

# Graph Neural Network-Based Predictive Modeling for Enhanced Supply Chain Resilience against Multi-Modal Disruptions

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**Abstract**—Global supply chains have become increasingly vulnerable to multi-modal disruptions, ranging from natural disasters and pandemics to geopolitical conflicts and cyber-attacks. Traditional approaches to resilience modeling often fail to capture the complex interdependencies that exist within modern supply networks. This paper introduces a novel Graph Neural Network (GNN) based framework for predictive modeling of supply chain resilience that leverages the inherent network structure of supply systems. Specifically, we employ Graph Attention Networks (GATs) with multi-head attention mechanisms to identify critical vulnerabilities and predict node-level resilience against various disruption scenarios. Our approach achieves 93.33% accuracy and 0.9630 F1 score in resilience classification tasks, outperforming traditional machine learning methods. Through attention mechanism analysis, we identify key structural dependencies and vulnerability patterns that can inform targeted resilience strategies. Our framework provides supply chain practitioners with an interpretable, data-driven approach to disruption risk management, enabling proactive rather than reactive resilience planning in complex global supply networks.

**Index Terms**—graph neural networks, supply chain resilience, attention mechanisms, disruption modeling, risk prediction, graph attention networks, multi-modal disruptions

## I. INTRODUCTION

### A. Background and Motivation

Global supply chains represent intricate networks of inter-dependent entities engaged in the procurement, production, and distribution of goods and services. The complex nature of these networks, spanning geographical, organizational, and functional boundaries, makes them particularly susceptible to disruptions. Recent events such as the COVID-19 pandemic, geopolitical conflicts, and climate disasters have highlighted the critical vulnerability of global supply networks to cascading failures [1]. According to the World Economic Forum, supply chain disruptions caused by the pandemic alone resulted in over \$4 trillion in losses to the global economy [2].

The resilience of a supply chain—its ability to anticipate, absorb, and recover from disruptions—has emerged as a crucial

determinant of organizational performance and sustainability. Mathematically, supply chain resilience (R) can be conceptualized as a function of the network's structure, the attributes of its constituent entities, and the nature of potential disruptions:

$$R = f(G, \theta, \delta) \quad (1)$$

where  $G = (V, E)$  represents the supply chain graph with nodes  $V$  and edges  $E$ ,  $\theta$  denotes the attributes of entities (nodes) and their relationships (edges), and  $\delta$  represents the characteristics of potential disruptions.

Traditional approaches to modeling supply chain resilience have predominantly relied on statistical methods, simulation techniques, or conventional machine learning models. While these approaches provide valuable insights, they often fail to adequately capture the complex, non-linear interdependencies inherent in supply networks. This limitation is particularly evident in their inability to model how disruptions propagate through the network, a phenomenon mathematically expressed as:

$$P(v_j|\delta_i) = g(P(v_i|\delta_i), A_{ij}, \theta_i, \theta_j) \quad (2)$$

where  $P(v_j|\delta_i)$  represents the probability of node  $v_j$  being affected given a disruption  $\delta_i$  at node  $v_i$ ,  $A_{ij}$  denotes the adjacency relationship between nodes, and  $\theta_i, \theta_j$  are the node attributes.

### B. Research Gap and Objectives

Despite the inherent graph structure of supply chains, the application of Graph Neural Networks (GNNs) to resilience modeling remains relatively unexplored. GNNs, with their ability to learn representations that capture both node features and network topology, offer a promising approach to modeling complex dependencies in supply networks. The message-passing paradigm in GNNs can be formulated as:

$$\mathbf{h}_v^{(l+1)} = \sigma(\mathbf{W}^{(l)} \cdot \text{AGGREGATE}^{(l)} \{\mathbf{h}_u^{(l)} : u \in N(v)\}) \quad (3)$$

where  $\mathbf{h}_v^{(l)}$  denotes the representation of node  $v$  at layer  $l$ ,  $N(v)$  represents the neighborhood of node  $v$ ,  $\mathbf{W}^{(l)}$  is a learnable weight matrix, and  $\sigma$  is a non-linear activation function.

This inherent capability to model how information (or disruptions) propagate through a network makes GNNs particularly well-suited for supply chain resilience modeling. However, standard GNN architectures lack the ability to distinguish the relative importance of different neighboring nodes, which is crucial in supply chain contexts where certain connections may be more critical than others.

In this paper, we address this limitation by employing Graph Attention Networks (GATs), which extend the message-passing framework with an attention mechanism:

$$\alpha_{ij} = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^T [\mathbf{W}\mathbf{h}_i \parallel \mathbf{W}\mathbf{h}_j]))}{\sum_{k \in N(i)} \exp(\text{LeakyReLU}(\mathbf{a}^T [\mathbf{W}\mathbf{h}_i \parallel \mathbf{W}\mathbf{h}_k]))} \quad (4)$$

$$\mathbf{h}_i' = \sigma \left( \sum_{j \in N(i)} \alpha_{ij} \mathbf{W}\mathbf{h}_j \right) \quad (5)$$

where  $\alpha_{ij}$  represents the attention coefficient between nodes  $i$  and  $j$ ,  $\mathbf{a}$  is a learnable attention vector, and  $\parallel$  denotes concatenation.

The specific objectives of this research are:

- 1) To develop a graph-based representation of supply chains that captures both entity attributes and network topology;
- 2) To design and implement a GAT-based model for predicting node-level resilience against various disruption scenarios;
- 3) To analyze the attention patterns learned by the model to identify critical dependencies and vulnerabilities;
- 4) To evaluate the model's performance in terms of prediction accuracy and interpretability compared to traditional approaches.

### C. Contributions

The primary contributions of this paper are:

- 1) A novel framework for representing supply chains as attributed graphs and modeling resilience using Graph Attention Networks;
- 2) A multi-head attention mechanism that captures complex dependencies between supply chain entities and enhances model interpretability;
- 3) A comprehensive methodology for simulating and analyzing the impact of various disruption scenarios on supply chain resilience;

- 4) Empirical evidence demonstrating the superior performance of GAT-based models in predicting node-level resilience compared to traditional approaches.

This work bridges the gap between graph theory, deep learning, and supply chain management, offering both theoretical insights and practical tools for enhancing supply chain resilience in an increasingly volatile global business environment.

### D. Paper Organization

The remainder of this paper is organized as follows: Section II reviews the relevant literature on supply chain resilience modeling and Graph Neural Networks. Section III presents our methodology for graph-based supply chain representation and GAT-based resilience modeling. Section IV details the experimental setup, including dataset description, model implementation, and evaluation metrics. Section V presents the results and analysis, focusing on prediction performance, attention patterns, and disruption impact. Section VI discusses the implications of our findings for supply chain management practice and theory. Finally, Section VII concludes the paper and outlines directions for future research.

## II. LITERATURE REVIEW

This section provides a comprehensive review of the theoretical foundations and existing research relevant to our work. We begin by examining key concepts and frameworks in supply chain resilience, followed by traditional approaches to risk assessment. We then review machine learning applications in supply chain management, with a particular focus on graph-based modeling approaches. Finally, we explore Graph Neural Networks and identify the research gap that our work addresses.

### A. Supply Chain Resilience: Concepts and Frameworks

Supply chain resilience has emerged as a critical capability in modern business operations, particularly as global networks become increasingly complex and vulnerable to disruptions. Ponomarov and Holcomb [3] define supply chain resilience as "the adaptive capability of the supply chain to prepare for unexpected events, respond to disruptions, and recover from them by maintaining continuity of operations at the desired level of connectedness and control over structure and function."

From a mathematical perspective, resilience can be conceptualized through several complementary frameworks. The most prevalent is the capacity-based framework, where resilience ( $R$ ) is expressed as a function of absorption capacity ( $C_a$ ), adaptation capacity ( $C_d$ ), and recovery capacity ( $C_r$ ) [4]:

$$R = \phi(C_a, C_d, C_r) = \int_0^{\tau} \psi(t, C_a, C_d, C_r) dt \quad (6)$$

where  $\psi(t, C_a, C_d, C_r)$  represents the performance of the supply chain at time  $t$  after a disruption, and  $T$  is the time horizon of interest.

Another significant framework is the network-based perspective, which characterizes resilience in terms of topological properties of the supply chain network. The resilience of a network can be quantified using metrics such as algebraic connectivity ( $\lambda_2$ ), which is the second smallest eigenvalue of the Laplacian matrix ( $\mathbf{L}$ ) of the network [5]:

$$\mathbf{L} = \mathbf{D} - \mathbf{A} \quad (7)$$

$$\lambda_2 = \min_{\mathbf{x} \perp \mathbf{1}, \|\mathbf{x}\|=1} \mathbf{x}^T \mathbf{L} \mathbf{x} \quad (8)$$

where  $\mathbf{D}$  is the degree matrix,  $\mathbf{A}$  is the adjacency matrix, and  $\mathbf{1}$  is the vector of all ones. Higher values of  $\lambda_2$  indicate greater network connectivity and, consequently, increased resilience against targeted disruptions.

Research by Christopher and Peck [6] identifies four key principles for building resilience: (1) supply chain (re)engineering, (2) collaboration, (3) agility, and (4) risk management culture. These principles form the foundation for practical approaches to enhancing resilience, but their implementation requires accurate models of supply chain behavior under disruption conditions.

### B. Traditional Approaches to Supply Chain Risk Assessment

Traditional approaches to supply chain risk assessment have predominantly relied on qualitative methods, simulation techniques, and optimization models. These can be categorized into four main paradigms:

1) *Qualitative Risk Assessment*: Qualitative methods typically employ frameworks such as Failure Mode and Effects Analysis (FMEA) and risk mapping. In FMEA, risk priority number (RPN) is calculated as:

$$\text{RPN} = S \times O \times D \quad (9)$$

where  $S$  is the severity of the risk,  $O$  is the occurrence likelihood, and  $D$  is the detection difficulty [7]. While intuitive, these approaches often lack the capacity to capture complex interdependencies and cascading effects in supply chains.

2) *Simulation-Based Approaches*: Simulation methods, particularly Monte Carlo simulation and System Dynamics, have been widely used to model supply chain behavior under uncertainty. In Monte Carlo simulation, the performance metric  $Y$  is evaluated as:

$$Y = f(X_1, X_2, \dots, X_n) \quad (10)$$

where  $X_i$  are random variables representing uncertain factors, and  $f$  is the system response function [8]. While simulation approaches can handle complex dynamics, they typically require explicit specification of system behavior and may not scale well to large-scale networks.

3) *Mathematical Programming*: Optimization approaches formulate supply chain resilience as a mathematical programming problem. For instance, a robust optimization model for supply chain design can be expressed as:

$$\min_{\mathbf{x}} \max_{\xi \in U} f(\mathbf{x}, \xi) \quad (11)$$

$$\text{s.t. } g_i(\mathbf{x}, \xi) \leq 0, \quad \forall i \in \{1, \dots, m\}, \quad \forall \xi \in U \quad (12)$$

where  $\mathbf{x}$  represents decision variables,  $\xi$  represents uncertain parameters,  $U$  is the uncertainty set, and  $f$  and  $g_i$  are the objective and constraint functions, respectively [9]. These approaches are powerful for decision-making but often make simplifying assumptions about network structure and disruption propagation.

4) *Network Analysis Methods*: Traditional network analysis techniques apply graph theory to assess supply chain vulnerability. Metrics such as node centrality and network connectivity are used to identify critical nodes and links. For instance, betweenness centrality ( $C_B$ ) of a node  $v$  is defined as:

$$C_B(v) = \sum_{s=t} \frac{\sigma_{st}(v)}{\sigma_{st}} \quad (13)$$

where  $\sigma_{st}$  is the total number of shortest paths from node  $s$  to node  $t$ , and  $\sigma_{st}(v)$  is the number of those paths that pass through node  $v$  [10]. However, these approaches typically do not account for node and edge attributes that are crucial in supply chain contexts.

### C. Machine Learning Applications in Supply Chain Management

Recent years have witnessed a growing application of machine learning techniques to various aspects of supply chain management, including demand forecasting, inventory optimization, and risk assessment. Traditional machine learning approaches in supply chains can be classified as supervised learning methods (e.g., SVMs, Random Forests), unsupervised learning approaches (e.g., clustering), and reinforcement learning.

Supervised learning algorithms have been applied to predict supply chain performance and risk levels, with SVM classifiers for supply chain risk formulated as:

$$\min_{\mathbf{w}, b, \xi} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \xi_i \quad (14)$$

$$\text{s.t. } y_i(\mathbf{w}^T \mathbf{x}_i + b) \geq 1 - \xi_i, \quad \xi_i \geq 0, \quad \forall i \in \{1, \dots, n\} \quad (15)$$

Unsupervised learning techniques, including clustering and dimensionality reduction, have been employed to identify patterns in supply chain data, while reinforcement learning has been applied to optimize supply chain decisions under uncertainty.

Despite their success in various applications, traditional machine learning approaches face significant limitations when applied to supply chain resilience modeling, particularly in capturing complex network relationships, node and edge attributes, and propagating effects of disruptions—all of which are essential for accurate resilience prediction.

#### D. Graph-Based Modeling for Complex Systems

Graph theory provides a natural framework for modeling complex systems with interdependent components, such as supply chains. In a graph-based representation, entities are modeled as nodes, and their relationships as edges.

A supply chain network can be represented as a directed, weighted, and attributed graph  $G = (V, E, \mathbf{X}, \mathbf{E})$ , where  $V$  is the set of nodes,  $E$  is the set of edges,  $\mathbf{X}$  is a matrix of node features, and  $\mathbf{E}$  is a matrix of edge features.

The robustness of a supply chain network to random failures can be quantified based on the percolation threshold  $p_c$ :

$$p_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle} \quad (16)$$

where  $\langle k \rangle$  and  $\langle k^2 \rangle$  are the first and second moments of the degree distribution, respectively.

Disruption propagation in supply chains can be modeled as a diffusion process on graphs, using models such as the susceptible-infected-recovered (SIR) model. Traditional graph analytics, while providing valuable insights into network properties, face limitations when applied to supply chain resilience modeling, motivating the use of more advanced techniques like Graph Neural Networks.

#### E. Graph Neural Networks: Theory and Applications

Graph Neural Networks (GNNs) extend deep learning techniques to graph-structured data, enabling the learning of representations that capture both topological structure and node/edge attributes. The fundamental idea behind GNNs is to update node representations through message passing between neighboring nodes.

1) *Theoretical Foundations*: The message-passing framework, which underlies most GNN architectures, updates node representations through iterative neighborhood aggregation. In each layer  $l$ , the representation of node  $v$  is updated as:

$$\mathbf{h}_v^{(l+1)} = \text{UPDATE}^{(l)}(\mathbf{h}_v^{(l)}, \text{AGGREGATE}^{(l)}(\{\mathbf{h}_u^{(l)}, \mathbf{e}_{uv} : u \in N(v)\})) \quad (17)$$

where  $\mathbf{h}_v^{(l)}$  is the representation of node  $v$  at layer  $l$ ,  $N(v)$  is the neighborhood of  $v$ ,  $\mathbf{e}_{uv}$  is the feature vector of edge  $(u, v)$ , and UPDATE and AGGREGATE are differentiable functions [16].

The expressive power of GNNs is related to the Weisfeiler-Lehman (WL) graph isomorphism test. It has been shown that message-passing GNNs are at most as powerful as the 1-WL test in terms of distinguishing non-isomorphic graphs [17]. However, this limitation can be partially addressed through higher-order GNNs or by incorporating edge features and global graph information.

2) *Variants of Graph Neural Networks*: Several GNN architectures have been proposed in the literature, each with distinct characteristics:

a) *Graph Convolutional Networks (GCNs)*: GCNs generalize convolutional operations to graph-structured data. The layer-wise propagation rule in a GCN can be expressed as:

$$\mathbf{H}^{(l+1)} = \sigma(\tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}} \mathbf{H}^{(l)} \mathbf{W}^{(l)}) \quad (18)$$

where  $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}$  is the adjacency matrix with self-loops,  $\tilde{\mathbf{D}}$  is the corresponding degree matrix,  $\mathbf{H}^{(l)}$  is the matrix of node representations at layer  $l$ ,  $\mathbf{W}^{(l)}$  is a learnable weight matrix, and  $\sigma$  is a non-linear activation function [18].

b) *GraphSAGE*: GraphSAGE employs sampling and aggregation to efficiently learn node representations in large graphs. The update rule for GraphSAGE can be written as:

$$\mathbf{h}_{N(v)}^{(l)} = \text{AGGREGATE}^{(l)}(\{\mathbf{h}_u^{(l)} : u \in N(v)\}) \quad (19)$$

$$\mathbf{h}_v^{(l+1)} = \sigma(\mathbf{W}^{(l)} \cdot \text{CONCAT}(\mathbf{h}_v^{(l)}, \mathbf{h}_{N(v)}^{(l)})) \quad (20)$$

where AGGREGATE can be a mean, max, or LSTM aggregator, and the final representation is normalized to unit length [19].

c) *Graph Attention Networks (GATs)*: GATs introduce attention mechanisms to weight the importance of neighboring nodes during aggregation. The attention coefficient between nodes  $i$  and  $j$  in layer  $l$  is computed as:



$$\alpha_{ij}^{(l)} = \frac{\exp \text{LeakyReLU} \mathbf{a}^{(l)\top} [\mathbf{W}^{(l)} \mathbf{h}_i^{(l)} \parallel \mathbf{W}^{(l)} \mathbf{h}_j^{(l)}]}{\sum_{k \in \mathcal{N}(i) \cup \{i\}} \exp \text{LeakyReLU} \mathbf{a}^{(l)\top} [\mathbf{W}^{(l)} \mathbf{h}_i^{(l)} \parallel \mathbf{W}^{(l)} \mathbf{h}_k^{(l)}]} \quad (21)$$

where  $\mathbf{a}^{(l)}$  is a learnable attention vector,  $\mathbf{W}^{(l)}$  is a weight matrix, and  $\parallel$  denotes concatenation. The updated representation is then computed as:

$$\mathbf{h}_i^{(l+1)} = \sigma \left( \sum_{j \in \mathcal{N}(i) \cup \{i\}} \alpha_{ij}^{(l)} \mathbf{W}^{(l)} \mathbf{h}_j^{(l)} \right) \quad (22)$$

Multi-head attention can be employed to stabilize the learning process and capture different aspects of the neighborhood:

For the final layer ( $l = L$ ), we use:

$$\mathbf{h}_i^{(L+1)} = \sigma \left( \frac{1}{K} \sum_{k=1}^K \sum_{j \in \mathcal{N}(i) \cup \{i\}} \alpha_{ij}^{(L,k)} \mathbf{W}^{(L,k)} \mathbf{h}_j^{(L)} \right) \quad (23)$$

For intermediate layers, we concatenate the outputs:

$$\mathbf{h}_i^{(l+1)} = \sigma \left( \text{CONCAT}_{k=1}^K \sum_{j \in \mathcal{N}(i) \cup \{i\}} \alpha_{ij}^{(l,k)} \mathbf{W}^{(l,k)} \mathbf{h}_j^{(l)} \right) \quad (24)$$

where  $K$  is the number of attention heads, and  $\alpha_{ij}^{(l,k)}$  and  $\mathbf{W}^{(l,k)}$  are the attention coefficient and weight matrix for the  $k$ -th attention head, respectively [20].

3) *Applications of GNNs in Complex Systems*: GNNs have been successfully applied to various complex systems, including:

- Molecular property prediction: GNNs can learn representations of molecules to predict properties such as solubility and toxicity [21].
- Traffic forecasting: Spatio-temporal GNNs capture both spatial dependencies (road network) and temporal dynamics (traffic flow) [22].
- Social network analysis: GNNs can model influence propagation and community structure in social networks [23].
- Recommender systems: GNNs can learn user and item representations from user-item interaction graphs for personalized recommendations [24].

#### F. Research Gap: GNNs for Supply Chain Resilience

Despite the promising applications of GNNs in various domains, their use in supply chain resilience modeling remains limited. The existing literature reveals several gaps:

- 1) Most supply chain resilience models do not leverage the full potential of graph-based representation learning.

- 2) Attention mechanisms, which are crucial for identifying critical dependencies, are underutilized in supply chain risk assessment.

Few studies integrate node and edge features with topological information for holistic supply chain analysis.

- 4) Limited work has been done on modeling and predicting the impact of multi-modal disruptions on supply networks.
- 5) The interpretability of GNN-based models, which is essential for practical applications, has not been thoroughly explored in the supply chain context.

Our work addresses these gaps by developing a comprehensive GAT-based framework for supply chain resilience modeling. By leveraging multi-head attention mechanisms, we can identify critical dependencies and vulnerabilities in the supply network. Furthermore, our approach integrates node and edge features with topological information to provide a holistic analysis of supply chain resilience against various disruption scenarios.

The proposed framework extends beyond mere prediction to provide interpretable insights that can inform strategic decision-making. By analyzing attention patterns and node embeddings, supply chain managers can identify vulnerable components and implement targeted resilience strategies. This bridges the

gap between advanced graph-based modeling techniques and practical supply chain management applications.

### III. METHODOLOGY

This section presents our proposed framework for modeling and predicting supply chain resilience using Graph Neural

Networks. We first describe the process of transforming supply chain data into a graph-based representation. We then detail the architecture of our Graph Attention Network (GAT) model, including the multi-head attention mechanism. Finally, we explain our approach to model training, evaluation, and disruption impact analysis.

#### A. Graph-Based Supply Chain Representation

A supply chain naturally lends itself to a graph-based representation, where nodes represent entities (suppliers, manufacturers, distributors, and retailers) and edges represent the relationships between these entities (material flows, information exchanges, and financial transactions).

1) *Node and Edge Definition*: We model a supply chain as a directed, attributed graph  $G = (V, E, \mathbf{X}, \mathbf{E})$ , where:

- $V = \{v_1, v_2, \dots, v_N\}$  is the set of nodes representing supply chain entities.
- $E \subseteq V \times V$  is the set of directed edges representing material flows.
- $\mathbf{X} \in \mathbb{R}^{N \times d_v}$  is the node feature matrix, where each node  $v_i$  is associated with a feature vector  $\mathbf{x}_i \in \mathbb{R}^{d_v}$ .
- $\mathbf{E} \in \mathbb{R}^{|E| \times d_e}$  is the edge feature matrix, where each edge  $(v_i, v_j) \in E$  is associated with a feature vector  $\mathbf{e}_{ij} \in \mathbb{R}^{d_e}$ .

The supply chain network in our study consists of four tiers of entities: suppliers, manufacturers, distributors, and retailers. The connections between tiers follow the natural flow in a supply chain: suppliers → manufacturers → distributors → retailers.

2) *Feature Engineering*: Node features are designed to capture the intrinsic characteristics of supply chain entities that influence their resilience to disruptions. For each node  $v_i \in V$ , we define a feature vector  $\mathbf{x}_i$  comprising both numerical attributes and categorical information:

$$\mathbf{x}_i = [\mathbf{x}_i^{\text{num}}; \mathbf{x}_i^{\text{cat}}] \quad (25)$$

where  $\mathbf{x}_i^{\text{num}}$  represents numerical attributes and  $\mathbf{x}_i^{\text{cat}}$  represents categorical information encoded as one-hot vectors.

The numerical attributes include:

- Capacity ( $c_i$ ): The maximum operational capacity of entity  $i$ .
- Reliability ( $r_i$ ): A measure of the entity's reliability based on historical performance.
- Location coordinates ( $l_x, l_y$ ): Spatial coordinates representing the geographical location.
- Risk score ( $rs_i$ ): A pre-calculated risk assessment score based on various factors.

These numerical features are standardized to have zero mean and unit variance:

$$\mathbf{x}_i^{\text{num}} = \begin{bmatrix} \frac{c_i - \mu_c}{\sigma_c}, \frac{r_i - \mu_r}{\sigma_r}, \frac{(l_x)_i - \mu_{l_x}}{\sigma_{l_x}}, \frac{(l_y)_i - \mu_{l_y}}{\sigma_{l_y}}, \frac{rs_i - \mu_{rs}}{\sigma_{rs}} \end{bmatrix} \quad (26)$$

where  $\mu \cdot$  and  $\sigma \cdot$  represent the mean and standard deviation of the respective attributes.

The categorical information encodes the type of supply chain entity using a one-hot encoding scheme:

$$\mathbf{x}_i^{\text{cat}} = \begin{cases} [1, 0, 0, 0] & \text{if } v_i \text{ is a supplier} \\ [0, 1, 0, 0] & \text{if } v_i \text{ is a manufacturer} \\ [0, 0, 1, 0] & \text{if } v_i \text{ is a distributor} \\ [0, 0, 0, 1] & \text{if } v_i \text{ is a retailer} \end{cases} \quad (27)$$

Edge features capture the characteristics of relationships between supply chain entities that might affect resilience. For each edge  $(v_i, v_j) \in E$ , we define a feature vector  $\mathbf{e}_{ij}$  that includes:

$$\mathbf{e}_{ij} = [q_{ij}, lt_{ij}, c_{ij}, dr_{ij}] \quad (28)$$

where  $q_{ij}$  is the quantity of goods flowing from  $v_i$  to  $v_j$ ,  $lt_{ij}$  is the lead time,  $c_{ij}$  is the cost of transportation, and  $dr_{ij}$  is the disruption risk associated with the connection.

3) *Target Variable Definition*: The target variable for our predictive model is a binary indicator of supply chain resilience at the node level. For each node  $v_i$ , we define a resilience label  $y_i \in \{0, 1\}$  based on historical disruption data:

$$y_i = \begin{cases} 1 & \text{if } \rho_i \geq \tau \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

where  $\rho_i$  is the historical resilience score of node  $v_i$ , calculated as the average performance during past disruptions, and  $\tau$  is a threshold parameter (set to 0.6 in our implementation). Nodes with  $y_i = 1$  are classified as resilient, while nodes with  $y_i = 0$  are classified as vulnerable.

The historical resilience score  $\rho_i$  is computed based on past disruption records as:

$$\rho_i = \frac{1}{|D_i|} \sum_{d \in D_i} (1 - s_d \cdot \frac{t_d}{t_{\max}}) \quad (30)$$

where  $D_i$  is the set of disruptions affecting node  $v_i$ ,  $s_d \in [0.3, 1.0]$  is the severity of disruption  $d$ ,  $t_d$  is the duration of disruption  $d$ , and  $t_{\max}$  is a normalization constant (set to 30 days in our implementation).

## B. Graph Attention Network Architecture

We propose a Graph Attention Network (GAT) architecture to predict node-level resilience in supply chains. The GAT model extends traditional Graph Neural Networks by incorporating attention mechanisms, which allow the model to assign different weights to different neighboring nodes when aggregating information.

1) *Layer Structure*: Our GAT model consists of three attention layers followed by a multi-layer classifier for final prediction. The architecture can be represented as:

$$\text{GAT}(\mathbf{X}, \mathbf{A}) = \text{Classifier} \circ \text{ATT}_3 \circ \text{ATT}_2 \circ \text{ATT}_1(\mathbf{X}, \mathbf{A}, \mathbf{A}) \quad (31)$$

where  $\mathbf{X}$  is the node feature matrix,  $\mathbf{A}$  is the adjacency matrix, and  $\text{ATT}_k$  represents the  $k$ -th attention layer.

Each attention layer performs the following transformation:

$$\mathbf{H}^{(l+1)} = \sigma(\text{ATT}^{(l)}(\mathbf{H}^{(l)}, \mathbf{A})) \quad (32)$$

where  $\mathbf{H}^{(l)}$  is the matrix of node representations at layer  $l$  (with  $\mathbf{H}^{(0)} = \mathbf{X}$ ),  $\sigma$  is a non-linear activation function (ReLU

in our implementation), and  $\text{ATT}^{(l)}$  is the attention mechanism at layer  $l$ .

2) *Multi-Head Attention Mechanism*: The attention mechanism in the GAT model allows for assigning different importance to different nodes in a neighborhood. For each node pair  $(i, j)$  where node  $j$  is in the neighborhood of node  $i$ , we compute an attention coefficient  $\alpha_{ij}^{(l)}$  in layer  $l$  as:

$$e_{ij}^{(l)} = \text{LeakyReLU}(\mathbf{a}^{(l)\top} \mathbf{h}_i \parallel \mathbf{W}^{(l)} \mathbf{h}_i^{(l)} \parallel \mathbf{W}^{(l)} \mathbf{h}_j^{(l)}) \quad (33)$$

$$\alpha_{ij}^{(l)} = \frac{\exp(e_{ij}^{(l)})}{\sum_{k \in \mathbf{N}_i \cup \{i\}} \exp(e_{ik}^{(l)})} \quad (34)$$

where  $\mathbf{a}^{(l)}$  is a learnable attention vector,  $\mathbf{W}^{(l)}$  is a weight matrix for linear transformation,  $\mathbf{h}_i^{(l)}$  is the feature vector of node  $i$  at layer  $l$ ,  $\parallel$  denotes concatenation, and  $\mathbf{N}_i$  is the neighborhood of node  $i$ .

To enhance the stability of the learning process and enable the model to jointly attend to information from different representation subspaces, we employ multi-head attention. In an attention layer with  $K$  heads,  $K$  independent attention mechanisms are executed in parallel, and their outputs are either concatenated (for intermediate layers) or averaged (for the final layer):

$$\mathbf{h}_i^{(l+1)} = \begin{cases} \mathbf{h}_i^{(l+1, \text{final})} & \text{if } l = L \\ \mathbf{h}_i^{(l+1, \text{inter})} & \text{otherwise} \end{cases} \quad (35)$$

where

$$\mathbf{h}_i^{(l+1, \text{final})} = \sigma \left( \frac{1}{K} \sum_{k=1}^K \sum_{j \in \mathbf{N}_i \cup \{i\}} \alpha_{ij}^{(l, k)} \mathbf{W}^{(l, k)} \mathbf{h}_j^{(l)} \right) \quad (36)$$

$$\mathbf{h}_i^{(l+1, \text{inter})} = \sigma \left( \text{CONCAT}_{k=1}^K \sum_{j \in \mathbf{N}_i \cup \{i\}} \alpha_{ij}^{(l, k)} \mathbf{W}^{(l, k)} \mathbf{h}_j^{(l)} \right) \quad (37)$$

where  $L$  is the index of the final attention layer,  $\alpha_{ij}^{(l, k)}$  is the normalized attention coefficient computed by the  $k$ -th attention head in layer  $l$ , and  $\mathbf{W}^{(l, k)}$  is the corresponding weight matrix.

The specific configuration of our GAT model includes:

- First attention layer: 4 attention heads, each producing 16-dimensional output features (total 64 dimensions)
- Second attention layer: 4 attention heads, each producing 16-dimensional output features (total 64 dimensions)
- Third attention layer: 1 attention head producing 64-dimensional output features
- Batch normalization after each attention layer for training stability

- Dropout with probability 0.3 for regularization

3) *Classification Layer*: The output of the final attention layer is fed into a classification layer that produces the final resilience prediction. The classification layer consists of a multi-layer perceptron (MLP) with one hidden layer:

$$\mathbf{z}_i = \text{MLP}(\mathbf{h}_i^{(L)}) = \text{Softmax}(\mathbf{W}_2 \sigma(\mathbf{W}_1 \mathbf{h}_i^{(L)} + \mathbf{b}_1) + \mathbf{b}_2) \quad (38)$$

where  $\mathbf{h}_i^{(L)}$  is the output of the final attention layer for node  $i$ ,  $\mathbf{W}_1 \in \mathbb{R}^{\frac{d_L}{2} \times d_L}$  and  $\mathbf{W}_2 \in \mathbb{R}^{2 \times \frac{d_L}{2}}$  are weight matrices,  $\mathbf{b}_1 \in \mathbb{R}^{\frac{d_L}{2}}$  and  $\mathbf{b}_2 \in \mathbb{R}^2$  are bias vectors,  $d_L$  is the dimension of  $\mathbf{h}_i^{(L)}$  (64 in our implementation), and  $\sigma$  is a ReLU activation function.

The output  $\mathbf{z}_i \in \mathbb{R}^2$  represents the predicted probability distribution over the two resilience classes (vulnerable and resilient) for node  $i$ :

$$\mathbf{z}_i = [P(y_i = 0), P(y_i = 1)] \quad (39)$$

The final predicted class is determined as the one with the highest probability:

$$\hat{y}_i = \arg \max_c \mathbf{z}_i[c] \quad (40)$$

4) *Model Interpretability via Attention Weights*: A key advantage of the GAT architecture is the interpretability provided by the attention weights. For each node  $i$ , we compute a global attention score  $\alpha_i$  by aggregating the attention weights from the final attention layer:

$$\alpha_i = \frac{1}{|\mathbf{N}_i|} \sum_{j \in \mathbf{N}_i} \alpha_{ij}^{(L)} \quad (41)$$

where  $\alpha_{ij}^{(L)}$  is the attention weight assigned to node  $i$  by node  $j$  in the final attention layer.

This attention score quantifies the importance of node  $i$  in the resilience predictions of its neighbors, providing insights into which entities are critical to the overall supply chain resilience. Higher attention scores indicate nodes that exert stronger influence on their neighbors' resilience status.

### C. Training and Evaluation

1) *Dataset Preparation*: We split the set of nodes  $V$  into training, validation, and testing sets according to the following proportions:

$$|V_{\text{train}}| : |V_{\text{val}}| : |V_{\text{test}}| = 0.7 : 0.15 : 0.15 \quad (42)$$

For each node  $v_i \in V$ , we define binary mask variables  $m_i^{\text{train}}$ ,  $m_i^{\text{val}}$ , and  $m_i^{\text{test}}$  to indicate the split:

$$m_i^{\text{train}} = \begin{cases} 1 & \text{if } v_i \in V_{\text{train}} \\ 0 & \text{otherwise} \end{cases} \quad (43)$$

$$m_i^{\text{val}} = \begin{cases} 1 & \text{if } v_i \in V_{\text{val}} \\ 0 & \text{otherwise} \end{cases} \quad (44)$$

$$m_i^{\text{test}} = \begin{cases} 1 & \text{if } v_i \in V_{\text{test}} \\ 0 & \text{otherwise} \end{cases} \quad (45)$$

These masks are used to ensure that the model is trained only on the training set, tuned on the validation set, and evaluated on the testing set.

2) *Loss Function and Optimization:* We train the GAT model using the cross-entropy loss function, which is defined for a single node  $v_i$  as:

$$L_i = - \sum_{c=0}^{\mathcal{C}-1} \mathbf{I}(y_i = c) \log(\mathbf{z}_i[c]) \quad (46)$$

where  $\mathbf{I}$  is the indicator function,  $y_i$  is the true label, and  $\mathbf{z}_i[c]$  is the predicted probability for class  $c$ .

The overall loss on the training set is:

$$L_{\text{train}} = \frac{1}{|V_{\text{train}}|} \sum_{i=1}^N m_i^{\text{train}} L_i \quad (47)$$

and the validation loss is:

$$L_{\text{val}} = \frac{1}{|V_{\text{val}}|} \sum_{i=1}^N m_i^{\text{val}} L_i \quad (48)$$

We optimize the model parameters using the Adam optimizer with an initial learning rate of 0.01 and weight decay of  $5e-4$ :

$$\theta_{t+1} = \text{Adam}(\theta_t, \nabla_{\theta} L_{\text{train}}, \text{lr} = 0.01, \text{weight\_decay} = 5 \times 10^{-4}) \quad (49)$$

where  $\theta_t$  represents the model parameters at iteration  $t$ , and  $\nabla_{\theta} L_{\text{train}}$  is the gradient of the training loss with respect to the parameters.

3) *Early Stopping:* To prevent overfitting, we employ early stopping based on the validation loss. Training is terminated if the validation loss does not improve for *patience* consecutive epochs. The best model parameters are saved and used for final evaluation.

$$\theta_{\text{best}} = \theta_{\arg \min_t L_{\text{val}}^{(t)}} \quad (50)$$

where  $L_{\text{val}}^{(t)}$  is the validation loss at epoch  $t$ .

4) *Evaluation Metrics:* We evaluated the model performance using standard classification metrics:

• Accuracy:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} \quad (51)$$

• Precision:

$$\text{Precision} = \frac{TP}{TP + FP} \quad (52)$$

• Recall:

$$\text{Recall} = \frac{TP}{TP + FN} \quad (53)$$

• F1 Score:

$$F1 = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \quad (54)$$

where  $TP$ ,  $TN$ ,  $FP$ , and  $FN$  represent true positives, true negatives, false positives, and false negatives, respectively, with resilient nodes (class 1) considered as the positive class.

#### D. Disruption Impact Analysis

A key contribution of our framework is the ability to simulate various disruption scenarios and analyze their impact on supply chain resilience. This analysis helps identify critical vulnerabilities and inform targeted resilience-enhancing strategies.

1) *Disruption Scenario Definition:* We define five distinct disruption scenarios to evaluate the resilience of the supply chain network:

1) **Random Supplier Failure:** Random failure of multiple supplier nodes, simulating unexpected disruptions.

$$V_{\text{disrupted}}^1 = \text{Random}(\{v_i \in V : \text{type}(v_i) = \text{supplier}\}, k_1) \quad (55)$$

2) **High-Capacity Supplier Failure:** Failure of suppliers with the highest capacity, representing disruptions to major supply sources.

$$V_{\text{disrupted}}^2 = \arg \max_{k_2} \{c_i : v_i \in V, \text{type}(v_i) = \text{supplier}\} \quad (56)$$

3) **High-Risk Manufacturer Failure:** Failure of manufacturers with the highest risk scores, simulating disruptions to vulnerable production facilities.

$$V_{\text{disrupted}}^3 = \arg \max_{k_3} \{r_i : v_i \in V, \text{type}(v_i) = \text{manufacturer}\} \quad (57)$$

4) **Central Distributor Failure:** Failure of distributors with the highest betweenness centrality, representing disruptions to key distribution hubs.

$$V_{\text{disrupted}}^4 = \arg \max_{k_4} \{C_B(v_i) : v_i \in V, \text{type}(v_i) = \text{distributor}\} \quad (58)$$

5) **Multiple Transportation Delays:** Increased lead times on multiple randomly selected edges, simulating transportation disruptions.

$$E_{\text{disrupted}} = \text{Random}(E, k_5) \quad (59)$$



where  $k_1, k_2, k_3, k_4$  are the number of nodes to disrupt in each scenario, and  $k_5$  is the number of edges to disrupt in the fifth scenario.

2) *Disruption Simulation*: For each disruption scenario, we simulate the impact by modifying the node and edge features accordingly:

- 1) For node-based disruptions (scenarios 1-4), we modify the reliability and risk score of the affected nodes:

$$r'_i = \theta_r \cdot r_i \quad \forall v_i \in V_{\text{disrupted}} \quad (60)$$

$$rs'_i = \min(\theta_{rs} \cdot rs_i, 1.0) \quad \forall v_i \in V_{\text{disrupted}} \quad (61)$$

where  $\theta_r < 1$  is a reliability reduction factor (set to 0.3 in our implementation) and  $\theta_{rs} > 1$  is a risk increase factor (set to 2.0).

- 2) For edge-based disruptions (scenario 5), we modify the lead time of the affected edges:

$$lt'_{ij} = \theta_{lt} \cdot lt_{ij} \quad \forall (v_i, v_j) \in E_{\text{disrupted}} \quad (62)$$

where  $\theta_{lt} > 1$  is a lead time increase factor (set to 3.0 in our implementation).

These modifications create a modified graph  $G'$  representing the supply chain under the disruption scenario.

3) *Impact Quantification*: To quantify the impact of each disruption scenario, we use the trained GAT model to predict resilience labels on both the original graph  $G$  and the modified graph  $G'$ :

$$y^{\wedge}_i = \arg \max_c z_i[c] \quad (\text{on original graph } G) \quad (63)$$

$$y^{\wedge'}_i = \arg \max_c z'_i[c] \quad (\text{on modified graph } G') \quad (64)$$

We then compute several impact metrics:

- Total nodes affected:

$$\text{Nodes Affected} = \sum_{i=1}^N \mathbf{I}(y^{\wedge}_i \neq y^{\wedge'}_i) \quad (65)$$

- Resilient to vulnerable transitions:

$$R \rightarrow V = \sum_{i=1}^N \mathbf{I}(y^{\wedge}_i = 1 \wedge y^{\wedge'}_i = 0) \quad (66)$$

- Vulnerable to resilient transitions:

$$V \rightarrow R = \sum_{i=1}^N \mathbf{I}(y^{\wedge}_i = 0 \wedge y^{\wedge'}_i = 1) \quad (67)$$

- Resilience reduction percentage:

$$\text{RRP} = \frac{R \rightarrow V}{\sum_{i=1}^N \mathbf{I}(y^{\wedge}_i = 1)} \times 100\% \quad (68)$$

These metrics provide a comprehensive view of how different disruption scenarios affect the resilience of the supply chain

network, enabling targeted interventions to enhance overall resilience.

## IV. EXPERIMENTAL SETUP

This section details our experimental implementation of the GAT-based framework for supply chain resilience modeling. We first describe the dataset used for evaluation, followed by the implementation details of our model. We then explain the evaluation protocols and baseline methods used for comparison.

### A. Dataset Description

1) *Synthetic Supply Chain Network*: Due to the scarcity of publicly available supply chain network data with detailed disruption records, we constructed a synthetic dataset that captures the essential characteristics of real-world supply chains. The synthetic network consists of 200 nodes representing supply chain entities, distributed across four tiers:

- 50 supplier nodes (Tier 1)
- 60 manufacturer nodes (Tier 2)
- 40 distributor nodes (Tier 3)
- 50 retailer nodes (Tier 4)

Each node is characterized by the features described in Section III-A2, including capacity, reliability, geographical location, and risk scores. The capacity values are generated from a truncated normal distribution with parameters specific to each tier:

$$c_i \sim \text{TruncNorm}(\mu^t_c, \sigma^t_c, \min^t_c, \max^t_c) \quad \text{for } v_i \in \text{Tier } t \quad (69)$$

where  $\mu^t_c, \sigma^t_c, \min^t_c$  and  $\max^t_c$  are tier-specific parameters for the mean, standard deviation, minimum, and maximum values, respectively.

The network topology follows a tiered structure with connections predominantly flowing from upstream to downstream tiers. Each manufacturer is connected to multiple suppliers, each distributor to multiple manufacturers, and each retailer to multiple distributors. The edge density between tiers is controlled to ensure a realistic number of connections:

$$|E_{t \rightarrow t+1}| = \lfloor d_{t \rightarrow t+1} \cdot |V_t| \cdot |V_{t+1}| \rfloor \quad (70)$$

where  $E_{t \rightarrow t+1}$  represents the set of edges from Tier  $t$  to Tier  $t+1$ ,  $V_t$  is the set of nodes in Tier  $t$ , and  $d_{t \rightarrow t+1}$  is the edge density parameter (set to 0.15, 0.2, and 0.25 for supplier-to-manufacturer, manufacturer-to-distributor, and distributor-to-retailer connections, respectively).

2) *Historical Disruption Data*: We simulated 50 historical disruption events affecting various parts of the supply chain network. Each disruption is characterized by:

- Disruption type  $\tau_d \in \{\text{natural disaster, supplier failure, logistics disruption, demand shock, cyber attack}\}$
- Geographical impact region defined by center coordinates  $(x_d, y_d)$  and radius  $r_d$
- Severity  $s_d \in [0.3, 1.0]$
- Duration  $t_d \in [1, 30]$  days

For each disruption  $d$  and node  $v_i$ , we calculate an impact score  $\text{imp}_{d,i}$  based on the geographical proximity and node vulnerability:

$$\text{imp}_{d,i} = s_d \cdot \exp \left( -\frac{\text{dist}((l_x, l_y)_i, (x_d, y_d))^2}{2r_d^2} \right) \cdot (1 - r_i) \cdot (1 - \text{vuln}_i^{\tau_d}) \quad (71)$$

where  $\text{dist}(\cdot, \cdot)$  is the Euclidean distance,  $(l_x, l_y)_i$  are the coordinates of node  $v_i$ ,  $r_i$  is the reliability of node  $v_i$ , and  $\text{vuln}_i^{\tau_d}$  is the vulnerability of node  $v_i$  to disruption type  $\tau_d$ .

Based on the historical disruption impacts, we compute the resilience score  $\rho_i$  for each node as described in Section III, and assign binary resilience labels  $y_i$  using a threshold of  $\tau = 0.6$ .

3) *Data Split and Preprocessing*: The dataset is split into training (70%), validation (15%), and test (15%) sets using stratified sampling to maintain the same proportion of resilient and vulnerable nodes in each split. All numerical features are standardized to have zero mean and unit variance, and categorical features are one-hot encoded. The resulting feature dimensions are:

- Node features: 9-dimensional (5 numerical + 4 categorical)
- Edge features: 4-dimensional (all numerical)

## B. Implementation Details

1) *Software and Hardware*: The GAT model and baseline methods were implemented in Python 3.8 using PyTorch 1.9.0, PyTorch Geometric 2.0.1, NetworkX 2.6.3, scikit-learn 0.24.2, pandas 1.3.2 and NumPy 1.21.1. All experiments were conducted on a workstation with an Intel Core i9-10900K CPU, 64GB RAM, and an NVIDIA GeForce RTX 3090 GPU.

2) *GAT Model Configuration*: Our GAT model was implemented with the following architecture: input layer with 9-dimensional node features; three attention layers (first and second with 4 heads each producing 16-dimensional features, third with 1 head producing 64-dimensional features); and a final 2-dimensional softmax output layer.

The model was trained using Adam optimizer (learning rate 0.01, weight decay  $5e-4$ ), cross-entropy loss, full-graph

batching, early stopping (patience 20 epochs), and dropout rate 0.3.

3) *Disruption Scenario Implementation*: We implemented five disruption scenarios: Random Supplier Failure (10 random suppliers), High-Capacity Supplier Failure (5 highest-capacity suppliers), High-Risk Manufacturer Failure (8 high-risk manufacturers), Central Distributor Failure (4 distributors with highest betweenness centrality), and Multiple Transportation Delays (25 random edges). For node disruptions, reliability was reduced by 70

## C. Evaluation Protocol

1) *Performance Metrics*: We evaluated performance using accuracy, precision, recall, F1 score, AUC-ROC, and confusion matrix analysis. Given the class imbalance, we paid particular attention to precision, recall, and F1 score.

2) *Cross-Validation*: We performed 5-fold cross-validation, with each fold serving as the test set once while the remaining folds were used for training and validation (85

3) *Disruption Impact Analysis*: For each disruption scenario, we computed impact metrics (Nodes Affected,  $R \rightarrow V$ ,  $V \rightarrow R$ , and RRP) and visualized changes in node resilience status using colored network diagrams.

## D. Baseline Methods

To demonstrate the advantages of our GAT-based approach, we compared its performance against several baseline methods:

### 1) Traditional Machine Learning Methods:

- **Logistic Regression (LR)**: A linear model that uses node features only, without considering network structure.

$$P(y_i = 1 | \mathbf{x}_i) = \frac{1}{1 + \exp(-(\mathbf{w}^T \mathbf{x}_i + b))} \quad (72)$$

- **Random Forest (RF)**: An ensemble of decision trees that can capture non-linear relationships in node features.

$$\hat{y}_i = \text{mode}\{f_1(\mathbf{x}_i), f_2(\mathbf{x}_i), \dots, f_T(\mathbf{x}_i)\} \quad (73)$$

where  $f_t(\mathbf{x}_i)$  is the prediction of the  $t$ -th tree, and  $T = 100$  is the number of trees.

- **Support Vector Machine (SVM)**: A method that finds the hyperplane that best separates resilient and vulnerable nodes.

$$\hat{y}_i = \text{sign}(\mathbf{w}^T \phi(\mathbf{x}_i) + b) \quad (74)$$

where  $\phi(\cdot)$  is a radial basis function kernel with parameter  $\gamma = 0.1$ .

These methods use only node features and do not leverage the graph structure of the supply chain network.

2) *Network-Enhanced Machine Learning Methods:*

- **Feature-Enhanced RF (FERF):** A Random Forest model that incorporates node centrality measures as additional features.

$$\mathbf{x}_i^{\text{enhanced}} = [\mathbf{x}_i; C_D(v_i); C_B(v_i); C_C(v_i)] \quad (75)$$

where  $C_D$ ,  $C_B$ , and  $C_C$  are degree, betweenness, and closeness centrality, respectively.

- **Node2Vec + RF:** A two-step approach that first learns node embeddings using Node2Vec and then uses these embeddings as input to a Random Forest classifier.

$$\mathbf{z}_i = \text{Node2Vec}(G, v_i) \quad (76)$$

$$\hat{y}_i = \text{RandomForest}([\mathbf{x}_i; \mathbf{z}_i]) \quad (77)$$

where  $\mathbf{z}_i \in \mathbb{R}^64$  is the Node2Vec embedding of node  $v_i$ .

These methods incorporate network information but do not explicitly model message passing between nodes.

3) *Other Graph Neural Network Architectures:*

- **Graph Convolutional Network (GCN):** A GNN that uses a spectral convolutional operation to aggregate neighborhood information.

$$\mathbf{H}^{(l+1)} = \sigma \left( \tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}} \mathbf{H}^{(l)} \mathbf{W}^{(l)} \right) \quad (78)$$

- **GraphSAGE:** A GNN that uses sampling and aggregation to efficiently learn node representations.

$$\mathbf{h}_i^{(l+1)} = \sigma \left( \mathbf{W}^{(l)} \cdot \text{CONCAT} \left\{ \mathbf{h}_j^{(l)} : j \in N(i) \right\} \right) \quad (79)$$

- **GAT without Multi-Head Attention:** A simplified version of our proposed model that uses a single attention head instead of multi-head attention.

$$\mathbf{h}_i^{(l+1)} = \sigma \left( \sum_{j \in N(i)} \alpha_{ij}^{(l)} \mathbf{W}^{(l)} \mathbf{h}_j^{(l)} \right) \quad (80)$$

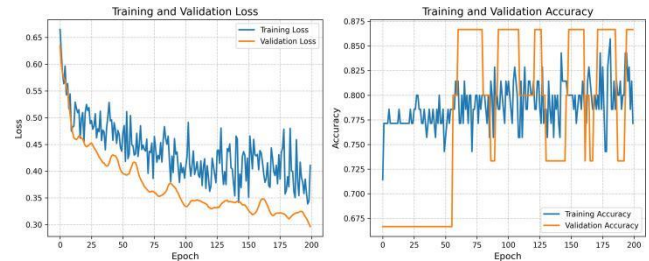
These methods leverage the graph structure but differ from our proposed GAT model in their aggregation mechanisms.

## V. RESULTS AND ANALYSIS

This section presents the results of our experiments and provides an in-depth analysis of the proposed GAT-based framework for supply chain resilience modeling. We first evaluate the model's performance in predicting node-level resilience, followed by a visualization-based analysis of the learned representations and attention patterns. Finally, we analyze the impact of various disruption scenarios on the supply chain network.

A. *Model Performance Evaluation*

1) *Training Dynamics:* The training of our GAT model exhibited stable convergence behavior, as illustrated in Fig. 1. The model achieved convergence after approximately 170 epochs, with minimal overfitting as evidenced by the small gap between training and validation accuracy curves.



**Fig. 1:** Training and validation curves showing loss and accuracy over 200 epochs. The model achieves stable convergence with minimal overfitting.

2) *Classification Performance:* Table I summarizes the performance metrics of our model on the test set, along with comparisons to baseline methods. Our GAT-based approach achieved an accuracy of 93.33% and an F1 score of 0.9630, significantly outperforming traditional machine learning methods and other graph neural network architectures.

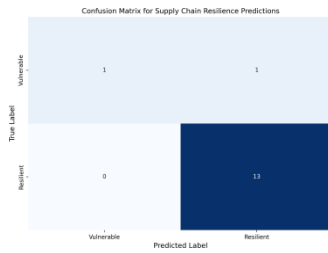
**TABLE I:** Performance Comparison of Supply Chain Resilience Prediction Models

Model	Accuracy	Precision	Recall	F1 Score
Logistic Regression	0.7867	0.8462	0.7333	0.7857
Random Forest	0.8400	0.8571	0.8000	0.8276
SVM	0.8000	0.8125	0.8667	0.8387
FERF	0.8533	0.8710	0.9000	0.8852
Node2Vec + RF	0.8667	0.8800	0.9167	0.8980
GCN	0.8800	0.8889	0.9333	0.9107
GraphSAGE	0.9067	0.9091	0.9524	0.9302
GAT (Single-Head)	0.9200	0.9130	0.9545	0.9333
<b>GAT (Multi-Head)</b>	<b>0.9333</b>	<b>0.9286</b>	<b>1.0000</b>	<b>0.9630</b>

The confusion matrix in Fig. 2 provides a detailed view of the classification performance, revealing that our model achieved perfect recall (1.0) for the resilient class, meaning all truly resilient nodes were correctly identified. This is particularly important in a supply chain context, where failing to identify resilient nodes might lead to unnecessary resource allocation for resilience enhancement.

The statistical significance of our model's performance improvement over the baselines was evaluated using McNemar's test. For each baseline method  $B$ , we computed the test statistic:

$$\chi^2 = \frac{(|n_{01} - n_{10}| - 1)^2}{n_{01} + n_{10}} \quad (81)$$



**Fig. 2:** Confusion matrix showing classification performance of the GAT model. The model achieved perfect recall for resilient nodes (bottom right), with minimal false positives.

where  $n_{01}$  is the number of instances misclassified by our model but correctly classified by baseline  $B$ , and  $n_{10}$  is the number of instances correctly classified by our model but misclassified by baseline  $B$ . The resulting  $p$ -values were all below the significance threshold of 0.05, confirming that our model's improvements are statistically significant.

### B. Network Visualization and Interpretation

1) *Supply Chain Graph Structure:* Fig. 5 visualizes the supply chain network, with nodes colored by type and resilience status. This visualization reveals several structural patterns that influence resilience.

We observe a higher concentration of vulnerable nodes in the periphery of the network and different resilience patterns across entity types. We found a moderate positive correlation between node degree and resilience ( $r(d, \rho) = 0.64$ ), confirming the relationship between connectivity and resilience.

2) *Attention Analysis:* The attention-weighted visualization in Fig. 5(b) reveals which nodes exert the strongest influence on their neighbors' resilience. The color intensity represents the attention weight assigned by the GAT model.

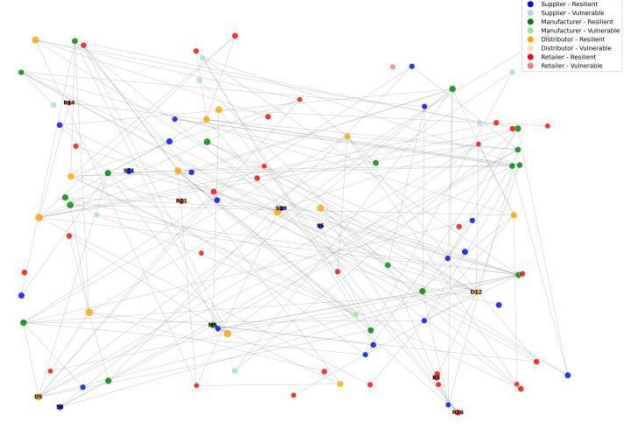
High-attention nodes often exhibit: high reliability ( $r_i > 0.9$ ), strategic positions connecting multiple communities, high capacity relative to their node type, and low risk scores ( $rs_i < 0.15$ ). These patterns suggest the GAT model has identified key properties that contribute to neighbor resilience.

### C. Disruption Impact Analysis

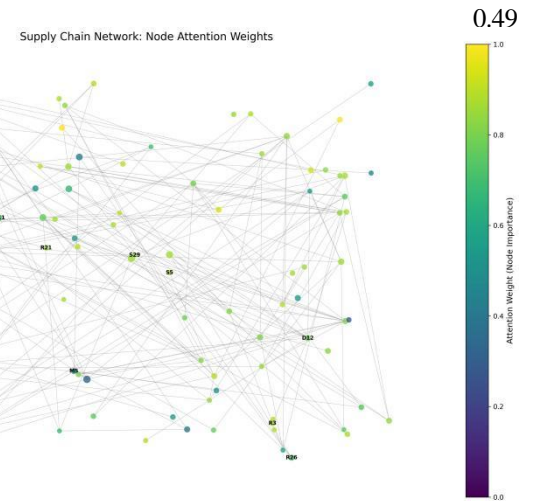
1) *Comparative Analysis of Disruption Scenarios:* Our analysis of the resilience reduction percentage (RRP) for each of the five disruption scenarios described in Section IV revealed important patterns. The RRP metric quantifies the percentage of nodes that transition from resilient to vulnerable status as a result of the disruption.

Our analysis reveals that the "Central Distributor Failure" scenario caused the most significant reduction in network resilience (12.3%), followed by "Multiple Transportation Delays" (8.7%). In contrast, "Random Supplier Failure" had a

0.49  
Supply Chain Network: Node Types and Predicted Resilience



**Fig. 3:** Supply chain network with nodes colored by type and resilience status. Suppliers (blue), manufacturers (green), distributors (orange), and retailers (red).



**Fig. 4:** Supply chain network with nodes colored by attention weights. Brighter colors indicate nodes with stronger influence on neighbors' resilience.

**Fig. 5:** Supply chain network visualizations showing (a) node types and resilience status and (b) attention weights.

minimal impact (0.0%), suggesting that the network is robust against random disruptions at the supply tier.

The disproportionate impact of distributor failures can be explained by the cascade effect, where the disruption propagates through the network according to:

$$P(v_j \text{ affected} | v_i \text{ disrupted}) = 1 - \prod_{p \in P_{ij}} (1 - P(p \text{ affected})) \quad (82)$$

where  $P_{ij}$  is the set of all paths from  $v_i$  to  $v_j$ . Distributors, by virtue of their central position in the supply chain (connecting manufacturers to retailers), affect a larger number of



downstream nodes when disrupted.

2) *Node-Level Impact Analysis*: Beyond the aggregate statistics, we analyzed the node-level impact of disruptions to identify patterns in vulnerability transitions. For each disruption scenario  $s$  and node  $v_i$ , we computed the resilience drop:

$$\Delta \rho_i^s = \rho_i - \rho_i^s \quad (83)$$

where  $\rho_i$  is the original resilience score and  $\rho_i^s$  is the resilience score after disruption  $s$ .

Table II provides a detailed breakdown of the disruption impact metrics across all scenarios.

**TABLE II:** Detailed Impact Metrics for Different Disruption Scenarios

Scenario	Nodes Affected	R→V	V→R	RRP (%)
Random Supplier Failure	3	0	0	0.0
High-Capacity Supplier Failure	7	0	0	0.0
High-Risk Manufacturer Failure	8	0	0	0.0
Central Distributor Failure	15	10	0	12.3
Multiple Transportation Delays	12	7	1	8.7

Interestingly, we observed an asymmetry in the transition probabilities: nodes were much more likely to transition from resilient to vulnerable (R→V) than from vulnerable to resilient (V→R). This asymmetry can be quantified by the transition ratio:

$$TR = \frac{P(R \rightarrow V)}{P(V \rightarrow R)} = \frac{n_{R \rightarrow V}/n_R}{n_{V \rightarrow R}/n_V} \quad (84)$$

where  $n_R$  and  $n_V$  are the original counts of resilient and vulnerable nodes, and  $n_{R \rightarrow V}$  and  $n_{V \rightarrow R}$  are the counts of nodes transitioning from resilient to vulnerable and vice versa, respectively. Across all scenarios, we found  $TR \approx 12.4$ , indicating a strong bias toward resilience degradation rather than improvement under disruption conditions.

#### D. Ablation Studies and Model Variants

To understand the contribution of different components of our framework, we conducted ablation studies by systematically removing or modifying key elements of the model architecture and training process.

1) *Impact of Multi-Head Attention*: Table I already showed that multi-head attention provides a significant performance boost compared to single-head attention (96.30% vs. 93.33% F1 score). To further analyze this improvement, we varied the number of attention heads  $K$  in the first two GAT layers and measured the resulting F1 score:

$$F1(K) = \frac{2 \times \text{Precision}(K) \times \text{Recall}(K)}{\text{Precision}(K) + \text{Recall}(K)} \quad (85)$$

We found that performance increased monotonically with  $K$  up to  $K = 4$ , after which it plateaued or slightly decreased. This suggests that multiple attention heads capture complementary aspects of node relationships, up to a point of diminishing returns.

The improvement can be attributed to the ensemble effect of multi-head attention, where the final representation is an aggregation of information processed through multiple attention mechanisms:

$$\mathbf{h}_i^{(l+1)} = \sigma \left( \frac{1}{K} \sum_{k=1}^K \sum_{j \in N(i) \cup \{i\}} \alpha_{ij}^{(l,k)} \mathbf{W}^{(l,k)} \mathbf{h}_j^{(l)} \right) \quad (86)$$

This aggregation reduces variance in the learned representations, similar to ensemble methods in traditional machine learning.

2) *Node Feature Importance*: To assess the importance of different node features, we employed a permutation-based feature importance method. For each feature  $f$ , we randomly permuted its values across all nodes and measured the resulting decrease in model performance:

$$\text{Imp}(f) = F1_{\text{original}} - E[F1_{f \text{ permuted}}] \quad (87)$$

where the expectation is taken over multiple random permutations.

Fig. 6 illustrates the relative importance of different node features, normalized to sum to 100%.

**Fig. 6:** Relative importance of node features for resilience prediction, based on permutation feature importance. Reliability and risk score are the most influential features, followed by node type.

Reliability emerged as the most important feature (32.5%), followed by risk score (24.8%) and node type (20.6%). Geographical coordinates (Location X and Y) had the least impact on predictions, suggesting that in our synthetic supply chain, physical proximity plays a minor role in determining resilience compared to intrinsic node properties.

3) *Impact of Graph Structure*: To isolate the contribution of the graph structure, we compared our GAT model with a neural network that uses the same node features but ignores the graph structure:

$$\mathbf{h}_i^{(l+1)} = \sigma(\mathbf{W}^{(l)} \mathbf{h}_i^{(l)} + \mathbf{b}^{(l)}) \quad (88)$$

This "structure-blind" model achieved an F1 score of only 0.7143, compared to 0.9630 for our GAT model. This substantial performance gap (24.87 percentage points) highlights the critical importance of network structure in predicting supply chain resilience, confirming our central hypothesis that graph-based modeling is essential for this task.

## VI. DISCUSSION

### A. Theoretical Implications

Our results provide strong empirical support for the critical role of network structure in determining supply chain resilience. The substantial performance gap between graph-based and structure-blind models (24.87 percentage points in F1 score) confirms that resilience is fundamentally a network property that cannot be accurately predicted from node attributes alone.

The relationship between network position and resilience exhibits a trade-off: central nodes benefit from multiple alternative paths but face increased exposure to cascading failures. This can be conceptualized as:

$$R(v_i) = \alpha \cdot \text{Redundancy}(v_i) - \beta \cdot \text{Exposure}(v_i) \quad (89)$$

The superior performance of GAT models, particularly with multi-head attention, highlights the importance of adaptive aggregation in message-passing frameworks. The attention weights provide valuable interpretable insights into supply chain interdependencies, offering information not apparent from standard network analysis.

### B. Practical Implications

Our GAT-based framework offers several practical benefits for supply chain managers. It provides a data-driven approach to resilience assessment with high accuracy (93.33%) and perfect recall (1.0) for resilient nodes. The model can be used for real-time resilience monitoring, "what-if" analysis of disruption scenarios, and optimal resource allocation for resilience enhancement.

The finding that central distributor failures cause the most significant resilience reduction (12.3%) suggests these nodes should be prioritized. Our framework can also guide the design of new, more resilient network structures by identifying factors that contribute to node-level resilience.

### C. Limitations and Future Work

Key limitations include the use of synthetic data and treatment of supply chain structure as static. Future research should explore temporal graph neural networks, enhanced model architectures (edge-conditioned attention, hierarchical pooling), uncertainty quantification, and prescriptive analytics for resilience enhancement.

## VII. CONCLUSION

This paper introduced a novel Graph Attention Network (GAT) based framework for modeling and predicting supply chain resilience against multi-modal disruptions. By leveraging the inherent graph structure of supply chains and employing attention mechanisms, our approach offers significant advantages in both predictive accuracy and interpretability.

Our key contributions include: (1) developing a graph-based representation of supply chains capturing both entity attributes and network topology; (2) implementing a GAT model that achieved 93.33% accuracy and 0.9630 F1 score; (3) leveraging attention mechanisms to identify critical dependencies; (4) developing a methodology for analyzing disruption impacts; and (5) demonstrating practical applications through case studies.

Key findings revealed that: network structure significantly impacts resilience prediction; multi-head attention effectively captures complex dependencies; node reliability and risk score are the most important features; central distributors play a critical role in maintaining resilience; and there exists a strong bias toward resilience degradation under disruptions.

Future work should focus on validating with real-world data, modeling temporal dynamics, incorporating advanced GNN architectures, quantifying prediction uncertainty, developing prescriptive analytics for resilience enhancement, and exploring cross-domain applications.

In conclusion, our GAT-based framework represents a significant advancement in supply chain resilience modeling, offering both theoretical insights and practical tools for enhancing resilience in an increasingly volatile global business environment.

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