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Suggested Method for Prediction Using Gaussian Process Regression Kernel Regression

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ABSTRACT

Accurately predicting children's weight is challenging due to measurement inconsistencies. To address this, a hybrid kernel function, combining the squared exponential kernel and the Gaussian kernel with a mixture parameter, is proposed for developing a fuzzy Gaussian process regression model. The integration of fuzzy set theory and a triangular membership function helps handle weight measurement inaccuracies by determining the degrees of membership for each element in the weight vector.

The model is estimated using the spider monkey optimization (SMO) algorithm and implemented in MATLAB Ver. 2023a. The fminunc function is used for optimization, while the fitrgp function applies fuzzy set theory for regression. A dataset consisting of 191 observations from Al-Elwiya Maternity Teaching Hospital in Baghdad (collected between June 1, 2022, and December 31, 2022) is used for evaluation. The dataset is divided into 70% training data (n_train = 129) and 30% testing data (n_test = 55). The standard deviation of explanatory variables is [illegible] = 0.9234, and the smoothing parameter l = 0.8 is selected through optimization.

Model performance is assessed using root mean square error (RMSE), mean square error (MSE), and mean absolute percentage error (MAPE). Results indicate a significant difference between actual and predicted values for the squared exponential kernel function, whereas the Gaussian kernel function produces closer predictions. The hybrid kernel function achieves the most accurate predictions, aligning more closely with actual child weight values than the individual kernel functions.

The proposed fuzzy Gaussian process regression model with a hybrid kernel function outperforms both the squared exponential and Gaussian kernel functions in predictive accuracy. By effectively handling measurement uncertainties through fuzzy set theory, the model provides a more reliable approach for predicting children's weights.

Keywords: Fuzzy Gaussian Process, Gaussian Kernel Function Regression, Hybrid Kernel Function, Fuzzy Set Theory, Alpha-Cut Method, Spider Monkey Optimization (SMO) Algorithm.

INTRODUCTION

Uncertainties in data refer to the lack of complete knowledge or precision about the values or characteristics of the data. These uncertainties can come from a variety of sources and can significantly affect how the data is analyzed, interpreted, and decided upon. A mathematical framework known as fuzzy logic allows for a gradual transition between true and incorrect values, so addressing ambiguity and imprecision. Various domains have effectively employed it to manage complicated and ambiguous data. Regression models can utilize fuzzy logic to manage uncertainty and imprecision related to input data, model parameters, or variable relationships. Fuzzy Gaussian Process Regression (Fuzzy GPR) is an extension of traditional Gaussian Process Regression (GPR) that incorporates fuzzy logic to handle uncertainties in the data. The kernel function plays an important role in capturing the underlying patterns and relationships in the data. The kernel function defines the similarity or correlation between different data points, influencing the overall behavior of the Gaussian process. Sollich and Christopher, 2004 use the Equivalent Kernel to Understand Gaussian Process Regression. Chu and Zoubin, 2005 used Gaussian Processes for Ordinal Regression with ordinal data. Rasmussen and Nickisch, Hannes, 2010 use Gaussian Processes for Machine Learning (GPML) Toolbox to find Gaussian Process Regression. Schulz a. et al 2018 summarize recent psychological experiments utilizing Gaussian processes. Said and Altaweel, 2021 use Bayesian Approach for Analyzing Computer Models using Gaussian Process Models. JEAN-LUC et al., 2022 introduces algorithms to select/design kernels in Gaussian process regression/kriging surrogate modeling techniques and apply it aerodynamics. Shi et al., 2022 use of the Combined Kernel Function Based Gaussian Process Regression Method in

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Engine Performance Predictio. J.-L. Akian & et al., 2022 introduces algorithms to select/design kernels in Gaussian process regression/kriging surrogate modeling techniques and apply it on aerodynamics.

The objective of the study is to develop a fuzzy Gaussian process regression model using a hybrid kernel function (a combination of the squared exponential and Gaussian kernels) to improve the accuracy of predicting children's weights. The model is estimated using the spider monkey optimization (SMO) algorithm, incorporating fuzzy set theory to handle inaccuracies in weight measurements. The study aims to evaluate the performance of the proposed hybrid kernel function by comparing it with standard kernel functions using error metrics such as root mean square error (RMSE), mean square error (MSE), and mean absolute percentage error (MAPE).

2. FUZZY INFORMATION

2.1. Crisp and Fuzzy Set

If $\tilde{A}$ is a subset of X, and X is a universal set, then x may or may not belong to set $\tilde{A}$. Let $\mu_{\tilde{A}}(x)$ be a characteristic function for set $\tilde{A}$ that gives each element in set X a degree of belonging to set A, and this function is dichotomous valued {0,1}, then crisp set as in equation (1), (Ibrahim and Mohammed, 2017)

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & \text{if } x \in \tilde{A} \\ 0, & \text{if } x \notin \tilde{A} \end{cases} \tag{1}$$

The set with ambiguous boundaries is called the fuzzy set. A membership function that assigns a degree of belonging to each element in the set within the interval [0, 1] defines the fuzzy set. Each component or item in the fuzzy set is allowed to have partial membership, with each piece having a certain degree of membership. (Pak, 2017, 504).

Let X be a universal set, then the partial fuzzy set $\tilde{A}$ of X is characterized by a membership function $\mu_{\tilde{A}}(x)$) that produces values between [0,1] for all values of x in the fuzzy sample space. The fuzzy set is the set of ordered pairs as in equation (2), (Danyaro et al., 2010)

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)), x \in X, i = 1,2,3, \dots, n, \ 0 \le \mu_{\tilde{A}}(x) \le 1\} \tag{2}$$

2.2. Membership Function

One of the key and important functions in fuzzy set theory is the membership function, which produces values in the interval [0, 1] to indicate the degree of belonging of each element in the traditional comprehensive set within the fuzzy set. It is employed to determine if elements in the fuzzy set belong. (Abboudi et al., 2020), It is a positive-valued function that converts an element's degree of importance (or degree of belonging) in the comprehensive set to the fuzzy set as in Figure 1.(Rutkowski, 2004) , and it define three main characteristics Core , Support, and Boundary, (Sivanandam et al., 2007)

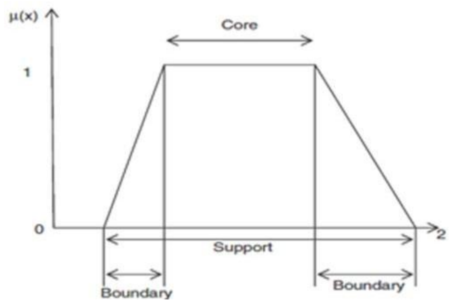

Figure1. Membership function and it characteristic

The scientist (Zadeh) proposed a set of membership functions to make appropriate decisions in cases of uncertainty.

2.3. Alfa-Cut

Let τ is the lowest degree of belonging to any element in the fuzzy set since the crucial belonging is restricted to two (a_l, a_r) on the fuzzy set's support line. $\tilde{A}$. Its value falls within the interval [0,1], which represents the degree of belonging of the important elements, (H. Garg et al, 2013). Zadeha (1971) first introduced the concept of cutting in the fuzzy set. The traditional set, whose elements are part of the fuzzy set $\tilde{A}$ and whose degree of membership is

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larger than or equal to τ, is defined as the alpha-cut of the fuzzy set. It may be expressed mathematically as in equation (3):

$$\tilde{A}^{\tau} = \{\tilde{x} \in X;\ \mu_{\tilde{A}}(x) \geq \tau\} \quad (3)$$

3. GAUSSIAN PROCESS REGRESSION (GPR)

The fundamental ideas behind a Gaussian process, such as the joint and conditional probability, kernels, non-parametric models, and multivariate normal distribution (Wang, 2022). GPR is a popular probabilistic supervised machine learning framework for regression and classification applications with the use of previous information (kernels), which may generate predictions and offer metrics of prediction uncertainty. (Rasmusse and Williams, 2006)

In GP regression with noisy observations, if we have the target variable or training variable $y_i \in R\_$, and the explanatory variables $X_{ij} \in R^q$, $where\ i = 1,2, \ldots, n$, j=1,2,...., q , then the GP model is as in equation (4) ,(Schulz et al., 2018)

$$Y_i = f(X_{ij}) + e_i \quad (4)$$

Where e_i is iid Gaussian noise , $e_i \sim N(0, \sigma^2)$, $Y_i \sim N(0, \sigma^2 I_n)$ and the function f is a random variable have Gaussian process with the multivariate normal distribution~ $MN(\mu_X, K(X, X')$ as in equation (5) ,

$$f \sim MN(\mu_X, K(X, X')) \quad (5)$$

The regression function $f(X_{ij})$ in (GPR) is unknown and is treated as a random variable have a probability distribution. Thus, the Bayesian approach will be achieved by determining the posterior distribution that represents the degree of belief in the values achieved after taking the sample which is formulated as in equation(6),

$$h(f|\ X, Y, \underline{\vartheta}) = \frac{L(Y|f,X,\vartheta) \times p(f,\vartheta)}{p(Y|X,\vartheta)} \quad (6)$$

Where $h(f|\ X,\ Y,\ \underline{\vartheta})$ is posterior distribution, $L(Y|f,\ X,\ \vartheta)$ Likelihood function, $p(f,\ \vartheta)$ is Prior distribution for the Gaussian process, $p(Y|X,\ \vartheta) = \int P(Y|\ f,\ X,\ \underline{\vartheta})p(f,\ \vartheta)df$ is marginal distribution , $\vartheta = [\sigma_f^2,\ l\ ]$ is a vector of Hyper-parameters, as they represent the variance of the model's outputs , σ_f^2 known as the scaling coefficient, which helps to detect outliers if its value is extremely big, (Rasmusse & I. Williams, 2006). If its value is modest, it helps to differentiate functions that are close to the average. If its value is large, it allows for more variance. l The length scale helps to characterize how much the function has been smoothed. Its high value suggests that the model is well-smoothed and that the function changes slowly, then the posterior distribution of GPR is as in equation(7) to equation (10) ,

$$P(Y|\ f, X, \underline{\vartheta}) = (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{1}{2\sigma}[(y-f)(y-f)']} \quad (7)$$

$$p(f, \vartheta) = (2\pi)^{-\frac{g}{2}} e|k|^{-\frac{1}{2}-\frac{1}{2}fK^{-1}f'} \quad (8)$$

$$L(Y|f, X, \vartheta) = \prod_{i=1}^{q} P(Y|\ f, X, \underline{\vartheta}) = (2\pi\sigma^2)^{-\frac{g}{2}} e^{-\frac{1}{2\sigma}[(y-f)(y-f)']} \quad (9)$$

Then,

$$L(Y|f, X, \vartheta) \times p(f, \vartheta) = (2\pi\sigma^2)^{-\frac{g}{2}} e^{-\frac{1}{2\sigma^2}[(y-f)(y-f)']} \times (2\pi)^{-\frac{g}{2}} e|k|^{-\frac{1}{2}-\frac{1}{2}fK^{-1}f'}$$
$$= (2\pi)^{-\frac{g}{2}}\sigma^{-q}\,|k|^{-\frac{1}{2}} \times e^{-\frac{1}{2}[\frac{1}{\sigma^2}(y-f)(y-f)'+fK^{-1}f']} \quad (10)$$

And the marginal is as in equation(11),

$$p(Y|X, \vartheta) = \int P(Y|\ f, X, \underline{\vartheta})p(f, \vartheta)df = (2\pi)^{-\frac{g}{2}}\sigma^{-q}\,|k|^{-\frac{1}{2}} \int e^{-\frac{1}{2}[\frac{1}{\sigma^2}(y-f)(y-f)'+fK^{-1}f']}\ df \quad (11)$$

Then the posterior distribution is as in equation(12),

$$h(f|\ X, Y, \underline{\vartheta}) \propto e^{-\frac{1}{2}[\frac{1}{\sigma^2}(y-f)(y-f)'+fK^{-1}f']}\ d \quad (12)$$

Then the distribution of GPR is as in equation(13) to equation (15) ,

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$$Y_i = f(X_{ij}) + e_i \sim N\left(0, \begin{bmatrix} K(X_1, X_1') & K(X_1, X_2') & \cdots & K(X_1, X_n') \\ K(X_2, X_1') & K(X_2, X_2') & \cdots & K(X_2, X_n') \\ \vdots & \vdots & \vdots & \vdots \\ K(X_n, X_1') & K(X_n, X_2') & \cdots & K(X_n, X_n') \end{bmatrix} + \sigma^2 I_n \right) \quad (13)$$

And,

$$E(f|Y,X,\vartheta) = K_{ji}(K_{ii} + \sigma^2 I_n)^{-1}Y \quad (14)$$

$$V(f|Y,X,\vartheta) = K_{11} - K_{ji}(K_{ii} + \sigma^2 I_n)^{-1}K_{ij} \quad (15)$$

Where K is kernel function (Ye and Guo, 2023) and (Y. Morita, et al., 2021)

4. SUGGESTED FUZZY GAUSSIAN PROCESS REGRESSION KERNEL

In the realm of machine learning, (GPR) is a significant data fitting technique has the potential to generate nonlinear models for any system. Bayesian techniques can help avoid over-fitting even in cases when the model space is indefinitely dimensional. These techniques yield maximum likelihood model based on restricted collection of data (Shi et al., 2022) (Saeid and AlTaweel, 2021) , but if that the target variable are imprecise and uncertain and are expressed in fuzzy numbers $\tilde{y} \in \tilde{Y}$ as in equation(16) , (Bashar & Mahdi, 2022),

$$\tilde{Y} = \{[-\infty, \infty), \mu_y(Y)\} \quad (16)$$

All the items expressed in the fuzzy set with a degree of membership greater than or equal to the alpha cut (α-cut), which indicates the degree of membership of the elements we are interested in, make up the typical observation vector that we may extract from the fuzzy set as in equation(17). $Y^{(\tau)}$, (Bashar & Mahdi, 2022)

$$Y^{(\tau)} = \{\tilde{y} = (-\infty, \infty) \in \tilde{Y}, \mu_Y(Y) = \tau \,;\, \mu_Y(Y) \geq \tau\} \quad (17)$$

Where $\mu_{\tilde{y}}(Y)$ the membership function, which can be any kind of membership function and generates a degree of membership for each observation in the sample space. Triangle functions are employed, and they as in equation(18), (Mahdi & Bashar, 2020)

$$\mu_{\tilde{Y}}(Y) = \begin{cases} \frac{y - a_1}{a_2 - a_1} & for\ a_1 \leq y \leq a_2 \\ \frac{a_3 - y}{a_3 - a_2} & for\ a_2 \leq y \leq a_3 \end{cases} \quad (18)$$

Then the Fuzzy Gaussian process regression Kernel is as in equation(19),

$$Y^{(\tau)} = f(X) + e \quad (19)$$

With fuzzy posterior distribution is as in equation(20) to equation (23),

$$h(f|X,\tilde{Y},\underline{\vartheta}) \propto e^{-\frac{1}{2}\left[\frac{1}{\sigma^2}(\tilde{y}-f)(\tilde{y}-f)' + fK^{-1}f'\right]} \, d \quad (20)$$

$$\tilde{Y}_i = f(X_{ij}) + e_i \sim N(0,\ K(X, X') + \sigma^2 I_n) \quad (21)$$

With,

$$E(f|\tilde{Y},X,\vartheta) = K_{ji}(K_{ii} + \sigma^2 I_n)^{-1}\tilde{Y} \quad (22)$$

$$V(f|\tilde{Y},X,\vartheta) = K_{11} - K_{ji}(K_{ii} + \sigma^2 I_n)^{-1}K_{ij} \quad (23)$$

5. KERNEL FUNCTIONS

Gaussian process regression (GPR) relies on kernel functions, which are also known as covariance functions or similarity functions. The kernel function, which defines the relationships between data points in the input space, captures the patterns' data. The choice of kernel function has a significant impact on the behavior of the GP model and its ability to depict complex relationships. (Kanagawa et al., 2018)

Gaussian process regression employs various types of kernel functions to model different types of relationships within the data, we used the following types of kernel functions:

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5.1. Squared Exponential Kernel Function K_{SE}

Also known as Gaussian kernel which it a popular choice in machine learning, particularly in the context of Gaussian processes and kernelized support vector machines. It is commonly used for capturing smooth and continuous patterns in data as in equation(24), (Shi et al, 2022) and (Biancolini, 2018)

$$K_{SE} = \sigma_f^2 e^{-\frac{(\|X_a - X_b\|^2)}{2l^2}} \quad (24)$$

Where X_a and X_b are input vectors, $\|X_a - X_b\|$ is the Euclidean distance between the vectors, σ_f^2 is overall variance, l length scale or characteristic length is a hyper parameter that determines the "width" of the kernel as in Figuar2.

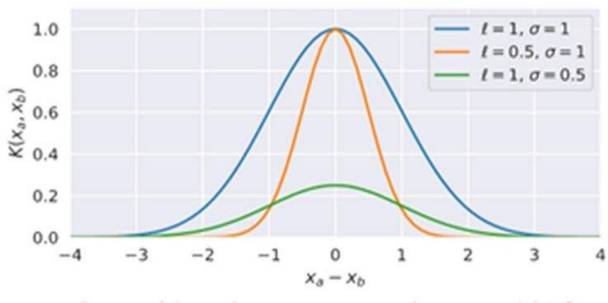

Figure 2. Squared Exponentiated quadratic kernel curve

5.2 The Gaussian Kernel Function K_{GK}

Similar to the Squared Exponential Kernel, the Gaussian RBF Kernel is frequently used in Gaussian process regression to represent smooth and non-linear interactions. When points are farther apart in the input space, the similarity of both kernels shows a gradual decline.

which it used in machine learning, it's a versatile kernel that allows for a broader range of smooth and non-smooth patterns in the data as in equation(25), (Bart M. Romeny, 2003)

$$K_{GK} = \frac{1}{(2\pi\sigma)^{\frac{9}{2}}} e^{-\frac{(\|X_a - X_b\|^2)}{2l^2}} \quad (25)$$

Where σ is the overall standard deviation, l the length scale or characteristic length is a hyper parameter that determines the "width" of the kernel as in Figuar3.

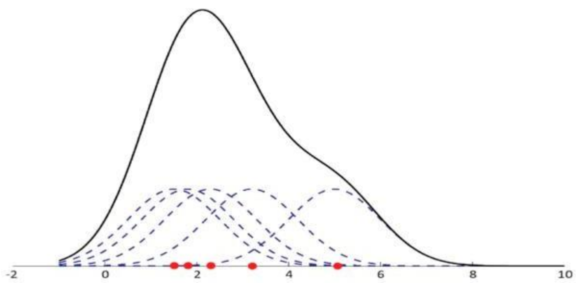

Figure 3 The Gaussian Kernel function curve.

5.3 Hybrid kernel function (K_{Hybrid}):

In a Gaussian process regression model, K_{Hybrid} is a companied of several kernel functions. In order to capture a variety of patterns and connections within the data, the concept behind employing a hybrid kernel is to take use of the capabilities of various kernel functions. A hybrid kernel is an attempt to create a more expressive and versatile representation of the underlying processes by fusing several kernels. In this paper we suggest the hybrid kernel function from two kernels functions K_{SE} and GK as in equation(26)

$$K_{Hybrid} = K_{SE} + \lambda K_{GK} \quad (26)$$

Where $0 < \lambda < 1$ scale-mixture parameter, Then the Hybrid Kernel is as in equation(27)

$$K_{\text{Hybrid}} = \sigma_f^2 e^{-\frac{(\|X_a - X_b\|_2^2)}{2l^2}} + \lambda \frac{1}{(2\pi\sigma)^{\frac{g}{2}}} e^{-\frac{(\|X_a - X_b\|_2^2)}{2l^2}} = e^{-\frac{(\|X_a - X_b\|_2^2)}{2l^2}} \left(\sigma_f^2 + \frac{\lambda}{(2\pi\sigma)^{\frac{g}{2}}}\right) \quad (27)$$

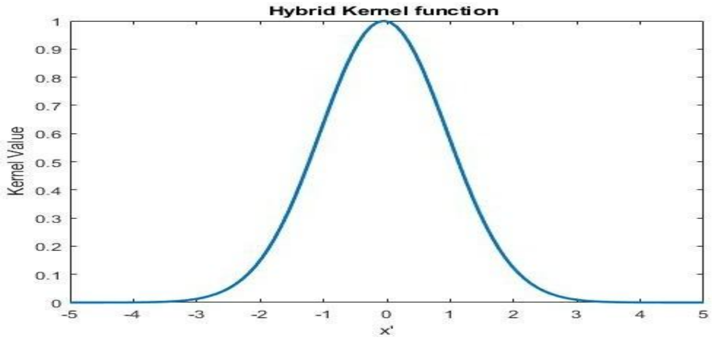

Figure 4. The Hybrid Kernel curve

6. SPIDER MONKEY OPTIMIZATION ALGORITHM (SMO)

Spider monkey optimization (SMO) algorithm is a recent addition to the list of swarm intelligence based optimization algorithms. This Algorithm developed by (Bansal, 2014) by monitoring the behavior of monkeys in their pursuit and search for food. (Sharma et al., 2014)

This algorithm has four main parameters: the MG parameter, which represents the maximum number of groups in the swarm, and the disturbance factor PR, which is responsible for the amount of disturbance in the current situation to find the approximate solution to the problem. The limits of the disturbance rate are within the period [0.1, 0.9], and the limits of the global best GBL are defined by the period [n]. /2,2n], and the limits of the local best LBL are defined by the dimension q+n. The SMO algorithm is an iterative algorithm subject to trial and error, and its steps are:

- Preparing the swarm size, i.e. determining the initial population size of the swarm as as in equation(28):

$$SM_{iq} = SM_{minq} + U(0,1) \times (SM_{maxq} + SM_{minq}) \quad (28)$$

Where SM_{iq} is Individual's location SM_{maxq} , SM_{minq} is maximum and minimum Individual's swarm , $U(0,1)$ is uniform distribution

- Determining the local best (the local leader) by determining the location of the individual SM_{iq} based on the local best and the information available from the members of the group by calculating the matching function for the new individual's location SM_{newiq} and the local individual's location. If it is ($f_{SM_{ig}} < f_{SM_{newig}}$) then the location of the new individual is determined as in equation(29):

$$SM_{newiq} = SM_{iq} + u(0,1) \times (LB_{Kq} - SM_{iq}) + U(-1,1) \times (SM_{rq} - SM_{iq}) \quad (29)$$

Where SM_{newiq} new individual's location, LB_{Kq} the local leader in k subgroups, SM_{rq} , random individual's location in group k, $U(-1,1)$ is uniform distribution

- The global best within the SMO algorithm, by updating the sites according to the information of the global best and the information of the group members as in equation(30)

$$SM_{newiq} = SM_{iq} + u(0,1) \times (GB_q - SM_{iq}) + U(-1,1) \times (SM_{rq} - SM_{iq}) \quad (30)$$

Where GB_q is the Global Best within individual's group at dimension q, (Sharma et al., 2020)

7. EXPERIMENT RESULTS

7.1 Diabetes Disease

The body's insufficient insulin utilization causes type 2 diabetes mellitus (T2DM), also known as non-insulin-dependent or adult-onset diabetes, according to the WHO. Over 95% of diabetics are type 2. This kind of diabetes is caused by obesity and inactivity. The symptoms may mirror type 1 diabetes but are usually milder. Thus, the illness may be Type 2 diabetes occurs when cells don't respond to insulin, according to the ADA. The pancreas produces insulin to help the body use blood glucose for energy. Type 2 diabetics have trouble absorbing glucose into

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cells, causing hyperglycemia. Type 2 diabetes causes frequent urination, thirst, tiredness, weight loss, and increased urine production. Type 2 diabetes is managed with a balanced diet, exercise, and hypoglycemic medication.(Murad AM et al ., 2023)

Double diabetes (DD) is a special kind of type 1 diabetes (DM) that is characterized by a more severe metabolic phenotype and a higher risk of complications in the macrovascular and microvascular systems. This kind showed clinical signs of insulin resistance (IR). The main goal of detecting double diabetes is to quickly provide the best treatment choices in order to reduce the increased risk of chronic problems and other negative metabolic features that are connected to this illness.(Hassan HJ et al ., 2023)

7.2 Data describe

The present work is performed at Al-Elwiya Maternity Teaching Hospital in Baghdad from June1, 2022 to December 31, 2022 were used with number of observations (n=191) that represent the crisp set. The target variable Y (training variable) represented the child's weight in gram .And the explanatory variables (X_1 age, X_2 mother's weight, X_3 No. of live births, X_4 No. of stillbirths, X_5 medical history For diabetes, X_6 level of education, X_7 type of diabetes (type 1 - type 2 - Gestational Diabetes(GD)- None), X_8 Diabetes control (insulin - diet - insulin + diet - none), X_9 type of insulin (soluble - lenty - lenty + soluble - none), X_{10} blood sugar level, X_{11} polyhydramnios, x_{12} macrosomia, X_{13} Use of contraceptive medications, X_{14} Type of delivery (natural - caesarean) , X_{15} Condition of the child at birth (alive - dead), X_{16} Birth malformations, X_{17} Postpartum sugar levels, X_{18} Postpartum complications .

7.3 Fuzziness of data

Considering that child weight measurements are inaccurate and include some errors in measurement, which can be addressed by using the principle of fuzzy sets and by using a triangular membership function in equation (21) to find the fuzzy set by finding the degrees of membership for each element in the weight vector as table 1. which consist of ordered pairs as equation (2) with $\{(y_i, \mu_{\tilde{y}}(y_i)) \ , \ i = 1,2, \dots , 191\}$. According to the nature of the phenomena studied and represented by the weights of children, the researchers suggested choosing alpha-cut factor $\tau = 0.5$, then the fuzzy set $\tilde{Y}^{\tau=0.5} = \{(y_i, \mu_{\tilde{y}}(y_i)), y \in Y, i = 1,2,3, \dots ,184, \ \tau = 0.5\}$ as Table 1.

7.4 The fuzzy Gaussian process regression model prediction

The fuzzy Gaussian process regression model for the children's fuzzy weights by (SMO) algorithm in paragraph (7) conducted with MatLab Ver2023a by using the optimization function (fminunc) and paragraphs (4) , (5) by use the fuzzy set in (7.3) by using (fitrgp) with 70% from data as training set (ntrian=129) , 30% as testing set (ntest=55) under $\sigma_f = 0.9234$ which represent the standard deviation of overall explanatory variables and $l = 0.8$ choosing by using optimization the parameter with SMO by using the optimization function (fminunc) to ensure greater data smoothing as listed in Table 2.

Table 1. The child's weight in grams and the membership function corresponding to each weight and fuzzy observations $\tilde{y}$ under, $\tau = 0.5$

i	y	$\mu_y(y_i)$	$\tilde{y}$	i	y	$\mu_y(y_i)$	$\tilde{y}$	i	y	$\mu_y(y_i)$	$\tilde{y}$	i	y	$\mu_y(y_i)$	$\tilde{y}$
1	300	0.00000	2500	51	3000	0.62791	3000	101	3400	0.72093	3500	151	3800	0.81395	4000
2	400	0.02326	2500	52	3000	0.62791	3000	102	3400	0.72093	3500	152	3900	0.83721	4000
3	400	0.02326	2600	53	3000	0.62791	3000	103	3400	0.72093	3500	153	3900	0.83721	4000
4	450	0.03488	2700	54	3000	0.62791	3000	104	3400	0.72093	3500	154	3900	0.83721	4000
5	500	0.04651	2750	55	3000	0.62791	3000	105	3400	0.72093	3500	155	3900	0.83721	4000
6	1500	0.27907	2800	56	3000	0.62791	3000	106	3400	0.72093	3500	156	3900	0.83721	4000
7	2000	0.39535	2800	57	3000	0.62791	3000	107	3500	0.74419	3500	157	4000	0.86047	4000
8	2500	0.51163	2800	58	3000	0.62791	3000	108	3500	0.74419	3500	158	4000	0.86047	4100
9	250	0.5116	290	59	300	0.627	300	10	350	0.744	350	15	400	0.860	420

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	0	3	0		0	91	0	9	0	19	0	9	0	47	0
1 0	260 0	0.534 88	290 0	60	300 0	0.627 91	300 0	11 0	350 0	0.744 19	350 0	16 0	400 0	0.860 47	420 0
11	270 0	0.5581 4	300 0	61	300 0	0.627 91	320 0	11 1	350 0	0.744 19	350 0	16 1	400 0	0.860 47	420 0
1 2	275 0	0.5697 7	300 0	62	300 0	0.627 91	320 0	11 2	350 0	0.744 19	350 0	16 2	400 0	0.860 47	420 0
1 3	280 0	0.5814 0	300 0	63	300 0	0.627 91	320 0	11 3	350 0	0.744 19	350 0	16 3	400 0	0.860 47	420 0
1 4	280 0	0.5814 0	300 0	64	300 0	0.627 91	320 0	11 4	350 0	0.744 19	350 0	16 4	400 0	0.860 47	420 0
1 5	280 0	0.5814 0	300 0	65	300 0	0.627 91	320 0	11 5	350 0	0.744 19	350 0	16 5	410 0	0.883 72	420 0
1 6	290 0	0.604 65	300 0	66	300 0	0.627 91	320 0	11 6	350 0	0.744 19	350 0	16 6	420 0	0.906 98	425 0
17	290 0	0.604 65	300 0	67	300 0	0.627 91	320 0	11 7	350 0	0.744 19	350 0	16 7	420 0	0.906 98	425 0
1 8	300 0	0.627 91	300 0	68	320 0	0.674 42	320 0	11 8	350 0	0.744 19	350 0	16 8	420 0	0.906 98	425 0
1 9	300 0	0.627 91	300 0	69	320 0	0.674 42	320 0	11 9	350 0	0.744 19	350 0	16 9	420 0	0.906 98	425 0
2 0	300 0	0.627 91	300 0	70	320 0	0.674 42	320 0	12 0	350 0	0.744 19	350 0	17 0	420 0	0.906 98	425 0
2 1	300 0	0.627 91	300 0	71	320 0	0.674 42	320 0	12 1	350 0	0.744 19	350 0	17 1	420 0	0.906 98	430 0
2 2	300 0	0.627 91	300 0	72	320 0	0.674 42	320 0	12 2	350 0	0.744 19	360 0	17 2	420 0	0.906 98	430 0
2 3	300 0	0.627 91	300 0	73	320 0	0.674 42	320 0	12 3	350 0	0.744 19	360 0	17 3	425 0	0.918 60	430 0
2 4	300 0	0.627 91	300 0	74	320 0	0.674 42	320 0	12 4	350 0	0.744 19	360 0	17 4	425 0	0.918 60	430 0
2 5	300 0	0.627 91	300 0	75	320 0	0.674 42	325 0	12 5	350 0	0.744 19	360 0	17 5	425 0	0.918 60	430 0
2 6	300 0	0.627 91	300 0	76	320 0	0.674 42	325 0	12 6	350 0	0.744 19	360 0	17 6	425 0	0.918 60	430 0
2 7	300 0	0.627 91	300 0	77	320 0	0.674 42	325 0	12 7	350 0	0.744 19	360 0	17 7	425 0	0.918 60	440 0
2 8	300 0	0.627 91	300 0	78	320 0	0.674 42	325 0	12 8	350 0	0.744 19	370 0	17 8	430 0	0.930 23	440 0
2 9	300 0	0.627 91	300 0	79	320 0	0.674 42	325 0	12 9	360 0	0.767 44	370 0	17 9	430 0	0.930 23	450 0
3 0	300 0	0.627 91	300 0	80	320 0	0.674 42	325 0	13 0	360 0	0.767 44	370 0	18 0	430 0	0.930 23	450 0
3 1	300 0	0.627 91	300 0	81	320 0	0.674 42	330 0	13 1	360 0	0.767 44	370 0	18 1	430 0	0.930 23	460 0
3 2	300 0	0.627 91	300 0	82	325 0	0.686 05	330 0	13 2	360 0	0.767 44	375 0	18 2	430 0	0.930 23	460 0
3 3	300 0	0.627 91	300 0	83	325 0	0.686 05	335 0	13 3	360 0	0.767 44	375 0	18 3	430 0	0.930 23	460 0
3 4	300 0	0.627 91	300 0	84	325 0	0.686 05	340 0	13 4	360 0	0.767 44	380 0	18 4	440 0	0.953 49	460 0
3 5	300 0	0.627 91	300 0	85	325 0	0.686 05	340 0	13 5	370 0	0.790 70	380 0	18 5	440 0	0.953 49	
3 6	300 0	0.627 91	300 0	86	325 0	0.686 05	340 0	13 6	370 0	0.790 70	380 0	18 6	450 0	0.976 74	
3 7	300 0	0.627 91	300 0	87	325 0	0.686 05	340 0	13 7	370 0	0.790 70	380 0	18 7	450 0	0.976 74	

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38	3000	0.62791	3000	88	3300	0.69767	3400	138	3700	0.79070	3800	188	4600	1.00000	
39	3000	0.62791	3000	89	3300	0.69767	3400	139	3750	0.80233	3800	189	4600	1.00000	
40	3000	0.62791	3000	90	3350	0.70930	3400	140	3750	0.80233	3800	190	4600	1.00000	
41	3000	0.62791	3000	91	3400	0.72093	3400	141	3800	0.81395	3800	191	4600	1.00000	
42	3000	0.62791	3000	92	3400	0.72093	3400	142	3800	0.81395	3800				
43	3000	0.62791	3000	93	3400	0.72093	3400	143	3800	0.81395	3800				
44	3000	0.62791	3000	94	3400	0.72093	3400	144	3800	0.81395	3800				
45	3000	0.62791	3000	95	3400	0.72093	3400	145	3800	0.81395	3900				
46	3000	0.62791	3000	96	3400	0.72093	3400	146	3800	0.81395	3900				
47	3000	0.62791	3000	97	3400	0.72093	3400	147	3800	0.81395	3900				
48	3000	0.62791	3000	98	3400	0.72093	3400	148	3800	0.81395	3900				
49	3000	0.62791	3000	99	3400	0.72093	3400	149	3800	0.81395	3900				
50	3000	0.62791	3000	100	3400	0.72093	3500	150	3800	0.81395	4000				

The prediction values of child's weight by fuzzy Gaussian process model under Squared exponential, Gaussian, Hybrid kernel functions as in Table2.

Table 2. The fuzzy child's weight prediction under K_{SE} , K_{GK} kernel functions and K_{Hybrid} kernel functions under mixture parameter λ=0.5

i	$\tilde{y}$	$\hat{y}_{K_{SE}}$	$\hat{y}_{GK}$	$\hat{y}_{Hybr}$	i	$\tilde{y}$	$\hat{y}_{K_{SE}}$	$\hat{y}_{GK}$	$\hat{y}_{Hybr}$	i	$\tilde{y}$	$\hat{y}_{K_{SE}}$	$\hat{y}_{GK}$	$\hat{y}_{Hybr}$	i	$\tilde{y}$	$\hat{y}_{K_{SE}}$	$\hat{y}_{GK}$	$\hat{y}_{Hybr}$
1	2500	2612	2582	2523	51	3000	3111	3108	3034	101	3500	3600	3569	3531	151	4000	4221	3946	4011
2	2500	2612	2582	2523	52	3000	3111	3108	3034	102	3500	3600	3599	3531	152	4000	4221	3946	4011
3	2600	2712	2754	2671	53	3000	3111	3108	3034	103	3500	3600	3599	3531	153	4000	4221	3946	4011
4	2700	2811	2769	2735	54	3000	3111	3108	3034	104	3500	3600	3599	3531	154	4000	4221	3946	4011
5	2750	2852	2890	2728	55	3000	3111	3108	3034	105	3500	3600	3599	3531	155	4000	4221	3946	4011
6	2800	2912	2918	2810	56	3000	3111	3108	3034	106	3500	3600	3599	3531	156	4000	4221	3946	4011
7	2800	2912	2918	2810	57	3000	3111	3108	3034	107	3500	3600	3599	3531	157	4000	4221	3946	4011
8	28	29	29	281	5	30	311	31	30	1	35	36	35	35	15	41	42	42	411

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	00	12	18	0	8	00	1	08	34	08	00	00	99	31	8	00	55	41	5
9	2900	3012	2996	2918	59	3000	3111	3108	3034	109	3500	3600	3599	3531	159	4200	4255	4221	4207
10	2900	3012	2996	2918	60	3000	3111	3108	3034	110	3500	3600	3599	3531	160	4200	4255	4221	4207
11	3000	3111	3108	3034	61	3200	3172	3164	3178	111	3500	3600	3599	3531	161	4200	4255	4221	4207
12	3000	3111	3108	3034	62	3200	3172	3164	3178	112	3500	3600	3599	3531	162	4200	4255	4221	4207
13	3000	3111	3108	3034	63	3200	3172	3164	3178	113	3500	3600	3599	3531	163	4200	4255	4221	4207
14	3000	3111	3108	3034	64	3200	3172	3164	3178	114	3500	3600	3599	3531	164	4200	4255	4221	4207
15	3000	3111	3108	3034	65	3200	3172	3164	3178	115	3500	3600	3599	3531	165	4200	4255	4221	4207
16	3000	3111	3108	3034	66	3200	3172	3164	3178	116	3500	3600	3599	3531	166	4250	4260	4258	4251
17	3000	3111	3108	3034	67	3200	3172	3164	3178	117	3500	3600	3599	3531	167	4250	4260	4258	4251
18	3000	3111	3108	3034	68	3200	3172	3164	3178	118	3500	3600	3599	3531	168	4250	4260	4258	4251
19	3000	3111	3108	3034	69	3200	3172	3164	3178	119	3500	3600	3599	3531	169	4250	4260	4258	4251
20	3000	3111	3108	3034	70	3200	3172	3164	3178	120	3500	3600	3599	3531	170	4250	4260	4258	4258
21	3000	3111	3108	3034	71	3200	3172	3164	3178	121	3500	3600	3599	3531	171	4300	4152	4378	4312
22	3000	3111	3108	3034	72	3200	3172	3164	3178	122	3600	3700	3589	3625	172	4300	4152	4378	4312
23	3000	3111	3108	3034	73	3200	3172	3164	3178	123	3600	3700	3589	3625	173	4300	4152	4378	4312
24	3000	3111	3108	3034	74	3200	3172	3164	3178	124	3600	3700	3589	3625	174	4300	4152	4378	4312
25	3000	3111	3108	3034	75	3250	3355	3278	3241	125	3600	3700	3589	3625	175	4300	4152	4378	4312
26	3000	3111	3108	3034	76	3250	3355	3278	3241	126	3600	3700	3589	3625	176	4300	4152	4378	4312
27	3000	3111	3108	3034	77	3250	3355	3278	3241	127	3600	3700	3589	3625	177	4400	4552	4499	4445
28	3000	3111	3108	3034	78	3250	3355	3278	3241	128	3700	3500	3675	3743	178	4400	4552	4499	4445
29	3000	3111	3108	3034	79	3250	3355	3278	3241	129	3700	3500	3675	3743	179	4500	4583	4567	4511
30	3000	3111	3108	3034	80	3250	3355	3278	3241	130	3700	3500	3675	3743	180	4500	4583	4567	4511
31	3000	3111	3108	3034	81	3300	3456	3389	3310	131	3700	3500	3675	3743	181	4600	4782	4696	4626
32	3000	3111	3108	3034	82	3300	3456	3389	3310	132	3750	3500	3678	3730	182	4600	4782	4696	4626
33	3000	3111	3108	3034	83	3350	3566	3559	3331	133	3750	3500	3678	3730	183	4600	4782	4696	4626

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34	3000	3111	3108	3034	84	3400	3567	3497	3422	134	3800	4000	3968	3812	184	4600	4782	4696	4626
35	3000	3111	3108	3034	85	3400	3567	3497	3422	135	3800	4000	3968	3812					
36	3000	3111	3108	3034	86	3400	3567	3497	3422	136	3800	4000	3968	3812					
37	3000	3111	3108	3034	87	3400	3567	3497	3422	137	3800	4000	3968	3812					
38	3000	3111	3108	3034	88	3400	3567	3497	3422	138	3800	4000	3968	3812					
39	3000	3111	3108	3034	89	3400	3567	3497	3422	139	3800	4000	3968	3812					
40	3000	3111	3108	3034	90	3400	3567	3497	3422	140	3800	4000	3968	3812					
41	3000	3111	3108	3034	91	3400	3567	3497	3422	141	3800	4000	3968	3812					
42	3000	3111	3108	3034	92	3400	3567	3497	3422	142	3800	4000	3968	3812					
43	3000	3111	3108	3034	93	3400	3567	3497	3422	143	3800	4000	3968	3812					
44	3000	3111	3108	3034	94	3400	3567	3497	3422	144	3800	4000	3968	3812					
45	3000	3111	3108	3034	95	3400	3567	3497	3422	145	3900	4130	3879	3917					
46	3000	3111	3108	3034	96	3400	3567	3497	3422	146	3900	4130	3879	3917					
47	3000	3111	3108	3034	97	3400	3567	3497	3422	147	3900	4130	3879	3917					
48	3000	3111	3108	3034	98	3400	3567	3497	3422	148	3900	4130	3879	3917					
49	3000	3111	3108	3034	99	3400	3567	3497	3422	149	3900	4130	3879	3917					
50	3000	3111	3108	3034	100	3500	3600	3569	3531	150	4000	4221	3946	4011					

For the training data we illustrate the true value and the predicted mean with confidence interval by using Squared exponential as in Figure 4, squared exponential kernel function as in Figure5 and Hybrid kernel Figure 6.

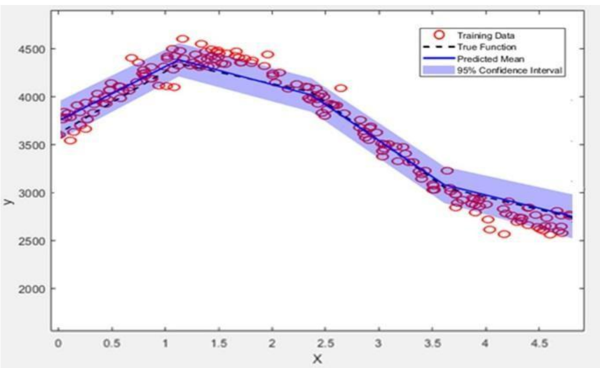

Figure 4. Fuzzy Gaussian process regression model for the children's fuzzy weights under SEQ Kernel function

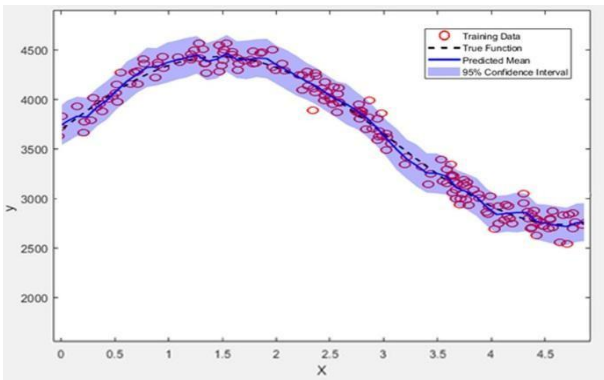

Figure 5. Fuzzy Gaussian process regression model for the children's fuzzy weights under GK Kernel function

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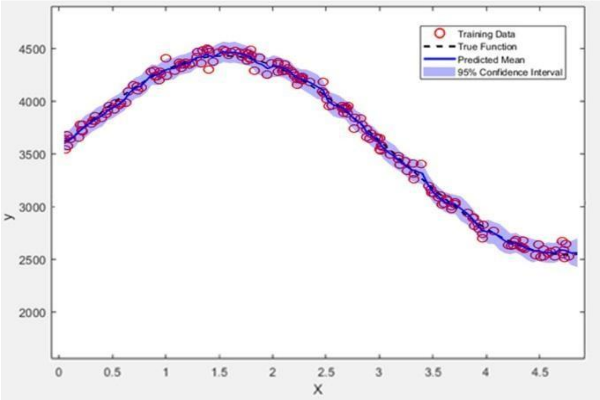

Figure 6. Fuzzy Gaussian process regression model for the children's fuzzy weights under Hybrid Kernel function

Model performance is assessed using root mean square error (RMSE), mean square error (MSE), and means absolute percentage error (MAPE). Results indicate a significant difference between actual and predicted values for the squared exponential kernel function, whereas the Gaussian kernel function produces closer predictions. The hybrid kernel function achieves the most accurate predictions, aligning more closely with actual child weight values than the individual kernel functions as in Table 3.

Table 3. Comparing fuzzy Gaussian process regression under kernels functions

Kernel	RMSE	MSE	MAPE
Squared Exponential Kernel	6.98099	48.73422	23.56743
Gaussian Kernel	5.78377	33.452	18.46633
Hybrid Kernel	1.98839	3.953695	4.356330

8. DISCUSSION

Table 2 showed that the prediction values of child's weight by fuzzy Gaussian process model under Squared exponential, Gaussian, Hybrid kernel functions respectively. We note that under Squared exponential there is a big difference between the real and predicted values , under Gaussian kernel that the predictive values are closer to the true values under this function and better than the squared exponential kernel function, under Hybrid kernel the predictive values are more fitting and consistent with the real values of children's weights, which indicates that the hybrid function gave more accurate predictions than the squared exponent function and the Gaussian function , Table 3. These results are confirmed because the root mean square error, mean square error, mean absolute percentage error of the fuzzy Gaussian process regression model at the hybrid kernel function is less than the rest of the kernel functions.

9. CONCLUSIONS

The study demonstrates that the fuzzy Gaussian process regression model with a hybrid kernel function effectively improves the accuracy of children's weight predictions. By integrating fuzzy set theory and a triangular membership function, the model successfully addresses measurement inconsistencies, enhancing predictive reliability. The spider monkey optimization (SMO) algorithm further refines the model by optimizing key parameters.

Performance evaluation using RMSE, MSE, and MAPE confirms that the hybrid kernel function produces more precise predictions compared to the squared exponential and Gaussian kernel functions. The findings highlight the superiority of the hybrid approach in aligning predicted values with actual child weights.

In conclusion, the proposed model provides a more reliable and accurate method for weight prediction, making it a valuable tool for pediatric health assessments. Future work could explore its application to larger datasets and other medical prediction challenges.

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