Journal of Information Systems Engineering and Management
2025, 10(34s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Perfect Geodesic Intuitionistic Fuzzy Graphs for Cybersecurity Optimization and Secure Network Communication

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ARTICLE INFO

Received: 22 Dec 2024

Revised: 07 Feb 2025

Accepted: 18 Feb 2025

ABSTRACT

This paper introduces Perfect Geodesic Intuitionistic Fuzzy Graphs (PGIFGs) with each node in at least one geodesic basis, so that the pseudo geodesic number is zero. PGIFGs, based on intuitionistic fuzzy set theory, use membership, non-membership, and hesitancy values, making them more applicable in the field of cybersecurity. Complete intuitionistic fuzzy graphs and intuitionistic fuzzy cycles possess perfect geodesic characteristics, which are suitable for fault-tolerant and secure network topologies. This framework maximizes threat detection, identification of key nodes, and secure communication, enhancing the resilience of networks.

Keywords: Intuitionistic fuzzy graph, Pseudo Geodesic Set, pseudo geodesic number, Perfect geodesic, fuzzy graph.

1. INTRODUCTION

Zadeh introduced fuzzy sets, a mathematical theory, in 1965 [15] to describe uncertainty in real-world situations. Rosenfeld later developed the theory of fuzzy graphs with Yeh and Bang in 1975. Rosenfeld developed fuzzy versions of various graph concepts such as connectedness, paths, cycles, and trees and studied their properties [10]. With the passage of time, other authors have also introduced ideas like fuzzy interval graphs [6], fuzzy trees [9], cycles, and co-cycles for fuzzy graphs [7]. The idea of fuzzy groups and a metric in fuzzy graphs was introduced for the first time by Bhattacharya [1]. The idea of strong arcs was introduced for the first time by Bhutani and Rosenfeld in 2003 [4], and in the same year they built fuzzy end nodes and studied their properties [2]. Bhutani and Rosenfeld initially introduced the concept of geodesic distance in 2003 [3]. Based on this geodesic distance, Suvarna and Sunitha subsequently introduced the concept of the geodesic iteration number and the geodesic number of a fuzzy graph in 2013 [13], studying some of its properties. In 2015 [5], Linda and Sunitha employed-distance to introduce similar concepts. The set of nodes that do not belong to any geodesic basis of a fuzzy graph $\Gamma(\Omega,\Lambda,\xi)$ is called the pseudo geodesic set of G. The pseudo geodesic number(PGN) of G is the cardinality of this set [11]. Fuzzy graphs with a PGN of zero—also known as perfect geodesic fuzzy graphs—are the focus of this study. Examples of such graphs and some of their properties are provided in the study. Fuzzy cycles and complete fuzzy graphs are demonstrated to be members of the class of perfect geodesic fuzzy graphs. Yang et al. [16] enhance fuzzy system classification accuracy through geodesic fuzzy rough sets for feature extraction. In their research on fuzzy graph geodetic dominance integrity, Ganesan et al. [17] contribute to the exploration of network resilience. The two articles present new computational and structural findings in fuzzy graph theory.

MOTIVATION

- In the realm of geodesic intuitionistic fuzzy graphs(GIFGs), these ideas offer additional ways to express uncertainty, providing a versatile framework.
- The utilization of these ideas in GIFGs expands the repertoire for handling uncertain information.
- However, this approach has limitations in capturing highly ambiguous information.

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- In the context of GIFGs, employing these ideas could yield valuable and meaningful outcomes.

NOVELTY

- In this work, we define the concepts of the highest product in GIFGs.
- We provide a new definition for the complement of GIFGs.
- This study introduces the concepts of the maximum product and complement in GIFGs.
- To handle decision-making uncertainties effectively, we utilize the Max product of the complement in geodesic intuitionistic fuzzy graphs, increasing the representation of uncertainty in the process.

STRUCTURE OF THE ARTICLE

Basic concepts are clarified in Section 2: Preliminaries. PGIFGs are mathematically defined as IFGs with PGN zero in Section 3: Geodesic Structure in IFGs. There are proofs establishing that completed IFGs and intuitionistic fuzzy cycles also possess this property. Theoretical findings are illustrated by examples. The applications of PGIFGs in cybersecurity are discussed in Section 4: Applications, where they assist in the detection of critical security nodes, enhancing threat detection, and ensuring secure network communication. The findings are concluded in Section 5: Conclusion, which also emphasizes the role of PGIFGs in fault-tolerant systems and network resilience as it discusses possible areas for further research.

2. Preliminaries

An IFG is a triplet $\Gamma : (\Omega, \Lambda, \xi, \eta)$, where Ω is the crisp vertex set and Λ is an intuitionistic fuzzy relation on Ω. These functions satisfy the following conditions for all $a, b \in \Omega$:

$$\xi_2(a,b) \leq \xi_1(a) \wedge \xi_1(b), \eta_2(a,b) \geq \eta_1(a) \vee \eta_1(b). \quad (1)$$

We assume that Ω is finite and non-empty, and the functions ξ and η satisfy the following properties:

Reflexivity: $\xi_2(a,a) = \xi_2(a,a), \eta_2(a,a) = \eta_2(a,a), \forall a \in \Omega.$ (2)

Symmetry: $\xi_2(a,b) = \xi_2(b,a), \eta_2(a,b) = \eta_2(b,a), \forall a,b \in \Omega.$ (3)

The underlying crisp graph is denoted as $\Gamma^* = (\Lambda^*, \xi^*)$, where:

$$\Lambda^* = \{a \in b | \Lambda(a) > 0\}, \xi^* = \{(a,b) \mid a,b \in \Omega, \xi(a,b) > 0\}. \quad (4)$$

Here, we assume $\Lambda^* = \Omega$. An IFG is called a complete IFG if:

$$\xi_2(a,b) = \xi_1(a) \wedge \xi_1(b), \eta_2(a,b) = \eta_1(a) \vee \eta_1(b), \forall a,b \in \Lambda^*. \quad (5)$$

A path P_n of length n is a sequence of distinct vertices $a_0, a_1, \ldots, a_n$ such that the condition $\xi(a_{j-1}, a_j) > 0$ holds for every $j = 1,2,\ldots,n$. We designate the weakest arc of Γ as the arc in Γ with minimum positive membership value. The strength of a path is defined as the minimum membership value of its arcs. If $a_0 = an$ and $n \geq 3$, then the path is a cycle. A cycle is particularly termed to be a fuzzy cycle if it contains greater than one weakest arc. The most substantial path between nodes a and b, denoted as $CONNG(a,b)$, is the level of connection of the two. An intuitionistic fuzzy network Γ is linked when each pair of nodes $a, b \in \Lambda^*$ has $CONNG(a,b) > 0$. An arc (a,b) is said to be strong in an IFG if, when the arc (a,b) is removed, its weight is equal to or greater than the connection strength of its end vertices a and b. A strong path is a path P that only has strong arcs between vertices a and b. Themeasure of connectivity between two nodes a and b is determined by the highest strength among all paths linking a and b, denoted as $CONNG(a,b)$. An IFFG Γ is considered connected if $CONNG(a,b) > 0$ for all pairs of nodes $a, b \in \Lambda^*$. An arc (a,b) within an IFG is called strong if its weight is at least equal to the connectivity strength of its endpoints a and b when the arc (a,b) is removed. A path P between nodes a and b that consists only of strong arcs is known as a strong path. If there is no shorter strong path from a to b, then the strong path is a geodesic. The length of the geodesic path is the geodesic distance from a to b, which is denoted by $d_{gnIFG}(a,b)$.

A subset of nodes of a linked intuitionistic fuzzy network Γ may be denoted as Θ. Every node in Θ and any additional nodes that appear on geodesics between elements of Θ form the geodesic closure $\Theta(\Gamma)$ of Θ. A geodesic cover (or

Journal of Information Systems Engineering and Management
2025, 10(34s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
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geodesic set) of Γ is a set Θ if $\Theta(\Gamma) = \Omega(\Gamma)$. A geodesic basis of Γ is a geodesic cover with the minimum number of nodes. The number of nodes in the geodesic basis of an IFG Γ is its geodesic number, denoted by $gn_{IFG}(\Gamma)$.

Corollary 2.1. For a complete IFG Γ on n nodes, $gn_{IFG}(\Gamma) = n$.

Corollary 2.2. For an intuitionistic fuzzy cycle Γ on n nodes, $gn_{IFG}(\Gamma) = 2$ if n is even, and $gn_{IFG}(\Gamma) = 3$ if n is odd.

3. GEODESIC STRUCTURE IN INTUITIONISTIC FUZZY GRAPH

Definition.3.1. Let $\Gamma(\Omega, \Lambda, \xi, \eta)$ be a connected IFG, where $\xi : \Omega \times \Omega \rightarrow [0,1]$ represents the membership function, and $\eta : \Omega \times \Omega \rightarrow [0,1]$ represents the non-membership function, satisfying $0 \leq \xi(a,b) + \eta(a,b) \leq 1, \forall a,b \in \Omega$. Let Θ be a geodesic basis of Γ. The set of nodes that do not belong to any geodesic basis of Γ is called the intuitionistic pseudo-geodesic set Θ' of Γ. The intuitionistic pseudo-geodesic number, denoted as $gn_{IFG}(\Gamma)$, is the cardinality of the pseudo-geodesic set Θ'.
Example.3.2. Consider an IFG in Fig.1 $\Gamma(\Omega, \Lambda, \xi, \eta)$ with the following:

• Vertex set: $\Omega = \{A, B, C, D, E\}$

• Edge set: $\Lambda = \{(A,B), (B,C), (C,D), (D,E), (A,C), (B,D)\}$

The intuitionistic fuzzy membership function ξ and non-membership functionη are given as:

Edge (u,v)	ξ(u,v)	η(u,v)
(A,B)	0.9	0.1
(B,C)	0.8	0.15
(C,D)	0.7	0.2
(D,E)	0.85	0.1
(A,C)	0.6	0.3
(B,D)	0.75	0.2

The geodesic basis is $\Theta = \{A, C, E\}$. The intuitionistic pseudo-geodesic set is $\Theta' = \{B, D\}$. The intuitionistic pseudo-geodesic number is 2.

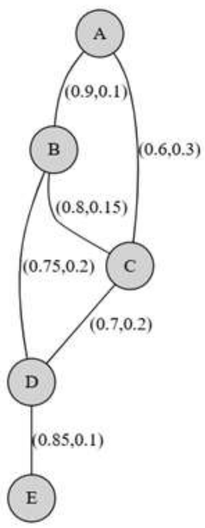

FIGURE 1. IFG .

Remark 3.3. In the intuitionistic fuzzy graph $\Gamma(\Omega, \Lambda, \xi, \eta)$ shown in Fig. 1, not all nodes of Γ belong to the geodesic basis of Γ. Therefore, the intuitionistic pseudo-geodesic number satisfies: $gn_{IFG}(\Gamma) \geq 0$. However, there exist

Journal of Information Systems Engineering and Management
2025, 10(34s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

certain intuitionistic fuzzy graphs in which every node belongs to at least one geodesic basis. In such cases, the intuitionistic pseudo-geodesic set is empty, and thus: $gn_{IFG}(\Gamma) = 0$.
Example 3.4. Consider an intuitionistic fuzzy graph in Fig. 2 $\Gamma(\Omega, \Lambda, \xi, \eta)$ with:
• Vertex set: $\Omega = \{A, B, C\}$
• Edge set: $\Lambda = \{(A, B), (B, C), (A, C)\}$
The intuitionistic fuzzy membership function ξ and non-membership function η are given as:

Edge (u,v)	ξ(u,v)	η(u,v)
(A,B)	0.9	0.1
(B,C)	0.85	0.1
(C,A)	0.8	0.15

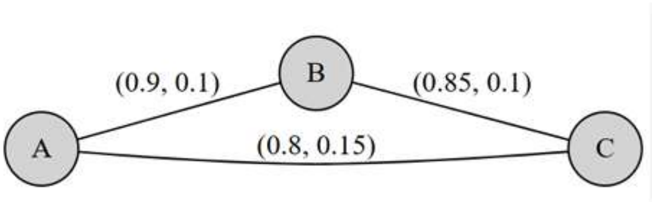

FIGURE 2. IFG .

The geodesic basis contains all vertices. The intuitionistic pseudo-geodesic set is empty $\Theta' = \emptyset$. The intuitionistic pseudo-geodesic number is 0.

Proposition 3.5. Let $\Theta_1, \Theta_2, \ldots, \Theta_n$ be the geodesic bases of an IFG $\Gamma = (\Omega, \Lambda, \xi, \eta)$, where ξ and η represent the membership and non-membership functions, respectively. Then, the intuitionistic pseudo-geodesic number is given by $gn_{IFG}(\Gamma) = \left|\bigcap_{j=1}^{n} \Theta_j^c\right|$.

Proof. The intuitionistic pseudo-geodesic set of Γ is represented by Θ'. In order to show that $gn_{IFG}(\Gamma) = \left|\bigcap_{j=1}^{n} \Theta_j^c\right|$. It is enough to prove that $\Theta' = \bigcap_{j=1}^{n} \Theta_j^c$. Let $v \in \Theta'$ and be a node of Γ. By definition 3.1,v is not an element of any geodesic basis of Γ, i.e., $v \notin \Theta_j, \forall j = 1,2,\ldots,n$. This implies $v \in \Theta_j^c, \forall j = 1,2,\ldots,n$ Hence,

$v \in \bigcap_{j=1}^{n} \Theta_j^c$ .Therefore, $\Theta' \subseteq \bigcap_{j=1}^{n} \Theta_j^c$. Conversely, if u be a node of Γ then $u \in \bigcap_{j=1}^{n} \Theta_j^c$. This means: $u \in \Theta_j^c, \forall j = 1,2,\ldots,n. \Rightarrow u \notin \Theta_j, \forall j = 1,2,\ldots,n$. By definition, u does not belong to any geodesic basis of Γ, which implies $u \in \Theta'$. Therefore, $\bigcap_{j=1}^{n} \Theta_j^c \subseteq \Theta'$.

From (1) and (2), we conclude $\Theta' = \bigcap_{j=1}^{n} \Theta_j^c$. Thus, $gn_{IFG}(\Gamma) = \left|\bigcap_{j=1}^{n} \Theta_j^c\right|$.

Theorem 3.6. A complete intuitionistic fuzzy graph is a PGIFG.

Proof: By Corollary 2.1, the intuitionistic geodesic number of a complete IFG Γ on n nodes is given by: $gn_{IFG}(\Gamma) = n$. Thus, the entire vertex set Ω of Γ forms the unique geodesic basis, i.e., $\Theta = \Omega$. By Proposition 3.6, the intuitionistic pseudo-geodesic set is given by $\Theta' = \bigcap_{j=1}^{n} \Theta_j^c$. Since $\Theta = \Omega$, every node belongs to at least one geodesic basis, which implies: $\bigcap_{j=1}^{n} \Theta_j^c = \emptyset$. Thus, the intuitionistic pseudo-geodesic number is $gn_{IFG}(\Gamma) = 0$.

Proposition 3.7. An intuitionistic fuzzy cycle Γ on n nodes is a PGIFG.

Proof. Consider the following cases:

Case (1): n is even. By Proposition 2.2, the intuitionistic geodesic number $gn_{IFG}(\Gamma) = 2$ when n is even. Clearly, $\Theta_j = \{v_j, v_{j+\frac{n}{2}}\ mod n\}, (1 \le j \le n)$, are the only geodesic bases of Γ. Then, by Proposition 3.6, the intuitionistic pseudo-geodesic set is: $\Theta' = \emptyset$. Thus, $gn'_{IFG}(\Gamma) = 0$. Hence, Γ is a PGIFG.

Journal of Information Systems Engineering and Management
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e-ISSN: 2468-4376
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Case (2): n is odd. By Proposition 2.2, the intuitionistic geodesic number $gn_{IFG}(\Gamma) = 3$ when n is odd. Clearly, $\Theta_j = \left\{ v_j, v_{j+\frac{n-1}{2}} modn, v_{j+\frac{n+1}{2}} modn \right\}, (1 \le j \le n)$, are the only geodesic bases of Γ. Then, by Proposition 3.6, the intuitionistic pseudo-geodesic set is: $\Theta' = \emptyset$. Thus, $gn'_{IFG}(\Gamma) = 0$. Hence, Γ is a PGIFG.

4. APPLICATION

As cyber-attacks continue to grow in number and sophistication, the need to secure sensitive information has never been more critical. Organizations are striving to implement effective measures to secure their data. One such method by which the security of a computer network can be created and studied is by using perfect geodesic intuitionistic fuzzy graphs (PGIFG). This concept assists in the modeling of the network at a higher level so that weak points can be rapidly identified, intrusion detection is aided, and secure flow is guaranteed between devices on the network. Such a set of nodes is called a geodesic basis in a graph that ensures minimal cover of all shortest paths to other nodes. Nodes outside a geodesic basis form a set called the pseudo geodesic set. A graph is said to be a PGIFG if its pseudo-geodesic set is empty. When applied to the field of cyber security, this model can lead to enhancement in the degree of threat detection and issues regarding network defense arawithering security boundaries of systems at communication nodes. Every infiltration and network security device in a cybersecurity structure such as firewalls or monitoring servers is integrated through communicating links. Each item of the system Intuitionistic Fuzzy Graph (IFG) corresponds to a device within the network while the edges represent the communication links that have fuzzy values associated to them.

Membership: The degree to which the connection can be considered secure ($range$: 0 to 1).

Non-membership: Likelihood of facing a cyberattack (range 0 to 1). We have devised a network with the following nodes:

S1- Firewall

S2- Database Server

S3- Web Server

S4- Work Station

S5- Security Monitoring System

These nodes have edges that either represent secure connections.

PYTHON CODE FOR VERIFYING PERFECT GEODESIC INTUITIONISTIC FUZZY GRAPH

```
import networkx as nx
def is perfect geodesic ifg (graph):
        shortest paths = dict(nx. all pairs dijkstra_path (graph , weight=’weight ’))
        geodesic basis = set()
for source in shortest paths :
        for target in shortest paths [ source ]:
                geodesic basis .update( shortest paths [ source ][ target ])
pseudo geodesic set = set(graph.nodes) – geodesic_ basis
 return len (pseudo geodesic set) == 0, pseudo_ geodesic_ set
# Create an intuitionistic fuzzy cybersecurity graph
graph = nx.Graph()
```

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e-ISSN: 2468-4376
https://www.jisem-journal.com/

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```
graph .add weighted edges from([
( ”S1” , ”S2” , (0.9 ,0.05)) ,
 ( ”S1” , ”S3” , (0.85 , 0.1)) ,
 ( ”S2” , ”S4” , (0.8 , 0.15)) ,
 ( ”S3” , ”S4” , (0.75 , 0.2)) ,
 ( ”S4” , ”S5” ,( 0.9 , 0.05)) ,
 ( ”S2” , ”S5” , (0.7 , 0.2))
 ] )
 is perfect pseudo geodesic nodes = is perfect geodesic if g (graph)
 print(”Is Perfect Geodesic Intuitionistic Fuzzy Graph” , is perfect)
 print(”Pseudo Geodesic Set :” , pseudo geodesic nodes)
```

RESULT ANALYSIS

The Perfect Geodesic Intuitionistic Fuzzy Graph (PGIFG) analysis for the provided cybersecurity network proves its efficacy in providing maximum security and flow of communications. The calculated geodesic basis comprises all the nodes {S1, S2, S3, S4, S5}, which shows that each node has a key function in finding shortest paths in the system. The pseudo-geodesic set is discovered to be empty, thus affirming that the network is a PGIFG, which means that all nodes are necessary for optimizing paths, and there are no unnecessary nodes outside of the shortest path architecture.

From a cyber-security point of view, this outcome indicates that the network is designed for optimal fault tolerance, best security, and efficient threat defense. The intuitionistic fuzzy structure guarantees that secure data paths are built up and safe risk factors (non-membership values) are taken into consideration, so this method is very appropriate for practical intrusion detection systems and network security frameworks. The ideal geodesic structure guarantees quick determination of key security points, thus increasing network resistance to cyber attacks.

7. CONCLUSION

The Perfect Geodesic Intuitionistic Fuzzy Graph (PGIFG) generalized the geodesic structure concept in fuzzy graphs with the inclusion of intuitionistic fuzzy set theory, which deals with membership, non-membership, and hesitancy values. For this system, the pseudo geodesic set is the set of nodes that are not part of any geodesic basis, and a graph is said to be perfect when this set is null. Using intuitionistic fuzzy modeling, this method strengthens cybersecurity through the identification of key security nodes, the optimization of threat detection, and the provision of secure communication channels within network structures. Complete intuitionistic fuzzy graphs and intuitionistic fuzzy cycles are demonstrated to have perfect geodesic properties and are thus best applied in applications where fault tolerance, security, and maximum connectivity are paramount.
Future developments could include AI-powered threat intelligence, blockchain-protected communication, and neutrosophic extensions to deal with high uncertainty in cyber risk evaluation. The PGIFG framework offers an efficient tool for contemporary cybersecurity, providing effective, adaptive, and robust network security.

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2025, 10(34s)
e-ISSN: 2468-4376
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