Journal of Information Systems Engineering and Management
2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

Coefficient Estimates for New Subclasses of m – fold Symmetric Functions Associated with Sakaguchi Type Function

Chinthamani. S[1], P. Lokesh [2], V. Sudha[3], C. Abirami*[4]

[1]Assistant Professor, Department of Mathematics
Stella Maris College (Autonomous), Chennai – 600086
chinthamani@stellamariscollege.edu.in

[2]Associate Professor, Department of Mathematics
Adhiparasakthi College of Engineering, G.B.Nagar, Kalavai, India - 632506
lokeshpandurangan@gmail.com

[3]Assistant Professor, Senior Grade, Department of Mathematics, Faculty of Engineering and Technology,
SRM Institute of Science and Technology, Vadapalani, Chennai – 600026
sudhamaths2024@gmail.com

*[4]Assistant Professor, Department of Mathematics, College of Engineering and Technology
SRM Institute of Science and Technology, Kattankulathur, Chennai – 603203
abiramic@srmist.edu.in

*Corresponding Author

ARTICLE INFO

Received: 18 Nov 2024
Revised: 28 Dec 2024
Accepted: 20 Jan 2025

ABSTRACT

This paper presents two new subclasses of the class of m - fold symmetric bi univalent Sakaguchi sort of functions on the open unit disc. For functions in these new subclasses, we derive initial coefficient estimates of the Fekete - Szego and Taylor - Maclaurin functional problems.

2010 Subject Classification for Mathematics: 30C45, 30C50.

Keywords: Analytic function, Bi Univalent functions, Fekete – Szego problem, Sakaguchi type functions, Subordination.

1 Introduction

Let A be the family of functions f which are analytic in the open unit disk $\Delta = \{z \in C : |z| < 1\}$ of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

The class of functions in A, which are univalent in Δ is denoted by S. Every function $f \in S$ has an inverse f^{-1} distinct by $f^{-1}[f(z)] = z, (z \in \Delta)$ and $f[f^{-1}(w)] = w, (|w| < r_0(f); r_0(f) \geq \frac{1}{4})$, where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \ldots .$$

If both f and f^{-1} are univalent in Δ, then a function $f \in A$ is considered bi-univalent in Δ. The symbol $\sum$ indicates the class of bi univalent functions.

It is stated that a function m - fold symmetric (refer[1]) if the normalized form that follows is present:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \quad (m \in \mathrm{N}, z \in \Delta). \tag{1}$$

The m-fold symmetric univalent function m class, which is represented by S_m, is normalized by the series expansions (1) mentioned above. The class's functions are symmetric one-fold. Analogues to the concept of $m-$ fold symmetric univalent functions, is defined the concept of $m-$ fold symmetric bi univalent functions. Each function f in the class Σ generates a $m-$ fold symmetric bi-univalent functions for each positive integer m. The normalized form of f is given in (1) and f^{-1} is given in the followings

$$\begin{aligned} g(w) &= w - a_{m+1} w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ &- \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \ldots \end{aligned} \tag{2}$$

where $f^{-1} = g$. The class of $m-$ fold symmetric bi-univalent functions is denoted by Σ_m.

Numerous authors have recently examined Fekete-Szego functional problems and coefficient estimates for subclasses of $m-$fold symmetric bi univalent functions (refer [2],[4],[7] – [11]). When two analytic functions, f and g, are considered subordinate to each other, as shown by $f(z) \prec g(z)$, If an analytical function w exists such that $w(0) = 0, |w(z)| < 1$ and $f(z) = g(w(z))$.

A function $f(z)$, given by (1), is said to be in the class $\mathrm{M}_{\Sigma,m}(\phi)$, if the following conditions are satisfied:

$$f \in \Sigma_m,\ f'(z) \prec \phi(z) \text{ and } g'(w) \prec \phi(w)$$

which defines the function g(w) in (2).

In order to determine whether classes of Sakaguchi type functions satisfy geometrical conditions, Frasin [3] looked into the coefficient inequalities.

$$\mathrm{Re}\left\{\frac{(s-t)z(f'(z))}{f(sz) - f(tz)}\right\} > \alpha \tag{3}$$

Journal of Information Systems Engineering and Management
2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

regarding complicated numbers s, t but s ≠ t and α (0 ≤ α < 1) . (refer[5],[6])

Two new function class subclasses are introduced in this article. $\sum_m$ of bi-univalent functions which both f and f^{-1} are $m-$fold symmetric Sakaguchi type of functions. Furthermore, the goal is to determine the initial Taylor-Maclaurin coefficient estimation $|a_{m+1}|$ and $|a_{2m+1}|$ and look at Fekete-Szego inequality for functions in every one of these newly created subclasses.

To validate our primary findings, we require the subsequent lemma.

Lemma 1.1. If $h \in P$, then $|p_n| \le 2$ for each n, where P is the family of all functions h analytic in Δ for which $\Re\{h(z)\} > 0$,

$$h(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots$$

2 Coefficient estimates of the function of the class $\mathrm{M}_{\Sigma_m}(\lambda, s, b : \alpha)$

Definition 2.1: For $\lambda \ge 0$, a complex numbers s, b but $s \ne b$, the function f is said to be in (1), if the following conditions are satisfied:

$$f'(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)^{\lambda} \prec \varphi(z), z \in \Delta \tag{4}$$

and

$$g'(w)\left(\frac{(s-b)w}{g(sw)-g(bw)}\right)^{\lambda} \prec \varphi(w), w \in \Delta \tag{5}$$

where the function $g = f^{-1}$ given in (2).

Theorem 2.2. Let the function f given by (1) in the class $\mathrm{M}_{\Sigma_m}(\lambda, s, b : \alpha)$, $\lambda \ge 0$, a complex numbers s, b but $s \ne b$ and $0 < \alpha \le 1$. Then

$$|a_{m+1}| \le \frac{2|\alpha|}{\sqrt{\left|\alpha\left[\begin{array}{l}(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)\\ +\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)\end{array}\right]-2(\alpha-1)\left((m+1)-\lambda(s+b)\right)^2\right|}}, \tag{6}$$

$$|a_{2m+1}| \le \frac{2\alpha}{(2m+1)-\lambda\left(s^2+sb+b^2\right)} + \frac{4(m+1)\alpha^2}{\left((m+1)-\lambda(s+b)\right)^2}, \tag{7}$$

2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

and for $\mu \in \Re$

$$\left|a_{2m+1}-\mu a_{m+1}^{2}\right| \le \begin{cases} \dfrac{2\alpha}{(2m+1)-\lambda\left(s^{2}+sb+b^{2}\right)}, \left|h(\theta)\right| \le \dfrac{1}{2\left((2m+1)-\lambda\left(s^{2}+sb+b^{2}\right)\right)} \\ 4\alpha\left|h(\theta)\right|, \left|h(\theta)\right| \ge \dfrac{1}{2\left((2m+1)-\lambda\left(s^{2}+sb+b^{2}\right)\right)} \end{cases}. \tag{8}$$

where

$$h(\theta)=\left\{\alpha\begin{bmatrix}(m+1)\left((2m+1)-\lambda\left(s^{2}+sb+b^{2}\right)\right)\\ +\left(\lambda(\lambda+1)(s+b)^{2}-2\lambda(m+1)(s+b)\right)\end{bmatrix}-2(\alpha-1)\left((m+1)-\lambda(s+b)\right)^{2}\right\}$$

Evidence. From (4) and (5), it follows that

$$f'(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)^{\lambda}=[p(z)]^{\alpha}, \tag{9}$$

and

$$g'(w)\left(\frac{(s-b)w}{g(sw)-g(bw)}\right)^{\lambda}=[q(w)]^{\alpha}. \tag{10}$$

where the $p(z)$ and $q(w)$ in P have the forms

$$p(z)=1+p_{m}z^{m}+p_{2m}z^{2m}+p_{3m}z^{3m}+\ldots \quad , \tag{11}$$

and

$$q(w)=1+q_{m}w^{m}+q_{2m}w^{2m}+q_{3m}w^{3m}+\ldots \tag{12}$$

Currently, we obtain by equating the coefficients of (9) and (10).

$$\left((m+1)-\lambda(s+b)\right)a_{m+1}=\alpha p_{m} \tag{13}$$

$$\left((2m+1)-\lambda\left(s^{2}+sb+b^{2}\right)\right)a_{2m+1} + \left(\frac{\lambda(\lambda+1)}{2}(s+b)^{2}-\lambda(m+1)(s+b)\right)a_{m+1}^{2}=\frac{1}{2}\alpha(\alpha-1)p_{m}^{2}+\alpha p_{2m}, \tag{14}$$

$$-\left((m+1)-\lambda(s+b)\right)a_{m+1}=\alpha q_{m}, \tag{15}$$

Journal of Information Systems Engineering and Management
2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
Research Article

$$\begin{aligned}&\left((m+1)a_{m+1}^2-a_{2m+1}\right)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)\\&+\left(\frac{\lambda(\lambda+1)}{2}(s+b)^2-\lambda(m+1)(s+b)\right)a_{m+1}^2=\frac{1}{2}\alpha(\alpha-1)q_m^2+\alpha q_{2m}\end{aligned} \quad (16)$$

We derive (13) and (15) as follows:

$$p_m=-q_m, \quad (17)$$

and

$$2\left((m+1)-\lambda(s+b)\right)^2a_{m+1}^2=\alpha^2\left(p_m^2+q_m^2\right). \quad (18)$$

Now, by using equations (14), (16), and (18), we get

$$\left\{\alpha\begin{bmatrix}(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)\\+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)\end{bmatrix}-2(\alpha-1)\left((m+1)-\lambda(s+b)\right)^2\right\}a_{m+1}^2=\alpha^2\left(p_m^2+q_m^2\right).$$

(19)

Following that, using Lemma 1.1 for the coefficients p_{2m} and q_{2m} We determine the coefficient bound for in the aforementioned equation $|a_{m+1}|$ as asserted in (6).

After deducting (16) from (14), we now have

$$\begin{bmatrix}2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{2m+1}\\-(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{m+1}^2\end{bmatrix}=\alpha\left(p_{2m}-q_{2m}\right)+\frac{1}{2}\alpha(\alpha-1)\left(p_m^2+q_m^2\right).$$

(20)

The deductions (17), (18), and (20) imply that

$$|a_{2m+1}|=\frac{\alpha\left(p_{2m}-q_{2m}\right)}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}+\frac{\alpha^2(m+1)\left(p_m^2+q_m^2\right)}{4\left((m+1)-\lambda(s+b)\right)^2}. \quad (21)$$

Next, for the coefficients, apply Lemma 1.1 once more. p_m, p_{2m}, q_m and q_{2m} Using the equation above, we can determine the bound for $|a_{2m+1}|$ as stated in (7).

The Fekete-Szego inequalities for the subclass's function are then sought to be provided.

Journal of Information Systems Engineering and Management
2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/ **Research Article**

Equations (19) and (21), taken together, provide

$$a_{2m+1}-\mu a_{m+1}^{2}=(1-\mu)\frac{\alpha^{2}(p_{2m}+q_{2m})}{\left\{\alpha\begin{bmatrix}(m+1)((2m+1)-\lambda(s^{2}+sb+b^{2}))\\+(\lambda(\lambda+1)(s+b)^{2}-2\lambda(m+1)(s+b))\end{bmatrix}-2(\alpha-1)((m+1)-\lambda(s+b))^{2}\right\}}$$
$$+\frac{\alpha(p_{2m}-q_{2m})}{2((2m+1)-\lambda(s^{2}+sb+b^{2}))}$$

$$=\alpha\left\{\left(h(\theta)+\frac{1}{2((2m+1)-\lambda(s^{2}+sb+b^{2}))}\right)p_{2m}+\left(h(\theta)-\frac{1}{2((2m+1)-\lambda(s^{2}+sb+b^{2}))}\right)q_{2m}\right\}$$

where

$$h(\theta)=\frac{(1-\mu)\alpha}{\left\{\alpha\begin{bmatrix}(m+1)((2m+1)-\lambda(s^{2}+sb+b^{2}))\\+(\lambda(\lambda+1)(s+b)^{2}-2\lambda(m+1)(s+b))\end{bmatrix}-2(\alpha-1)((m+1)-\lambda(s+b))^{2}\right\}}$$

The inequality stated in (8) is therefore obtained by applying Lemma 1.1.

This concludes the Theorem 2.2 proof. □

3 Estimates of the Class Function's Coefficients $K_{\Sigma_m}(\lambda,s,b:\beta)$

Definition 3.1 A function f, given by (1), is in the class $K_{\Sigma_m}(\lambda,s,b:\beta)$ if The following prerequisites are met:

$$f\in\Sigma_m \text{ and } \Re\left(f'(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)^{\lambda}\right)>\beta,(\lambda\geq 0,0<\beta\leq 1,z\in\Delta), \quad (22)$$

and

$$\Re\left(g'(w)\left(\frac{(s-b)w}{g(sw)-g(bw)}\right)^{\lambda}\right)>\beta,\ (\lambda\geq 0,0<\beta\leq 1,w\in\Delta). \quad (23)$$

where (2) gives the function g.

Theorem 3.2 Let the function f given by (1) be in the class $K_{\Sigma_m}(\lambda,s,b:\beta)$ $(\lambda\geq 0 and 0<\beta\leq 1)$, then

$$\left|a_{m+1}\right| \leq \sqrt{\frac{4(1-\beta)}{(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)}},$$

(24)

$$\left|a_{2m+1}\right| \leq \frac{2(1-\beta)}{(2m+1)-\lambda\left(s^2+sb+b^2\right)}+\frac{4(m+1)(1-\beta)^2}{\left((m+1)-\lambda(s+b)\right)^2}, \tag{25}$$

and for $\eta \in \Re$

$$\left|a_{2m+1}-\eta a_{m+1}^2\right| \leq \begin{cases} \dfrac{2(1-\beta)}{(2m+1)-\lambda\left(s^2+sb+b^2\right)}, & |h(\zeta)| \leq \dfrac{1}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)} \\ 4h(\zeta)(1-\beta), & |h(\zeta)| \geq \dfrac{1}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)} \end{cases}.$$

(26)

where $h(\zeta)=(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)$

Proof. It follows from (22) and (23) that there exists $p(z), q(w) \in \mathrm{P}$ such that

$$f'(z)\left(\frac{(s-b)z}{f(sz)-f(bz)}\right)^{\lambda}=\beta+(1-\beta)p(z), \tag{27}$$

and

$$g'(w)\left(\frac{(s-b)w}{g(sw)-g(bw)}\right)^{\lambda}=\beta+(1-\beta)q(w). \tag{28}$$

where $p(z)$ and $q(w)$ have the forms (11) and (12), respectively. Equating the coefficients in (27) and (28)

$$\left((m+1)-\lambda(s+b)\right)a_{m+1}=(1-\beta)p_m, \tag{29}$$

$$\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{2m+1}+\left(\frac{\lambda(\lambda+1)}{2}(s+b)^2-\lambda(m+1)(s+b)\right)a_{m+1}^2=(1-\beta)p_{2m},$$

(30)

$$-\left((m+1)-\lambda(s+b)\right)a_{m+1}=(1-\beta)q_m, \tag{31}$$

and

$$\begin{aligned}&\left((m+1)a_{m+1}^2 - a_{2m+1}\right)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)\\&+\left(\frac{\lambda(\lambda+1)}{2}(s+b)^2-\lambda(m+1)(s+b)\right)a_{m+1}^2=(1-\beta)q_{2m}\end{aligned} \quad (32)$$

From (29) and (31), we obtain

$$p_m = -q_m \quad (33)$$

and

$$2\left((m+1)-\lambda(s+b)\right)^2 a_{m+1}^2 = (1-\beta)^2\left(p_m^2+q_m^2\right). \quad (34)$$

From (30) and (32), we find that

$$\begin{aligned}&(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{m+1}^2\\&+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)a_{m+1}^2=(1-\beta)\left(p_{2m}+q_{2m}\right)\end{aligned}, \quad (35)$$

Then, by applying Lemma 1.1 for the coefficients p_{2m} and q_{2m} in last equation, we obtain the desired bound for $|a_{m+1}|$ as asserted in (24).

Now, to find $|a_{2m+1}|$, subtracting (32) from (30), we obtain

$$2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{2m+1}-(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)a_{m+1}^2=(1-\beta)\left(p_{2m}-q_{2m}\right),$$

Or, equivalent

$$a_{2m+1}=\frac{(1-\beta)\left(p_{2m}-q_{2m}\right)}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}+\frac{(m+1)a_{m+1}^2}{4}.$$

Substituting the values of a_{m+1}^2 from (34), we have

$$a_{2m+1}=\frac{(1-\beta)\left(p_{2m}-q_{2m}\right)}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}+\frac{(m+1)(1-\beta)^2\left(p_m^2+q_m^2\right)}{4\left((m+1)-\lambda(s+b)\right)^2}. \quad (36)$$

Next, for the coefficients, apply Lemma 1.1 once more p_m, p_{2m}, q_m and q_{2m} We derive the intended bound for in the previous equation $|a_{2m+1}|$ as asserted in (25).

Journal of Information Systems Engineering and Management
2025, 10(3)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

Research Article

The Fekete-Szego inequalities for the subclass function should then be obtained. We obtain from equations (35) and (36),

$$a_{2m+1}-\eta a_{m+1}^2=(1-\eta)\frac{(1-\beta)^2(p_{2m}+q_{2m})}{(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)},$$

$$+\frac{(1-\beta)(p_{2m}-q_{2m})}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}$$

$$=(1-\beta)\left\{\left(h(\xi)+\frac{1}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}\right)p_{2m}+\left(h(\xi)-\frac{1}{2\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)}\right)q_{2m}\right\}$$

where

$$h(\xi)=\frac{(1-\mu)(1-\beta)}{(m+1)\left((2m+1)-\lambda\left(s^2+sb+b^2\right)\right)+\left(\lambda(\lambda+1)(s+b)^2-2\lambda(m+1)(s+b)\right)}.$$

Afterwards, we get the inequality stated in (26), by applying Lemma 1.1.
With this, Theorem 3.2 is fully proved. □

4. Concluding Corollaries for the subclasses

Setting the parameters $\lambda=1$ and $m=1$ in the Theorem 2.2 and Theorem 3.2, We derive the subsequent corollaries.

Corollary 4.1. Let the function f is given by (1) in the sub class $M_{\Sigma_m}(\lambda,s,b:\alpha)$. Then

$$|a_2|\le\frac{\sqrt{2}|\alpha|}{\sqrt{\left|\alpha\left[\left(3-\left(s^2+sb+b^2\right)\right)-(s+b)(2-(s+b))\right]-(\alpha-1)(2-(s+b))^2\right|}},$$

$$|a_3|\le\frac{2\alpha}{3-\left(s^2+sb+b^2\right)}+\frac{8\alpha^2}{(2-(s+b))^2}$$

and for $\eta\in\Re$

$$|a_3-\mu a_2^2|\le\begin{cases}\dfrac{2\alpha}{3-\left(s^2+sb+b^2\right)},|h(\theta)|\le\dfrac{1}{2\left(3-\left(s^2+sb+b^2\right)\right)}\\ 8\alpha|h(\theta)|,|h(\theta)|\ge\dfrac{1}{2\left(3-\lambda\left(s^2+sb+b^2\right)\right)}\end{cases}$$

where

$$h(\theta)=\left\{\alpha\left[\left(3-\left(s^2+sb+b^2\right)\right)-(s+b)(2-(s+b))\right]-(\alpha-1)(2-(s+b))^2\right\}$$

Corollary 4.2. Let the function f is given by(1) in the sub class $K_{\Sigma_m}(\lambda, s, b : \beta)$. Then

$$|a_2| \le \sqrt{\frac{2(1-\beta)}{\left(3-\left(s^2+sb+b^2\right)\right)-(s+b)(2-(s+b))}},$$

$$|a_3| \le \frac{2(1-\beta)}{3-\left(s^2+sb+b^2\right)} + \frac{8(1-\beta)^2}{(2-(s+b))^2},$$

and for $\eta \in \Re$

$$|a_3 - \eta a_2^2| \le \begin{cases} \dfrac{2(1-\beta)}{3-\left(s^2+sb+b^2\right)}, & |h(\zeta)| \le \dfrac{1}{2\left(3-\left(s^2+sb+b^2\right)\right)} \\ 8h(\zeta)(1-\beta), & |h(\zeta)| \ge \dfrac{1}{2\left(3-\left(s^2+sb+b^2\right)\right)} \end{cases}.$$

where

$$h(\zeta) = \left(3-\left(s^2+sb+b^2\right)\right)-(s+b)(2-(s+b)).$$

References:

[1] S. Bulut, Coefficient estimates for a new subclass of m-fold symmetric analytic bi-univalent functions, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. 68 (2019), no. 2, 1401–1410.

[2] G. Dong, T. Huo, A. En and X. Liang-Peng, Coefficient estimates for a class of m-fold symmetric bi-univalent function defined by subordination, Commun. Math. Res. 35 (2019), no. 1, 57–64.

[3] B.A. Frasin , Coefficient inequalities for certain classes of Sakaguchi Type functions,

Int. J.of Nonlinear Science, Vol.10, No.2,pp 206-211.(2010).

[4] T.R.K. Kumar, S. Karthikeyan, S. Vijayakumar and G. Ganapathy, Initial coefficient estimates for certain subclasses of m-fold symmetric bi-univalent functions, Adv. Dyn. Syst. Appl. 16 (2021), no. 2, 789–800.

[5] P.Lokesh, B.Srutha Keerthi ,Results On Bazilevic Sakaguchi Type Of Functions, Advances in Mathematics: Scientific Journal, ISSN: 1857-8365, Volume – 9, No. 8, 5763 – 5774, 2020.

[6] P.Lokesh, S.Prema, Quasi – subordination for a Bazilevic Sakaguchi function associated with k-analogue of Bessel function, AIP Conference Proceedings 2718, 050007 (2023)

[7] A. Motamednezhad, S. Salehian and N. Magesh, The Fekete-Szego problems for a subclass of m-fold symmetric bi-univalent functions, TWMS J. Appl. Eng. Math. 11 (2021), no. 2, 514–523.

[8] A.K. Wanas and A.H. Majeed, On subclasses of analytic and m-fold symmetric bi-univalent functions, Iran. J.Math. Sci. Inf. 15 (2020), no. 2, 51–60.

[9] A.K. Wanas and ´A.O. Pall-Szao, Coefficient bounds for new subclasses of analytic and m-fold symmetric bi univalent functions, Stud. Univ. Babes-Bolyai Math. 66 (2021), no. 4, 659–666.

[10] A.K. Wanas and H. Tang, Initial coefficient estimates for classes of m-fold symmetric bi-univalent functions involving Mittag-Leffler function, Math. Morav. 24 (2020), no. 2, 51–61.

[11] A.K. Wanas and S. Yalcın, Initial coefficient estimates for a new subclasses of analytic and m-fold symmetric bi-univalent functions, Malaya J. Mate. 7 (2019), no. 3, 472–476.