

A Fuzzy Set and Application of Fuzzy Multi Criteria Decision Making on Vendor Selection

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ABSTRACT

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Vendor selection plays a pivotal role in supply chain management, influencing the efficiency and effectiveness of operations. Traditional vendor selection methods often rely on crisp decision-making frameworks, which may oversimplify complex decision scenarios and neglect uncertainties inherent in real-world situations. This paper explores the application of fuzzy set theory as a novel approach to vendor selection, aiming to enhance decision-making processes in supply chain management.

Drawing from the principles of fuzzy set theory, which allow for the representation of vague or imprecise information, this study proposes a framework that accommodates the inherent uncertainties and ambiguities associated with vendor selection criteria. By employing fuzzy logic and membership functions, the proposed framework enables decision makers to express the degree of membership of vendors to predefined criteria in a more flexible and nuanced manner.

In conclusion, this paper contributes to the ongoing discourse on vendor selection methodologies by introducing a novel approach grounded in fuzzy set theory. By embracing uncertainty and ambiguity, the proposed framework offers a more realistic and adaptable decision-making tool for supply chain practitioners, paving the way for further advancements in the field. A numerical example has been presented to support the proposed model.

Keywords: Multi Criteria Decision Making; Vendor selection; Fuzzy set; Membership function

1. Introduction

In the dynamic landscape of supply chain management, vendor selection stands as a critical process influencing the overall performance and competitiveness of organizations. Effective vendor selection entails identifying and partnering with suppliers that align closely with organizational objectives, offering optimal quality, reliability, and cost-effectiveness. Traditionally, vendor selection has been approached through crisp decision-making frameworks, wherein suppliers are evaluated based on precise, predetermined criteria. However, such deterministic approaches often fail to capture the inherent uncertainties and complexities of real-world decision scenarios, leading to suboptimal outcomes and missed opportunities for improvement.

In recent years, scholars and practitioners alike have recognized the limitations of conventional vendor selection methods and have sought alternative approaches capable of accommodating

uncertainty and ambiguity. Fuzzy set theory has emerged as a promising paradigm for addressing these challenges, offering a flexible and nuanced framework for decision making in uncertain environments. Rooted in the work of Lotfi A. Zadeh in the 1960s, fuzzy set theory as an extension of classical set theory by allowing for the representation of vague or imprecise information through the concept of membership functions. By quantifying the degree of membership of elements to sets, fuzzy set theory enables decision makers to model and reason with uncertain, subjective, or incomplete data characteristic particularly relevant to vendor selection processes.

This paper seeks to explore the application of fuzzy set theory in the context of vendor selection, with the aim of enhancing decision-making processes and improving the overall efficiency and effectiveness of supply chain management. By embracing uncertainty and ambiguity, fuzzy set theory offers a more realistic and adaptable approach to vendor evaluation, enabling decision makers to capture the nuances and complexities inherent in supplier selection criteria. Through a comprehensive review of existing literature on vendor selection methods and fuzzy set theory, this paper establishes the theoretical foundation for the proposed framework. Building upon this theoretical framework, the paper outlines a methodology for applying fuzzy set theory in vendor selection, encompassing data collection, membership function definition, and decision-making processes.

2. Literature Review:

Vendor selection is a critical component of supply chain management, influencing the overall performance and competitiveness of organizations across various industries. Traditional vendor selection methods typically rely on deterministic decision-making frameworks, wherein suppliers are evaluated based on crisp, predefined criteria such as cost, quality, delivery time, and reliability. While these methods offer a structured approach to supplier evaluation, they often overlook the inherent uncertainties and complexities present in real-world decision scenarios.

Numerous studies have highlighted the limitations of traditional vendor selection methods, particularly in contexts characterized by ambiguity, subjectivity, and incomplete information. For instance, Lee and Kim (2017) emphasized the need for more flexible and adaptive decision-making approaches that can accommodate uncertainties in supplier performance and market dynamics. Similarly, Chen et al. (2019) argued that conventional vendor selection models fail to capture the qualitative aspects of supplier relationships, such as trust, communication, and cultural fit, which are essential for long-term collaboration and value creation.

To address these challenges, scholars and practitioners are increasingly exploring alternative decision-making approaches, such as fuzzy set theory, for handling the complexities of vendor selection in uncertain environments. First introduced by Zadeh in 1965 [1], fuzzy set theory offers mathematical tools to represent and reason with vague or imprecise information. By facilitating gradual transitions between membership and non-membership in sets, fuzzy set theory empowers decision-makers to effectively model and analyze intricate and uncertain decision scenarios.

Fuzzy set theory's application to supply chain management and vendor selection has been the subject of numerous studies. For several parameters in VSP, Zimmerman [4] (1986) used a weighted linear technique. A multi-objective MIP was developed by Weber and Current [12] (1993) for vendor selection and order distribution among chosen vendors. For supplier evaluation, Min (1994) presented an MCDM approach based on utility theory. The analytical hierarchy process (AHP) was employed by Barbarosoglu and Yazgac (1997)[11] and Narasimha (1983) in their vendor selection processes. A fuzzy multi-criteria decision-making approach was presented by Kumar and Vrat [10] (2017) for vendor selection, taking reputation, financial stability, and product quality into account. Similar to this, Ghodsypour and O'Brien (2019) created a fuzzy analytic hierarchy process (AHP) model that incorporates linguistic variables and subjective assessments for vendor evaluation.

Despite the growing interest in fuzzy set theory and its applications in vendor selection, there remains a need for further research to explore its potential benefits and limitations in real-world settings. This paper seeks to contribute to this ongoing discourse by investigating the effectiveness of fuzzy set theory in enhancing decision-making processes in vendor selection, with a focus on its practical implications for supply chain management.

3. Theoretical Framework:

Fuzzy set theory offers a robust framework for handling uncertain, ambiguous, or imprecise information, which is especially pertinent to vendor selection in supply chain management. Fundamentally, fuzzy set theory enhances classical set theory by permitting gradual transitions between membership and non-membership in sets. This capability allows decision-makers to address the subtleties and complexities present in real-world decision-making situations.

The key components of fuzzy set theory include Linguistic variable, fuzzy sets, membership functions, convex fuzzy set, fuzzy number and fuzzy logic.

3.1 Linguistic variable:

Linguistic variables are those whose values are sentences in real or synthetic languages. For example, phrases like tall, very tall, very very tall, somewhat tall, not very tall, tall but not very tall, very tall, and more or less tall could be included in the values of the fuzzy variable "height". It is generally accepted that "tall" is a primary term or linguistic value.

3.2 Definition of fuzzy sets:

According to conventional set theory, an element can either belong to or not belong to a set. On the other hand, fuzzy set theory introduces the concept of "fuzziness" or "degree of membership," which softens this sharp boundary. A fuzzy set is a group of elements that are identified by a membership function that gives each element a degree of membership that indicates how much of the set's attributes it possesses.

Mathematically, a fuzzy set \tilde{A} in a universe of discourse X is defined by its membership function $\mu_{\tilde{A}}(x)$, where x represent an element of X and $\mu_{\tilde{A}}(x)$ denotes the degree of membership of x in \tilde{A} .

i.e $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X \text{ where } \mu_{\tilde{A}}(x), \text{ is termed 'the degree of membership' of } x \text{ in } \tilde{A}\} [1]$.

3.3 Membership function :

Membership function play a crucial role in fuzzy set theory as they determine the degree to which an element belongs to a fuzzy set. The functions map elements from the universe of discourse to real number in the interval $[0,1]$ representing the degree of membership. Various types of membership function can be employed including triangular, trapezoidal, Guassian, sigmoidal functions, each suited to different modelling scenarios based on the characteristics of the underlying data.

Features of the membership function:

The three primary fundamental components of the characteristic membership function are

1. Core: It is the collection of all points where $\{x: \mu_{\tilde{A}}(x) = 1\}$. It could be a null set.
 2. Support: The collection of all positions where $\{x: \mu_{\tilde{A}}(x) > 0\}$
 3. Boundary: The set of all points such that $\{x: 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$.
- The boundary elements have partial membership in the fuzzy set \tilde{A} .

3.4 Concepts of Fuzzy numbers

Let $X = \{x\}$ denote a space of objects. Then a fuzzy set \tilde{A} in X is a set of ordered pairs $\tilde{A} = \{x, \mu_{\tilde{A}}(x)\}$, $x \in X$ where $\mu_{\tilde{A}}(x)$ is termed 'the grade of membership' of x in \tilde{A} . We shall assumed that $\mu_{\tilde{A}}(x) \rightarrow [0,1]$ with grade 1 and 0 representing respectively full membership and non-membership in the fuzzy set \tilde{A} . A fuzzy number is defined as a normal and convex fuzzy set $\tilde{A} \subset \mathbb{R}$ i.e a fuzzy set \tilde{A} satisfying the two following properties:

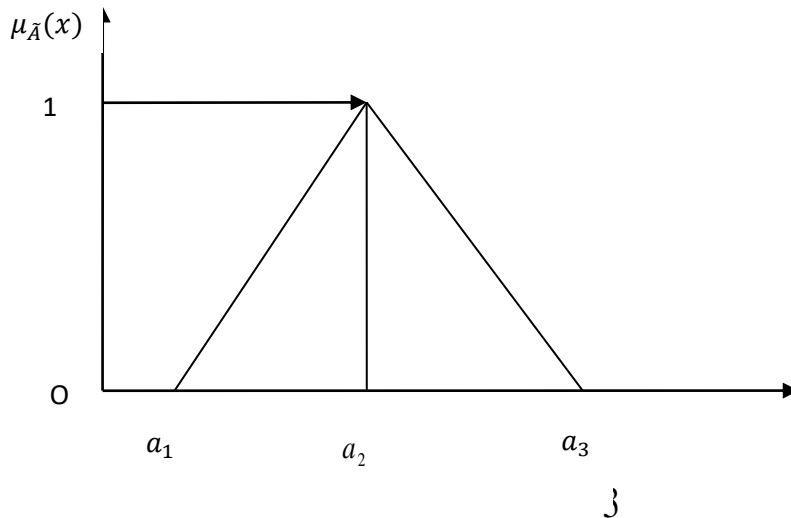
- i) $\mu_{\tilde{A}}(x) = 1$, for at least one $x \in \mathbb{R}$
- ii) Every ordinary subset $A_\alpha = \{x, \mu_{\tilde{A}}(x) \geq \alpha\}$, $\alpha \in [0,1]$ is convex
i.e $\forall x_1, x_2 \in \mathbb{R}$ if $\mu_{\tilde{A}}(x)[\lambda x_1 + (1 - \lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \cap \mu_{\tilde{A}}(x_2) \forall \lambda \in [0,1]$

A fuzzy set is considered normal when at least one of its elements reaches the maximum possible membership grade. i.e $\forall x \in \mathbb{R}, \mu_{\tilde{A}}(x) = 1$ where V stands for maximum.

A convex and normalized fuzzy set defined on \mathbb{R} with a piecewise continuous membership function is called a fuzzy number.

3.5 Triangular fuzzy numbers: A fuzzy number \tilde{A} is a triangular fuzzy number (TFN) denoted by (a_1, a_2, a_3) such that $(a_1 \leq a_2 \leq a_3)$ and if its membership function $\mu_{\tilde{A}}$ is given by :

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$



3.6 Defuzzification : It is the procedure in fuzzy systems that translate the aggregated fuzzy output into a specific crisp value.

Some widely used Methods of defuzzification include:

- 1) Max membership principle
- 2) Centroid method
- 3) Weighted average method
- 4) Mean max membership
- 5) Signed Distance Method.

In the context of vendor selection, fuzzy set theory can be applied to represent and evaluate supplier performance across multiple criteria, such as cost, quality, delivery time, and reliability. By defining appropriate membership functions for each criterion and aggregating, decision makers can make more informed and robust decisions, considering the inherent uncertainties and trade-offs involved in supplier selection.

This paper proposes a theoretical framework for applying fuzzy set theory in vendor selection, drawing upon the principles of fuzzy set theory, membership functions, and fuzzy logic. Building upon this theoretical foundation, the paper outlines a methodology for integrating fuzzy set theory into the vendor selection process, encompassing data collection, membership function definition, and decision-making processes. Through empirical validation and case studies, the effectiveness and practical implications of the proposed framework will be explored, offering insights into its potential benefits and limitations for supply chain management practitioners.

1. Model Development:

When there is only one decision maker, a finite set of vendors $S_1, S_2, S_3, \dots, S_n$ and a variety of decision criteria $G_1, G_2, G_3 \dots G_m$ each with different levels of importance, the task is to choose the best vendor. The decision maker evaluates how well each vendor meets the criteria using descriptive language. The procedure can be summarized in the following steps.

- (a) Express the linguistic terms provided by the decision maker as triangular fuzzy numbers as $\tilde{R}_{i_1}, \tilde{R}_{i_2}, \tilde{R}_{i_3} \dots \tilde{R}_{i_n}$ for each Vendor $S_i, i=1,2,\dots,n$

(b) Given that the different criteria have varying levels of importance, which are also described using linguistic terms such as most important objective, important objective, less important, not at all important, etc., these should be expressed as triangular fuzzy numbers as $\tilde{t}_1, \tilde{t}_2, \tilde{t}_3 \dots \tilde{t}_m$

(c) Calculate the ratio of each triangle fuzzy number to the composite triangular number to find the relative weights. For instance, if $\tilde{t}_1 = (s_1, s_2, s_3)$

$\tilde{t}_2 = (p_1, p_2, p_3) \dots \tilde{t}_m = (q_1, q_2, q_3)$ are m triangular fuzzy number (TFN), then the composite TFN is given by

$$\tilde{T} = \tilde{t}_1 + \tilde{t}_2 + \dots + \tilde{t}_m$$

Or

$$\tilde{T} = (s_1 + p_1 + \dots + q_1, s_2 + p_2 + \dots + q_2, s_3 + p_3 + \dots + q_3)$$

Then relative weights are

$$w_1 = \frac{\tilde{t}_1}{\tilde{T}}$$

$$w_2 = \frac{\tilde{t}_2}{\tilde{T}}$$

$$w_3 = \frac{\tilde{t}_3}{\tilde{T}}$$

Since division operations on triangular fuzzy numbers (TFNs) may not yield another TFN, we can approximate the results as needed.

$$W^*_1 = \left(\frac{s_1}{s_3 + p_3 + \dots + q_3}, \frac{s_2}{s_2 + p_2 + \dots + q_2}, \frac{s_3}{s_1 + p_1 + \dots + q_1} \right)$$

Similarly we find $W^*_2, W^*_3, W^*_4 \dots W^*_m$

(d) Calculate the defuzzified value for each relative weight. If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number, then its defuzzified value is obtained using the sign distance defuzzification method and is given by $\tilde{A} = \frac{a_1 + 2a_2 + a_3}{4}$

Let $\tilde{W}_1, \tilde{W}_2, \dots, \tilde{W}_m$ be the corresponding defuzzified value to each relative weights .

(e). Find

$$G^{\tilde{W}_1} = \{\tilde{R}_{1_1}, \tilde{R}_{2_1}, \dots, \tilde{R}_{n_1}\}$$

$$G^{\tilde{W}_2} = \{\tilde{R}_{1_2}, \tilde{R}_{2_2}, \dots, \tilde{R}_{n_2}\}$$

$$G^{\tilde{W}_3} = \{\tilde{R}_{1_3}, \tilde{R}_{2_3}, \dots, \tilde{R}_{n_3}\}$$

.....

$$G_m^{\tilde{W}_m} = \{ \tilde{R}_{1m}^{\tilde{W}_m}, \tilde{R}_{2m}^{\tilde{W}_m}, \dots, \tilde{R}_{n_m}^{\tilde{W}_m} \}$$

Then decision model is

$$D = G_1^{\tilde{W}_1} \cap G_2^{\tilde{W}_2} \dots G_m^{\tilde{W}_m}$$

The decision D represents a fuzzy subset of the possible systems, and the membership function $\mu_D(x)$ shows how well each system satisfies the set of goals.

- (f) The value triangular fuzzy numbers obtained is defuzzified using centroid method
- (g) Perform linear ordering of the weighted choice values and select the system S_i with the highest weighted choice value.

4. Application and Results:

The developed framework for applying fuzzy set theory in vendor selection is implemented in the selected case studies. Using the defined membership functions, supplier performance is evaluated across multiple criteria, including cost, quality, delivery time, reliability etc. The application of the framework enables decision makers to model and analyze complex decision scenarios, considering the uncertainties and trade-offs inherent in supplier selection processes.

Let $X = \{S_1, S_2, S_3, S_4, S_5\}$ be a set of Vendors, fuzzy set theory makes it easier for four experts X_1, X_2, X_3, X_4 to evaluate anything based solely on linguistic manner in vendor selection. The different criteria are as follows[8]

- G_1 : Ordering cost, G_2 : On time delivery cost
- G_3 : Order delivery time, G_4 : Transportation Cost
- G_5 : Inventory holding cost, G_6 : Financial position Cost
- G_7 : Flexibility in service G_8 : percentage of warranty claim
- G_9 : Average response time G_{10} : Percentage of sustainable

Next, we translate these language terms into triangular fuzzy numbers as follows, coupled with the language terms indicating the significance of the criteria for the vendors.

Table 1: Linguistic terms and their corresponding fuzzy numbers

Linguistic terms	Membership Value	Triangular fuzzy numbers
VP(Very poor)	0	(0,0,0.2)

P(poor)	0.1	(0.1,0.2,0.3)
BA(below average)	0.3	(0.2, 0.3,0.5)
A(Average)	0.5	(0.4, 0.5, 0.6)
AA(Above average)	0.7	(0.5, 0.6, 0.8)
G(Good)	0.9	(0.7, 0.8,.9)
VG(Very good)	1	(0.8, 0.9, 1)

Table 2: Linguistic terms associated with the importance of the Criteria and their corresponding fuzzy numbers.[2]

Linguistic terms	Triangular fuzzy numbers
MI(Most important)	$(0.7, 0.9, 1) = \tilde{t}_1$
VI(Very important)	$(0.6, 0.8, 0.9) = \tilde{t}_2$
I(Important)	$(0.4, 0.5, 0.6) = \tilde{t}_3$
LI(less important)	$(0.2, 0.3, 0.4) = \tilde{t}_4$
NI(Not so important)	$(0.1, 0.2, 0.3) = \tilde{t}_5$

Table-3: Linguistic variables and fuzzy numbers assigned by the experts in connection with the criteria

Criteria	EXPERT			
	X ₁	X ₂	X ₃	X ₄
G ₁	MI(0.7,0.9,1)	VI(0.6,0.8,0.9)	MI(0.7,0.9,1)	I(0.4,0.5,0.6)
G ₂	MI(0.7,0.9,1)	MI(0.7,0.9,1)	MI(0.7,0.9,1)	VI(0.6,0.8,0.9)
G ₃	VI(0.6,0.8,0.9)	VI(0.6,0.8,0.9)	VI(0.6,0.8,0.9)	VI(0.6,0.8,0.9)
G ₄	MI(0.7,0.9,1)	I(0.4,0.5,0.6)	I(0.4,0.5,0.6)	NI(0.4,0.5,0.6)
G ₅	MI(0.7,0.9,1)	MI(0.7,0.9,1)	I(0.4,0.5,0.6)	I(0.4,0.5,0.6)
G ₆	I(0.4,0.5,0.6)	LI(0.2,0.3,0.4)	LI(0.2,0.3,0.4)	I(0.4,0.5,0.6)
G ₇	NI(0.1,0.2,0.3)	LI(0.2,0.3,0.4)	I(0.4,0.5,0.6)	LI(0.2,0.3,0.4)
G ₈	I(0.4,0.5,0.6)	NI(0.1,0.2,0.3)	LI(0.2,0.3,0.4)	I(0.4,0.5,0.6)
G ₉	MI(0.7,0.9,1)	I(0.4,0.5,0.6)	I(0.4,0.5,0.6)	I(0.4,0.5,0.6)
G ₁₀	NI(0.1,0.2,0.3)	NI(0.1,0.2,0.3)	LI(0.2,0.3,0.4)	LI(0.2,0.3,0.4)

Table-4: Normalized weights associated with the criteria given by the experts

Criteria	EXPERT			Defuzzified value	Normalised weight
	Average fuzzy number				
G ₁	0.6	0.775	0.875	0.756	0.1345314(W1)
G ₂	0.675	0.875	0.975	0.85	0.152152 (W2)

G ₃	0.6	0.8	0.9	0.775	0.13844 (W3)
G ₄	0.475	0.6	0.7	0.593	0.106208 (W4)
G ₅	0.55	0.7	0.8	0.687	0.123046 (W5)
G ₆	0.3	0.4	0.5	0.4	0.071085 (W6)
G ₇	0.225	0.325	0.425	0.325	0.057373 (W7)
G ₈	0.275	0.375	0.475	0.375	0.066515 (W8)
G ₉	0.475	0.6	0.7	0.593	0.106208 (W9)
G ₁₀	0.15	0.25	0.35	0.25	0.04366 (W10)

Table -5: Linguistics variables for alternatives.[8]

Criteria	Vendors	Experts			
		X1	X2	X3	X4
G1	S1	G	G	VG	G
	S2	VG	G	G	VG
	S3	A	AA	A	A
	S4	A	A	AA	AA
	S5	AA	A	A	A
G2	S1	VG	VG	G	VG
	S2	G	G	VG	G
	S3	G	VG	G	G
	S4	G	G	G	VG
	S5	G	G	G	G
G3	S1	VG	G	VG	G
	S2	G	VG	G	G
	S3	G	AA	AA	G
	S4	A	A	A	AA
	S5	G	G	AA	AA
G4	S1	VG	G	VG	G
	S2	G	AA	G	G
	S3	A	A	A	AA
	S4	G	VG	G	AA
	S5	G	G	G	AA
G5	S1	G	G	G	VG
	S2	A	G	A	AA
	S3	G	G	G	G
	S4	VG	VG	G	G
	S5	VG	G	VG	G
G6	S1	VG	VG	VG	G
	S2	A	A	P	A
	S3	A	A	A	A

	S4	VG	VG	G	VG
	S5	VG	G	VG	VG
G7	S1	G	VG	VG	G
	S2	P	A	A	P
	S3	P	A	A	P
	S4	G	VG	G	VG
	S5	VG	VG	G	G
G8	S1	G	G	G	G
	S2	A	AA	VG	G
	S3	G	G	P	AA
	S4	VG	VP	VG	G
	S5	VG	G	P	VG
G9	S1	VG	P	AA	G
	S2	A	G	VP	A
	S3	A	G	BA	AA
	S4	AA	A	A	A
	S5	AA	AA	G	A
G10	S1	G	G	G	P
	S2	AA	BA	G	G
	S3	AA	VP	AA	A
	S4	P	G	VP	G
	S5	AA	BA	AA	G

Table 6: Average fuzzy score and defuzzified score:

Criteria	Vendors	Average fuzzy score			Defuzzified score
G1	S1	0.55	0.755	1	0.765
	S2	0.6	0.75	1	0.775
	S3	0.05	0.316	0.575	0.31425
	S4	0.1	0.4	0.65	0.3875
	S5	0.05	0.316	0.575	0.31425
G2	S1	0.65	0.925	1	0.875
	S2	0.55	0.775	1	0.775
	S3	0.55	0.775	1	0.775
	S4	0.55	0.775	1	0.775
	S5	0.5	0.7	1	0.725
G3	S1	0.6	0.85	1	0.825
	S2	0.55	0.775	1	0.755
	S3	0.35	0.6	0.7	0.5625
	S4	0.2	0.35	0.575	0.36875

	S5	0.35	0.6	0.7	0.5625
G4	S1	0.625	0.85	1	0.83125
	S2	0.425	0.65	0.95	0.66875
	S3	0.05	0.35	0.575	0.33125
	S4	0.475	0.725	0.95	0.71875
	S5	0.05	0.35	0.575	0.33125
G5	S1	0.55	0.725	1	0.75
	S2	0.175	0.45	0.7	0.44375
	S3	0.5	0.7	1	0.725
	S4	0.6	0.85	1	0.825
	S5	0.6	0.85	1	0.825
G6	S1	0.65	0.925	1	0.875
	S2	0	0.25	0.425	0.23125
	S3	0	0.3	0.5	0.275
	S4	0.65	0.925	1	0.875
	S5	0.65	0.925	1	0.875
G7	S1	0.6	0.85	1	0.825
	S2	0	0.2	0.35	0.1875
	S3	0	0.2	0.35	0.1875
	S4	0.6	0.85	1	0.825
	S5	0.6	0.85	1	0.825
G8	S1	0.5	0.7	1	0.725
	S2	0.35	0.625	0.825	0.60625
	S3	0.3	0.45	0.75	0.4875
	S4	0.475	0.675	0.775	0.65
	S5	0.475	0.7	0.8	0.66875
G9	S1	0.35	.575	0.75	0.5625
	S2	0.125	0.325	0.525	0.325
	S3	0.125	0.375	0.65	0.38125
	S4	0.05	0.35	0.575	0.33125
	S5	0.225	0.5	0.775	0.5
G10	S1	0.375	0.55	0.8	0.56875
	S2	0.3	0.475	0.775	0.50625
	S3	0	0.325	0.6	0.3125
	S4	0.25	0.375	0.575	0.39375
	S5	0.3	0.425	0.6	0.4375

Now let us find

$$\tilde{G}_1^{W_1} = \{(0.955685, 0.97325, 0.989072), (0.960263, 0.977354, 0.992796), (0.886384, 0.913181, 0.941088), (0.8935543, 0.9191883, 0.9509699), (0.886384, 0.913181, 0.9410883)\}$$

$$\tilde{G}_2^{W_2} = \{(0.960453, 0.97908, 0.9960002), (0.950367, 0.9700066, 0.987734), (0.950367, 0.970006, 0.9877345), (0.9503675, 0.970006, 0.98773), (0.9451029, 0.96529, 0.98345)\}$$

$$\tilde{G}_3^{W_3} = \{0.9592, 0.9767, 0.99261\}, (0.954567, 0.972568, 0.988791), (0.92880, 0.949736, 0.976775), (0.88362, 0.911042, 0.93430), (0.928803, 0.949736, 0.9767758)\}$$

$$\tilde{G}_4^{W_4} = \{0.968, 0.98215, 0.9943\}, (0.9533, 0.968, 0.98531), (0.90953, 0.93108, 0.95338), (0.95737, 0.97214, 0.98839), (0.953383, 0.9686, 0.985311)\}$$

$$\tilde{G}_5^{W_5} = \{(0.959620, 0.975645, 0.992100), (0.914992, 0.93662, 0.95962), (0.955314, 0.97180, 0.986586), (0.963799, 0.979385, 0.993447), (0.96379, 0.979385, 0.993447)\}$$

$$\tilde{G}_6^{W_6} = \{(0.98101, 0.99001, 0.99809), (0.91897, 0.93769, 0.9527), (0.93342, 0.9492, 0.96232), (0.9810187, 0.990010, 0.9980983), (0.98101, 0.99001, 0.998098)\}$$

$$\tilde{G}_7^{W_7} = \{(0.98247, 0.990059, 0.996851), (0.91831, 0.9375, 0.95210), (0.98247, 0.990059, 0.9968519), (0.982471, 0.990059, 0.99685), (0.982471, 0.990059, 0.9968519)\}$$

$$\tilde{G}_8^{W_8} = \{(0.9751, 0.98436, 0.99258), (0.96457, 0.975128, 0.98650), (0.96167, 0.97262, 0.982162), (0.961677, 0.972627, 0.982162), (0.964572, 0.975128, 0.984367)\}$$

$$\tilde{G}_9^{W_9} = \{0.931083, 0.949248, 0.964990\}, (0.897005, 0.915313, 0.94051), (0.90953, 0.935895, 0.96124), (0.90953, 0.931083, 0.953383), (0.93108, 0.949248, 0.9721485)\}$$

$$\tilde{G}_{10}^{W_{10}} = \{0.97185, 0.97963, 0.986354\}, (0.9696, 0.97780, 0.987900), (0.951098, 0.95995, 0.97589), (0.954237, 0.96258, 0.977803), (0.95995, 0.96743, 0.9796368)\}$$

Now

$$\begin{aligned} \tilde{G} &= \tilde{G}_1^{W_1} \cap \tilde{G}_2^{W_2} \cap \tilde{G}_3^{W_3} \cap \tilde{G}_4^{W_4} \cap \dots \cap \tilde{G}_{10}^{W_{10}} \\ &= \{(0.9310, 0.94924, 0.96499), (0.897005, 0.915313, 0.940518), (0.886384, 0.913181, 0.94108), (0.88362, 0.91104, 0.934301), (0.8863, 0.913181, 0.94108)\} \end{aligned}$$

$$\text{Associated ordinary number } (S_1) = \frac{0.9310+2 \times 0.94924+0.96499}{4} = 0.9486$$

$$\text{Associated ordinary number } (S_2) = \frac{0.897005+0.915313+0.940518}{4} = 0.91703$$

$$\text{Associated ordinary number } (S_3) = \frac{0.886384+0.913181+0.94108}{4} = 0.91345$$

$$\text{Associated ordinary number } (S_4) = \frac{0.88362+0.91104+0.9343}{4} = 0.91004$$

$$\text{Associated ordinary number } (S_5) = \frac{0.8863+0.9131+0.9410}{4} = 0.91356$$

Final ranking:

Vendors	S_1	S_2	S_3	S_4	S_5
Final scores	0.9486	0.91703	0.91345	0.91004	0.91356
Rank	1	2	4	5	3

5.2 Results:

Using the fuzzy decision approach, the ranking of the vendors is as follows: $S_1 > S_2 > S_5 > S_3 > S_4$. The result here obtained is the best Vendor for the company.

The results of the case study analysis demonstrate the effectiveness of the proposed framework in enhancing decision-making processes in vendor selection. Sensitivity analysis and scenario testing reveal the robustness and reliability of the framework under different conditions, highlighting its flexibility and scalability in real-world settings.

5. Discussion:

6.1 Comparison with Existing Literature:

The results of the study are compared with existing literature on vendor selection methods and fuzzy set theory applications. Consistencies, discrepancies, and novel insights are identified, contributing to the ongoing discourse on decision-making paradigms in supply chain management. The discussion contextualizes the findings within the broader theoretical and practical landscape, elucidating the contributions and implications of the research for academia and industry.

6.2 Theoretical Implications:

The study contributes to the theoretical advancement of vendor selection methodologies by integrating fuzzy set theory into the decision-making process. Theoretical implications include extending classical decision-making paradigms to accommodate uncertainty and ambiguity, enhancing the realism and applicability of decision models in dynamic and uncertain environments. The study also underscores the importance of interdisciplinary approaches in supply chain management research, bridging the gap between theoretical insights and practical applications.

6. Conclusion:

Conventional outcomes often fail to encompass the complexities of human cognition. Consequently, experts frequently prefer to express judgments in a fuzzy rather than a precise manner. Vendor selection dilemmas typically entail uncertain and imprecise data, rendering fuzzy set theory well-suited for tackling such challenges. The utilization of linguistic variables in decision-making proves particularly beneficial when performance metrics cannot be precisely quantified. Future research efforts might concentrate on refining methodologies for more efficient resolution of vendor selection issues and the development of group decision support systems in fuzzy environments.

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