

# Redundant Mathematical Solution for Complex Homotopy Structures using Graph Theory based on Bipartite Chromatic Polynomial for Solving Distance Problems

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## ARTICLE INFO

Received: 18 Dec 2024

Revised: 10 Feb 2025

Accepted: 28 Feb 2025

## ABSTRACT

**Introduction:** Increasing network communication area has lot of unstructured routing to create complex structures. The communication structure is non-linear to create connective edges to degrade the communication performance. Many non-linear solutions and distance theory models contains maximum non-liability of variables are taken to solve the problems. But structure difference and dynamic variables are constantly applied to make solution which leads errors and complex solutions. To resolve this problem, to propose a Redundant mathematical solution for complex homotopy structures using Multinomial-Cordial Graph Theory (MCGT) based on Bipartite Chromatic Polynomial Distribution Theory (BCPDT) for solving distance problems. To apply neighbor-based distance coverage model with cordial labeling variable structure to reduce the complexity variable structure problems, this paper explores a novel strategy for encapsulating the non-linear complex homotopy in its entirety by employing graph theory and the concept of cordial labeling. By establishing a connection between algebraic topology and graph theoretical constructs, we formulate a redundant solution that illuminates the intricacies of complex homotopy but also provides practical methodologies for solving distance-related issues prevalent in various mathematical and applied fields as well, compared to the previous models.

**Keywords:** Graphs and labels, bipartite graph, cordial labelling, graph theory, nonlinear solutions, polynomial distribution, homotopy network, neighbor distance.

## INTRODUCTION

The study of complex homotopy theory, traditionally rooted in algebraic topology, has historically encountered limitations when addressing various distance problems inherent in both discrete and continuous spaces. This paper ventures to explore a novel strategy for encapsulating complex homotopy in its entirety by employing graph theory and the concept of cordial labeling. By establishing a connection between algebraic topology and graph theoretical constructs, we formulate a redundant solution that not only illuminates the intricacies of complex homotopy but also provides practical methodologies for solving distance-related issues prevalent in various mathematical and applied fields. Homotopy theory, a fundamental part of algebraic topology, concerns the classification of topological spaces based on their deformation properties. Traditionally, homotopy groups and continuous mappings have furnished a robust framework for this classification. However, when addressing problems that involve spatial configurations and distances, the intrinsic geometric structures can complicate direct applications of classical homotopy approaches.

In contrast, graph theory presents a versatile language for representing complex systems, combining the elements of topology, combinatorial structures, and numerical distance metrics. The intersection of these two fields facilitates innovative solutions to longstanding mathematical challenges. This paper positions itself at this confluence, proposing a redundant solution to complex homotopy problems by formalizing concepts from graph theory, particularly through the lens of cordial labeling.

Cordial labeling of graphs, a relatively recent development in combinatorial graph theory, introduces a balanced labeling scheme wherein the counts of vertices receiving two distinct labels differ by at most one. This property can

be harnessed to aid in addressing spatial distance problems through structural representations of topological constructs.

### LITERATURE SURVEY

They present a new type of set labels called "set affinity labels" to explore the properties of graphs that allow set relation labels [1]. A design evaluation or configuration label graph of  $G$  is an embedding function  $f: V(G) \rightarrow P(X)$ . Let's assume  $P(X)$  - power set,  $X$ - set,  $G$ - graph,  $f$  - Prime cordial labeling. They propose a constructive, proof-related development of graph theory to perform inference operations from labelled graphs and adequately represent the planarity of connected multigraphs defined by integral embeddings on surfaces [2]. The novel [3] combines graph theory and rational homotopy theory by  $\mathbb{Q}[x_1, \dots, x_n]$  describing  $m$ -absorbing ideals formed by sequences of homogeneous symmetric polynomials. Moreover, they introduce correlation markers ( $m = 3, 4$ ) to connect the trajectories of the two replicas to the  $m$  pan map. In addition, [4] they also assessed pathway-associating cordial labelling of  $r$  copies of the 3-pan graph. SD prime cordial labeling:  $|m_{n^*(1)} - m_{n^*(0)}| \leq 1$ , where  $m_{n^*(1)} - m_{n^*(0)}$  - number of edge label. Many vertices are evaluated and labelled  $n$ , labelling by 1 and 0, respectively [5]. Furthermore, SD was estimated based on prime cordial label maps.

Additionally, graph beta evaluation labelling introduces a weaker version of elegant and consistent labelling. The separator analyzes cordial markers through ladder diagrams, Möbius ladder diagrams, total pathway and cycle graphs [6]. The author [7] obtained effective and promising results based on well-known graph operations on Star, Bistar, and related graphs. If the graph allows one, a  $G_g(V_g, E_g)$  is indicated to be a bisector cordial graph. To evaluate the Cordial labelling condition for graph  $G$ -function, analyze the Cordial graph  $f: V(G) \rightarrow \{0, 1\}$  and verify that vertex transitions are cordial in jewel graphs [8]. However, labelling fields in graph theory encounter various challenge when dealing with vertices, edges, or non-negative integers assigned to graphs.

Graph theory plays a vital role in QSPR data analysis by encoding the topology of molecular graphs. Therefore [9], many new topological indexers combine graph labelling and topological coding to analyze labelled graphs.

**Definition 2.1:** The duality of the Baranovski and Sajdanovic complex is defined as equivalent to the generalized version in the configurational space of points on a one-dimensional manifold based on Kriz's model defined in [10] the  $n$ -point configuration space  $\text{Conf}(M, \Gamma)$ .

$$\text{Conf}(M, \Gamma) = \{(x_1, \dots, x_n) \in M^n; x_i \neq x_j \text{ if } e_{i,j} \in M(\Gamma)\}$$

Let's assume  $\Gamma$  - complete graph,  $\text{Conf}(M, \Gamma)$  - Configuration classical space,  $n$  - points in a manifold,  $M$ -manifold.

$$\text{Conf}(M, n) = \{(x_1, \dots, x_n) \in M^n; x_i \neq x_j \text{ for } i \neq j\}$$

They suggest a tri-topological network, a three-logical algebraic construction model based on homotopy under a network's nonlinear structure employing connected component graph  $(T^3 - C^2G)$ . Furthermore, [11] improves communication efficiency by using Homotopy Algebraic Invariance Linear Queueing Theory (HAILQT) to solve the link failure routing propagation problem. The graph category evaluates a type 2 system with a homotopy of 2 units. These construct and illustrate the homotopic transparency of finite graphs regarding spider moves [12].

Both up-to- $k$  degenerate graphs and  $k$ -trees are generalizations of trees [13]. They define many properties and illustrate many relationships to problems in graph theory and other general graph classes. The author suggested an output sensing technique using delay  $O(m \cdot \omega(G))$  for any  $n$ -vertex chord graph with  $m$  edges [14]. The author described the problem of cataloguing the subgraphs arising from maximal  $k$  decompositions of a sub graph with the largest group size  $\omega(G) \leq n$ . According to the proposed dg Lie algebra model of homotopic automorphisms, constraints on differential isomorphisms of manifolds are removed. Furthermore, [15] demonstrated that the logical homotopy class of a homogeneously connected manifold self-group classification space requires a minimum of five dimensions. They define a new rational equivalent  $H^*(G)$  algebraic variation for undirected, simple, connected finite graph  $G$ . Similarly, some rational homotopy theorems allow us to compute their variants using well-known theorems [16].

The theory of algebras and homotopy algebras offers a comprehensive understanding of higher-order homotopy properties. It utilizes the general concept of figures called infinite morphologies [17] to analyze algebraic varieties with infinite structures based on a simple description of homotopy varieties.

The author [18] examined CI-lower and CI-upper approximations in topological space, utilizing key expressions and performance maps derived from Cordial Incidence (CI).

**Definition 2.2:** The function  $f: V(G) \rightarrow \{0,1\}$  is referred to as the binary vertex label of graph  $G$ , with  $f(v)$  representing the label assigned to vertex  $v$  in  $G$  by  $f$ .

The author introduced replication functions to analyze the primary connectivity structure of rotationally symmetric graphs obtained from generalized Peterson graphs  $P(n,k)$ . Additionally [19], it has been demonstrated that the resulting graph is prime-cordial when a Petersen graph is validated with a specific path graph. The term optimal label is frequently used in comprehensive harmonious analysis evaluations [20]. Graph labelling has introduced smart, flexible annotation types and cordial labelling evaluations, targeting different motivations. A cordial graph is characterized by its properties, which are examined through the structures related to the sixteenth sprockets, cordial, and peripheral cordial graphs. A binary vertex labelling of a graph  $G$  is called a cordial labeling if  $|vf(0) - vf(1)| \leq 1$  and  $|ef(0) - ef(1)| \leq 1$ ,  $G$ -is the cordial labeling [21].

## ESSENTIALS OF COMPLEX HOMOTOPY

Complex homotopy theory extends classical homotopy concepts into the realm of complex manifolds and higher dimensional spaces. The fundamental idea is to explore facts about paths and loops within these spaces, with the focus on continuous transformations and equivalences.

Let  $X$  and  $Y$  be two connected topological spaces. We denote by  $\pi_n(X, x_0)$   $n$ -th homotopy group of  $X$  based at a point  $x_0$ . The fundamental group,  $\pi_1(X, x_0)$  encapsulates information about loops based at  $x_0$ . Higher homotopy groups provide insights into higher-dimensional analogs of loops, essentially capturing the ways in which spheres of various dimensions can be transformed, embedded, or navigated within the manifold  $X$ .

These homotopical structures can be associated with distances on topological spaces through discussions of simplicial complexes, where vertices represent points in space and edges represent connections or paths between these points. However, the challenge arises when attempting to discern and quantify distances in a cohesive manner, particularly in complex space where traditional Euclidean notions may break down.

## GRAPH THEORY: A TOOL FOR REPRESENTING COMPLEX HOMOTOPY IN BIPARTITE NODE COVERAGE

Graph theory provides a potent framework for visualizing and solving problems associated with complex structures. A graph  $G$  is defined as a pair  $G = (V, E)$  where  $V$  represents the set of vertices and  $E$  the set of edges that connect these vertices. By appropriately transforming topological features into graph constructs, we can harness the power of combinatorial reasoning.

### Distance Problems in Graph Theory

Distance problems in graph theory typically involve calculating the shortest path between nodes, measuring the diameter of graphs, or analyzing network connectivity. In the context of complex homotopy, we are particularly interested in how distances between points in a topological space can affect the algebraic properties of the space itself. For instance, given a graph  $G$  that represents a topological space, we can define the distance  $d(u, v)$  between two vertices  $u$  and  $v$  in terms of the number of edges in the shortest path connecting them. This metric provides essential insights into connectivity and can be aligned with homotopy classes, where homotopic paths yield equivalent distance properties within the structure of the graph.

### Cordial Labeling as a Solution

Cordial labeling may be employed to enhance our understanding of these distances by introducing a nuanced labeling system. A graph  $G$  is declared to have cordial labelling if vertices can be assigned labels 0 or 1 as follows:

- The absolute difference between the number of vertices labeled 0 and vertices labeled 1 is at most one.
- The edges between labeled vertices present certain parity conditions.

By assigning these labels, we structure the graph in a way that facilitates the calculation of specific distances. The balance inherent in cordial labeling aids not only in the formal representation of distances but also allows for redundancy in the encoding of vertex properties, which can be exceptionally beneficial when dealing with complicated topological metrics.

The nonlinear chromatic polynomial  $\lambda^4 - 3\lambda^3 + 3\lambda^2$  is not linear connective to the communication point by connecting similar edges.

For this chromatic polynomial  $P_n(\lambda) = \lambda^4 - 3\lambda^3 + 3\lambda^2$ , the graph  $G$  has 4 vertices and 3 edges.

Hence,  $P_4(\lambda) = \lambda^4 - 3\lambda^3 + 3\lambda^2$

Case 1. Let the graph  $G$  is connected, the number of edges = number of vertices -1.

$G$  represents a tree

Hence,  $P_4(\lambda) = \lambda(\lambda - 1)^3$

If  $G$  is tree with  $n$  vertices,  $n \geq 2$  then

$P_n(\lambda) = \lambda(\lambda - 1)^{n-1} = \lambda^4 - 3\lambda^3 + 3\lambda^2 - \lambda$  which is contradiction.

Case 2. If graph  $G$  is not connected

then  $G = K_3 \cup K_1$

$$\therefore P_4(\lambda) = P_3(\lambda) \cdot P_1(\lambda)$$

$$= [\lambda(\lambda - 1)(\lambda - 2)]\lambda$$

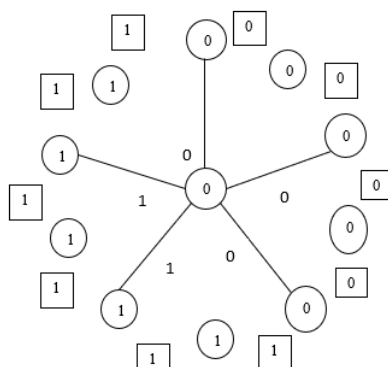
$$= \lambda^4 - 3\lambda^3 + 2\lambda^2$$

which is contradiction.

Hence,  $\lambda^4 - 3\lambda^3 + 3\lambda^2$  cannot be any graph's chromatic polynomial.

### Example 1:

The gear graph  $G_5$  and the total edge product Cordial Labeling is illustrated in Figure1



### Theorem 1.

The complete graph  $Q_n$  allows total edge cordial labeling for  $n > 2$ .

#### Proof:

Let us consider the complete graph  $Q_n$  with  $|V(Q_n)| = n$  and  $|E(Q_n)| = \frac{n(n-1)}{2}$ . The total number of elements in  $Q_n$  is  $\frac{n(n+1)}{2}$ . When  $m < n$ ,  $Q_m$  is denoted as a subgraph of  $Q_n$ . Furthermore, use the equation  $\left\lfloor \frac{n(n+1)}{4} \right\rfloor \leq \frac{m(m+1)}{2}$  to find the smallest integer  $m$ . Assign 0 to  $\frac{m(m-1)}{2} - l$  edges of subgraph  $Q_m$  and denote  $\frac{m(m+1)}{2} - \left\lfloor \frac{n(n+1)}{4} \right\rfloor$  by  $l$ . Additionally, assign label 1 to all remaining edges of the super graph  $Q_n$ .

Then,  $v_f(0) = m$ ,  $e_f(0) = \frac{m(m-1)}{2} - l$ ,  $v_f(1) = n - m$  and  $e_f(1) = \frac{n(n-1)}{2} - \frac{m(m-1)}{2} + l$ . Therefore  $\left| (v_f(0) + e_f(0)) - (v_f(1) + e_f(1)) \right|$

$$\begin{aligned} &= \left| \left( m + \frac{m(m-1)}{2} - l \right) - \left( n - m + \frac{n(n-1)}{2} - \frac{m(m-1)}{2} + l \right) \right| \\ &= \left| \left( \frac{m(m+1)}{2} - l \right) - \left( \frac{n(n+1)}{2} - \frac{m(m+1)}{2} + l \right) \right| \\ &= \left| \left\lfloor \frac{n(n+1)}{4} \right\rfloor - \left( \frac{n(n+1)}{2} - \left\lfloor \frac{n(n+1)}{4} \right\rfloor \right) \right| \\ &= \left| \left\lfloor \frac{n(n+1)}{4} \right\rfloor - \left\lceil \frac{n(n+1)}{4} \right\rceil \right| \leq 1 \end{aligned}$$

The cordial labelling  $Q_n$  can represent the total edge product for  $n > 2$ .

### Theorem 2

$Q_{m,n}$  denotes a complete bipartite graph, representing the total edge product cordial graph while excluding  $Q_{1,1}$  and  $Q_{2,2}$ .

**Proof:** Analyzing the complete bipartite graph  $Q_{m,n}$ ,  $|V(Q_{m,n})| = m + n$  and  $|E(Q_{m,n})| = mn$ . As a result, the total number of elements in  $Q_{m,n}$  is  $m + n + mn$ . Moreover,  $m \leq n$  is defined without loss of generality. Sorted as vertices of one subset  $v_1, v_2, \dots, v_m$  and vertices of another subset  $u_1, u_2, \dots, u_n$ , the above two specific cases are analyzed.

Case 1: When  $m = 1$  and  $n > 1$

Let's assume  $Q_{1,n}$  - tree with an even number of sizes. They demonstrated that all trees with a degree greater than 2 are cordial graph of edge product.

Case 2: When  $m > 2$

A subgraph of  $Q_{m,n}$  is  $Q_{m,l}$  where  $l < n$ . Next, identify the largest integer  $l$  in which  $m + l + ml \leq \left\lfloor \frac{m+n+mn}{2} \right\rfloor$  where  $r = \left\lfloor \frac{m+n+mn}{2} \right\rfloor - (m + l + ml)$ . We defined  $f: E(Q_{m,n}) \rightarrow \{0,1\}$  as follows.

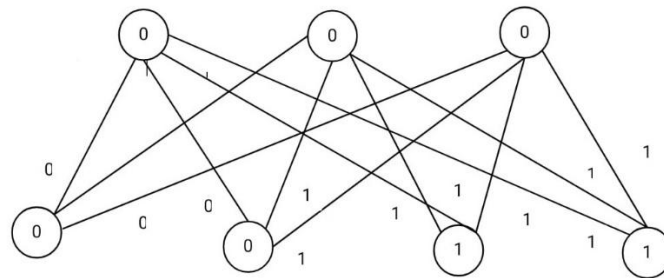
$$\begin{aligned} f(v_i, u_j) &= 0; & 1 \leq i \leq m \text{ and } 1 \leq j \leq l \\ f(v_i, u_{l+1}) &= 0; & 1 \leq i \leq r - 1 \\ f(v_i, u_{l+1}) &= 1; & r \leq i \leq m \\ f(v_i, u_j) &= 1; & 1 \leq i \leq m \text{ and } l + 2 \leq j \leq n \end{aligned}$$

In the analysis of the above-defined labelling pattern  $v_f(0) + e_f(0) = \left\lfloor \frac{m+n+mn}{2} \right\rfloor$  and  $v_f(1) + e_f(1) = \left\lfloor \frac{m+n+mn}{2} \right\rfloor$ . Thus  $\left| (v_f(0) + e_f(0)) - (v_f(1) + e_f(1)) \right| \leq 1$ .

Thus, a complete bipartite graph  $K_{m,n}$  is a complete marginal product adjacency graph excluding  $K_{1,1}$  and  $K_{2,2}$ .

Here  $m = 3, n = 4$ . Hence  $l = 1$  and  $r = 2$ . For which  $v_f(0) = 5, e_f(0) = 4, v_f(1) = 2$  and  $e_f(1) = 8$ .

Therefore,  $\left| (v_f(0) + e_f(0)) - (v_f(1) + e_f(1)) \right| = 1$ .



The figure 2 shows the complete bipartite graph  $K_{3,4}$  and its total edge product cordial labelling.

### THE REDUNDANT SOLUTION

By synthesizing the concepts laid forth, we propose a framework where complex homotopy is modeled via graphical representations endowed with cordial labels. This redundancy in representation serves several purposes:

**Redundant Paths:** The utilization of various paths  $n \rightarrow p$  within the labeled graph allows for a multiplicity of routes between points  $n(p)$ , capturing the core concept of homotopy, namely the idea paths that can be continuously deformed into each other are equivalent.

**Distance Calculation:** With the structured nature of cordial labeling  $c(n)$  at carrying node  $n$  between the connective edges  $n(x, y)$ , one can derive distance metrics that reflect the algebraic properties of the underlying topological space without losing the connections that exist through homotopy equivalence.

**Automated Solutions:** The graph-theoretical representation  $G \rightarrow n(V, E)$  opens avenues for computational models that automate distance calculations, providing robust solutions even in complicated scenarios which are often found in research problems.

### APPLICATION TO DISTANCE PROBLEMS

Applying our redundant solution in real-world contexts can yield a range of effective methodologies. For instance, in optimization problems involving network connectivity or in geographical information systems where points of interest are modeled as vertices, we can leverage cordial labeling to enhance the quality of distance measurements and streamline pathfinding algorithms. Additionally, the methods outlined could be useful in determining parameters in fields such as robotics, where navigating complex spaces is often a necessity, or even in computer graphics, where efficient rendering of spatial relations is critical.

Consider the Modular adjacency graph ‘ $m$ ’ points at the center  $mq$  where  $q$  is the dimensional samples  $\{I_1, \dots, I_n\}$  and  $\{J_1, \dots, J_r\}$  that are separately taken from graph  $G$  and  $n$  &  $r$  are the number of nodes correspondingly. In  $\{J_y\}_{y \neq x}$ , the distance between  $I_x$  and  $J_y$  its closest neighbour is defined as,

$$\rho_n = \min_{y=1, \dots, n, y \neq x} \|I_x - J_y\|, \quad (1)$$

Let,  $\|\cdot\|$  is the  $D^2$  normalize in  $\mathbb{R}^q$ . Given at the samples  $\{I_x\}$  and  $\{J_y\}$ , we aim to estimate the divergence between  $x$  and  $y$ . The estimation of divergence makes use of the relationship between these two groups of samples. Specifically, we employ the distance between  $I_x$  and its closest neighbour in  $\{J_y\}$  in addition to  $\rho_n(x)$  as previously defined.



$$u_r(x) = \min_{y=1, \dots, r} \|I_x - J_y\|. \quad (2)$$

The  $q$  open ball with radius  $\rho_n(x)$  and center at  $I_x$  is now considered with unique distance in another divergence node  $n$  at the point  $u_r(x)$  it is represented by the symbol  $C(I_x, \rho_n(x))$ . However, only one  $J_y$  of the samples  $\{J_y\}_{y \neq x}$  is included in the closure of  $C(I_x, \rho_n(x))$ . At  $I_x$ , the density estimates of  $g$  empirically is,

$$g(I_x) = \frac{1/(n-1)}{k_1(q)\rho_n^q(x)} \quad (3)$$

Let  $k_1(q) = \pi^{q/2} \Gamma(q/2 + 1)$ , and  $k_1(q)\rho_n^q(x)$  is the volume of  $C(I_x, \rho_n(x))$  at  $k$  point of dynamic iterations in closure point  $C$ . Similarly, the density estimates of  $n$  assessed at  $I_x$  is given by  $\{J_y\}_{y=1, \dots, r}$ , where only one  $J_y$  is contained in the closure of  $C(I_x, u_r(x))$ .

$$n(I_x) = \frac{1/r}{k_1(q)u_r^q(x)} \quad (4)$$

By following we perform NN divergence estimator,

$$\hat{Q}_{n,r}(g||h) = \frac{1}{n} \sum_{x=1}^n \log \frac{\frac{1/(n-1)}{k_1(q)\rho_n^q(x)}}{\frac{1/r}{k_1(q)u_r^q(x)}} = \frac{q}{n} \sum_{x=1}^n \log \frac{u_r(x)}{\rho_n(x)} + \log \frac{r}{n-1} \quad (5)$$

The ‘ $n$ ’ represents the high pointing node covering the least area to the communication node with shortest distance.

## CONCLUSION

Through the synthesis of complex homotopy theory with graph theoretical constructs, particularly through the innovative use of cordial labeling, this paper presents a redundant solution to address distance problems. The combination of algebraic topology and combinatorial structures not only enriches our understanding of space but also provides practical strategies to mathematically confront associated challenges. Future work may explore the extension of these concepts into hybrid domains, examining the implications of this approach within applied mathematics and computer science. The interweaving of topological constructs with graph theoretical methods represents a promising frontier for ongoing research and exploration in mathematical theory and its myriad applications.

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