

# Copula Based Flood Frequency Analysis in Upper Teesta River Basin

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## ABSTRACT

Due to recent climate change, the Teesta River Basin is severely affected by extreme hydrological events like floods. The recent flood events in this river affected millions of people residing in this catchment area across India and Bangladesh. In this present study, the univariate and copula-based multivariate flood frequency model is analysed to estimate the flood magnitude with different return periods using long-term time series data. The result shows that the univariate model overestimates the flood magnitude compared to the copula-based flood frequency model as it is associated with different flood variables like flood peak, volume, and duration. Four copula families are chosen for multivariate flood frequency modelling. From the statistical analysis, Gumbel-Hougaard and Frank copula families are chosen for the best fit joint distribution of flood variables.

**Keywords:** Flood frequency analysis, Teesta River Basin, Copula, Multivariate analysis, Univariate modelling

## INTRODUCTION

Flood is a natural extreme hydrological event. In this high stage, the river overflows its banks and inundates the low-lying areas, causing damage to the loss of property, human life, agriculture, and economic development. The flood peak values are required to design the dam, spillway, weir, barrages, and other hydraulic structures. On 4th Oct 2023, the Teesta River basin was affected by a severe flash flood due to a glacial lake outburst claiming at least 94 lives, sweeping away several bridges, and damaging Chunthang hydropower projects. Similarly, on 28th June 2020, heavy rainfall caused a massive landslide in the Dzongu Region and caused a severe flood. In terms of landslides, flash floods, and erosion, the Teesta basin is among the most susceptible river basins in the Himalayan region. (Agarwal and Narain 1991). The previous flood recorded at Teesta River basin in 1950, 1968, 1973, 1975, 1976, 1978, 1993, 1996, 2000, 2015. (Pal et al. 2016). The basin received the highest amount of rainfall, 200-400 mm, in every monsoon season. The higher elevation ranges from 300 m to 8500 m above mean sea level, causing significant variability of rainfall. The glacial lake outburst is a significant threat to the Teesta River basin, as reported by several researchers in recent years. (Banerjee & Bhuiyan 2023, Islam & Patel 2022). Several dams and barrages like Chungthang, Dickchu, Teesta Middle, Teesta Lower, and Gajaldoba have been constructed in the Teesta River basin to prevent the flash flood scenario. In the year 2015, a severe cloud burst in the Sikkim Himalayan region raised the discharge level to 5500 m<sup>3</sup>/s at the Gajaldoba barrage and flooded the lower catchment areas. The Teesta River is a high Himalayan River mainly fed by snow melt water, glacial lakes (Gurudombar Lake, Tsolamu Lake), orographic precipitation as well as groundwater (Mondal & Chakroborty, 2016; Wiecejczka et al., 2014). The flash floods in the Teesta River basin were mainly due to glacial lake outbursts or heavy rainfall in the upper catchment area, except due to a landslide in 1968 (Agarwal et al., 2016). The floodwater significantly increases the reservoir storage level above the high flood level because of a sudden release of water from the reservoir, causing flash floods in the downstream section of the basin. Sarkar et al. 2011a, 2011b examined the effect of climate change on the existing Teesta River barrage in the India region. Mondal and Islam (2017) and Ferdous and Mallick (2019) studied design flow characteristics with the lowest and highest value of flow trends in the Teesta River basin to assess the historical and future floods based on the Global Climate Model data along with rainfall runoff inundation model. The flood peak, volume, and duration are strongly dependent on each other and are affected by climate change. So, it is necessary to identify the change

pattern. These changes create non-stationarity in univariate modelling and affect the flood variables' joint distribution. For proper modelling of flood frequency analysis in a particular watershed, it is essential to identify these changes in the joint distribution of flood variables (Akbari & Reddy, 2019).

Still, no study has been reported in the Upper Teesta basin to predict the flood peak, as data availability is limited. So, there is a significant gap to understanding the flood dynamics in the Upper Teesta Basin. This study seeks to assess the following key points: (i) Evaluating the efficiency of parametric and non-parametric probability distributions in different flood variables. (ii) measuring dependency between the variables and (iii) analysing different Archimedean copula to identify the best fit. This study presents the recent application of copula-based flood frequency with key findings in section 2. The study area and methodology are described in section 3. The results and discussion with different flood variables, flood magnitude, volume, and duration with their dependency and best-fit copula are in section 4. The detailed conclusion is given in section 5.

### RECENT APPLICATION OF COPULA IN FLOOD FREQUENCY ANALYSIS

This section mainly comprises key insights on the recent application of copula-based flood frequency analysis. This summary includes the recent publication of papers published after 2022, covering the key findings and different methodologies.

**Table 1: Key findings of Recent Literature for copula-based flood frequency analysis**

Author/Year	Study Area	Methods	Key Findings
Marzieh Khajehali et al. (2025)	Kan River Basin, Iran	Tri-variant hierarchical Archimedean copula	A heterogeneous asymmetric copula provides greater flexibility in capturing different levels of asymmetry across various regions of the distribution, resulting in more precise modelling. Climate change significantly increases the trivalent joint return period.
Zhao et al. (2025)	Tongguan Station, Yellow River, China	Clayton and Vine copula model	The Clayton copula function stands out as the optimal bivariate joint distribution function for SSC (peak sediment concentration)- $Q$ and SSC- $V$ pairs. The D wine copula is the most suitable for the SSC- $Q$ - $V$ triplets.
Xinting Yu et al. (2025)	Shifeng Creek watershed, China	reduced-dimension vine copula construction approach	Vine copula models effectively capture complex variable dependencies, emphasizing strong spatial connections crucial for precise flood risk assessment during intense rainfall events.
Mahsa Biustani et al. (2025)	Hamidiyeh, Abdolkhan, and Paye-Pol, Karkhe river, Iran	D-Vine and C-Vine copula model	The Arithmetic Optimization Algorithm (AOA) and the Genetic Algorithm (GA) are evaluated for parameter estimation in R-vine models. AOA consistently outperforms GA in both D-vine and C-vine models.
Wang et al. (2024)	Tangnaihai station Yellow River Basin, China	Mixed D-Vine copula	A stochastic model is developed to simulate monthly streamflow using a mixed D-Vine copula, which demonstrates superior performance to bivariate copulas in capturing interannual and seasonal discharge variability.

Jiang et al. (2024)	Yangtze River Basin, China	Clayton Copula	Both negative and positive trends in coincident and antecedent local temperatures influence daily precipitation extremes, extending from southwest to northeast, with the most significant magnitudes occurring in the western region.
Wang et al. (2024)	Downtown area of Zhengzhou City, North China	Copula Method	Flood risk assessment model develops with multiple time scale using joint distribution of flood volume and its primary drivers like land use change, rapid urbanization in addition with copula method.
Salehi et al. (2024)	Kardeh and Radkan stations, Kashfrod River, Iran	Clayton Copula	The Clayton copula, combined with two marginal functions—Generalized Logistic and Generalized Extreme—provides the most accurate estimate of the joint distribution of data from the Kardeh and Radkan stations, its performance in the goodness-of-fit test
Mesfin Mano Hale et al. (2023)	Gunder Basin, Ethiopia	Archimedean Copula	The Gumbel-Hougaard copula provides the best fit for estimating the joint return period in flood frequency analysis, compared to univariate modelling.
Mohamad Haytham Khalo et. Al (2022)	Dez Dam, Karum river in Iran	Bi variant and tri-variate Archimedean copula	For short term period the bivariate return period is more reliable but in long term duration the tri-variate model performs better.

Based on the above literature, regional flood frequency analysis is the best choice for estimating extreme hydrological events like floods. There are several basic assumptions in multivariate flood frequency analysis, like 1) same marginal distribution for all the variables, 2) the normality of the flood variables, and 3) the mutual independence of the variables. It is challenging to formulate mathematical functions with increasing numbers of flood variables. Copula functions analyse the dependency structure of variables and their margins independently, without relying on the assumptions used in multivariate frequency analysis.

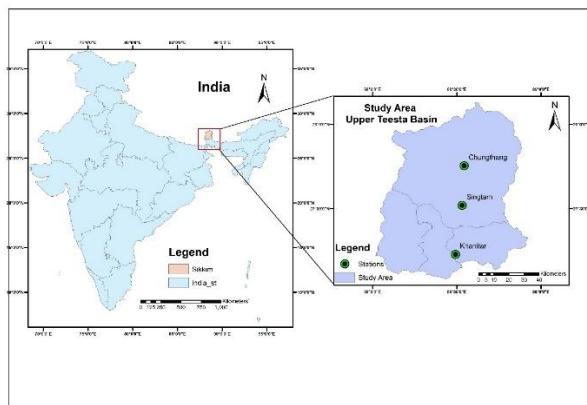
Conducting flood frequency analysis can be complex in interconnected rivers where two distinct river flows, each with its probability distribution, converge. Dodangeh et al. (2020) suggest utilizing bivariate distributions compare to univariate modelling for analysing peak flows at the junction of interconnected rivers. The probability of flood frequency at confluence points and uncertainty in bivalent joint analysis play a vital role in flood frequency analysis. Therefore, in the present study, the confluence points of two main tributaries, Lachen Chu and Lachung Chu, of the Teesta River at Chungthang are taken for multivariate flood frequency analysis.

## **STUDY AREA AND METHDOLOGY**

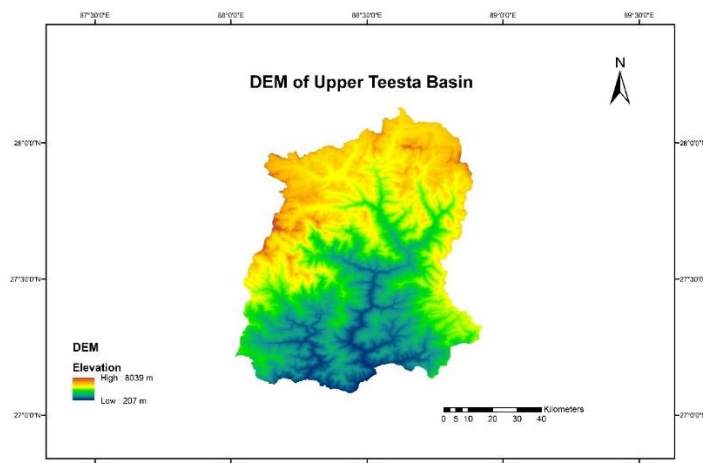
### **Study Region**

The Teesta River, 414 km long along with a drainage area of 12540 km<sup>2</sup>, is an international river in both India and Bangladesh originating from the Khangtse glacier at about 5400 m above mean sea level. The river flows in the Sikkim Himalayan region, touching Chngthang, Mangan, Dickchu, and Rangpo. After meeting Rangeet River at Teesta Bazar, it reaches the planes of Sevoke 20 km north of Siliguri. After Sevoke, it moves in the southern direction with the confluence of small Leesh, Gheesh, Neora and Chel Rivers. It touches the Gajoldoba Barrage. After Haldibari and Mekhaliganj, it enters Bangladesh and meets with the Brahmaputra River. The variation of rainfall in hilly areas is about 4000-6000 mm in monsoon and 1000-2000 mm near the Rangeet Valley.

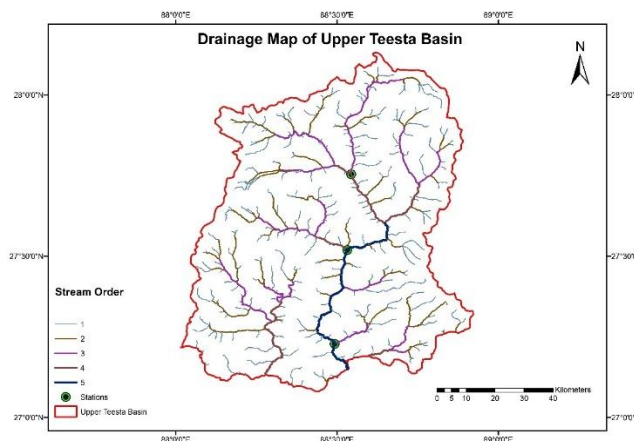
The study area is limited to the upper Teesta Basin up to Rangpo. Because of the high relief ratio (88.2-14.8), the Teesta River basin has an excellent capacity for outflow. (Chaubey et al. 2023, Chaubey et al. 2021). There is a sharp structural ridge that slopes south-westward and runs through the basin. The univariate flood frequency analysis was carried out at four different locations, Chungthng, Singtam and Khanitar, covering both higher and lower elevation ranges with the conventional Gumbel method.



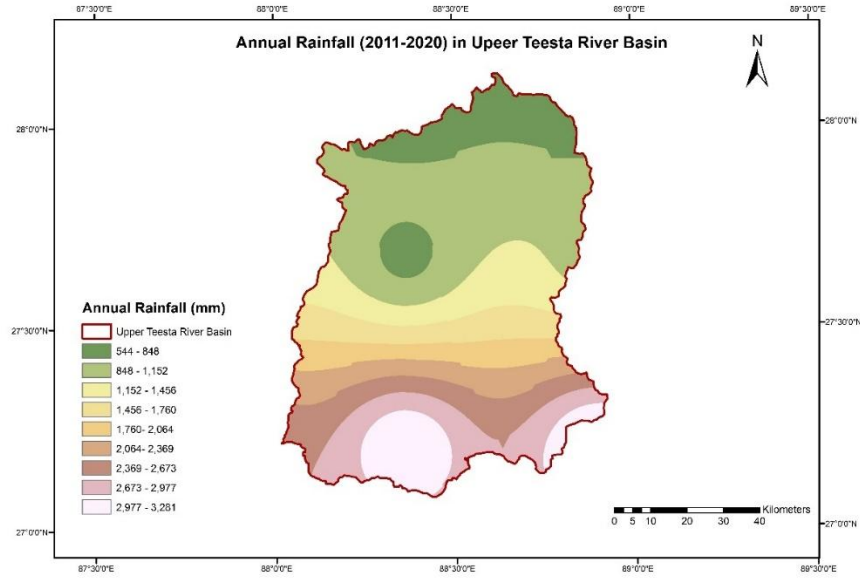
**Fig1: Study area of Upper Teesta River Basin**



**Fig 2: DEM of Upper Teesta River**



**Fig 3: Drainage Map of Upper Teesta River Basin**



**Fig 4: Annual Rainfall Map of Upper Teesta River Basin**

## Data & Methods

The annual maximum discharge data of four different locations Chungthang, Khanitar and Singtam from the period 1972-2022 was collected from Central Water Commission which collects the discharge data on Teesta River basin at regular intervals. These peak discharge data were utilized by flood frequency analysis by univariate and multivariate copula methods. Before flood frequency analysis the time series data was checked by turning point test and Kandell rank correlation test to check randomness and trend at 5% significant level.

The rainfall records were collected from Indian Meteorological Department, Gangtok. The extreme value distribution developed by Gumbel (1941) was known as Gumbel distribution widely used as to predict the flood peaks by univariate methods.

The Gumbel equation for practical use is:

$$x_T = \bar{x} + K\sigma_{n-1} \quad (1)$$

$x_T$  is the value of the Variable  $X$  with return period  $T$ .

$\bar{x}$  is the sample mean.

$K$  is the frequency factor expressed as  $K = \frac{y_T - \bar{y}_n}{S_n}$

In which  $y_T = -\left[\ln \ln \frac{T}{T-1}\right]$

$\bar{y}_n$  = reduced mean a function dependent upon sample size

$S_n$  = reduced standard deviation depending on sample size

Reduced mean and reduced standard deviation was calculated from the Gumbel extreme value distribution table.

Log-Pearson type III distribution was widely used in US water resource council and India. Firstly, the data were transformed into log scale (base 10) then analysed for univariate flood frequency analysis.

$$z = \log x \quad (2)$$

Where  $x$  is the value of flood peak  $X$ . Then

$$z_T = \bar{z} + K_z \sigma_z \quad (3)$$

$K_z$  = a frequency factor depending on return period  $T$  and coefficient of skew  $C_s$

The values of frequency factor were calculated form Log-Pearson type III table with different return period and coefficient of skewness.

### The Copula Function

Sklar (1996) was the first to use the Latin noun copula, which meaning "a link, tie, or bond," in a mathematical or statistical context. A copula is a mathematical function that enables the connection of univariate distributions to produce a joint distribution with a certain dependent structure. There are two correlated random variables,  $X$  and  $Y$ , with their respective univariate marginal density functions,  $F(x)$  and  $F(y)$ ; the Sklar (1996) theorem provides the connection between copulas and the joint distribution. The Sklar's theorem states that

Let's  $F_{XY}$  be joint distribution with margins  $F_X$  and  $F_Y$  then there exist a function  $C: [0, 1] \times [0, 1] \rightarrow [0, 1]$  such that,

$$F_{XY}(X, Y) = C(F_X(X), F_Y(Y)) \quad (4)$$

If  $X$  and  $Y$  are continuous then  $C$  is unique;  $C$  is uniquely determined on the range of  $X$  and range of  $Y$ .

A copula is a joint distribution function of standard uniform random variables. A bivariate copula can be represented as:

$$C: [0,1]^2 \rightarrow [0,1] \quad (5)$$

This equation has to full fill the following conditions.

- i)  $C(u,0) = C(v,0) = 0$
- ii)  $C(1,u) = C(u,1) = 0$
- iii)  $C(u_1, u_2) + C(v_1, v_2) - C(u_1, v_2) - C(v_1, u_2) \geq 0$  for all  $v_1 \leq v_2, u_1 \leq u_2$  (6)

The second condition ensures that the probability corresponding to any rectangle in the unit square is nonnegative.

Nelsen provides an overview of the various copula families (1999). The Archimedean, elliptical, extreme value copulas are some extensively employed kinds of copula functions. The Archimedean copula family is more favourable for hydrologic analyses, because it can be simply produced a great variety of copula families belong to this family, and it can be applied whether the correlation amongst hydrologic variable is positive or negative (Nelsen, 1999). Because of this, it was common practice to determine the joint probability distribution of associated flood variables using one-



parameter Archimedean copulas. In this present study Ali-Mikhail-Haq, Gumbel-Hougaard and Cook Johnson copulas are applied in analysis.

In multivariate statistics, Kendall's coefficient of correlation ( $\tau$ ) is a widely used nonparametric indicator of dependency or linkage. The observation's Kendall's  $\tau$  is calculated (estimated) from (Karmarkar and Simonovic 2009, Nelson 2006)

$$\tau_n = \binom{n}{2}^{-1} \sum_{i < j} \text{sign}[(x_i - x_j)(y_i - y_j)] \quad (7)$$

Where if  $[(x_i - x_j)(y_i - y_j)] > 0$  Then sign = 1, if  $[(x_i - x_j)(y_i - y_j)] < 0$  Then sign = -1;  $i, j = 1, 2, \dots, n$

The other nonparametric determination is Spearman's rho ( $\rho$ ). According to Myers and Wells 2013, the Spearman's rho ( $\rho$ ) is given by.

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \quad (8)$$

Where,  $d_i = (x_i - y_i)$  = the difference between the ranks of corresponding data set and  $n$  = the total number of values in each data set.

### Copula Parameter Estimation

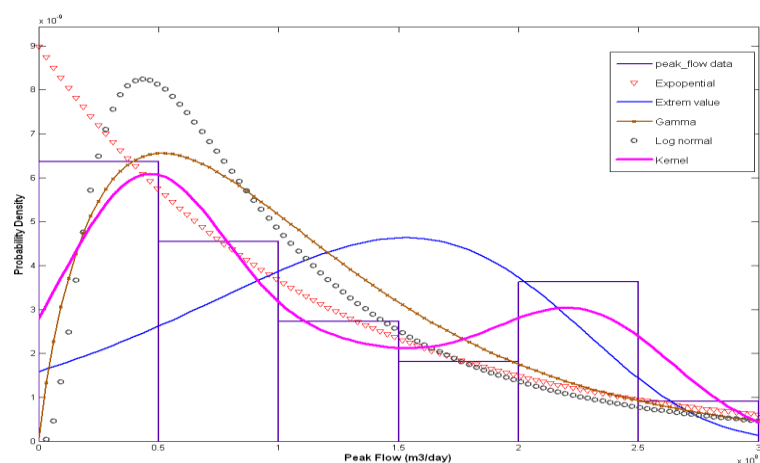
The copula parameter has been estimated by using two methods (Reddy and Ganguly 2012), method of moments based on inversion of Kendall's  $\tau$  (Genest et al. 1993) and maximum pseudo likelihood estimator. (Genest et al. 1995)

## RESULTS

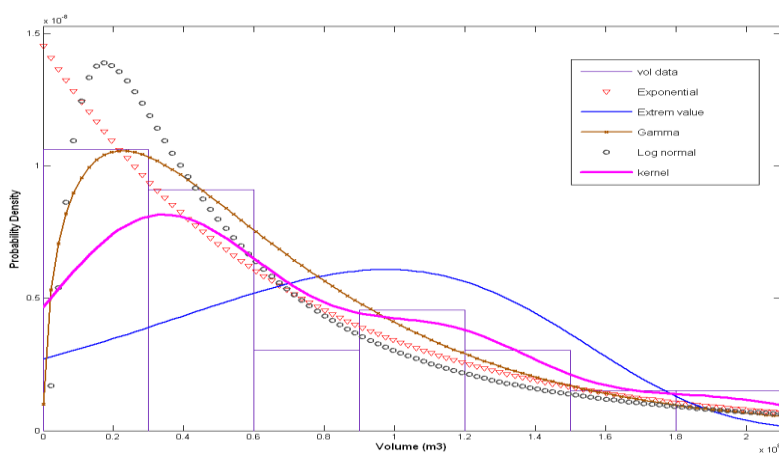
The Gumbel methods are used in this study area with three different locations Chungthang, Khanitar, Singtam and the flood magnitude is presented in the Table no: 2 with different return periods.

**Table 2: Flood Peak with Different Return Period by Univariate Model**

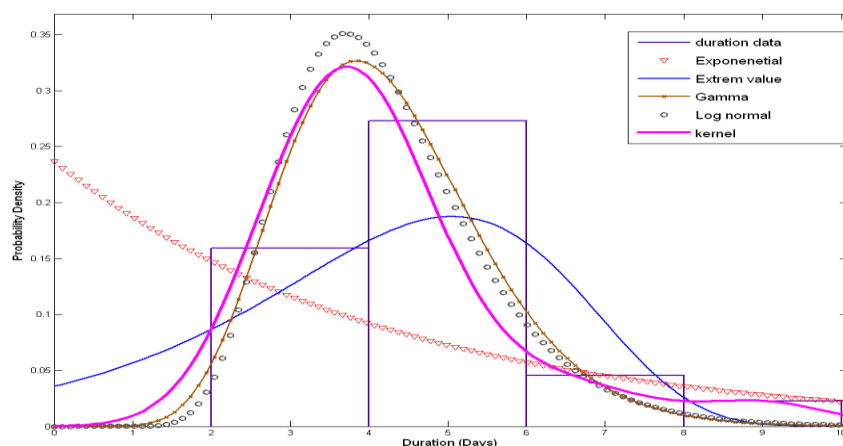
Sr. No.	Location	Return Period 2 Yr (Cusec.)	Return Period 10 Yr (Cusec.)	Return Period 25 Yr (Cusec.)
1	Chungthang	386	591	694
2	Singtam	93	166	204
3	Khanitar	2094	3481	4178



**Fig 5: Distribution Fitting Peak Flow**



**Fig 6: Distribution Fitting Volume**



**Fig 7: Distribution Fitting Duration**

Figures 5, 6, and 7 show the peak flow, volume, and duration histograms with parametric and non-parametric density functions fitted to the data. A comparison between the probability density functions and observed frequencies for all flood variables reveals that non-parametric distributions are the best fit. The duration data histogram shows a bimodal pattern, which standard parametric distributions cannot accurately capture. The kernel distribution method effectively captures bimodality. In this study, the Root Mean Square Error (RMSE) and Akaike Information Criterion (AIC) are utilized as goodness-of-fit metrics to assess the suitability of parametric and non-parametric probability distribution



functions for the potential margins of P, V, and D. Table 3 presents the RMSE and AIC values for all parametric and non-parametric distribution functions applied to peak flow, volume, and duration.

**Table 3: Comparison of AIC, RMSE and Chi values of flood variables for different marginal distributions**

Distribution	AIC			RMSE			Chi Value		
	P	V	D	P	V	D	P	V	D
Exponential	-97.3352	-107.129	- 23.6368	0.094	0.074	0.543	0.0852 (1.6354 )	0.0519	0.3942
Extreme value	-97.1032	- 98.9848	- 70.0805	0.090	0.086	0.171	0.0077 (1.1455)	0.048 8	0.0152
Gamma	- 96.6236	-123.093	- 99.3505	0.102	0.056	0.095	0.0019 (1.1455)	0.034 5	0.0619
Lognormal	-96.262	-127.962	-101.274	0.102	0.050	0.091	0.0318 (1.1455)	0.029 3	0.066 4
Kernel	-128.95	-135.918	-117.711	0.053	0.046	0.069	0.0337 (2.1673 )	0.005 2	0.029
Bracket value shows critical value of chi square									

The peak flow (P), volume (V), and duration (D) are fitted using the kernel estimation function. The AIC and RMSE values for all three variables are the lowest for the non-parametric distribution. The acceptance of the marginal distribution is calculated based on the Chi-square test. Table 2 indicates that the Chi-square value for the kernel function is lower than the critical Chi-square value, leading to its acceptance as the marginal distribution for all flood variables at a 99.5% significance level. This study uses the kernel function as the marginal distribution for further analysis.

In the preset study of Upper Teesta River basin, at Chungthang station the nature of mutual dependence among flood variables Flood peak (P), Flood volume (V) and Flood duration (D) are estimated using Pearson's linear correlation and Kendall's tau. Kendall's coefficient of correlation is a more robust way to test dependence as it is a rank-based procedure (Genest et. al., 2007). Table 4 shows the dependence among the P, V and D:

**Table 4: Correlation coefficient for flood variables**

Sr. No.	Flood variables	Pearson's linear Correlation coefficient	Kendall's coefficient of correlation
1	Peak flow + Volume	0.8273	0.7056
2	Peak flow + Duration	0.1851	0.1439
3	Volume + Duration	0.1513	-0.0103

The Pearson linear correlation coefficient shows positive correlation among P, V and D, but Kendall's Tau indicated V-D negatively correlated between them. The Kendall's tau effects the only selection of copula family. The correlation coefficients for the flood characteristics presented in Table 4 are all positive and consistent, indicating that using

Archimedean copulas to model the dependence structure of these flood variables is appropriate. In this study copula families are decided based on the range of Kendall's tau for joint distribution. The following figure shows joint distribution function for  $P$ - $V$ ,  $P$ - $D$  and  $V$ - $D$  using different families of copula.

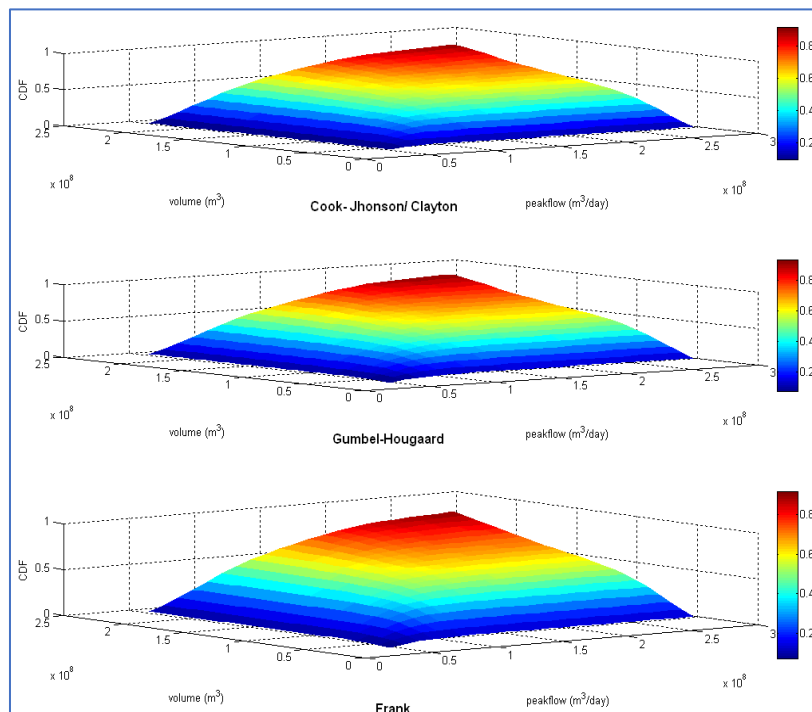


Figure 8: Joint Distribution P-V

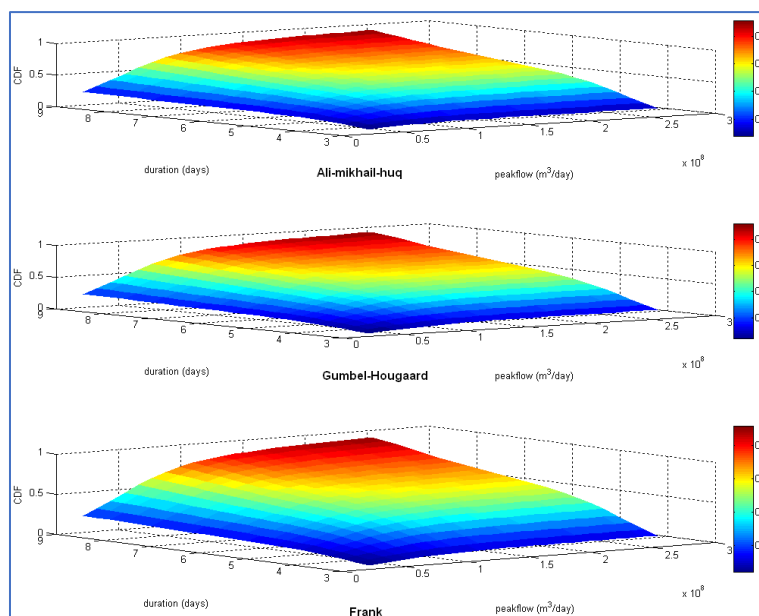
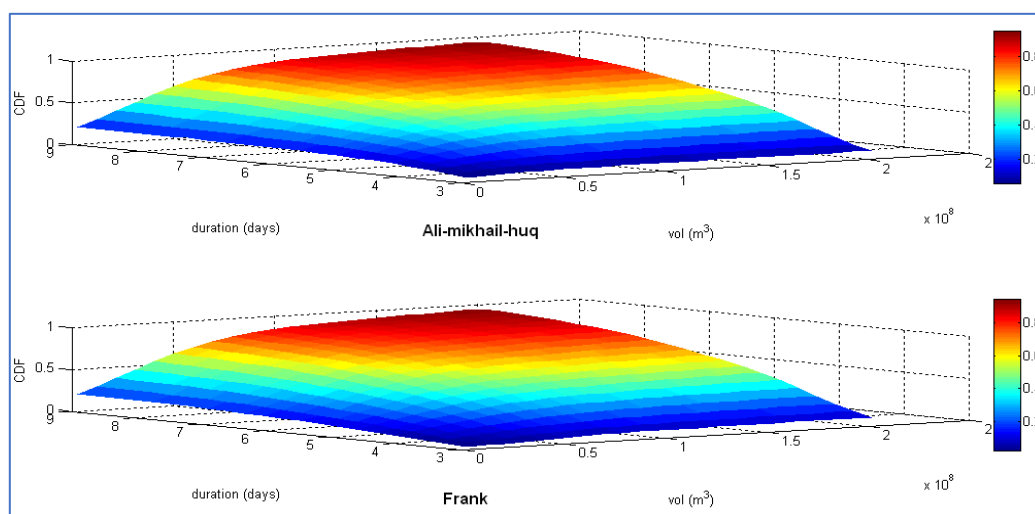


Figure 9: Joint Distribution P-D

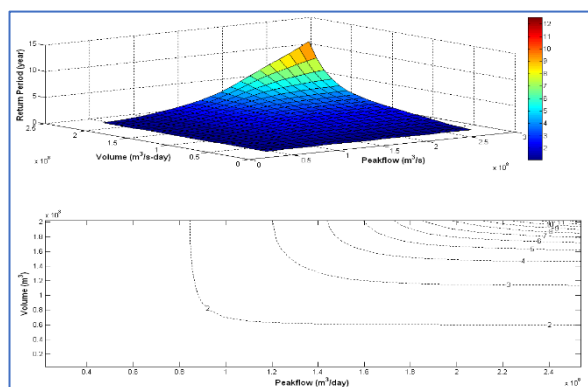
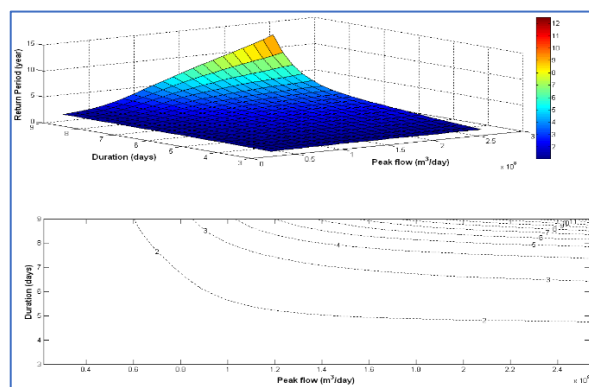
**Figure 10: Joint Distribution V-D**

The AIC and RMSE statistics are employed to assess how well the sample data fits the theoretical joint distribution derived from the copula functions. Table 5 presents the AIC and RMSE values for the joint distributions calculated using various copula functions for peak flow-volume (P-V), volume-duration (V-D), and peak flow-duration (P-D).

**Table 5: Comparison of AIC and RMSE for different families of copula**

Sr. No	Copula family	AIC			RMSE		
		P-V	P-D	V-D	P-V	P-D	V-D
1	Ali-Mikhail-huq	-	-62.82	-54.13	-	0.165	0.210
2	Cook-Jhonson	-100.22	-	-	0.058	-	-
3	Gumbel-Hougaard	-100.14	-75.12	-	0.058	0.117	-
4	Frank	-108.04	-72.96	-72.853	0.047	0.124	0.124

It can be evaluated from goodness of fit tests that the frank copula family is best fitted for joint distribution of *P-V* and *V-D*. The Gumbel-Hougaard is best model for *P-D* joint distribution as the AIC (-75.12) and RMSE (0.117) shows minimum. The following figures show the joint return period for peak flow- volume, peak flow- duration and volume duration.

**Figure 11: Joint Return Period: P & V****Figure 12: Joint Return Period: P & D**

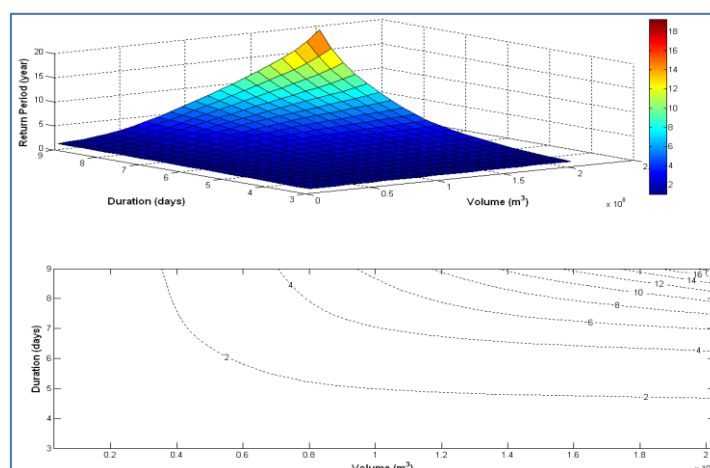


Figure 13: Joint Return Period: V &amp; D

Table 6: Magnitude of Flood in Different Return Period by Multivariate Model

Sr. No.	Location	Flood Peak Return Period 2 Yr (Cusec.)	Flood Peak Return Period 10 Yr (Cusec.)	Flood Peak Return Period 25 Yr (Cusec.)
1	Chungthang	365	542	603
2	Singtam	81	131	188
3	Khanitar	1963	3204	4031

Table 6 shows the flood magnitude with different return periods for multivariate modelling. The flood magnitude is lower than in univariate modelling. As Khanitar station is located downstream of the other two stations, the flood peak value is higher than the other stations.

### CONCLUSION

The univariate flood flow modelling focuses only on the flood flow magnitude. In contrast, they cannot capture the interrelation between other flood characteristics such as volume, duration, and peak flows. So, this modelling has a limitation with the normality of the data set and different marginal distributions. Extreme hydrological events like floods are complex and often simultaneously impacted by multiple variables. The Copula method is an effective way to efficiently model the flood variable without assuming their identical marginal distributions. Multivariate flood flow modelling can significantly improve our understanding of flood risk, enabling better predictions and more effective management strategies. This copula-based multivariate modelling in the Upper Teesta Basin allows for a more robust understanding of how these flood characteristics interact, which is critical for accurate flood frequency analysis. The findings of this study are summarized as follows.

- The flood magnitude is lower in multivariate modelling than in univariate modelling.
- Based on AIC and RMSE, this present study indicates that the Gumbel–Hougaard and Frank copulas best fit and demonstrate a systematic approach to joint distribution modelling.
- The different combinations of peak flow, volume, and duration with different return period produces more significant insights for water resource management and planning to comprehensive understanding of flood behaviour.

The recent floods in the Teesta River Basin have caused substantial damage to critical infrastructure like the Chungthang and Dikchu Dams. By focusing on multivariate flood frequency modelling, this research aims to enhance the understanding of flood events in the Upper Teesta Basin, which helps the better design and management strategies for hydraulic structures and flood control studies. Moreover, given the limited studies on flood frequency analysis at different confluence point at Upper Teesta Basin, it is recommended to apply this method for regional river analysis across different climates and incorporate more stations for a comprehensive assessment.

### CONFLICT OF INTEREST

The authors declare no conflict of interest

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