios, such as computer systems, telecommunications, transportation, manufacturing, and service industries. The primary purpose of queuing theory is to analyze and optimize the performance. Numerous researchers are interested in this topic. Many additions have been made to the fundamental queuing models, giving rise to new ideas such as vacation-queuing, correlated queuing, retrial queuing, impatient queuing. Of these, impatient customers' queuing is especially important to the business sector because it severely hinders a company's ability to generate income. When a consumer joins the line just when he anticipates a little wait and stays in line if his wait is short enough, the customer is considered impatient. Three common varieties of impatience exist. Three behaviors are evident in the first instance: balking, which involves arrival of customer in a queue but decides not to join; reneging, which involves joining and then reluctantly remaining in the line; and jockeying between lines, which occurs when many concurrent service channels each have their own queue.

1.2 Related Works: Wang et al. [23] offer a thorough analysis of queuing systems with impatient patrons. In order to assess different queuing systems, they look at a variety of factors, such as customer impatience behaviors, queuing models with impatient customers' solutions, and link optimization elements.

Recently, researchers interested in models who are waiting in line with demanding clients. Haight's [6,7] work served as the catalyst for the early studies on consumers' impatience in queuing theory. Ancker and Gafarian [2] investigated a queuing model with a finite capacity that included balking and reneging. Expanding on their own work, Ancker and Gafarian [3] looked at a pure balking mechanism. Montazer-Haghighi et al. [20] calculated a queuing model with numerous servers that balked and reneged in a stationary state. Researchers Rykov and Efrosinin [5] have keenly observed the behaviour of attendants in a queuing system with extra service levels. The behaviour of queuing model with many servers and impatient clients in a transient state was provided by Al-Seedy et al. in [1].

Anxious customers lead to unsatisfaction which is a major cause for downfall or negative repute of company. Kumar and Sharma [9] studied a queuing model having limited capacity, one server, impatience, and client retention with this idea in mind. They looked at the behavior of the model in steady state. Kumar [10] calculated a multi-serverqueuing model's time-dependent probability using the matrix technique. The probability generating function technique was employed by Madheshwari et al. [19] to ascertain the probabilities of a queuing system with many trials in steady state. In a discrete-time line, Lee [18] proposed the idea ofkeeping devoted customers. A limitless capacity for several servers Vijayalaxmi and Kassahun [22] discovered that a stationary probability was associated with a Markovian feedback queue that had repeated reneging, balking, and retention of reneged consumers.

The M/M/1/N queuing model was examined by Kumar and Sharma [8] with the addition of retained consumers. Kumar and Sharma [9] have examined a queuing system with a range of servers and irate clients being held back in a time-dependent manner. Kumar and Sharma's research indicates that a company can use particular persuasive strategies to convince disgruntled customers to stick with them. Kumar and Sharma [11] looked into the ephemeral conduct of the M/M/C queuing system with balking and the retention of reneged clients. The retention of reneged clients and an M/M/c/N queueing model were investigated by Kumar and Sharma [12]. Kumar and Sharma [13] investigated the ephemeral investigation of a queuing model with two different servers while accounting for the retention of reneged consumers.

The scanning of a catastrophic queuing system with state-dependent services along environment changes in transient state as presented by N.K. Jain and D.K. Kanethia [21.] Darvinder Kumar [4] conducted research on how the limited capacity queuing system is affected by environmental changes and disasters.

1.3 Briefing: Here in the model present in this paper, another factor that is change of environment is added that will affect the state of the queuing model. The scenario of environment change can be seen in case of banks, offices etc. Factor of environment changes affects the arrival rate as well as service rate.

1.4 Structure of the paper: The layout of this paper contains: section 2 involves model description and its assumption, section 3 provides description of the model mathematically, section 4 includes two parts one of them is the solution of the probabilities of the present model in steady state using matrix approach and second is the analysis of the model in transient state including comparison with other models, section 5 involves some special cases and in the end, conclusion of the paper is displayed in the section 6.

MODEL'S DESCRIPTION AND ASSUMPTION

The following presumptions under in the model:

1. There are two states R and S in the queuing system. The system follows a Poisson process with α rate to go from state S to state R and β rate to go from R to S.
2. There is a negative exponential distribution for both the inter-arrival and service times. While in state S, rate of arrival is 0 (i.e., $\lambda = 0$) and the rate of service is μ_2, the queuingsystem in state R has rate of arrival λ_1 and rate of service μ_1. Only one of states R and S is operational at any given time.
3. When the system is in state R, each client who joins the queue must wait for his service to startfor a certain amount of time T, say. He may either stay in the line for his duty with complementaryprobability or renounce with probability p if it doesn't start by then. It has been speculated that time variable T has exponential distribution in adverse with parameter ξ. There is no reneging whenthe system is in state S.
4. The system has a finite function for N, and the queue discipline is FCFS "first to arrive will be be the first to have service provided."

FORMULATION

Let in state R, system have probability denoted by $P_n(t)$ and at time t there are n number of clients. Let in state S, system have probability denoted by $Q_n(t)$ and at time t number of clients are same as that in state R.

Initially when the system is empty and is in environmental state R then we have

$$P_n(0) = \begin{cases} 1; & n = 0 \\ 0; & \text{otherwise} \end{cases}$$

$Q_n(0) = 0$; for all n

The system has following set of differential-difference equations:

$$\frac{dP_0(t)}{dt} = -(\lambda_1 + \beta)P_0 + \mu_1 P_1 + \alpha Q_0; \qquad n=0 \qquad (1)$$

$$\frac{dP_n(t)}{dt} = -(\lambda_1 + n\mu_1 + \beta)P_n + \lambda_1 P_{n-1} + (n+1)\mu_1 P_{n+1} + \alpha Q_n; 1 \le n < c \qquad (2)$$

$$\frac{dP_n(t)}{dt} = -(\lambda_1 + c\mu_1 + (n-c)\xi p + \beta)P_n + \lambda_1 P_{n-1} + \left(c\mu_1 + \left((n+1)-c\right)\xi p\right)P_{n+1} + \alpha Q_n;$$

$$c \le n \le N-1 \qquad (3)$$

$$\frac{dP_N(t)}{dt} = \lambda_1 P_{N-1} - (c\mu_1 + (N-c)\xi p + \beta)P_N + \alpha Q_N; \qquad n = N \qquad (4)$$

$$\frac{dQ_0(t)}{dt} = \beta P_0 + \mu_2 Q_1 - \alpha Q_0; \qquad n=0 \qquad (5)$$

$$\frac{dQ_n(t)}{dt} = \beta P_n + (n+1)\mu_2 Q_{n+1} - (n\mu_2 + \alpha)Q_n; \qquad 1 \le n < c \qquad (6)$$

$$\frac{dQ_n(t)}{dt} = \beta P_n + c\mu_2 Q_{n+1} - (c\mu_2 + \alpha)Q_n; \qquad c \le n \le N-1 \qquad (7)$$

$$\frac{dQ_N(t)}{dt} = \beta P_N - (c\mu_2 + \alpha)Q_N; \qquad n=N \qquad (8)$$

STEADY STATE ANALYSIS

The model's steady state probabilities are derived in this part by the use of the matrix decomposition method, yielding the model's steady state probabilities. From the equations (1) -(6) steady state equations are as follows:

$$0 = -(\lambda_1 + \beta)P_0 + \mu_1 P_1 + \alpha Q_0; \qquad n=0 \qquad (9)$$

$$0 = -(\lambda_1 + n\mu_1 + \beta)P_n + \lambda_1 P_{n-1} + (n+1)\mu_1 P_{n+1} + \alpha Q_n; 1 \le n < c \qquad (10)$$

$$0 = -(\lambda_1 + c\mu_1 + (n-c)\xi p + \beta)P_n + \lambda_1 P_{n-1} + \left(c\mu_1 + \left((n+1)-c\right)\xi p\right)P_{n+1} + \alpha Q_n;$$

$$c \le n \le N-1 \qquad (11)$$

$$0 = \lambda_1 P_{N-1} - (c\mu_1 + (N-c)\xi p + \beta)P_N + \alpha Q_N; \qquad n = N \qquad (12)$$

$$0 = \beta P_0 + \mu_2 Q_1 - \alpha Q_0; \qquad n=0 \qquad (13)$$

$$0 = \beta P_n + (n+1)\mu_2 Q_{n+1} - (n\mu_2 + \alpha)Q_n; \qquad 1 \le n < c \qquad (14)$$

$$0 = \beta P_n + c\mu_2 Q_{n+1} - (c\mu_2 + \alpha)Q_n; \qquad c \le n \le N-1 \qquad (15)$$

$$0 = \beta P_n - (c\mu_2 + \alpha)Q_n; \qquad n=N \qquad (16)$$

Let $P = (P_0, P`, Q_0, Q`)$ be the vector of probabilities in steady state, where $Q' = (Q_1, Q_2, \ldots, Q_n)$ and $P' = (P_1, P_2 \ldots P_n)$. In matrix form, the steady- state equations from [9-16] are as follows:

$$ZP = 0$$

Where 0 is column vector of Zeros

$$\text{and} \quad \mathbf{Z} = \begin{bmatrix} -(\lambda+\beta) & Z_{12} & \alpha & Z_{14} \\ Z_{21} & Z_{22} & 0 & \alpha I_{NxN} \\ \beta & Z_{32} & -\alpha & Z_{34} \\ Z_{41} & \beta I_{NxN} & Z_{43} & Z_{44} \end{bmatrix}$$

is a square matrix of order **(2N + 2) × (2N + 2)**. Each entry of the square matrix **Z** is given below:

$$\mathbf{Z_{12}} = [\mu_1, 0, 0, \ldots, 0]_{\mathbf{1xN}}, \qquad \mathbf{Z_{14}} = [0,0,0,\ldots,0]_{\mathbf{1xN}}, \qquad \mathbf{Z_{21}} = [\lambda_1, 0, 0, \ldots, 0]`_{\mathbf{1xN}}$$

$$Z_{22} = \begin{bmatrix} -(\lambda_1+\mu_1+\beta) & 2\mu_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \lambda_1 & -(\lambda_1+2\mu_1+\beta) & 3\mu_1 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_1 & -(\lambda_1+3\mu_1+\beta) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -(\lambda_1+(c-1)\mu_1+\beta) & c\mu_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & \lambda_1 & -(\lambda_1+c\mu_1) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -(\lambda_1+c\mu_1+(N-1-c)\xi p+\beta) & (c\mu_1+(N-c)\xi p) \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \lambda_1 & -(c\mu_1+(N-1)\xi p+\beta) \end{bmatrix}_{NxN}$$

$$\mathbf{Z_{32}} = [0,0,0,\ldots,0]_{1xN}, \quad \mathbf{Z_{34}} = [\mu_2, 0, 0, \ldots, 0]_{1xN}, \quad \mathbf{Z_{41}} = [0,0,0,\ldots,0]`_{1xN}, \quad \mathbf{Z_{43}} = [0,0,0,\ldots,0]`_{1xN}$$

$$Z_{44} = \begin{bmatrix} -(\mu_2+\alpha) & 2\mu_2 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \lambda_1 & -(2\mu_2+\alpha) & 3\mu_2 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_1 & -(3\mu_2+\alpha) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -((c-1)\mu_2+\alpha) & c\mu_2 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & \lambda_1 & -(c\mu_2+\alpha) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -(c\mu_2+\alpha) & c\mu_2 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & -(c\mu_2+\alpha)) \end{bmatrix}_{NxN}$$

$$-(\lambda_1+\beta)P_0 + B_{12}P` + Q_0\alpha = 0 \tag{17}$$

$$Z_{21}P_0 + Z_{22}P` + \alpha I_{NxN}Q` = 0 \tag{18}$$

$$\beta P_0 - \alpha Q_0 + Z_{34}Q` = 0 \tag{19}$$

$$\beta I_{NxN}P` + Z_{44}Q` = 0 \tag{20}$$

from equation (20) we get

$$Q` = -\beta I_{NxN} p` Z_{44}{}^{-1}$$

and substituting value of Q` in equation (18), we get

$$P` = -\frac{Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}} P_0$$

Again, substituting value of P` in equation (17), we get

$$Q_0 = \alpha^{-1}\left[(\lambda_1+\beta) + \frac{Z_{12}Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}}\right] P_0$$

On substituting value of Q_0 in equation (19), we get

$$Q` = Z_{34}{}^{-1}\left[\lambda_1 + \frac{Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}}\right] P_0$$

We know

$P_0 + P`\mathbf{I} + Q_0 + Q`\mathbf{I} = 1$

Where **I** is the unit matrix of order Nx1.

$$P_0 = \frac{1}{\left[1 - \frac{Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}} + (\lambda_1 + \beta) + \frac{Z_{12}Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}} + \lambda_1 + \frac{Z_{21}}{Z_{22} - \alpha\beta Z_{44}{}^{-1}}\right]}$$

CONCLUSION

Through this paper, we have investigated a multi-server queue with environment factor and retention of reneged customers. The steady analysis of the queuing system is established. In future work will be done on transient state.

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