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Maximal Prime Ideals in Generalized Almost Distributive Fuzzy Lattices

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ABSTRACT

The Maximal Prime Ideals (MPIs) in Generalized Almost Distributive Fuzzy Lattices are presented in this article (GADFL). In GADFL, we have also deduced certain properties and characteristic theorems of MPIs. Further, we have also derived the following theorems: let I_f and J_f be two ideals of GADFL L(R_f,A_f). Then I_f∧J_f is a MPI belonging to both I_f and J_f and finally, every ideal of GADFL L(R_f,A_f) is the union of all MPIs containing it.

AMS subject classification: 06D72, 06B10, 06D75.

Keywords: Generalized Almost Distributive Fuzzy Lattices (GADFL), Ideals, Prime Ideal (PI), Prime Filter (PF), Maximal Prime Ideals (MPIs) and Maximal Prime Filters (MPFs).

1. INTRODUCTION:

G.C. Rao, Ravi Kumar Bandaru, and N. Rafi proposed the definition of Generalized Almost Distributive Lattices (GADFL) as a generalisation of Almost Distributive Lattices (ADLs). L.A. Zadeh [7, 12, 13, 14] developed the notion of a fuzzy set in 1965. According to L.A. Zadeh [8], a fuzzy ordering is a transitive fuzzy relation that is a generalisation of the concept of ordering. N. Ajmal and K.V. Thomas [1] developed a fuzzy lattice as a fuzzy algebra in 1994, and fuzzy sub lattices were established in 1995. In 2009, I. Chon [4] developed a unique notion of fuzzy lattices and examined the level sets of fuzzy lattices based on fuzzy order theory. He also created the concepts of distributive and modular fuzzy lattices, as well as several basic fuzzy lattice properties. Berhanu et al. [2, 9] proposed ADFLs as a generalisation of DFLs, and used I. Chon's fuzzy partial order relations and fuzzy lattices to characterise some elements of an ADL. Berhanu and Yohannes [3] define GADFLs as a generalisation of ADFLs.

In this article, we are presented the MPIs in GADFL. Further we have derived some properties and characteristic theorems of MPIs in GADFL. Also, we have derived the theorems are, let I and J be two ideals of GADFL $L(R,A)$. Then $I \wedge J$ is a MPI belonging to both I and J and finally, every ideal of GADFL $L(R,A)$ is the union of all MPIs containing it .

2. PRELIMINARIES:

A few fundamental definitions are discussed.

Definition [15]: 2.1. Let $(R,\vee,\wedge)$ be a fuzzy poset and (R,A) be an algebra type (2,2). If (R,A) meets the following axioms, we call it a GADFL.

1. $A\big((a \wedge b) \wedge c, a \wedge (b \wedge c)\big) = A(a \wedge (b \wedge c), (a \wedge b) \wedge c) = 1;$

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2. $A(a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)) = A((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)) = 1;$
3. $A(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)) = A((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)) = 1;$
4. $A(a \wedge (a \vee b), a) = A(a, a \wedge (a \vee b)) = 1;$
5. $A((a \vee b) \wedge a, a) = A(a, (a \vee b) \wedge a) = 1;$
6. $A((a \wedge b) \vee b, b) = A(b, (a \wedge b) \vee b) = 1 \; for \; all \; a, b, c \in R$.

Example 2.2. Let $R = \{a, b, c\}$. Define two binary operations $\vee$ and $\wedge$ on R as follows.

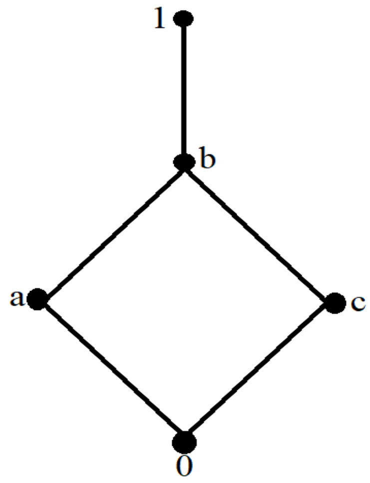

Figure: 2.1

Define a fuzzy relation $A: R \times R \to [0, 1]$ as follows:
$A(a, a) = A(b, b) = A(c, c) = 1, A(b, a) = A(b, c) = A(c, a) = A(c, b) = 0, A(a, b) = 0.2$
$and \; A(a, c) = 0.4$
Clearly (R, A) is a GADFL.

Definition 2.3. [17]: Let (R, A) be a GADFL. A non-empty subset I of R is said to be an ideal of (R, A), if it satisfies the following conditions:
1) If $x \in R, y \in I$ and $A(x, y) > 0$, then $x \in I$;
2) If $x, y \in I$ then $x \vee y \in I$.

Definition 2.4. [16]: If $I \neq R$, an Ideal I of (R, A) is termed proper. If $F \neq R$, a filter F of (R, A) is considered proper. For each any $x, y \in R, x \wedge y \in P (x \vee y \in P) \Longrightarrow x \in P \; or \; y \in P$, a suitable ideal (filter) P of R is said to be prime. If $R - P$ is PF, it is obvious that a subset P of R is a PI.

3. MAXIMAL PRIME IDEALS (MPIS) IN GADFL

In this section we study many interesting and important properties of MPIs and MPFs of $L(R_f, A_f)$.

Definition: 3.1.

Let I_f be an ideal of $L(R_f, A_f)$. A PI P_f is said to be a MPI of GADFL belonging to an ideal I_f if

1. $I_f \supseteq P_f$ and
2. There is no PI Q_f such that $I_f \supseteq Q_f \supset P_f$. That is P_f is maximal among the PIs of $L(R_f, A_f)$ containing I_f.

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Example: 3.2.

Consider the poset $(P_f, \leq)$
Define a fuzzy relation $A_f: R_f \to [0,1]$.

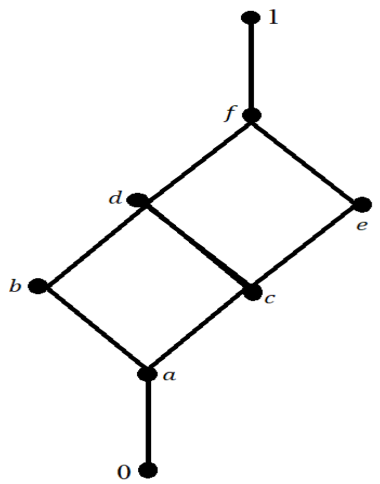

Figure: 3.1

Hasse diagram of the GADFL $L_f(R_f, A_f) = \{0, a, b, c, d, e, f, 1\}$.

Now, we defined by $A_f(0,0) = A_f(a,a) = A_f(b,b) = A_f(c,c) = 1$

and $A_f(d,d) = A_f(e,e) = A_f(f,f) = 0.5$

Then A_f is a maximal prime ideal of P_f.

Theorem: 3.3.

Let I be an ideal of $L(R_f, A_f)$. Let P_f be a PI containing I_f. Then P_f is MPI belonging to I_f if and only if for each $a_f \in P_f$ there is $b_f \notin P_f$ such that $a_f \wedge b_f \in I_f$.
Proof:

Let P_f be a MPI belonging to I_f.
Then prove that $R_f - P_f$ is a PF which is negligible in terms of the attribute of not meeting I_f.
Let $a_f \in P_f$. Then $a_f \notin R_f - P_f$
Let $E_f = (R_f - P_f) \vee [a_f)$
Suppose $E_f \cap I_f = \emptyset$
Then the PI P_f of R_f is a MPI *iff* for each $a_f \in P_f$, there is
$b_f \notin P_f$ such that $a_f \wedge b_f = 0$, there is a PF H_f such that $H_f \supseteq E_f$ and $H_f \cap I_f = \emptyset$.
Therefore, $R_f - H_f$ is a PI and $I_f \supseteq R_f - H_f$
Since $H_f \supseteq E_f \supseteq R_f - P_f$, we get $P_f \supseteq R_f - H_f$ and hence $R_f - H_f = P_f$
That is $H_f = R_f - P_f$ so that $a \in R_f - P_f$.
This is a contradiction.
Therefore $E_f \cap I_f = \emptyset$.

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Choose $r_f \in [(R_f - P_f) \vee [a_f)] \cap I_f$
Then $t_f \in I_f$ and $t_f \in (R_f - P_f) \vee [a_f)$
Therefore $t_f = b_f \wedge y_f$ where $b_f \in R_f - P_f$ and $y_f \in [a_f)$
Now, $t_f = A_f(b_f \wedge (y_f \vee a_f), 0)$
$= A_f((b_f \wedge y_f) \vee (b_f \wedge a_f), 0)$
$= A_f(t_f \vee (b_f \wedge a_f), 0)$
$= A_f(t_f \vee 0, 0)$
$= A_f(0, 0) \geq 0$. (Since $b_f \wedge a_f = 0$ and $a_f \wedge b_f = 0$).
Therefore, $b_f \wedge a_f \in I_f$.
That is for every $a_f \in P_f$, there is $b_f \notin P_f$ such that $b_f \wedge a_f \in I_f$.
Let K_f be any PI belonging to I_f and $P_f \supseteq K_f$.
Let $a_f \in P_f$. Then from our assumption there is $b_f \notin P_f$ such that $a_f \wedge b_f \in P_f$.
Now $P_f \supseteq K_f \Rightarrow a_f \wedge b_f \in K_f$ and hence $a_f \in K_f$
Since $b_f \notin K_f$. Therefore P_f is a PI belonging to I_f.
Hence the proof.

Definition: 3.4.

A set S_f of $L(R_f, A_f)$ GADFL is said to be multiplicatively closed subset of $L(R_f, A_f)$ if $S_f \neq \emptyset$ and for any $a_f, b_f \in S_f$ implies $A(a_f \wedge b_f, 0) > 0$ and $a_f \wedge b_f \in S_f$.

Theorem: 3.5.

Let I_f be an ideal and S_f be a multiplicatively closed subset of GADFL $L(R_f, A_f)$ such that $I_f \cap S_f = \emptyset$. Then there is a MPI T_f of $L(R_f, A_f)$ such that $L - S_f \supseteq T_f \supseteq I_f$.

Proof:

Let I_f be an ideal and S_f be a multiplicatively closed subset of GADFL $L(R_f, A_f)$ such that $I_f \cap S_f = \emptyset$.
Then there exists a PI P_f of $L(R_f, A_f)$ such that $P_f \supseteq I_f$ [By the definition of 2.5] and $S_f \cap P_f = \emptyset$.
Since P_f is a PI of $L(R_f, A_f)$, $L - P_f$ is a PF of $L(R_f, A_f)$.
Now, we prove that $L - P_f \supseteq S_f$.
Let $x_f, y_f \in S_f$ then $x_f \wedge y_f \in S_f$, [By the definition of 2.4]
Now $x_f, y_f \in S_f \Rightarrow A_f(x_f \vee (x_f \wedge y_f), y_f \vee (x_f \wedge y_f), 0) > 0$
$\Rightarrow A_f\left((x_f \vee x_f) \wedge (x_f \vee y_f), (y_f \vee x_f) \wedge (y_f \vee y_f), 0\right) > 0$
$\Rightarrow A_f(x_f \wedge y_f, 0) > 0$
$\Rightarrow A_f(0, 0) > 0$
$\therefore x_f, y_f \in S_f \Rightarrow x_f \wedge y_f \in S_f$
Similarly, we have to prove that $x_f \wedge y_f \in L - P_f$
$\therefore$ we get $L - P_f \supseteq S_f$.
Also, $I_f \cap (L - P_f) = \emptyset$
Now let $\mathcal{G} = \{F_f | F_f$ is a filter of $L(R_f, A_f), F_f \supseteq S_f$ and $I_f \cap F_f = \emptyset\}$
Also, let $\mathcal{G} = \{x_f \in R | A_f(x_f \vee y_f, 0) > 0 \; \forall \; y_f \in F_f\}$
clearly, $L - P_f \notin \mathcal{G}$ and hence $\mathcal{G} \neq \emptyset$.

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Therefore, there exists a maximal element g_f and $G_f \in \mathcal{G} \,\forall\, g_f \in G_f$.
To prove that G_f is a PF of $L(R_f, A_f)$.
Assume$G_f \in \mathcal{G}, g_f \in G_f$ and $r_f \in L(R_f, A_f)$.
Since g_f is maximal element, $A_f(g_f \vee x_f, g_f, 0) > 0 \,\forall\, x_f \in R$.
Hence $A_f(x_f, (r_f \vee g_f) \wedge x_f, 0) = A_f(x_f, (g_f \wedge r_f) \wedge x_f, 0)$

$$= A_f(x_f, (g_f \wedge x_f) \vee (r_f \wedge x_f), 0)$$

$$\geq sup_{K\in R} \min\left\{A_f(x_f, k_f), A_f\left(k, (g_f \wedge x_f) \vee (r_f \wedge x_f)\right), 0\right\} > 0$$

$$\geq \min\left\{A_f(x_f, g_f \wedge x_f), A_f\left(g_f \wedge x_f, (g_f \wedge x_f) \vee (r_f \wedge x_f)\right), 0\right\} > 0$$

Hence $A_f(x_f, (r_f \vee g_f) \wedge x_f, 0) > 0$ and it follows that $r_f \vee g_f \in \mathcal{G}$ and $G_f \in \mathcal{G}$ where $g_f \in \mathcal{G}$ and $r_f \in L(R_f, A_f)$.
Thus G_f is a PF of $L(R_f, A_f)$, and hence $L - G_f$ is a PI of $L(R_f, A_f)$.
Also, $L - G_f \supseteq I_f$ and $S_f \cap (L - G_f) = \emptyset$.
Clearly, $G_f \supseteq S_f$, now let, T_f be any other PI of $L(R_f, A_f)$.
To prove that T_f is a MPI of $L(R_f, A_f)$ such that $L - S_f \supseteq T_f \supseteq I_f$.
Let T_f be PI of $L(R_f, A_f)$ such that $T_f \supseteq I_f$ and $L - G_f \supseteq I_f$.
This gives $I_f \cap (L - T_f) = \emptyset$ and also $L - T_f \supseteq G_f \supseteq S_f$,
but, $g_f \in G_f$ is maximal element of $\mathcal{G}$.
Therefore, we get $(L - T_f) = G_f$ this gives $T_f = L - T_f$.
Thus, $L - S_f \supseteq T_f \supseteq I_f$ (since $L - T_f \supseteq G_f \supseteq S_f$).
Therefore, we get $T_f$is a MPI of $L(R_f, A_f)$, such that $L - S_f \supseteq T_f \supseteq I_f$.
Hence the proof.

Theorem: 3.6.
Let I_f, J_f be any two ideals of GADFL $L(R_f, A_f)$. Then J_f is a MPI belonging to I_f.

Proof:
Let I_f, J_f be any two ideals of GADFL $L(R_f, A_f)$.
To prove that J is MPI belonging to I_f.
Now $I_f \cap J_f = \emptyset$ implies $J_f \supseteq I_f$.
Let $e_f, h_f \in L(R_f, A_f)$ such that $e_f \notin J_f$ and $h_f \notin J_f$ then $e_f, h_f \in L - J_f$.
We have to prove that $e_f \wedge h_f \in L - J_f$.
Now, let $e_f, h_f \in L - J_f \Rightarrow A_f(e_f \vee (e_f \wedge h_f), h_f \vee (e_f \wedge h_f), 0) > 0$

$$\Rightarrow A_f(e_f \wedge (e_f \vee h_f), (e_f \vee h_f) \wedge h_f, 0) > 0$$

$$\Rightarrow A_f\left((e_f \vee h_f), (e_f \wedge h_f), 0\right) > 0$$

$$\Rightarrow A_f(e_f \wedge h_f, 0) > 0$$ [Since $e_f \vee h_f = 1$ and $e_f \wedge h_f = 0$ by the GADFL condition]

$$\Rightarrow A_f(0,0) > 0$$

$\therefore e_f, h_f \in L - J_f \Rightarrow e_f \wedge h_f \in L - J_f$
Hence $e_f \wedge h_f \in L - J_f$.
This gives$e_f \wedge h_f \notin L - J_f$
Therefore $J_f$is a PI of GADFL $L(R_f, A_f)$ and $J_f \supseteq I_f$.
Let K_f be any other PI of GADFL $L(R_f, A_f)$ such that $K_f \supseteq I_f$ and $J_f \supseteq K_f$.
Then $L - K_f$ is a MPI of $L(R_f, A_f)$.
and $I_f \cap (L - K_f) = \emptyset$.
Therefore, we get $L - J_f \supseteq L - K_f$ and hence $K_f \supseteq J_f$.
This gives $K_f = J_f$.

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Therefore J_f is a MPI of GADFL $L(R_f, A_f)$.
Hence J_f is a MPI belonging to I_f.
Hence the proof.

Now we prove the characterization theorems for Maximal Prime Ideals (MPIs).

Theorem: 3.7.

Let $L(R_f, A_f)$be a GADFL. Then the following are equivalent.

1. $L(R_f, A_f)$ be an ADFL.
2. Every MPI is an ideal.
3. Every PI P_f with $P_f \cap D_f = \emptyset$ is an ideal.

Proof:

Assume $(1) \Rightarrow (2)$. Let $L(R_f, A_f)$ be an ADFL.
To prove that every MPI is an ideal.
Let P_f is any non empty subset and $P_f{}^*$ is a MPI, such that $P_f{}^* \supseteq P_f$.
The set $P_f{}^* = \{x_f \in R_f | A_f(x_f \wedge p_f, 0) > 0 \text{ for all } p_f \in P_f\}$, now let $a_f, b_f \in P_f{}^*$.
Then $AA_f(a_f \wedge p_f, 0) > 0$ and $A_f(b_f \wedge p_f, 0) > 0 \; \forall \; p_f \in P_f$, on the other hand, since $L(R_f, A_f)$ be a GADFL,
$A_f(0, b_f \wedge p_f) > 0$ and $A_f(0, a_f \wedge p_f) > 0 \; \forall \; p_f \in P_f$
$\Rightarrow a_f \wedge p_f = 0$ and $b_f \wedge p_f = 0 \; \forall \; p_f \in P_f$.

$$\text{Hence } A_f\left(\left(a_f \vee b_f\right) \wedge p_f, 0\right) = A_f\left(\left(a_f \wedge p_f\right) \vee \left(b_f \wedge p_f\right), 0\right) = A_f(0 \vee 0, 0) = A_f(0,0) > 0)$$

Thus, $a_f \vee b_f \in P^*$
Again let $a_f \in P_f{}^*$ and $x_f \in R_f$
Then $A_f(a_f \wedge p_f, 0) > 0 \; \forall \; p_f \in P_f$

$$\text{Now, } A_f\left(\left(a_f \wedge x_f\right) \wedge p_f, 0\right) = A_f\left(\left(x_f \wedge a_f\right) \wedge p_f\right), \text{o)}$$
$$= A_f\left(\left(x_f \wedge a_f\right) \wedge p_f\right), 0)$$
$$\geq sup_{y \in R} \min\{A_f(x_f \wedge (a_f \wedge p_f), y_f), A_f(y_f, 0)\}$$
$$\geq \min\{A_f(x_f \wedge (a_f \wedge p_f), a_f \wedge p_f), A_f(a_f \wedge p_f, 0\} > 0 \; \forall \; p_f \in P_f$$

Hence $(a_f \wedge x_f) \wedge p_f \in P^* \; \forall \; p_f \in P_f$.
Thus P^* is an ideal of $L(R_f, A_f)$ GADFL.
Assume $(2) \Rightarrow (3)$
Assume that every MPI is an ideal.
To prove that every PI P_f with $P_f \cap D_f = \emptyset$ is an ideal.
Let D_f be a PI of $L(R_f, A_f), L - D$ is a filter of $L(R, A)$ and $L - D_f \supseteq P_f$.
Also $P_f \cap D_f = \emptyset$ is an ideal.
Assume $(3) \Rightarrow (1)$
Let $n_f \in R_f$, P_f is a PI with $P_f \cap D_f = \emptyset$ is an ideal.
Let $n_f, z_f \in R$ since $A_f(n_f \wedge z_f, (n_f \wedge z_f) \wedge z_f) = A_f\left((n_f \wedge z_f) \wedge z_f, n_f \wedge z_f\right) = 1$
then $(n_f, n_f \wedge z_f) \in P_f$. Also $A_f(z_f \wedge z_f, z_f \wedge z_f) = 1$
Hence $A_f(z_f, z_f) \in P_f$.
Since P_f is a PI with $P_f \cap D_f = \emptyset$ is an ideal of $L(R_f, A_f)$.

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Hence $A_f\left(\left(n_f \vee z_f\right) \wedge z_f, \left[\left(n_f \wedge z_f\right) \vee z_f\right] \wedge z_f\right)$

$$= A_f\left(\left[\left(n_f \wedge z_f\right) \vee z_f\right] \wedge z_f, \left(n_f \vee z_f\right) \wedge z_f\right) = 1$$

$\Rightarrow A_f\left(\left(n_f \vee z_f\right) \wedge z_f, z_f \wedge z_f\right) = A_f\left(z_f \vee z_f, \left(n_f \vee z_f\right) \wedge z_f\right) = 1$

$\Rightarrow A_f\left(\left(n_f \vee z_f\right) \wedge z_f, z_f\right) > 0$ and $A_f\left(z_f, \left(n_f \vee z_f\right) \wedge z_f\right) > 0$

$\therefore L(R_f, A_f)$is an Almost Distributive Fuzzy Lattice.

Hence $L(R_f, A_f)$ is an ADFL. Hence the proof.

Definition: 3.8.

Let F_f be a filter of $L(R_f, A_f)$. A PF G_f is named to be a MPF of GADFL be in the right place to a filter F_f if

1. $F_f \supseteq G_f$ and
2. there is no PF H such that $F_f \supseteq H \supset G_f$. That is G_f is Maximal among the PFs of $L(R, A)$ covering F_f.

Theorem: 3.9.

Every PI of a GADFL $L\left(R_f, A_f\right)$ contains a MPI.

Proof:

Let P be a PI of $L(R_f, A_f)$. Let $F_f = L - P_f$. Then F_f is a PF of $L(R_f, A_f)$ such that $F_f \cap L - P_f \neq \emptyset$ there is a maximal filter G in $L(R_f, A_f)$.

Now $F_f \supseteq G_f$

To prove $L - G_f \supseteq L - F_f$

Let $x_f, y_f \in L - G$ then there exist $a_f, b_f \in L(R_f, A_f)$ and $c_f, d_f \in G$ such that $a_f \wedge c_f = 0$ and $b_f \wedge d_f = 0$.

Then $x_f \wedge y_f \in L - G_f$

Now $x_f, y_f \in L - G_f \Rightarrow A_f(x_f \wedge y_f, (a_f \wedge c_f) \vee (b_f \wedge d_f), 0)$

$$= A_f\left(x_f \wedge y_f, \left(\left(a_f \wedge c_f\right) \wedge \left(x_f \wedge y_f\right)\right) \vee \left(\left(b_f \wedge d_f\right) \wedge \left(x_f \wedge y_f\right)\right), 0\right)$$

$$= A_f\left(x_f \wedge y_f, \left(0 \wedge \left(x_f \wedge y_f\right)\right) \vee \left(0 \wedge \left(x_f \wedge y_f\right)\right), 0\right)$$

$$= A_f\left(x_f \wedge y_f, 0, 0\right)$$

$$= A_f\left(x_f \wedge y_f, 0\right) > 0$$

$\therefore x_f \wedge y_f \in L - G_f$ implies $x_f \wedge y_f \in L - G_f$

Similarly, $x_f \wedge y_f \in L - F_f$

Hence $L - G_f \supseteq L - F_f = P_f$

Therefore P_f is a MPI of $L(R_f, A_f)$ if and only if is a MPF. Therefore, we get $L - G_f$ is a MPI contained in P_f.

Theorem: 3.10.

Every ideal of GADFL $L(R_f, A_f)$ is the union of all MPIs containing it.

Proof:

Let I_f be an ideal of GADFL $L(R_f, A_f)$

Then $I_f = \{x_f \in R_f | A_f\left(x_f \wedge s_f, 0\right) > 0 \,\forall\, s_f \in I_f\}$

Now let ${I_f}_0 = U\{P_f | P_f$ is a MPI containing I_f.

Clearly $I_f \supseteq {I_f}_0$ --------(1)

Conversely, Let $a_f \notin I_f$

Then $x \wedge s_f \neq 0$ for all $s_f \in I_f$

Then there exists a MPI P_f such that $x_f \wedge s_f \notin P_f$.

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Hence $x_f \notin P_f$ and $s_f \notin P_f$.
SinceP_f is prime $[s_f)^* \supseteq P_f \ \forall \ s_f \in I_f$
Therefore $I_f \supseteq P_f$
Thus P_f is a MPI containing I_f and $x_f \notin P_f$. Therefore, we get $x_f \notin I_{f_0}$ which yields that
$I_{f_0} \supseteq I_f$ ---------(2)
From equations (1) and (2) we get $I_f = I_{f_0}$.

4. CONCLUSION:

The ideas of Maximum prime ideals in GADFL are described in this work and numerous features of Maximal prime ideals are examined. Examine the characteristics of Maximal prime ideals provided in this study in further depth. Finding the S – Ideals in Dual of GADFLs is an exciting future project. On S – Ideals in Dual of GADFL, we will also derive certain characterization theorems.

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