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On the Forcing edge Steiner Global Domination Number of a Graph

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ABSTRACT

Let W be the minimum edge Steiner global dominance set of a connected graph G. If W is the only minimum edge Steiner global dominating set that contains T, then a subset T   W is referred to as a forcing subset for W. A minimum forcing subset of W is a forcing subset for W with minimum cardinality. The cardinality of a minimal forcing subset of W is its forcing edge Steiner global dominance number, represented by f_(¯γ se) (W). f_(¯γ se) (G) = min{f_(¯γ se) (W)}, is the forcing edge Steiner global domination number of G, represented by f_(¯γ se) (G), where the minimum is obtained across all minimal edge Steiner global dominating sets W in G. The forcing Steiner and edge Steiner global dominance number of a graph is given some realisation findings in this article.

Keywords: Forcing edge Steiner global domination number, edge Steiner number, edge Steiner domination number.

AMS subject classification: 05C12.

1. INTRODUCTION

This paper discusses a simple, connected undirected graph, $G = (V, E)$. Let n and m stand for size and order, respectively. For a fundamental reference to graph theory, see [2]. Two vertices, u and v, are considered nearby if uv is an edge of G. Vertex $v \in V$ has a degree of $deg(v) = |N(v)|$, and u is v's neighbour if $uv \in E(G)$. The collection of v's neighbours is represented by $N(v)$. A vertex v is called a universal vertex if $deg(v) = n - 1$. With $V(G[S]) = S$ and $E(G[S]) = \{uv \in E(G): u, v \in S\}$, the subgraph created by a set S of vertices of a graph G is represented as $G[S]$. If $G[N(v)]$ is complete, then a vertex v is an extreme vertex. If there is a universal vertex in $N(v)$ in the subgraph created by its neighbours, then a vertex v is a semi-extreme vertex of G.

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One of the fundamental ideas of graph theory is distance [3]. The length of the shortest $u - v$ path in a connected graph G is the distance $d(u, v)$between two vertices u and v. A $u - v$ geodesic is a $u - v$ route of length $d(u, v)$. The Steiner distance $d(W)$ of a nonempty set W of vertices in a connected graph G is the smallest size of a connected subgraph of G that contains W. In [2], the Steiner distance was examined. Every subgraph is a tree, and they are referred to as Steiner trees with regard to W or Steiner W-trees. The set of all vertices on Steiner W-trees is represented by $S(W)$. $S(W) = W$ if is linked. A Steiner W-tree is a shortest $u - v$ path or a $u - v$ geodesic if W has precisely two vertices, u and v. If every vertex in a set $W \subseteq V(G)$ or if $S(W) = V(G)$, then the set is referred to as a Steiner set of G. The Steiner number $S(G)$ of G is the cardinality of a Steiner set of minimal cardinality, often known as a minimum Steiner set or just an s-set. A graph's Steiner number was first shown in [2] and then examined in [5]. If a set of vertices W in G is both a dominating set of G and an edge Steiner set, then W is referred to as an edge Steiner dominating set of G. The edge Steiner domination number, represented as $\gamma_{\mathrm{se}}(G)$, is the lowest cardinality of an edge Steiner dominating set of G. A γ_{se}-set of G is defined as an edge Steiner dominant set of G size $\gamma_{\mathrm{se}}(G)$.

If each vertex of $V \setminus D$ has at least one neighbour in D, then D is a dominant set in G. The domination number of G, represented as $\gamma(G)$, is the lowest cardinality of a dominating set of G. A γ-set of G is a dominant set of cardinality $\gamma(G)$. If D is a dominating set of both G and $\bar{G}$, then a subset $D \subseteq V$ is referred to be a global dominating set in G. The smallest cardinality of a minimal global dominating set in G is the global domination number $\bar{\gamma}(G)$. In [4], these ideas were examined.

If a set S is both a Steiner set and a global dominating set of G, then $S \subseteq V$ is a Steiner global dominating set of G. The Steiner global domination number of G, represented by $\bar{\gamma}_s(G)$, is the lowest cardinality of a Steiner global dominating set of G. A $\bar{\gamma}_s$-set of $G$is a Steiner global dominating set of cardinality $\bar{\gamma}_s(G)$. If a vertex v is present in every $\bar{\gamma}_s$-set of G, then it is considered a Steiner global dominance vertex of G. If an edge Steiner set S is both an edge Steiner set and a global dominating set of a linked graph G, then S is an edge Steiner global dominating set of G. An edge Steiner global dominating set's minimal cardinality is the edge $\bar{\gamma}_{se}(G)$ is the Steiner global dominance number of G.

The Steiner global dominant vertices of G are all of its extreme and universal vertices. There exist, in fact, Steiner global dominant vertices that are neither universal nor extreme vertices of G. If a vertex v is present in every $\bar{\gamma}_{se}$-set of G, it is considered an edge Steiner global dominating vertex of G. All of G's universal and semi-extreme vertices are edge Steiner global dominating vertices. In actuality, certain edge Steiner global dominant vertices are neither universal nor semi-extreme vertices of G. In [6], these ideas were examined.

Numerous authors have examined the notion of force in [1][3]. Let S be a Steiner global dominating set of G that is at least minimal. If S is the only minimal Steiner global dominating set that contains T, then a subset $T \subseteq S$ is referred to be a forcing subset for S. A minimum forcing subset of S is a forcing subset for S with minimum cardinality. The cardinality of a minimal forcing subset of S is its forcing Steiner global dominance number, represented by $f_{\bar{\gamma}_s}(S)$. $f_{\bar{\gamma}_s}(G) = min\left\{f_{\bar{\gamma}_s}(S)\right\}$, is the forced Steiner global domination number of G, represented by $f_{\bar{\gamma}_s}(G)$, where the minimum is calculated across all minimal Steiner global dominating sets S in G. These ideas have been examined in [1][5].

The sequel use the following theorem.

Theorem 1.1. [5] Let G be a connected graph. Then

(i) Each extreme vertex and each universal vertex of G belongs to every Steiner global dominating set of G.

(ii) $f_{\overline{\gamma}_s}(G) \leq \overline{\gamma}_s(G) - |Z|$, where Z is the set of all Steiner global dominating vertices of G.

2. THE FORCING EDGE STEINER GLOBAL DOMINATION NUMBER OF A GRAPH

Definition 2.1. Let W be the least edge Steiner global dominance set of a connected graph G. If W is the only minimum edge Steiner global dominating set that contains T, then a subset $T \subseteq W$ is referred to as a forcing subset for W. A minimum forcing subset of W is a forcing subset for W with minimum cardinality. The cardinality of a minimal forcing subset of W is the forcing edge Steiner global domination number, represented as $f_{\overline{\gamma}se}(W)$. $f_{\overline{\gamma}se}(G) = min\{f_{\overline{\gamma}se}(W)\}$ is the forcing edge Steiner global domination number of G, represented by $f_{\overline{\gamma}se}(G)$, where the minimum is obtained across all minimal edge Steiner global dominating sets W in G.

Example 2.2. As shown in Figure 2.1, the graph G is represented as $W_1 = \{v_1, v_2, v_5\}$ and $W_2 = \{v_1, v_4, v_7\}$ are the only two $\overline{\gamma}_{se}$-sets of G such that $f_{\overline{\gamma}se}(W_1) = f_{\overline{\gamma}se}(W_2) = 1$ so tha $f_{\overline{\gamma}se}(G) = 1$.

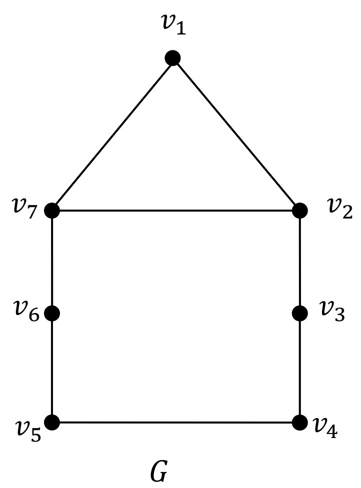

Figure 2.1

Theorem 2.3. For every connected graph G, $0 \leq f_{\overline{\gamma}se}(G) \leq \overline{\gamma}_{se}(G) \leq n$.

Theorem 2.4. Assume that G is a connected graph. Consequently,

(i) $f_{\overline{\gamma}se}(G) = 0$ if and only if G possesses a distinct Steiner global dominating set with a minimal edge.

(ii) $f_{\overline{\gamma}se}(G) = 1$ if and only if G contains a minimal edge Steiner global dominating set with at least two elements, one of which is a special minimum edge Steiner global dominating set that contains one of its components.

(iii) If and only if no minimal edge Steiner global dominating set of G is the sole minimum edge Steiner global dominating set that contains any of its appropriate subsets, then $f_{\overline{\gamma}se}(G) = \overline{\gamma}_{se}(G)$.

Observation 2.5. Let G be a connected graph, and W the set of all edge Steiner global dominating set. Then $f_{\overline{\gamma}se}(G) \leq \overline{\gamma}_{se}(G) - |W|$

The forcing edge Steiner global dominance number of standard graphs is determined below.

Observation 2.6. (i) For the path $G = P_n$ $(n \geq 2)$, $f_{\overline{\gamma}se}(G) = 0$.

(ii) For the complete graph $G = K_n$ $(n \geq 2)$, $f_{\overline{\gamma}se}(G) = 0$.

(iii) For the star graph $G = K_{1,n-1}$ $(n \geq 2)$, $f_{\overline{\gamma}se}(G) = 0$.

Theorem 2.7. For every positive integer $a \geq 0$, there exists a connected graph G such that $f_{\overline{\gamma}s}(G) = f_{\overline{\gamma}se}(G) = a$.

Proof. Let $P: x, y, z$ be a three-vertice route. Consider a replica of the path on two vertices, $P_i: u_i, v_i$ $(1 \leq i \leq a)$. Let H be the graph that is produced by adding the edges yu_i and zv_i $(1 \leq i \leq a)$ to P and $P_i (1 \leq i \leq a)$. Let G be the graph that was created from H by adding the edges zz_i $(1 \leq i \leq b - a - 1)$ and the additional vertices $z_1, z_2, \dots, z_{b-a-1}$. Figure 2.2 shows the graph G.

We establish by demonstrating that $\overline{\gamma}_s(G) = b$. Consider the set of end vertices of G to be $Z = \{x, z_1, z_2, \dots, z_{b-a-1}\}$. Since Z is a subset of each Steiner global dominating set of G according to Theorem 1.1 (i), $\overline{\gamma}_{se}(G) \geq b - a - 1 + 1 = b - a$. Z is not a Steiner global dominating set of G as $S(W) \neq V(G)$. Let $H_i = \{u_i, v_i\}$ $(1 \leq i \leq a)$. Every Steiner global dominating set has at least one vertex from each H_i $(1 \leq i \leq a)$, as can be readily shown, and so $\overline{\gamma}_s(G) \geq b - a + a = b$. Now $W = Z \cup \{u_1, u_2, \dots, u_a\}$ is a Steiner global dominating set of G and so $\overline{\gamma}_s(G) = b$.

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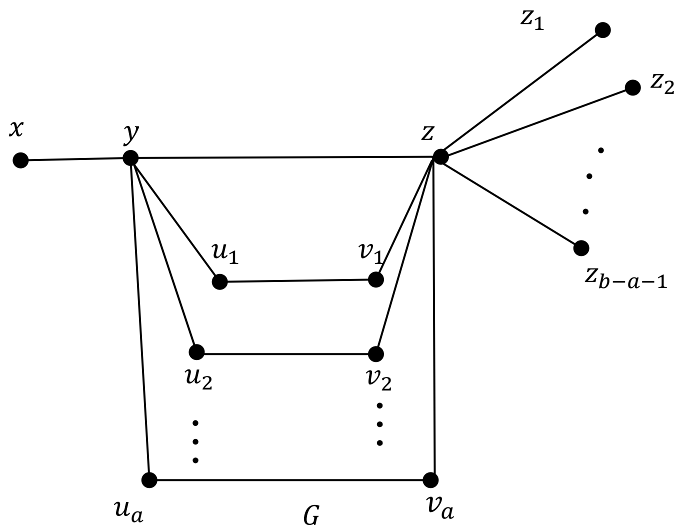

Figure 2.2

Following that, we demonstrate that $f_{\overline{\gamma}s}(G) = a$. $f_{\overline{\gamma}s}(G) = \overline{\gamma}_s(G) - |Z| = b - (b-a) = a$.according to Theorem 1.1 (ii). Now since $\overline{\gamma}_s(G) = b$ and every $\overline{\gamma}_s$-set of Now since $\overline{\gamma}_s(G) = b$ and every $\overline{\gamma}_s$-set of G contains Z, it is easily seen that $\overline{\gamma}_s$-set of G is of the form $W = Z \cup \{c_1, c_2, \dots, c_a\}$, where $c_i \in H_i$. Let T be any proper subset of W with $|T| < a$. Then there is a vertex c_j $(1 \le i \le a)$ such that $c_j \notin T$. Let b_j be a vertex of H_j distinct from c_j. Then $W_1 = (W - \{c_j\}) \cup \{b_j\}$ is a $\overline{\gamma}_s$-set of G properly containing T. Thus T is not a forcing subset of M. This is true for all minimum $\overline{\gamma}_s$-set of G and so it is follows that $f_{\overline{\gamma}s}(G) = a$.

Theorem 2.8. For every integer $a \ge 0$, there exists a connected graph G, such that $f_{\overline{\gamma}s}(G) = a$ and $f_{\overline{\gamma}se}(G) = 0$.

Proof. Let $P_i: x_i, y_i$ $(1 \le i \le a)$ be a replica of the route on two vertices, and let $P: x, y, z$ be a path of three vertices. Let Q be the graph that is created by adding the edges yx_i, xx_i and zy_i $(1 \le i \le a)$ to P and $P_i (1 \le i \le a)$. Figure 2.3 shows the graph G.

We start by demonstrating that $f_{\overline{\gamma}s}(G) = a$. Let $H_i = \{x_i, y_i\}$ $(1 \le i \le a)$ and $Z = \{x, z\}$. Since every Steiner global dominating set of G has at least one vertex from each H_i $(1 \le i \le a)$, Z is a subset of all Steiner global dominating sets of G according to Theorem 1.1 (i), meaning that $\overline{\gamma}_s(G) \ge a + 2$. Suppose that $W = Z \cup \{x_1, x_2, \dots, x_a\}$. Consequently, $S(W) = V(G)$.The Steiner global dominant set of G is thus S. Since W is a dominating set in G and $\overline{G}$. W is a global dominating set of G. Therefore W is an edge Steiner global dominating set of G so that $\overline{\gamma}_s(G) = a + 2$.

Following that, we prove $f_{\overline{\gamma}s}(G) = a$. $f_{\overline{\gamma}s}(G) = \overline{\gamma}_s(G) - |Z| = a + 2 - 2 = a$ according to Theorem 1.1 (ii). It is now clear that the $\overline{\gamma}_s$-set of G is of the type $W = Z \cup \{c_1, c_2, \dots, c_a\}$, where $c_i \in H_i$ $(1 \le i \le a)$, since $\overline{\gamma}_s(G) = a + 2$ and every $\overline{\gamma}_s$-set of G includes Z. Let T be any proper subset of W with $|T| < a$. Then there is a vertex c_j $(1 \le i \le a)$ such that $c_j \notin T$. Let b_j be a vertex of H_j distinct

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from c_j. Then $W_1 = (W - \{c_j\}) \cup \{b_j\}$ is a $\overline{\gamma}_s$-set of G properly containing T. Thus T is not a forcing subset of W. This is true for all minimum $\overline{\gamma}_s$-set of G and so it is follows that $f_{\overline{\gamma}s}(G) = a$. Such that we demonstrate $f_{\overline{\gamma}se}(G) = 0$. According to Theorem 1.1, Z is a subset of all Steiner global dominating sets of G, and since only the vertex x_i appears in every Steiner global dominating set, so $\overline{\gamma}_{se}(G) \geq a + 2$. It follows that $f_{\overline{\gamma}se}(G) = 0$ and $\overline{\gamma}_{se}(G) = a + 2$ since $W = Z \cup \{x_1, x_2, \ldots, x_a\}$ is the unique $\overline{\gamma}_{se}$-set of G.

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