Journal of Information Systems Engineering and Management
2025, 10(37s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

Modeling Friendship Dynamics in Social Networks: A Graph-Theoretic and Computational Approach

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ARTICLE INFO

Received: 28 Dec 2024

Revised: 18 Feb 2025

Accepted: 26 Feb 2025

ABSTRACT

This paper explores the application of graph theory in analyzing social networks, focusing on the relationships between members represented by vertices (individuals) and edges (connections). It examines how members establish friendships, including mutual connections that arise when members share common connections, as well as the potential for information sharing within these networks. The study investigates methods for predicting mutual friends based on the number of individual friends, utilizing linear regression to analyze trends and residual analysis to assess model quality. Additionally, it delves into the probability of forming relationships using the binomial distribution and employs the McCulloch-Pitts neuron model to study about un-friendships between members. The findings highlight the trends between the number of friends and mutual friendships, with residuals used to evaluate the accuracy of the predictive model. Overall, this research contributes to the understanding of friendship dynamics within social networks by integrating statistical modeling and graph theory. Understanding friendship structures is important for analyzing how information spreads in social networks. People who are closely connected form groups that help share content. Over time, some users become more influential by gaining many followers. These influential users work as a key role in spreading information within the network. This study examines how user connections, especially those linked to influential people with many followers, affect the spread of information. It moves from studying friendships to analyzing information diffusion. The Susceptible-Infected-Recovered (SIR) model is used to study real Instagram data. The research focuses on 200 Instagram channels to understand how network structure, content, and user behavior shape information flow. These results show how content spreads and influences user interactions. This study will be helpful in insights for marketers, influencers, and policymakers.

Keywords: Social networks, graph theory, mutual friends, influential users, information diffusion, SIR model, Instagram.

INTRODUCTION

The rapid growth of social networks like Facebook, Instagram, and Twitter has revolutionized how people communicate and form relationships in the digital era. These platforms allow users to connect, share, and interact, leading to complex social structures where relationships vary from casual acquaintances to close friendships. As social networks expand, understanding the friendships formation, mutual friendships, and the factors influencing relationship formation has become an essential area of study.

Social Network Analysis (SNA) provides a robust framework for exploring these dynamics by modeling users as nodes and relationships as edges in a graph. Directed edges represent one-sided relationships, such as follows on Twitter, while undirected edges represent mutual friendships, as seen in Facebook connections. Analyzing these relationships using mathematical models offers valuable insights into how friendships are established, maintained, and dissolved over time. In particular, mutual friendships—where two users share one or more common friends—

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play an important role in strengthening social ties and enhancing user engagement. Predicting the likelihood of mutual friendships, however, poses a significant challenge due to the intricacies of social interactions, the influence of network structure, and individual behavioral patterns.

This paper seeks to explore the dynamics of friendships and mutual friendships by employing probabilistic and machine learning techniques. In the realm of social networks, understanding the factors that influence user engagement is crucial for both individuals and brands. Followers influence scores, the frequency of posts, and engagement rates are fundamental metrics that shape how users interact with content and each other. This study focuses on employing linear regression analysis to examine how these variables correlate and contribute to a user's overall influence within their network. A binomial and Bernoulli distributions approach is used to model friendship formation as a random process and use linear regression to predict mutual friendships based on the number of direct connections. To further analyze relationship decay, the McCulloch-Pitts neuron model is used to simulate the dynamics of "unfriending," providing insights into factors contributing to the dissolution of friendships.

Social network platforms have grown quickly in recent years. They are valuable tools for communication and information sharing. Platforms like Facebook and Instagram help users share content with large audiences. Social networks allow people to exchange information, ideas, behaviors, and influence. The way information spreads in these networks is similar to how people share ideas in real life (Jin et al., 2013).Diffusion is the process of sharing new ideas or information. It happens through different channels and involves social interactions. One person shares knowledge and others pass it along. This spread usually follows a pattern. It starts slowly with a few early adopters. Then, more people join, and the process speeds up. Eventually, it slows down as fewer individuals remain to adopt the idea (Guille and Hacid, 2012).Social networks play an important role in many areas. They are used in marketing, health campaigns, and efforts to stop misinformation. Connections between users can be one-to-one or one-to-many. These links can represent personal relationships, online interactions, or shared experiences in a group. A person's influence in a social network depends on their position, the strength of their connections, and the type of information they share (Cinelli et al., 2021). Both a user's position and the structure of the network affect how information spreads. Information spreads through many channels. Digital platforms such as social media, blogs, and instant messaging also play a big role.

The diffusion process has three key components:
a) The Sender – Shares information with one or more receivers.
b) The Receiver – Can pass the information to others, creating a chain reaction.
c) The Medium – The channel through which the information flows, such as social media, personal connections, or face-to-face interactions.

This paper aims to improve the understanding of friendship dynamics in social networks. It also presents models to help predict relationship trends. The insights from this study can improve social platform design, enhance friend recommendation systems, and helps in a better understanding of online interactions. The main objective of this paper is to study and predict how friendships and mutual friendships form, evolve, and dissolve in social networks. To achieve this, the paper uses a mix of probabilistic models, supervised learning methods, and neural modeling techniques.The specific goals of this research are as follows:

- To model friendship formation using Bernoulli trials and the binomial distribution, treating friend requests as random events that can either be accepted or rejected.

- To apply linear regression to predict mutual friendships based on the number of existing friends and other related features, and to evaluate the model's accuracy using residual analysis.

- To simulate unfriendship dynamics by using the McCulloch-Pitts neuron model, which helps to understand how declining interaction or trust can lead to the end of a friendship over time.

- To examine the role of network structure—including directed and undirected relationships—in shaping the patterns of friendship formation, mutual connections, and unfriendship in large-scale online social networks.

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This paper also explores how user behavior and influence contribute to the spread of information, aiming to improve social media design, recommendation systems, and the overall understanding of online social interactions.

LITERATURE REVIEW

The study of social networks has become a central topic in understanding human interaction in online spaces. Various scholars have contributed to the literature by focusing on friendship models, the dynamics of mutual friendships, and the use of advanced statistical and machine learning techniques to predict and analyze these relationships. This section explores key research findings, offering insights into the structural and dynamic aspects of social networks, especially focusing on friendship models and mutual connections.

Graph Theory and Social Networks

Graph theory provides a fundamental framework for analyzing social networks, where individuals are represented as nodes and their relationships as edges in a graph. Wasserman and Faust (1994) laid the groundwork for Social Network Analysis (SNA), which visualizes and analyzes relationships through directed and undirected graphs. Directed graphs often represent asymmetric relationships (e.g., followers on Twitter), whereas undirected graphs signify symmetric, mutual friendships (e.g., Facebook friendships). Newman (2003) further extended the use of graph theory, presenting techniques to analyze complex networks, including social networks.

Barabási (1999) introduced the concept of scale-free networks, which are characterized by a few nodes (hubs) that possess a disproportionately high number of connections. This model is crucial for predicting how relationships form and spread in social networks, as hubs play a key role in creating mutual friendships by connecting users with many common friends.

Mutual Friendships in Social Networks

Mutual friendships are a key component of social network dynamics, with extensive research exploring their role in strengthening social ties and promoting information diffusion. Leskovec et al. (2008) studied the formation of mutual friendships, demonstrating that individuals with shared mutual friends are more likely to form new connections. This aligns with Granovetter's (1973) theory of strong and weak ties, where mutual friendships often represent strong ties that foster lasting relationships and network cohesion.

Miritello et al. (2013) focused on social triads and the role of mutual friends in forming stable connections. They found that social networks tend to evolve through the formation of new edges in these triads, where the presence of mutual friends between any two users encourages the establishment of new connections, further reinforcing network stability.

Statistical Models for Friendship Prediction

Predicting friendships in social networks has been a focus of numerous studies, with researchers employing various statistical models. Liben-Nowell and Kleinberg (2007) proposed models based on graph-based metrics, such as common neighbors and Jaccard similarity, to predict future friendships. Their findings revealed that shared connections significantly increase the likelihood of new friendships forming.

Crandall et al. (2010) employed machine learning techniques, showing that mutual friendships and shared interests are strong indicators of future friendships. Their research highlighted that statistical models using features such as mutual friends and interaction frequency could provide accurate predictions of future relationships.

Binomial Distribution in Social Networks

The binomial distribution has been applied to model friendship formation, treating each friend request as a probabilistic event. Katz and Powell (1957) pioneered the use of probabilistic models in group dynamics, laying the foundation for the application of binomial distribution in social network analysis.

Yu and Liu (2015) applied binomial distribution to predict friendship formation, modeling each friend request as an independent Bernoulli trial. By aggregating these trials, they provided a probabilistic framework for understanding how likely a user is to form new friendships.

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Recent Approaches to Mutual Friendship Analysis

Olorituna et al. (2013) focused on close friendships in social networks, particularly how mutual friends can provide emotional support and foster deeper connections. The study introduced metrics such as the Mean Duration per Dyad (MDD) and applied the Quadratic Assignment Procedure (QAP) with logistic regression for statistical significance testing. This approach provided insights into the strength of mutual friendships within sensed social networks.

Sanja Krakan et al. (2018) further explored friendship intensity in social networks. Their model proposed that the greater the weight of friendship between two individuals, the stronger their real-life connection. They argued that the strength of online friendships often correlates with trusted relationships in offline contexts, including family and like-minded groups.

Community Structure and Dynamic Networks

Altotaibi et al. (2021) provided a comprehensive review of community structures in evolving social networks. Their work discussed how community detection methods help identify disjoint or overlapping communities in dynamic social networks. The ability of social networks to evolve through the creation and dissolution of friendships plays a significant role in shaping community structures, which in turn influence the formation of mutual friendships.

Sehaj Pal Singh et al. (2020) examined the dynamic nature of social networks by focusing on how mutual friendships are created and dissolved over time. They analyzed real-life scenarios in social networks like Facebook, where users may add or remove friends, reflecting the fluidity of social interactions in online environments. Their findings emphasized the importance of capturing these dynamic changes when modeling and predicting social relationships.

Supervised Learning and Neural Networks for Predicting Friendships

Supervised learning techniques, such as decision trees and neural networks, have been widely applied to predict user interactions and friendships. Tang, Aggarwal, and Liu (2016) applied supervised learning models to forecast user relationships, using features like user demographics, content interactions, and previous friendship history. Their research demonstrated the effectiveness of machine learning techniques in predicting future friendships and interactions in large-scale networks.

In particular, the McCulloch-Pitts neuron model, an early form of artificial neural network, has been utilized to simulate unfriending dynamics in social networks. By modeling neurons as binary units that "fire" under specific conditions, researchers have been able to simulate how users may decide to "unfriend" others based on factors like declining interactions or trust.

Information Diffusion and Engagement

Models of information diffusion in social networks, including the SIR (Susceptible-Infected-Recovered) framework, have adopted epidemiological approaches (Cifuentes-Faura and Campan, 2022) to study the spread of content across platforms. Bakshy et al. (2015) further examined how social ties influence the likelihood and speed of information diffusion.

The literature on social networks provides a comprehensive understanding of friendship models, mutual connections, and the methods used to predict these relationships. Graph theory, probabilistic models, and machine learning techniques have proven to be effective tools in analyzing social networks. The studies reviewed demonstrate the importance of mutual friendships in network stability and the potential of supervised learning methods in predicting relationship trends. This paper builds on these foundations, utilizing binomial distribution and supervised learning techniques to analyze and predict friendship and mutual friendship dynamics.

PRINCIPLES OF SOCIAL NETWORK ANALYSIS AND FUNDAMENTAL CONCEPTS

Basic Principles in Social Networks

- Interdependence of Nodes: Nodes and their activities are interdependent rather than independent, emphasizing the relational nature of social networks (Wasserman & Faust, 1994; Scott, 2017).
- Resource Flow through Relationships: Relationships (linkages) between actors serve as channels for the flow of resources, both material and non-material (Borgatti & Halgin, 2011).
- Individual-Focused Network Models: An individual-focused network model considers the environment of a network structure as an opportunity or constraint for individual behavior (Granovetter, 1973; Burt, 1992).
- Conceptualization of Structure: Network models conceptualize structure (social, economic, political, etc.) as permanent patterns of relationships among actors (Newman, 2010; Scott, 2017).

Terminology Used in Social Network Analysis

- Actor/Node: An actor (node) is a discrete individual, corporate entity, or collective social unit, such as humans in a group, departments in a corporation, or public enterprises in cities (Wasserman & Faust, 1994).
- Relational Ties: Actors are connected through social ties, which establish linkages between pairs of actors (Scott, 2017).
- **Dyad**: A connection involving two individuals, referring to a pair of participants and the relationship that links them (Wasserman & Faust, 1994).
- **Triad**: A grouping of three individuals and the interconnections among them (Wasserman & Faust, 1994).
- **Subgroup**: Any collection of selected individuals along with all the connections that exist among them (Wasserman & Faust, 1994).
- **Group**: The complete set of individuals within a network for whom relational data is gathered (Wasserman & Faust, 1994).
- **Relation**: A specific type of connection shared among the members of a network (Scott, 2017).

Three Levels of Social Network Analysis

- Microscopic Analysis: At the micro-level, we analyze interactions between pairs of nodes, tracing patterns at the dyadic level to observe properties such as homophily and reciprocity. Triadic interactions among three nodes are also examined (Borgatti & Halgin, 2011).
- Mesoscopic Analysis: This level focuses on substructures of the network, particularly communities or clusters formed by frequent interactions among identical nodes. Nodes within a community may exhibit distinct behaviors compared to nodes in other communities (Newman, 2010).
- Macroscopic Analysis: At the macro level, the entire network is considered to understand micro-dynamics by exploring global properties such as connectivity, average path length, and degree distribution (Scott, 2017).

Analyzing Relationships to Understand People and Groups

- Binary and Valued Relationships: Relationships can be binary (e.g., "X follows Y on Twitter") or valued (e.g., "Y retweeted 6 tweets from X"). While quantifying relationships on platforms like Twitter is straightforward, measuring the quality of interpersonal relationships in softer social contexts can be challenging (Wasserman & Faust, 1994).
- Symmetric and Asymmetric Relationships: Some relationships are naturally asymmetric (e.g., teacher/student), while others, such as friendships on Facebook, are more balanced. Understanding the directionality of relationships is crucial (Granovetter, 1973).
- Multimode Relationships: Relationships can exist between different types of actors (e.g., companies hiring people, investors buying stocks from corporations). These relationships are described as bimodal (Burt, 1992).

ESTABLISHMENT OF RELATIONSHIPS IN SOCIAL NETWORKS

All major online social networks, such as Myspace, Facebook, LiveJournal, and Orkut, are fundamentally built around the concept of friendship. When a user sends a friend request to another member of the social network, there are two possible outcomes: the request can either be accepted or rejected. These interactions can be treated as random experiments regarding the formation of friendships in social networks.

Journal of Information Systems Engineering and Management
2025, 10(37s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
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As an extension of relationships in social networks, friendship prediction can be approached through a probability-based model. This model learns from case studies within the social network, where friend requests can be sent, and there are chances of acceptance or rejection.

Graph theory in social network

Graph theory is a powerful tool in social network analysis (SNA) for modeling, understanding, and analyzing relationships, information flow, and community structures within networks. By representing users and interactions as nodes and edges, graph theory provides a systematic approach to studying complex social structures. Below is an exploration of foundational graph theory concepts, their relevance in SNA, and examples of their application.

Graph Basics

- **Graph (G)**: Consists of nodes (vertices) and edges (links) connecting them. *Example*: In a social network, each node represents an individual, and each edge represents a relationship (e.g., friendship).
- **Directed Graph**: Edges have a direction, such as one user following another.
- **Undirected Graph**: Edges lack direction, commonly used for mutual friendships.

Types of Graphs in SNA

- **Weighted Graphs**: Edges have weights, representing the strength of a relationship (e.g., frequency of interaction).
- **Bipartite Graphs**: Nodes are divided into two sets, with edges only between sets. Example*:* Connecting users to groups they belong to.

Graph Representation

- **Adjacency Matrix**: Matrix representation where each cell (i, j) shows an edge between nodes i and j. *Example*: In a friendship network, if U1 and U2 are friends, adj_matrix[U1][U2] = 1.
- **Edge List**: A list of pairs representing connections, effective for sparse networks.
- **Adjacency List**: Each node is linked to a list of adjacent nodes, ideal for sparse graphs.

Matrix Representation of a Graph

Adjacency Matrix of a Graph (or Connected Matrix)

Let G be a graph with n vertices and no parallel edges. The matrix A = [aij]nxn is called the Adjacency Matrix of G if:

aij = 1, if vi and vj are connected by an edge.
aij = 0, if there is no edge between vi and vj.

If there is a self-loop at vi, it is considered that vi and vi are connected by an edge.

Table 1: Representation of Relationship in Matrix

	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	1	0
C	0	1	0	1	0

Journal of Information Systems Engineering and Management
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	A	B	C	D	E
D	1	1	1	0	1
E	1	0	0	1	0

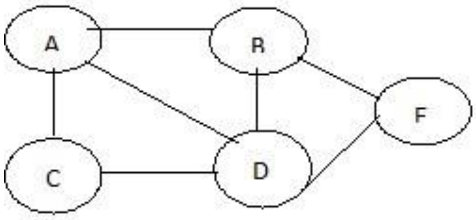

Figure 1: Representation of Relationship in Social Network using Graph

Adjacency Matrix of a Di-Graph

Let G be a graph with n vertices and no parallel edges. The matrix A = [aij]nxn is called the Adjacency Matrix of G if:

aij = 1, if there is an edge directed from vi to vj
aij = 0, otherwise.

Table 2: Adjacency Matrix of a Di-Graph

	N1	N2	N3	N4
N1	1	0	1	0
N2	1	0	1	0
N3	0	1	0	1
N4	0	0	0	0

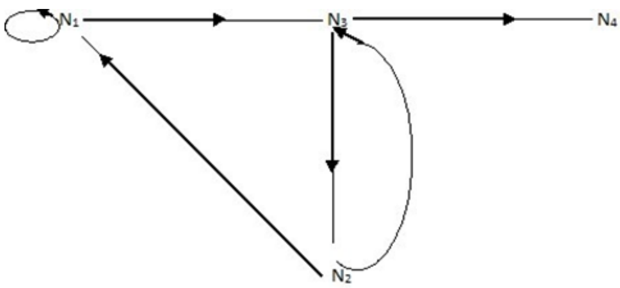

Figure 2: Representation of Directional Relationship in Social Network using Graph and Adjacency Matrix

(A directed graph with 4 vertices N1, N2, N3, and N4, where edges direct N1->N3, N1->N2, N2->N3, N3->N4)

Importance of Adjacency Matrix

The Adjacency Matrix can represent two disjoint groups in social networks.

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Table 3.: First Disjoint Group in Social Network

	N1	N2	N3	N4	N5	N6
N1	0	1	0	0	0	0
N2	1	0	0	0	0	0
N3	0	1	0	0	0	0
N4	0	0	0	0	1	0
N5	0	0	0	0	0	1
N6	0	0	0	1	0	0

Table 4: Second Disjoint Group in Social Network

	N1	N2	N3
N1	0	1	0
N2	1	0	0
N3	0	1	0

(Two separate graphs:

- The first graph has 6 vertices N1, N2, N3, N4, N5, and N6, with edges connecting N1-N2, N2-N3, N4-N5, and N5-N6.
- The second graph has 3 vertices N1, N2, and N3, with edges connecting N1-N2 and N2-N3)

The Probabilistic Approach to Studying the Establishment of Friendship in Social Networks

The Bernoulli distribution is a discrete probability distribution where each random experiment results in one of two possible outcomes: 'success' or 'failure.' More formally, the random variable X takes the value 1 (success) with probability p, and 0 (failure) with probability 1 - p. This type of experiment is known as a Bernoulli trial. When the number of trials n equals 1, the binomial distribution simplifies into the Bernoulli distribution.In social networks like Facebook, consider a scenario where a node (user) sends a friend request to another user. If the friend request is accepted, a friendship is established, which we classify as a success. Thus, the probability of success, p, can be modeled as p = 1/2. If X is a random variable following a Bernoulli distribution, the probability of friendship is:

$$P(X = 1) = p = 1/2.$$

Binomial Distribution Approach to Establishing Friendship in Social Networks

The binomial distribution formula is for any random variable X, given by;

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$$P(x:n,p) = {}^{n}C_{x}\, p^{x}\, (1-p)^{n-x} \text{ Or } P(x:n,p) = {}^{n}C_{x}\, p^{x}\, (q)^{n-x}$$

Where,

Where:

n is the number of independent trials (friend requests sent)

x is the number of successful trials (friendships formed)

p is the probability of success (acceptance of a friend request)

1 - p = q is the probability of failure (rejection of a friend request)

Properties of the Binomial Distribution and Their Application to Social Networks

- Binary Outcomes: The binomial distribution is based on experiments with two possible outcomes: success (friendship) or failure (no friendship). Similarly, when a user sends a friend request, the response is either accepted or rejected.
- Fixed Number of Trials: In the binomial model, there are n independent trials. A user in a social network can send n friend requests to other users.
- Probability of Success and Failure: Each friend request has its own probability of success or failure, and the probabilities can differ from one request to another depending on the factors influencing friendship.
- Independence of Trials: Every friend request is independent of others, meaning that the outcome of one request does not affect the outcome of another. This property aligns with the binomial distribution's assumption of independent trials.
- Counting the Number of Successes: The number of accepted friend requests (successes) is what contributes to the total friend count in a user's social network. Therefore, only accepted requests are recorded.

Mutual Friends in Social Networks and Predictive Modeling using Linear Regression

Simple Linear Regression is a statistical method for summarizing and studying the relationship between two quantitative variables:

- The independent variable (predictor) is denoted as X
- The dependent variable (response) is denoted as Y

Note : This dataset, showing the relationship between the number of friends (X) and mutual friends (Y), was collected informally from Facebook for demonstration purposes, though formal documentation is not currently available.

Table 5: Friends and Mutual Friends dataset

X (No of friends)	Y (MUTUAL FRIEND)
152	62
173	82
139	55
185	92
129	46
135	58
180	75
162	73
153	61
130	49

Journal of Information Systems Engineering and Management
2025, 10(37s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
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The linear regression model between the number of friends (X) and mutual friends (Y) resulted in the following accuracy metrics:

- R^2 Score (Coefficient of Determination): 0.915, which indicates that about 91.5% of the variance in mutual friends can be explained by the number of friends.
- Root Mean Squared Error (RMSE): 4.10, which measures the average deviation of the predicted values from the actual values.

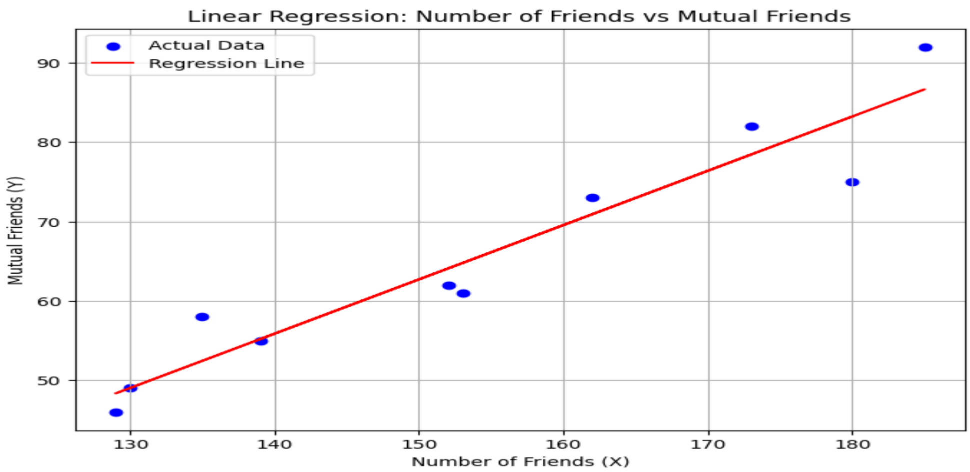

Figure 3: The results of the linear regression analysis between the number of friends (X) and mutual friends (Y)

Model Evaluation and Interpretation

Model Fit and R^2 Score:

- The R^2 score of 0.915 indicates that approximately 91.5% of the variation in mutual friends (Y) is explained by the number of friends (X). This reflects a strong linear relationship, suggesting that as the number of friends increases, the number of mutual friends also rises proportionally.
- A high R^2 value confirms that the model effectively captures the trend, supporting the assumption that a linear approach is suitable for this dataset.

Root Mean Squared Error (RMSE)

- With an RMSE of 4.10, the model's predictions typically deviate from the actual values by about 4 mutual friends.
- Given the dataset's range, this relatively small error indicates that the model provides accurate predictions.

Scatter Plot and Regression Line:

- The scatter plot, where blue dots represent actual data points and the red line represents the regression line, demonstrates a clear positive trend.
- Most data points are closely aligned with the regression line, reinforcing that the model fits well and effectively represents the relationship between the number of friends and mutual friends.

. Interpretation of the Regression Equation:

- The simple linear regression model follows the equation: $\mathbf{Y = \beta_0 + \beta_1 X}$, where **Y** represents mutual friends, **X** denotes the number of friends, and β_0 and β_1 are parameters.
- The slope (β_1) quantifies the expected increase in mutual friends for each additional friend, enabling predictions of mutual friends based on the number of friends within a similar dataset.

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Limitations and Assumptions:

- The model assumes a linear relationship, but this may not hold if the actual pattern is non-linear.
- Since the dataset is relatively small, analyzing a larger dataset could provide deeper insights into this relationship.
- The model assumes that the errors follow a normal distribution and have a constant spread. This needs to be checked using larger and more detailed datasets.

Overall, this linear regression model provides a strong and reliable estimate of mutual friends based on the number of friends. The high R^2 score and low RMSE confirm that users with more friends generally have a higher number of mutual friends, following a linear pattern.

Social Network Metrics in Friendship Dynamics

Several key metrics are used to measure and analyze friendship dynamics in social networks:

- Degree Centrality: This metric measures the number of direct connections (friends or followers) a user has. Highly central users have more connections and typically play a more influential role in the network.
- Clustering Coefficient: This measures the degree to which a user's friends are also friends with each other. A high clustering coefficient indicates that friendships are forming within close-knit groups, fostering a sense of community and shared interaction.
- Betweenness Centrality: This metric identifies users who act as bridges between different parts of the network. Users with high betweenness centrality are critical to the formation of new friendships, as they connect disparate groups and facilitate the spread of information across the network.

Relationship Dynamics Over Time and the McCulloch-Pitts Neuron Model in a Friendship Context

Modeling Friendship Decay Over Time

Friendships generally weaken gradually rather than ending abruptly, largely depending on the level of interaction. This process can be mathematically represented using an exponential decay function:

$$F(T) = e^{(-\lambda T)}$$

where:

- **F(T):** Friendship strength at time **T**
- **λ (lambda):** Rate at which friendship decays (a higher **λ** leads to a faster decline)
- **T:** Time elapsed since the last interaction
- This function captures the following trends:
- When **T = 0**, friendship remains at full strength (**F(T) = 1**).
- As **T** increases, **F(T)** declines, indicating a weakening friendship.
- A larger **λ** results in a more rapid decline in friendship, while a smaller **λ** preserves the connection for a longer period.

Example of Friendship Decay

For **λ = 0.1**, the friendship strength over time follows this pattern:

Table 6: Friendship strength with time

Days Without Interaction (T)	**Friendship Strength (F(T))**
0 days	1.00 (Full friendship)
2 days	$e^{(-0.1 \times 2)} = 0.82$ (Slight decay)
5 days	$e^{(-0.1 \times 5)} = 0.61$ (Moderate decay)
10 days	$e^{(-0.1 \times 10)} = 0.37$ (Significant decay)
20 days	$e^{(-0.1 \times 20)} = 0.13$ (Friendship nearly lost)

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This table demonstrates how friendships diminish over time in the absence of interaction.

Friendship Status and Unfriendship Scenarios

Friendship in social networks is dynamic and can change over time. The decision to unfriend may happen in different ways:

- One member removes the other (**one-sided unfriendship**).
- The second member later reciprocates (**delayed unfriendship**).
- Both members remove each other simultaneously (**mutual unfriendship**).

The friendship decision process can be structured in a table:

Table 7: Friendship status among two users

Person A (N1)	Person B (N2)	Friendship Status
0	0	0 (No friendship)
0	1	0 (No friendship)
1	0	0 (No friendship)
1	1	1 (Friendship exists)

The McCulloch-Pitts Neuron Model for Friendship Dynamics

The McCulloch-Pitts model, one of the earliest artificial neural network models, can be applied to friendship decisions. It operates on binary inputs using a threshold-based activation function, with two types of inputs:

- **Excitatory inputs:** Contribute positively to the decision.
- **Inhibitory inputs:** Contribute negatively to the decision.

- **Input Representation**
- In this model:

$$\mathbf{X_1 = 1} \rightarrow \text{Friendship exists}$$
$$\mathbf{X_2 = 1} \rightarrow \text{Unfriendship occurs}$$

F(T) represents friendship strength over time, calculated using the exponential decay function.

- The net input function is given by:

$$\textbf{Net Input} = \mathbf{W_1X_1 + W_2X_2 + W_3F(T)}$$

where:

$$\mathbf{W_1 = 1} \text{ (weight assigned to friendship status)}$$
$$\mathbf{W_2 = -1} \text{ (weight assigned to unfriendship)}$$
$$\mathbf{W_3 = 1} \text{ (weight assigned to friendship strength over time)}$$

- Threshold-Based Decision

To determine whether the friendship persists or ends, a threshold value ($\boldsymbol{\theta = 1.5}$) is set:

$$\text{If } \textbf{Net Input} \geq \mathbf{1.5} \text{ the friendship remains intact } (\mathbf{1}).$$
$$\text{If } \textbf{Net Input} < \mathbf{1.5} \text{ the friendship dissolves } (\mathbf{0}).$$

Example Calculation

By considering different time intervals, the impact of diminishing friendship strength on unfriendship decisions can be examined:

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Table 8: example of Friendship status among two users after passage of time

Friendship (X_1, X_2)	Time Since Last Interaction (T)	Friendship Strength (F(T))	Net Input Calculation	Output (Friendship Status)
(1,1)	3 days	e^(-0.1 × 3) = 0.74	1 - 1 + 0.74 = 0.74	0 (Unfriendship)
(1,0)	2 days	e^(-0.1 × 2) = 0.82	1 + 0 + 0.82 = 1.82	1 (Friendship holds)

This analysis highlights how the passage of time and the decay of friendship strength influence the decision to maintain or end a friendship.

Key Observations

- If two members interact regularly (low T), their friendship remains strong.
- If there is no interaction for a long time (high T), friendship strength decreases, leading to unfriendship.
- When F(T) becomes too low, the net input does not reach the threshold, causing unfriendship.
- The decay rate (λ) controls how quickly friendships fade.

Table 9: use of different types of online platform in social networks

Platform	Bernoulli Trial Interpretation	Binomial Model	Platform Dynamics
Facebook	A user sends a friend request, and the recipient either accepts (success) or rejects (failure).	If a user sends n friend requests, the number of accepted requests follows a binomial distribution.	- Strong social ties (family, close friends) lead to a higher acceptance rate ($p > 0.5$). - Random friend requests or those sent to influencers may have lower acceptance probabilities ($p < 0.5$). - Privacy settings restrict who can send/accept friend requests, influencing p.
Twitter (X)	A user follows another, and the followed user either follows back (success) or does not (failure).	If a user follows n accounts, the number of follow-backs follows a binomial distribution.	- No mutual acceptance required for connections; asymmetry affects modeling. - Follow-back probability depends on factors like user engagement, content relevance, and follower count (p varies widely).
Instagram	A user follows another, and the followed user either follows back or ignores.	If n users are followed, the number of follow-backs follows a binomial distribution.	- Influencers and celebrities have a much lower follow-back probability ($p < 0.5$). - Engagement (likes, comments) can increase follow-back probability. - The "Suggested Friends" feature may influence p.
LinkedIn	A user sends a connection request, which is either	The number of accepted connection requests follows	- Professional relationships increase likelihood of acceptance

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	accepted or ignored.	a binomial distribution.	($p > 0.5$) for industry peers. - Cold outreach (random requests) has a lower acceptance rate ($p < 0.5$). - Profile strength and mutual connections impact p.

SIR SIMULATION BASED ON INFLUENTIAL USER ACTIVITY

This section applies the Susceptible-Infected-Recovered (SIR) model to simulate the diffusion of information driven by influential users within a social network. Influential nodes are identified based on engagement metrics, and their activity serves as a trigger for the spread of content across the network.

Degree Centrality in Social Networks

Degree centrality is a measure of the number of connections or edges a node (individual, organization, etc.) has in a social network. It's a simple yet effective way to identify influential nodes.

Mathematical Definition:

Degree Centrality (DC) = Number of edges connected to a node / (Total number of nodes - 1)

Influence Score

Influence score is a more comprehensive measure that considers not only the number of connections but also their quality and relevance.

Influence Score Factors:

- Number of followers
- Quality of followers (e.g., their influence score)
- Engagement (e.g., likes, comments, shares)
- Relevance of connections
- Authority of the node (e.g., expertise, credibility)

Followers

Followers are individuals who subscribe to or follow a node's updates, posts, or activities in a social network. Having a large number of followers can indicate popularity, but it's not the only factor in determining influence.

These metrics help analyze social networks, identify key influencers, and understand information diffusion patterns.

Implementation of SIR model

Mapping the SIR Model to the Dataset:

- Susceptible (S): Users who haven't interacted with the content yet — represented by the Followers (m).
- Infected (I): Users who are actively interacting with the content — represented by Average Likes (m).
- Recovered (R): Users who have already interacted with the content and are no longer engaging — estimated using Total Likes (b) / Posts (k).

Parameters for the Model

Infection Rate (β): This determines how quickly the content spreads (e.g., influenced by Influence Score).
Recovery Rate (γ): Rate at which users stop interacting with the content.
Steps to Implement the SIR Model:

STEP 1: Set the Initial Conditions

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S_o : Set to the number of followers (m).

I_o : Set to the average likes (m).

R_o : Estimated as Total Likes (b) / Posts (k).

Table 10: Representation of component for S,I,R model

Component	Representation
Susceptible (S)	Followers (m)
Infected (I)	Average Likes (m)
Recovered (R)	Total Likes (b) / Posts (k)

STEP 2 : Model Equations:

Susceptible:

$$S(t+1) = S(t) - \beta . S(t) . I(t)$$

Infected:

$$I(t+1) = I(t) + \beta . S(t) . I(t) - \gamma . I(t)$$

Recovered:

$$R(t+1) = R(t) + \gamma . I(t)$$

Table 11: Model Parameter

Parameter	Description
Infection Rate (β)	Content spread rate, influenced by Influence Score
Recovery Rate (γ)	Rate at which users stop interacting with content

- Calculate β (infection rate) as Influence Score / 100.
- Calculate γ (recovery rate) as 60-Day Engagement Rate (%) / 100.

Step 3: SIR Model Simulation

Set initial conditions:

S_o (susceptible) = Number of Followers (m).
I_o (infected) = Average Likes (m).
R_o (recovered) = Total Likes (b) / Posts (k).

Simulate day-by-day changes in S, I, and R populations using model equations:

$$S(t+1) = S(t) - \beta \ S(t) \ I(t).$$
$$I(t+1) = I(t) + \beta \ S(t) \ I(t) - \gamma \ I(t).$$
$$R(t+1) = R(t) + \gamma \ I(t).$$

Step 4: Metrics Calculation and Summary

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- Calculate metrics for each channel
- Time to peak.
- Total recovered.
- Diffusion rate (β).

Step 5: Visualization

Plot metrics:
- Peak infected population per channel.
- Time to peak per channel.
- Diffusion rate (β) per channel.
- Save and display plots.

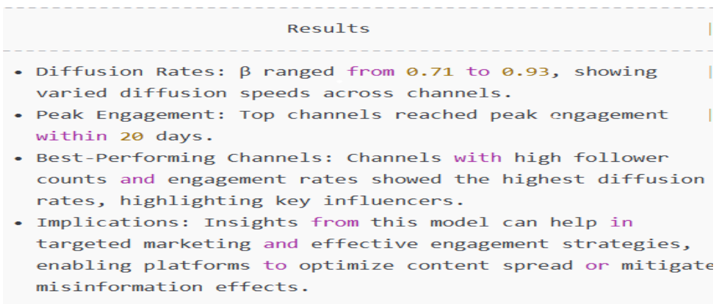
The link https://www.kaggle.com/code/chitaxiang/instagram-influencer-data-analysis/input directs to a dataset featuring the top 200 Instagram users ranked by their Influence Score. This dataset offers comprehensive insights into leading Instagram accounts, including metrics such as Influence Scores, number of followers, average likes, total posts, 60-day engagement rates, average likes on recent posts, overall likes accumulated, and the users' countries or regions.

Table12: Top 200 Instrgram Influencer

[https://www.kaggle.com/code/chitaxiang/instagram-influencer-data-analysis/input]

rank	Influence_score	posts	followers	avg_likes	60_day_eng_rate	new_post_avg_like	total_likes	country
1	92	3.3k	475.8m	8.7m	1.39%	6.5m	29.0b	Spain
2	91	6.9k	366.2m	8.3m	1.62%	5.9m	57.4b	United States
3	90	0.89k	357.3m	6.8m	1.24%	4.4m	6.0b	
4	93	1.8k	342.7m	6.2m	0.97%	3.3m	11.5b	United States
200	80	4.2k	32.8m	232.2k	0.30%	97.4k	969.1m	Indonesia

Results:


Results

- Diffusion Rates: β ranged from 0.71 to 0.93, showing varied diffusion speeds across channels.
- Peak Engagement: Top channels reached peak engagement within 20 days.
- Best-Performing Channels: Channels with high follower counts and engagement rates showed the highest diffusion rates, highlighting key influencers.
- Implications: Insights from this model can help in targeted marketing and effective engagement strategies, enabling platforms to optimize content spread or mitigate misinformation effects.

Figure 4: Results of implementing SIR model

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e-ISSN: 2468-4376
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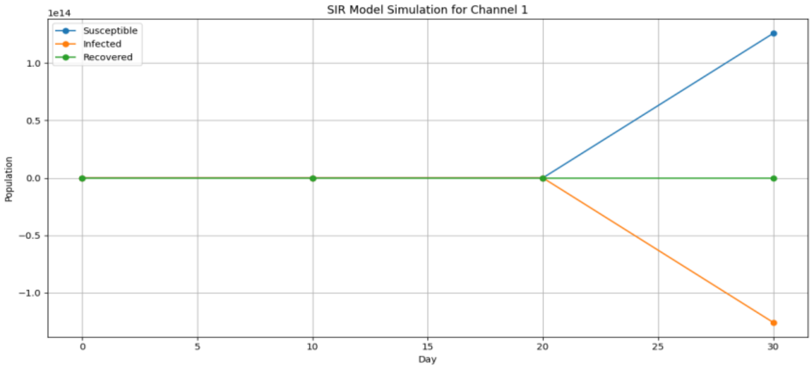

Figure 5:SIR Model For Channel 1

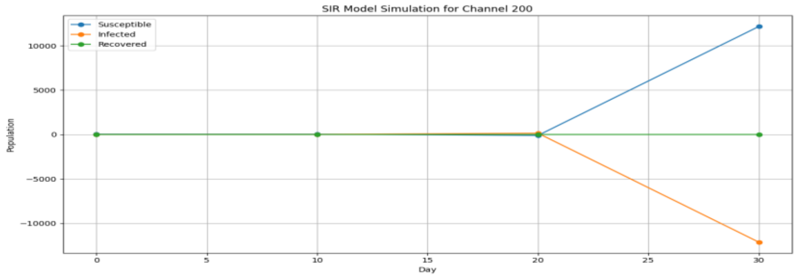

Figure6:SIR Model For Channel 200

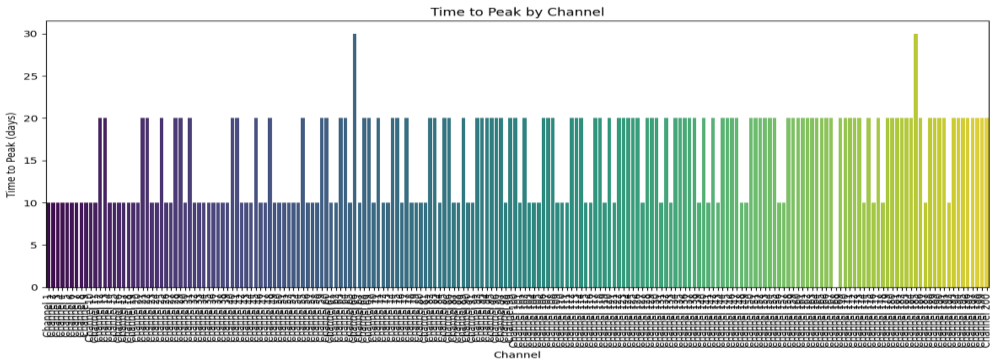

Figure7: Time to Peak by Channel

CONCLUSION AND FUTURE SCOPE

This paper has discussed a combined approach to understand how friendships form and change in social networks, and how information spreads through these platforms. It applies methods from probability, graph theory, supervised learning, neural networks, and epidemic modeling. The Bernoulli and Binomial models show that making friends online can be seen as a random event, where each friend request has a chance of being accepted or rejected. Using linear regression, the study finds a strong relationship between a user's friends count and mutual friends. This helps in predicting mutual friendships. The McCulloch-Pitts neuron model is used to explain how friendships fade over time, especially when users stop interacting. It helps simulate when people may choose to unfriend others. This research also uses the SIR model to study how content spreads on Instagram. In this model, followers are seen as susceptible users, average likes show current engagement, and total likes per post represent users who have already interacted. The study uses influence scores and engagement rates to measure how fast and widely content spreads.

Together, these models help explain how friendships grow, decline, and how information flows in social media networks. The results can support better friend recommendation systems, improve content marketing strategies, and help predict online behavior patterns.

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Future Scope

- Larger Datasets: Future research should include diverse, real-time datasets for validation.
- Non-linear Models: Exploring non-linear relationships can capture complex dynamics in large networks.
- Advanced Machine Learning: Implementing deep learning techniques could reveal deeper interaction patterns.
- Temporal Dynamics: Studying how external events affect friendships over time would enhance understanding.
- Influencer Impact: Analyzing how influencers shape friendship networks could provide valuable insights.

These directions will enhance comprehension of social network dynamics and the intricate nature of user relationships. Furthermore, incorporating more refined metrics—such as content virality and various interaction types—can contribute to greater precision in predictive modeling. Future studies may also benefit from larger sample sizes and the exploration of advanced non-linear approaches, including decision trees or neural networks, to more effectively estimate influence scores.

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