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Research Article

Cordial Labeling in the Context of Some Graphs

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ABSTRACT

Let $G = (V, E)$ be the graph. A mapping $f: V \to \{0,1\}$ is called Binary vertex labeling and $f(v)$ is called the label of the vertex v of G under f . For an edge $e = uv$, the induced edge labeling $f^*: E \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $f: V \to \{0,1\}$ and for each edge uv, assign the label $|f(x) - f(y)|$. Then the binary vertex labeling f of a graph G is said to be cordial labeling if $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. In this paper, some graphs are proved for cordial labeling and known to be cordial graph.

Keywords: Cordial labeling, Wheel graph, Star graph, Closed Helm graph, Square of cycle graph.

AMS classification: $05C50$; $05C78$.

1. Introduction

Let G be a graph with vertices denoted by V and edge by E. Graph theory is a branch of mathematics that studies the relationships and connections between graphs. It is an area of mathematics that focuses on graphs, or diagrams using lines and points.

Graph labeling is a process that involves assigning integers to vertices, edges, or both as per particular requirements. Graph labeling approaches were developed from a concept first introduced by Rosa in 1967[8]. In 1987, Cahit[1] pioneered the idea of cordial labeling and it was found to be the lower variant of graceful and harmonious.

Let $G = (V, E)$ be the graph. A mapping $f: V \to \{0,1\}$ is called Binary vertex labeling and $f(v)$ is called the label of the vertex v of G under f . For an edge $e = uv$, the induced edge labeling $f^*: E \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $f: V \to \{0,1\}$ and for each edge uv, assign the label $|f(x) - f(y)|$. Then the binary vertex labeling f of a graph G is said to be cordial labeling[4] if $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

In [2] Cahit proved that all trees are cordial, $K_{m,n}$ is cordial for all m and n, All fans are cordial. In [6] Pariksha Gupta, et.al proved a Cordial labeling pattern for star of bistar graph. In [5], Jeba Jesintha, et.al proved that families of blade graph such as blade graph and one-point union of blade graph is cordial. In [9], Sarang Sadawarte, et.al determined that a graph formed by connecting two sets of a 3-pan graph via a path of distinct length, a graph formed by connecting two sets of a 4-pan graph via a path of distinct length, and any path union of r copies of a 3-pan graph possesses cordial labeling. In [3], Deva Kirubanithi, et.al proved that the star glued with subdivided shell graph and super subdivision of circular ladder graph admits cordial labeling. In [7], Amit. H. Rohad, et.al determined that shadow graph of star $K_{1,n}$, splitting graph of star $K_{1,n}$, degree splitting graph of star $K_{1,n}$ is cordial and also showed that Jewel graph J_n, Jellyfish graph $J_{n,n}$ are cordial graphs.

Labeled graph are used in Social Sciences, Optimization, military surveillance, Neutral Networks, astronomy, communication network addressing and cryptography. Cordial labeling is useful in DNA code word design problem and in noisy communication channels.

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In this paper, Wheel gripped star graph, the Path union of square of cycle graph and the Star of square of cycle graph are analyzed for cordial labeling.

2. Preliminary Definitions

This section contains definitions for a few terms that are used in the main results.

Definition 2.1. [11] A wheel graph is one that connects a single apex vertex to all of the vertices in a cycle.

Definition 2.2. The Star graph $K_{1,n}$ is a tree on $n+1$ nodes with one vertex having degree n called the central and the remaining n nodes are of degree 1.

Definition 2.3. The Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ is obtained by attaching the star graph S_m to every vertex in the wheel graph W_n excluding the central vertex.

Definition 2.4. [12] Let $G_1, G_2, G_3, \ldots, G_n, n \geq 2$, be copies of a graph G. Let $v_i \epsilon V(G_i), i = 1,2, \ldots, n$, be the vertex corresponding to the vertex $v \epsilon V(G)$ in the i^{th} copy of G_i. We denote by $P(n.G)$ the graph attained by including the edge $v_i v_{i+1}$ to G_i and $G_{i+1}, 1 \leq i \leq n-1$, and we call $P(n.G)$ as path union of n copies of G.

Definition 2.5. [10] Square of a graph G denoted by G^2 has the exact same vertex as G and two vertices are adjacent in G^2 if they are at a distance of 1 or 2 from G.

Definition 2.6. A graph obtained by replacing each vertex of a star graph $K_{1,n}$ by a graph G is called star of G. We denote it as G^*.

3. Main Results

In this section, few theorems are proved.

Theorem 3.1. The Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ is Cordial when same Star graph S_m is linked to each vertex of wheel graph W_n and n is odd.

Proof: Let $G_{n,m}(W_n, S_m)$ be the wheel gripped star graph where n is the number of vertices of the wheel graph W_n and m is the number of vertices of the Star graph S_m and $m \geq 2$. At first label the vertices of the Wheel graph W_n and then label the vertices of the attached star graph S_m. Let the vertices of the wheel graph be labeled as a_i where $i = 0,1,2,\ldots,n-1$. Let the vertices of the star graph be labeled as b_j where $j = 1,2,3,\ldots,v$. Here v is the total number of vertices of all star S_m attached to the wheel graph W_n. Fix the central vertex of the wheel graph W_n as $a_0 = 0$.

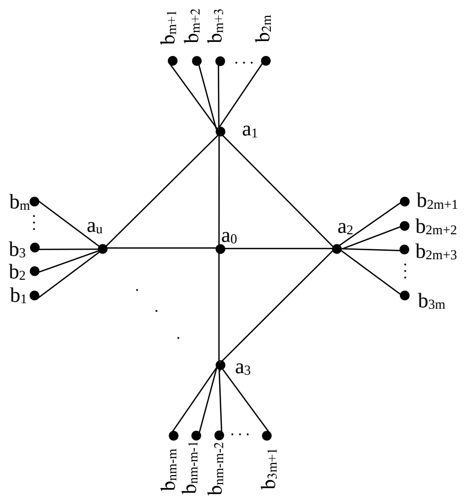

Figure 1: Generalized Wheel Gripped Star graph $(G_{n,m}(W_n, S_m))$.

Here, $\left|V\left(G_{n,m}(W_n, S_m)\right)\right| = nm + n - m$ and $\left|E\left(G_{n,m}(W_n, S_m)\right)\right| = nm + 2n - m - 2$.

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Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, if\ i \equiv 0,3(mod4) \\ 1, if\ i \equiv 1,2(mod4) \end{cases}$$

$$f(b_j) = \begin{cases} 0, if\ j \equiv 0(mod2) \\ 1, if\ j \equiv 1(mod2) \end{cases}$$

Case 1: If $n \equiv 1(mod4)$ then the group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{nm+n-m}{2} \right\rfloor + 1; \quad V_f(1) = \left\lfloor \frac{nm+n-m}{2} \right\rfloor$$

Case 2: If $n \equiv 3(mod4)$ then the group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{nm+n-m}{2} \right\rfloor; \quad V_f(1) = \left\lfloor \frac{nm+n-m}{2} \right\rfloor + 1$$

For both Case 1 and Case 2, the group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \frac{nm+2n-m-2}{2}; \quad e_f(1) = \frac{nm+2n-m-2}{2}$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Therefore, the Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ admits cordial labeling when same star graph S_m is linked to the wheel graph W_n.

The Illustration of the theorem is given for Case 1.

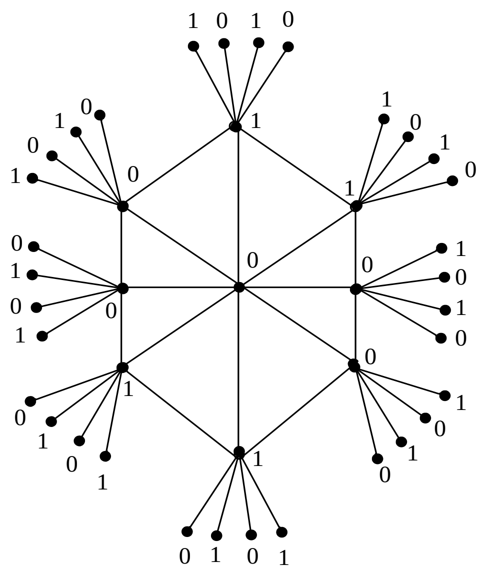

Figure 2: Cordial labeling of $G_{9,4}(W_9, S_4)$.

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In the above figure, $n = 9, m = 4, |V| = 41, |E| = 48, V_f(0) = 21, V_f(1) = 20, e_f(0) = 24, e_f(1) = 24$. Hence it admits cordial labeling.

The Illustration of the theorem is given for Case 2.

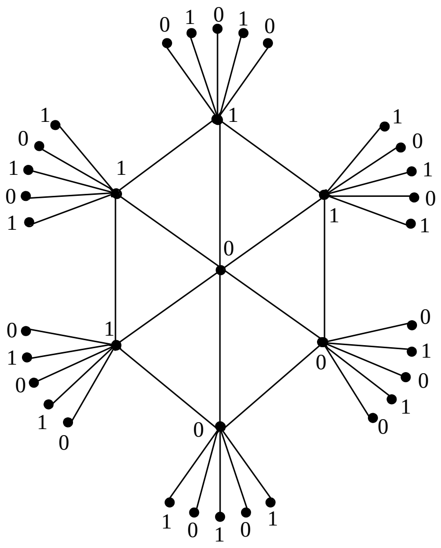

Figure 3: Cordial labeling of $G_{7,5}(W_7, S_5)$.

In the above figure, $n = 7, m = 5, |V| = 37, |E| = 42, V_f(0) = 18, V_f(1) = 19, e_f(0) = 21, e_f(1) = 21$. Hence it admits cordial labeling.

Theorem 3.2. The Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ is Cordial when different Star graph S_m is linked to each vertex of wheel graph W_n and n is odd.

Proof: Let $G_{n,m}(W_n, S_m)$ be the wheel gripped star graph where n is the number of vertices of the wheel graph W_n and m is the number of vertices of the Star graph S_m and $m \geq 2$. At each vertex, m increases either oddly or evenly. At first label the vertices of the Wheel graph W_n and then label the vertices of the attached star graph S_m. Let the vertices of the wheel graph be labeled as a_i where $i = 0,1,2,\ldots,u$. Here $u = n - 1$. Let the vertices of the star graph be labeled as b_j where $j = 1,2,3,\ldots,v$. Here v is the total number of vertices of all star S_m attached to the wheel graph W_n. Fix the central vertex of the wheel graph W_n as $a_0 = 0$.

Case 1: If m is oddly increased ie, $m \equiv 1(mod2)$.

Here, $\left|V\left(G_{n,m}(W_n, S_m)\right)\right| = n^2 - n + 1$, $|E(G_{n,m}(W_n, S_m))| = n^2 - 1$

Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, if\ i \equiv 0,3(mod4) \\ 1, if\ i \equiv 1,2(mod4) \end{cases}$$

$$f(b_j) = \begin{cases} 0, if\ j \equiv 1(mod2) \\ 1, if\ j \equiv 0(mod2) \end{cases}$$

Subcase (i) : If $n \equiv 1(mod4)$ then the group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{n^2 - n + 1}{2} \right\rfloor + 1; \quad V_f(1) = \left\lfloor \frac{n^2 - n + 1}{2} \right\rfloor$$

Subcase (ii): If $n \equiv 3(mod4)$ then

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The group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{n^2 - n + 1}{2} \right\rfloor; \quad V_f(1) = \left\lfloor \frac{n^2 - n + 1}{2} \right\rfloor + 1$$

For both subcase (i) and subcase (ii), the group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \frac{n^2 - 1}{2}; \quad e_f(1) = \frac{n^2 - 1}{2}$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Therefore, the Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ admits cordial labeling when different star graph S_m is linked to the wheel graph W_n and m is increased oddly.

Case 2: If m is evenly increased ie, $m \equiv 0(mod2)$.

Here, $\left|V\left(G_{n,m}(W_n, S_m)\right)\right| = n^2$, $\left|E\left(G_{n,m}(W_n, S_m)\right)\right| = n^2 + n - 2$

Define the vertex labeling as follows

$$f(a_i) = \begin{cases} 0, if\ i \equiv 0,3(mod4) \\ 1, if\ i \equiv 1,2(mod4) \end{cases}$$

$$f(b_j) = \begin{cases} 0, if\ j \equiv 0(mod2) \\ 1, if\ j \equiv 1(mod2) \end{cases}$$

Subcase (i): if $n \equiv 1(mod4)$ then

The group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{n^2}{2} \right\rfloor + 1; \quad V_f(1) = \left\lfloor \frac{n^2}{2} \right\rfloor$$

Subcase (ii): if $n \equiv 3(mod4)$ then

The group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \left\lfloor \frac{n^2}{2} \right\rfloor; \quad V_f(1) = \left\lfloor \frac{n^2}{2} \right\rfloor + 1$$

For both Subcase (i) and Subcase (ii), the group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \frac{n^2 + n - 2}{2}; \quad e_f(1) = \frac{n^2 + n - 2}{2}$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Therefore, the Wheel Gripped Star graph $G_{n,m}(W_n, S_m)$ admits cordial labeling when different star graph S_m is linked to the wheel graph W_n and m is increased evenly.

The Illustration of the theorem is given for Case 1.

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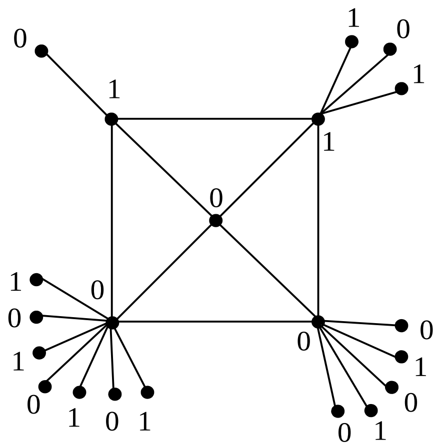

Figure 4: Cordial labeling of $G_{5,m}(W_5, S_m)$.

In the above figure, $n = 5, n \equiv 1(mod4)$, $m \equiv 1(mod2)$, $|V| = 21$, $|E| = 24$, $V_f(0) = 11$, $V_f(1) = 10$, $e_f(0) = 12$, $e_f(1) = 12$. Hence it admits cordial labeling.

The Illustration of the theorem is given for Case 2.

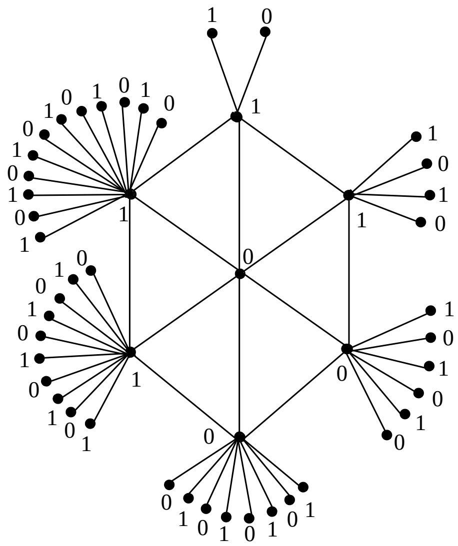

Figure 5: Cordial labeling of $G_{7,m}(W_7, S_m)$.

In the above figure, $n = 7$, $n \equiv 3(mod4)$, $m \equiv 0(mod2)$, $|V| = 49$, $|E| = 54$, $V_f(0) = 24$, $V_f(1) = 25$, $e_f(0) = 27$, $e_f(1) = 27$. Hence it admits cordial labeling.

Theorem 3.3. The Path union of square of cycle graph $P(m\,.\,C_n^2)$ is cordial.

Proof: Let $G = P(m\,.\,C_n^2)$ be the path union of square of cycle graph C_n^2. Let a_j^i $(1 \le i \le m, 1 \le j \le n)$ denote the vertices of C_n^2.

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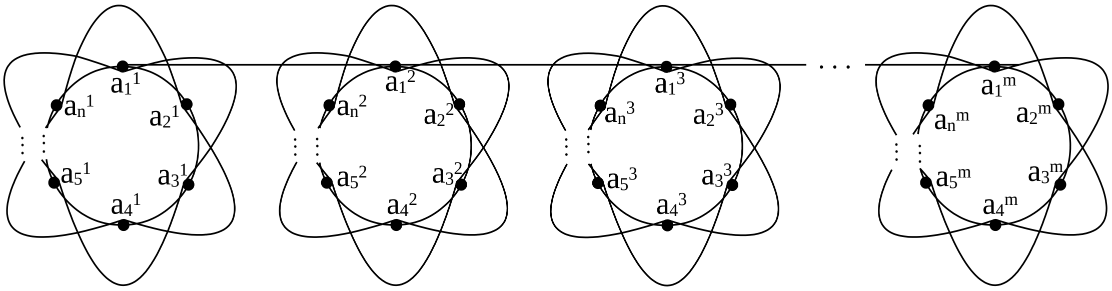

Figure 9: Generalized Path union of square of cycle graph $P(m\,.\,C_n^2)$

let $|V(P(m\,.\,C_n^2))| = mn$ and $|E(P(m\,.\,C_n^2))| = 2mn + m - 1$.

Case:1 If $i \equiv 1,2(mod4)$

The vertex labeling is given as

$$f(a_j^i) = \begin{cases} 0, if\ j \equiv 0(mod2) \\ 1, if\ j \equiv 1(mod2) \end{cases}$$

Case:2 If $i \equiv 0,3(mod4)$

The vertex labeling is given as

$$f(a_j^i) = \begin{cases} 0, if\ j \equiv 1(mod2) \\ 1, if\ j \equiv 0(mod2) \end{cases}$$

For Case 1 and Case 2, the group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \frac{mn}{2};\quad V_f(1) = \frac{mn}{2}$$

If $m \equiv 0(mod2)$

The group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \left\lfloor \frac{2mn + m - 1}{2} \right\rfloor + 1;\quad e_f(1) = \left\lfloor \frac{2mn + m - 1}{2} \right\rfloor$$

If $m \equiv 1(mod2)$

The group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \frac{2mn + m - 1}{2};\quad e_f(1) = \frac{2mn + m - 1}{2}$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Therefore, Path union of square of cycle graph $P(m\,.\,C_n^2)$ admits cordial labeling.

The Illustration of the theorem is given for Case 1.

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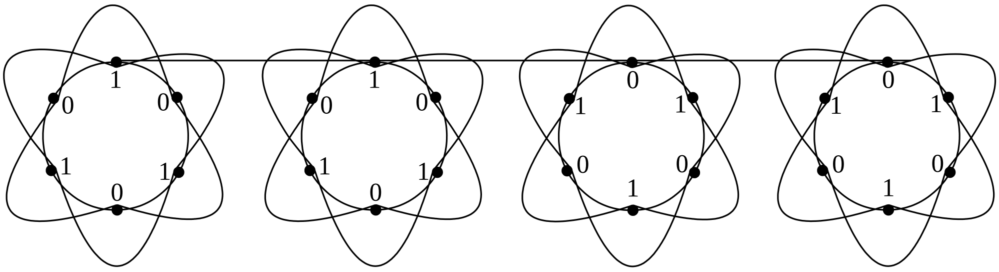

Figure 10: Cordial labeling of Path union of square of cycle graph $P(4\,.\,C_6^2)$

In the above figure, $m = 4, n \equiv 0(mod2), |V| = 24, |E| = 51, V_f(0) = 12, V_f(1) = 12,\ e_f(0) = 26, e_f(1) = 25$. Hence it admits cordial labeling.

The Illustration of the theorem is given for Case 2.

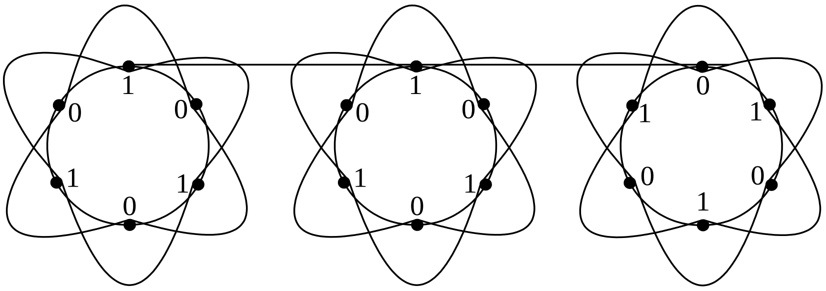

Figure 11: Cordial labeling of Path union of square of cycle graph $P(3\,.\,C_6^2)$

In the above figure, $m = 3, n \equiv 1(mod2), |V| = 18, |E| = 38, V_f(0) = 9, V_f(1) = 9,\ e_f(0) = 19,\ e_f(1) = 19$. Hence it admits cordial labeling.

Theorem 3.4. The Star of square of cycle graph $C_n^{2^*}$ is cordial.

Proof: Let $G = C_n^{2^*}$ be the star of square of cycle graph. Label the vertices of central C_n^2 graph as a_i where $(1 \leq i \leq n)$, n is even. Label the vertices of other C_n^2 graph as a_i^j where , $1 \leq i \leq n, 1 \leq j \leq m$.

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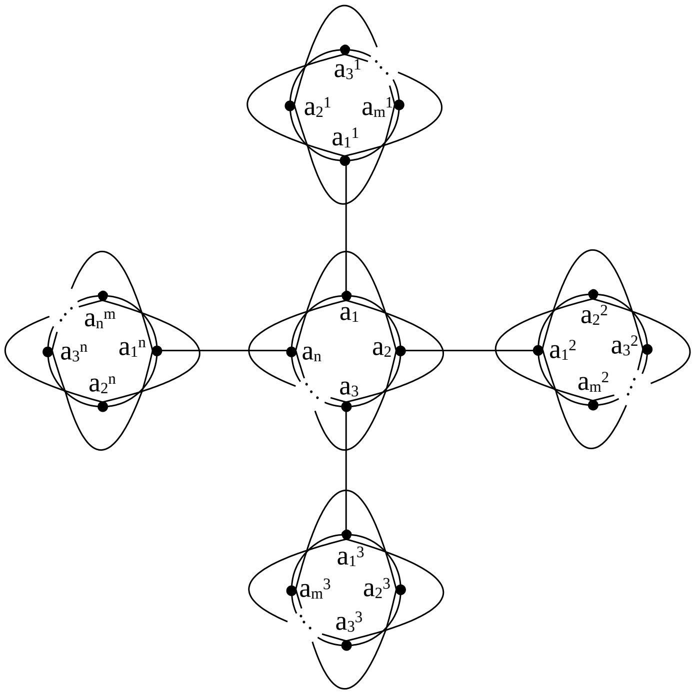

Figure 12: Generalized Star of square of cycle graph $C_n^{2^*}$.

Let $|V(C_n^{2^*})| = mn + n$ and $|E(C_n^{2^*})| = 2nm + 2n + n$.

The vertex labeling of central C_n^2 graph is given by

$$f(a_i) = \begin{cases} 0, if\ i \equiv 0(mod2) \\ 1, if\ i \equiv 1(mod2) \end{cases}$$

The vertex labeling of other than the central C_n^2 graph is given as follows

$$f(a_i^j) = \begin{cases} 0, if\ i \equiv 0(mod2) \\ 1, if\ i \equiv 1(mod2) \end{cases}$$

The total group of vertices labeled with 0 and 1 is defined as follows

$$V_f(0) = \frac{mn+n}{2};\quad V_f(1) = \frac{mn+n}{2}$$

The total group of edges labeled with 0 and 1 are defined as follows

$$e_f(0) = \frac{2nm+2n+n}{2};\quad e_f(1) = \frac{2nm+2n+n}{2}$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Therefore, Star of square of cycle graph $C_n^{2^*}$ admits cordial labeling.

The Illustration of the theorem is given as follows.

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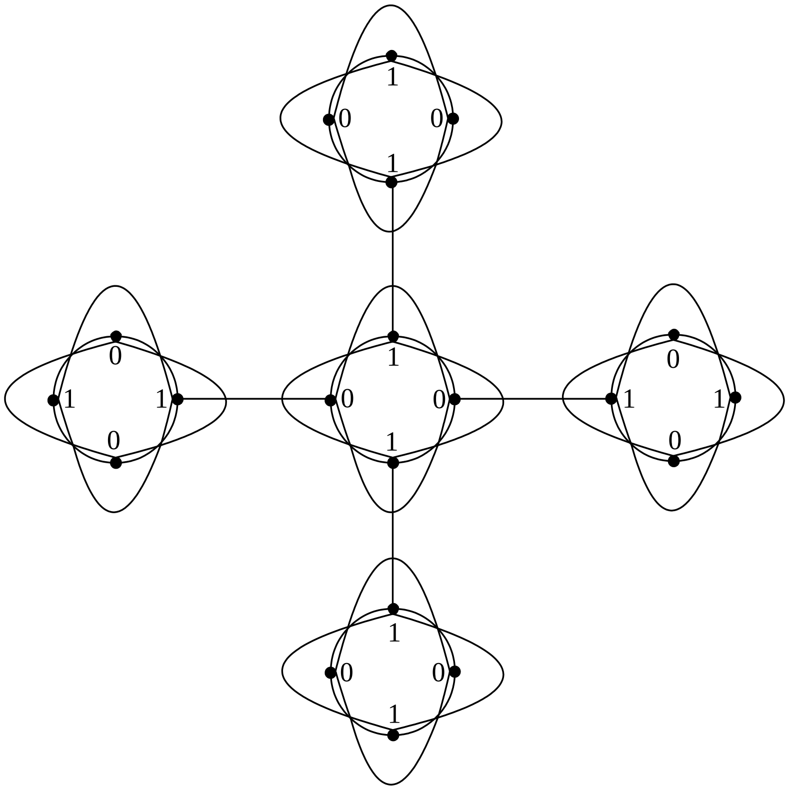

Figure 13: Cordial labeling of Star of square of cycle graph $C_4^{2^*}$.

In the above figure, $|V| = 20, |E| = 44, V_f(0) = 10, V_f(1) = 10,\ e_f(0) = 22,\ e_f(1) = 22$. Hence it admits cordial labeling.

4. Conclusion

An essential part of graph theory is the labeling of graphs. In light of this, a few of its many demanding applications include astronomy, communication network addressing, databases management, radar codes, coding theory, and the fields of X-ray crystallography, the social sciences, optimization, military surveillance, neutral networks, and cryptography. Cordial Labeling is used in the DNA Code Word Design problem, Noisy Communication Channel, Error correcting codes, scheduling problems, design of experiments and graph colouring. In this paper we have proved that Wheel gripped star graph, the Path union of square of cycle graph and the Star of square of cycle graph admit cordial labeling.

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