

# A Novel Bhattacharya and Cosine Operator Based Enhanced Trapezoidal Bipolar Fuzzy AHP with TOPSIS for selecting the best Software as a Service (SaaS) provider

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## ABSTRACT

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Cloud computing (CC) provides dynamic computation services to offer a reliable and cost-effective cloud service to the customer. The selection of the best software as a service (SaaS) cloud services provider by ensuring considering various MCDM criteria (Cost, Ease of Use, Features and Functionalities, Security, Scalability, and Customer Support). For the selection of the best SaaS providers, we have considered Zoho CRM, Google Workspace, Microsoft 365, Slack, Asana, Dropbox, Shopify, Zoom, and AWS for evaluating the service consistency and high accuracy. The Multi-Criteria Decision Making (MCDM) provides a platform for selecting the best SaaS based on Quality of Service(QoS). Therefore, the proposed work has introduced a novel Bhattacharya and Cosine operator-based enhanced Trapezoidal Bipolar Fuzzy Analytic Hierarchy Process (AHP) with the Technique for Order of Preference by Similarity to Ideal Solution (Topsis) based decision-making for selecting the best SaaS. Thus, the proposed method minimizes the complexity of decision-making in uncertain scenarios. The Python-based experimental tool is utilized to evaluate the effectiveness of the proposed with the conventional techniques. Moreover, the case study and the sensitivity analysis prove the SaaS adoption while ensuring the robustness and stability of the suggested work.

**Keywords:** SaaS Provider Selection, Multi-Criteria Decision Making, Trapezoidal Bipolar Fuzzy AHP, Bhattacharya and Cosine Similarity Operators, TOPSIS Decision-Making.

## 1. INTRODUCTION

Cloud computing (CC) is a web-based system that offers computing services like storage, processing, networking, and software [1]. CC allows organizations and individuals to use scalable, affordable, and elastic resources on-demand without on-premise infrastructure [2]. CC enhances productivity, collaboration, and security of data, and reduces the costs of maintenance as well as operational complexity [3].

Cloud computing has transformed how firms use and gain access to computer programs via Software as a Service (SaaS) platforms that provide cheap, elastic, and versatile solutions [4]. SaaS vendors deliver software via the Internet, removing on-site installations and minimizing maintenance requirements. However, choosing the best SaaS vendor remains a complicated decision-making issue since different service vendors provide comparable features but differ in significant Quality of Service (QoS) aspects such as price, ease of use, security, scalability, customer support, and feature sets [5][6]. MCDM techniques have been widely researched for many years in an attempt to make more dependable decisions. The MCDM technique is meant to pick the optimum choice from among alternative criteria [7][8]. To assist decision-makers in making the optimum SaaS provider choice, MCDM methods are now more and more being applied to assess and score cloud services methodically [9].

Despite developments in SaaS selection frameworks, current MCDM techniques have numerous drawbacks. Traditional AHP, TOPSIS [10][11], and other MCDM approaches frequently struggle with uncertainty, subjectivity, and the ambiguity of decision-making criteria. These models usually rely on discrete values, which fail to convey the inherent ambiguity found in expert judgments and QoS metrics. Additionally, previous methods do not adequately integrate uncertainty metrics with similarity-based ranking strategies, resulting in inconsistent decision outcomes.

Furthermore, most comparison assessments of SaaS providers are either qualitative or lack resilience in dealing with dynamic QoS variations [12]. The lack of an integrated fuzzy-based strategy that incorporates both bipolarity and trapezoidal membership functions creates a research gap in ensuring an accurate, adaptive, and scalable SaaS selection process [13].

To solve these limitations, this work proposes a new Bhattacharya and Cosine Operator-Based Enhanced Trapezoidal Bipolar Fuzzy AHP with TOPSIS for SaaS provider selection under uncertainty. By adding the Bhattacharya and Cosine similarity operators, the model increases the accuracy of measuring similarity between choice criteria, addressing vagueness and uncertainty in expert opinion. The Trapezoidal Bipolar Fuzzy AHP promotes weight allocation through positive and negative decision attributes to provide an enhanced ranking process while addressing the intrinsic uncertainty in SaaS provider decision-making. The TOPSIS method solidifies decision-making through the selection of the best SaaS provider based on similarity with the ideal solution, reducing ambiguity and improving decision trustworthiness. An experimental Python-based tool is utilized to analyze performance, creating a systematic, clear, and reliable selection tool able to address uncertainty in decision-making situations.

### **Objectives of the paper**

To analyze cloud computing service providers for software as a service (SaaS) based on the Trapezoidal Bipolar Fuzzy Analytic Hierarchy Process (AHP) and TOPSIS. The research intends to rank essential criteria such as cost, scalability, reliability, and security in order to find the optimal provider. This approach provides a systematic decision-making framework for SaaS adoption.

### **Contribution of the paper**

- This is the first study to introduce Trapezoidal Bipolar Fuzzy in the context of SaaS provider selection, as no prior research has utilized this approach.
- Unlike traditional Trapezoidal Bipolar Fuzzy, this paper proposes an improvised Trapezoidal Bipolar Fuzzy model to enhance decision-making accuracy.
- While many studies have used Bipolar Fuzzy and Trapezoidal Fuzzy separately, limited research has integrated Trapezoidal Bipolar Fuzzy, this study represents a substantial advancement in the profession.
- Most existing papers rely on the Euclidean operator for similarity measurement; this study replaces Euclidean operators with Bhattacharya and Cosine operators to improve similarity measurement accuracy.
- The proposed approach combines Bhattacharya and Cosine operators, leveraging their mutual strengths to enhance Trapezoidal Bipolar Fuzzy AHP with TOPSIS, ensuring a more robust and reliable decision-making strategy.

## **2. LITERATURE REVIEW**

Chakraborty et al. (2021) [14] introduced Trapezoidal Bipolar Neutrosophic Numbers (TrBNN) to enhance decision-making under uncertainty. They classified TrBNNs into three categories based on membership dependencies and developed the Debipolarization scheme, a ranking method using the removal area technique. Applied to MCGDM, their approach improved accuracy, reliability, and robustness, effectively capturing uncertainty for more precise and ethical decision-making.

Liu and Wang (2023) [15] developed a VIKOR technique using trapezoidal fuzzy numbers for multi-attribute group decision-making under uncertainty. Expert weights were established through distance measurement, whereas criterion weights employed deviation maximization. Also used in an emergency alternative choice problem, followed by sensitivity analysis and contrast tests, the method improved judgment accuracy and objectivity, demonstrating its validity for complex, uncertain decision-making situations.

Mustafa et al. (2021) [16] investigated bipolar fuzzy Multi-Criteria decision-making to assist students in selecting the most suitable university. To study factors that affect admissions, the authors constructed a hierarchical structural model with bipolar fuzzy and soft expert sets. A new algorithm was designed to enhance decision accuracy. The model was tested based on university choice cases, confirming its ability to grade institutions and advise candidates. The findings established that the approach produced greater clarity and accuracy in decision-

making, thus being suitable for multi-criteria educational decision issues and helping students select the optimal university under varying influencing factors.

Suresh et al. (2021) [17] aimed to enhance neutrosophic trapezoidal fuzzy number ranking, which is essential in addressing uncertain and vague decision-making situations.

To rank the numbers systematically, the authors proposed a centroid approach rooted in Euclidean distances. Some details of the ranking function were investigated and comparison cases employed to verify the method. Finally, the ranking system was tested on a problem of a purchase decision in order to show its usefulness. The results indicated that the proposed method effectively managed uncertainty, enhanced ranking accuracy, and provided a systematic decision-making methodology, which is useful in a large variety of real-world applications.

Kamacı et al. (2021) [18] proposed a bipolar trapezoidal neutrosophic set structure to deal with complex multicriteria decision problems. Its mathematical behavior was explained by the authors, and they developed Dombi-based aggregation operators, such as weighted averaging and weighted geometric operators, for neutrosophic data processing. Two different decision-making approaches were provided for different situations. Effectiveness was measured through sensitivity analysis and multiple comparison testing. The results showed that the suggested strategies improved decision accuracy and reliability, thus making them suitable for handling uncertainty in complex decision-making situations, especially when inputs are given in a bipolar trapezoidal neutrosophic environment.

Mostafa (2021) [19] developed an efficient and completely consistent strategy for selecting cloud service providers (CSPs) by tackling computational complexity and inconsistency in multi-criteria decision-making (MCDM). The proposed Best-Only Method (BOM) was presented and compared to the Analytical Hierarchical Process (AHP) and the Best-Worst Method (BWM) in terms of efficiency, consistency ratio (CR), and total deviation. The findings indicate that BOM needs considerably less comparisons, having a CR of 0% and a TD of 0, outdoing AHP (37.92%, 21.26) and BWM (13.35%, 8.65). The research compares BOM's effectiveness and reliability. With the help of an example use-case scenario, showing its excellence over traditional MCDM methods.

Liu et al. (2021) [20] developed a comprehensive MCDM model to assess and choose cloud services against quality of service (QoS) factors when solving uncertainty and linguistic assessment problems. The proposed solution applies the cloud model for converting qualitative ideas into quantitative scores and suggests an improved distance measure algorithm with the use of cloud droplet distribution. To make the best decisions, we employ a dynamic expertise weighting approach and an enhanced TOPSIS supported by grey relational analysis (GRA). Tested using a real mining enterprise's cloud service choice, the results support the scheme's strength and practicability, presenting a sound theory base for cloud service assessment.

Alhalameh and Al-Tarawneh (2022) [21] developed a hybrid MCDM approach for effective service brokering in cloud-IoT networks, where the optimal selection of data centers is made based on cost and performance criteria. The methodology is based on the TOPSIS technique and tested in two modes: Integrated-TOPSIS, where the criteria weights are predefined by the user, and Entropy-TOPSIS, where the weights are determined using the Entropy method. The findings, implemented through an open-source simulation platform, reveal that the approach can effectively optimize brokering performance by taking into account diverse service provider traits. The findings demonstrate the feasibility of optimizing service brokering for cloud-IoT contexts.

Zhang and Bai (2024) [22] suggested UBQoS\_ESDM, a robust SaaS decision-making approach, to optimize Quality of Service (QoS) selection in the scenario of enormous, fuzzy big QoS data. The process employs a cloud model to optimize QoS specification defects, a Skyline query to minimize the search space, and TOPSIS-based algorithms to evaluate SaaS options. Besides, reverse QoS cloud generators and an adaptive QoS cloud model adjustment mechanism are implemented to add flexibility to dynamic QoS changes. Theoretical analysis and experiments demonstrate the superiority of the approach in accurately selecting QoS-optimized SaaS, enhancing decision-making efficiency, and adhering to user expectations.

## **Problem Statement**

As cloud computing is on the rise, organizations and individuals are scrambling to determine the most suitable Software as a Service (SaaS) provider that best meets their unique requirements. Cost, scalability, reliability, security, simplicity, and customer support are some of the key deciding factors in the usage of SaaS solution. But doubts and subjectivism in assessing those characteristics render decision-making challenging. Conventional approaches such as AHP and TOPSIS are challenged by imprecision and subjectivity in the opinion of experts, resulting in inappropriate rankings. This paper resolves these limitations and enhances decision consistency and weight allocation precision by introducing a Novel Bhattacharya and Cosine Operator-Based Enhanced Trapezoidal Bipolar Fuzzy AHP with TOPSIS. With cloud computing on the rise, organizations and individuals are struggling to determine the most suitable Software as a Service (SaaS) provider that best meets their unique requirements. Cost, scalability, reliability, security, ease, and consumer support are some of the key determining factors in SaaS solution adoption. The proposed method enhances scalability and cross-industry applicability, leading to a more robust, data-driven, and systematic assessment framework. This research fills these gaps, offering a more precise and adaptable approach to SaaS provider selection, which is advantageous to businesses in dynamic cloud computing environments.

### 3. PRELIMINARIES

#### Definition 1: Fuzzy set: [24]

A set  $\tilde{E}$ , defined as  $\tilde{E} = \{(B, \tau_{\tilde{E}}(B)) : B \in E, \tau_{\tilde{E}}(B) \in [0, 1]\}$  and usually denoted by the pair as  $(B, \tau_{\tilde{E}}(B))$ ,  $B \in E$  and  $\tau_{\tilde{E}}(B) \in [0, 1]$ , then,  $\tilde{E}$  is said to be a fuzzy set.

#### Definition 2: Bipolar fuzzy number [25]

A bipolar fuzzy number  $B = \langle K, M \rangle = \langle [k_1, k_2, k_3, k_4], [m_1, m_2, m_3, m_4] \rangle$  is a bipolar fuzzy subset of a real line  $\mathbb{R}$  with satisfaction degree  $\lambda_K$  and dissatisfaction degree  $\lambda_M$  satisfying the following postulates:

- $\lambda_K$  is a piecewise continuous function from the real line to  $[0, 1]$ , while  $\lambda_M$  is a piecewise continuous function from real line to  $[-1, 0]$ .
- $\lambda_K(y) = 0$ , for all  $y \in (-\infty, k_1]$ ,  $\lambda_M(y) = 0$  for all  $y \in (-\infty, m_1]$
- $\lambda_K(y)$  is strongly increasing on  $[k_1, k_2]$  and  $\lambda_M(y)$  is strongly decreasing on  $[m_1, m_2]$
- $\lambda_K(y) = 1$  for all  $y \in [k_2, k_3]$ ,  $\lambda_M(y) = -1$  for all  $y \in [m_2, m_3]$
- $\lambda_K(y)$  is strongly decreasing on  $[k_3, k_4]$ , and  $\lambda_M(y)$  is strongly increasing on  $[m_3, m_4]$
- $\lambda_K(y) = 0$ , for all  $y \in [k_4, \infty)$ ,  $\lambda_M(y) = 0$ , for all  $y \in [m_4, \infty)$ .
- For ease of reference, the satisfaction and dissatisfaction degrees can be defined as

$$\lambda_K(y) = \begin{cases} \lambda_K^L(y), & \text{if } y \in [k_1, k_2] \\ 1, & \text{if } y \in [k_2, k_3] \\ \lambda_K^R(y), & \text{if } y \in [k_3, k_4] \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

and

$$\lambda_M(y) = \begin{cases} \lambda_M^L(y), & \text{if } y \in [m_1, m_2] \\ -1, & \text{if } y \in [m_2, m_3] \\ \lambda_M^R(y), & \text{if } y \in [m_3, m_4] \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where  $\lambda_K^L(y): [k_1, k_2] \rightarrow [0, 1]$ ,  $\lambda_K^R(y): [k_3, k_4] \rightarrow [0, 1]$   
 $\lambda_M^L(y): [m_1, m_2] \rightarrow [-1, 0]$ ,  $\lambda_M^R(y): [m_3, m_4] \rightarrow [-1, 0]$

$\lambda_{K(y)}^L$  and  $\lambda_{M(y)}^L$  denote left membership functions for  $\lambda_K(y)$  and  $\lambda_M(y)$ , respectively. Similarly,  $\lambda_{K(y)}^R$  and  $\lambda_{M(y)}^R$  denote the right membership functions for  $\lambda_K(y)$  and  $\lambda_M(y)$ , respectively.

A BFN is represented graphically in Figure 1, where  $\lambda_K$  and  $\lambda_M$  are satisfaction and dissatisfaction degrees respectively.

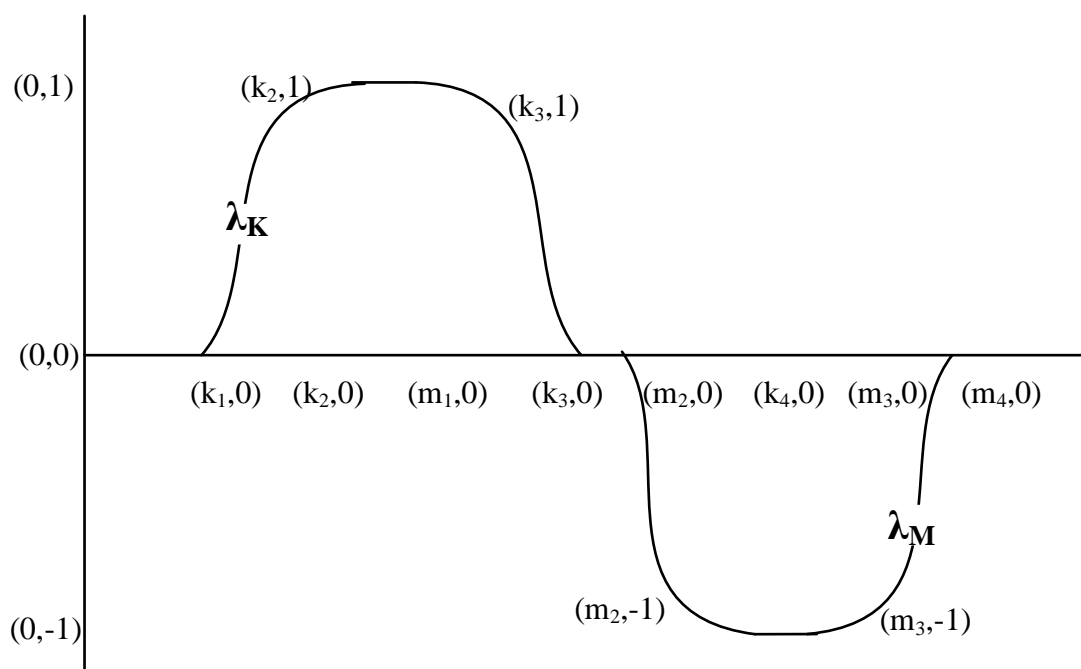


Figure 1: Graphical representation of BFN

### Definition 3: Trapezoidal Bipolar Fuzzy Number [25]

A BFN  $B = \langle K, M \rangle = \langle [k_1, k_2, k_3, k_4], [m_1, m_2, m_3, m_4] \rangle$  is a trapezoidal bipolar fuzzy number (TrBFN), denoted by  $\langle (k_1, k_2, k_3, k_4), (m_1, m_2, m_3, m_4) \rangle$ , if its satisfaction degree  $\lambda_K$  and dissatisfaction degree  $\lambda_M$  are given as:

$$\lambda_K = \begin{cases} \frac{y-u_1}{u_2-u_1}, & \text{if } y \in [k_1, k_2] \\ 1, & \text{if } x \in [k_2, k_3] \\ \frac{u_4-y}{u_4-u_3}, & \text{if } x \in [k_3, k_4] \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

and

$$\lambda_M = \begin{cases} \frac{m_1-y}{m_2-m_1}, & \text{if } y \in [m_1, m_2] \\ -1, & \text{if } y \in [m_2, m_3] \\ \frac{y-m_4}{m_4-m_3}, & \text{if } y \in [m_3, m_4] \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

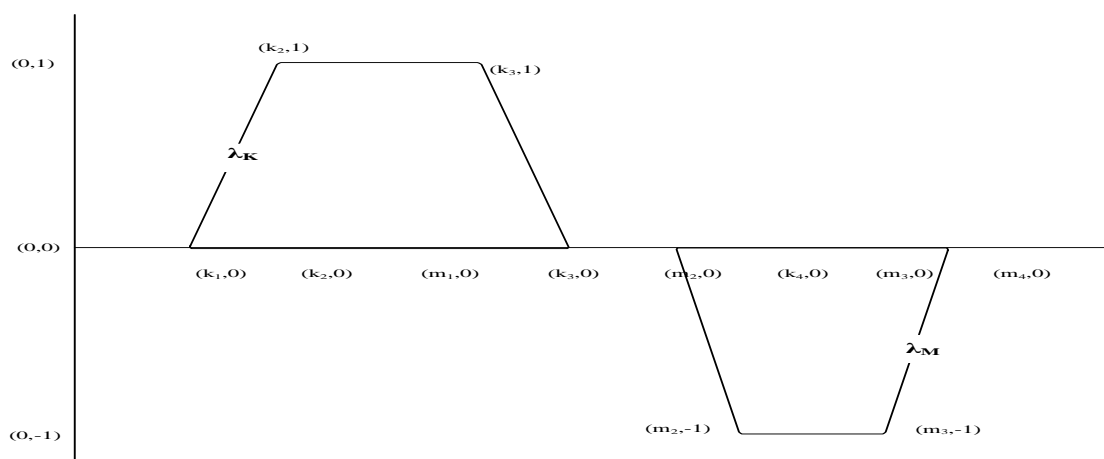


Figure 2 : Graphical representation of TrBFN

### Example 1:

If we consider the trapezoidal bipolar fuzzy number with the following parameters:  $\lambda_K = (2,4,6,8)$  and  $\lambda_M = (3,5,7,9)$ . Now, by using the satisfaction and dissatisfaction degree formulas:

$$\lambda_K = \begin{cases} \frac{y-2}{4-2}, & \text{if } y \in [2 \text{ to } 4] \\ 1, & \text{if } x \in [4 \text{ to } 6], \\ \frac{8-y}{8-6}, & \text{if } x \in [6 \text{ to } 8] \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

and

$$\lambda_M = \begin{cases} \frac{3-y}{5-3}, & \text{if } y \in [3 \text{ to } 5] \\ -1, & \text{if } y \in [5 \text{ to } 7], \\ \frac{y-9}{9-7}, & \text{if } y \in [7 \text{ to } 9] \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

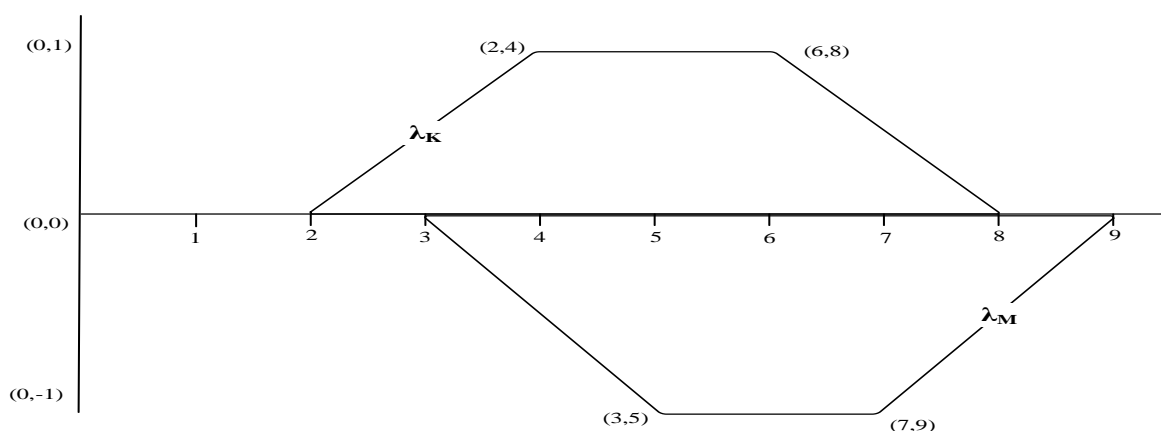


Figure 3 : Graphical representation of TrBFN



#### Definition 4: Bipolar Fuzzy Linguistic Variables [25]

Linguistic variables are defined by sentences or words in natural or artificial language, rather than numerical quantities. For example, the variable "Categorization Levels" with values or phrases set as {very low, low, medium low, medium, medium high, high, very high} is linguistic.

#### 4. Formulation of Trapezoidal Bipolar Fuzzy AHP

In Trapezoidal Bipolar Fuzzy AHP (TBF-AHP), we integrate both positive and negative membership values to evaluate decision alternatives. Below is a step-by-step derivation of how weights are computed.

##### Step 1: Define Trapezoidal Bipolar Fuzzy Numbers (TBFNs)

We are considering a Trapezoidal Bipolar Fuzzy Number (TBFN), represented as  $\lambda_j$  and  $\lambda_M$  for the positive impact (membership function) and the negative impact (non-membership function). This trapezoidal representation helps handle uncertainty by capturing favorable and unfavorable aspects in decision-making.

##### Step 2: Construct a Pairwise Comparison Matrix

For each criterion  $C_i$ , a pairwise comparison matrix is formed using decision-maker judgments.

Given criteria  $C_1, C_2, \dots, C_n$ , we construct a trapezoidal fuzzy pairwise matrix:

$$\tilde{B} = \begin{bmatrix} \langle K_{11}, M_{11} \rangle & \langle K_{12}, M_{12} \rangle & \langle K_{13}, M_{13} \rangle & \dots & \langle K_{1n}, M_{1n} \rangle \\ \langle K_{21}, M_{21} \rangle & \langle K_{22}, M_{22} \rangle & \langle K_{23}, M_{23} \rangle & \dots & \langle K_{2n}, M_{2n} \rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \langle K_{n1}, M_{n1} \rangle & \langle K_{n2}, M_{n2} \rangle & \langle K_{n3}, M_{n3} \rangle & \dots & \langle K_{nn}, M_{nn} \rangle \end{bmatrix} \quad (7)$$

where each element  $\langle K_{ij}, M_{ij} \rangle$  is a bipolar trapezoidal fuzzy number.

##### Step 3: Normalize the Fuzzy Pairwise Matrix

Each element of the pairwise comparison matrix is normalized by:

$$K_{ij}^N = \frac{K_{ij}}{\sum_{k=1}^n K_{ik}}, M_{ij}^N = \frac{M_{ij}}{\sum_{m=1}^n M_{im}} \quad (8)$$

Where:  $K_{ij}^N$  represents the normalized satisfaction degree, and  $M_{ij}^N$  represents the normalized dissatisfaction degree.

For each criterion  $C_i$ , we compute the row-wise sum and normalize.

##### Step 4: Compute the Bipolar Fuzzy Weight Vector

The fuzzy weight vector for each criterion is computed as:

$$\tilde{W}_i = \langle W_i^+, W_i^- \rangle = \left( \frac{\sum_{j=1}^n K_{ij}^N}{n}, \frac{\sum_{j=1}^n M_{ij}^N}{n} \right)$$

where:  $W_i^+$  represents the positive weight (satisfaction degree  $\lambda_K$ ),  $W_i^-$  represents the negative weight (dissatisfaction degree  $\lambda_M$ ).

Thus, each criterion has a bipolar fuzzy weight.

#### 5. Group Decision Making by Using TrBF-TOPSIS

In this section, we present a multi-criteria group decision-making model based on the TOPSIS method in a bipolar fuzzy environment. The excellence of using this approach is to assess alternatives and criteria using BFNs instead of BFSs. The steps for the decision-making model are given in the following:

**Step 1:** The bipolar fuzzy MCDM problem is expressed in the form of a matrix as follows:

$$\begin{matrix} & \begin{matrix} C_1 & C_2 & \cdot & \cdot & C_s \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_t \end{matrix} & \begin{bmatrix} y_{11m} & y_{12m} & \cdot & \cdot & y_{s1m} \\ y_{21m} & y_{22m} & \cdot & \cdot & y_{s2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & - & \cdot & \cdot & \cdot \\ x_{t1m} & x_{t2m} & \cdot & \cdot & y_{tsm} \end{bmatrix} \end{matrix} \quad (9)$$

$$W_m = [w_{1m} \quad w_{2m} \quad \cdots \quad w_{nm}]^T \quad (10)$$

where  $y_{ijm} = \langle (k_{ijm}^1, k_{ijm}^2, k_{ijm}^3, k_{ijm}^4), (m_{ijm}^1, m_{ijm}^2, m_{ijm}^3, m_{ijm}^4) \rangle$   $i = 1, 2, \dots, m$   $j = 1, 2, \dots, n$  are TrBFNs representing the BFLVs with the domain as interval  $[0, 1]$  and  $w_{jm}$  represents fuzzy values. Here  $y_{ijm}$ , is the performance rating of  $i^{th}$  alternative  $A_i$  for  $j^{th}$  criterion  $c$  and  $w_{jm}$ , is the weight of the  $j^{th}$  criterion assigned by the  $m^{th}$  decision-maker  $F_m, m=1, 2, \dots, q$ .

**Step 2 :** The aggregated performance rating  $y_{ij} = \langle (k_{ij}^1, k_{ij}^2, k_{ij}^3, k_{ij}^4), (m_{ij}^1, m_{ij}^2, m_{ij}^3, m_{ij}^4) \rangle$  of alternative  $A_i$  for criterion  $C_j$  assessed by  $q$  decision-makers can be evaluated as:

$$\begin{aligned} k_{ij}^1 &= \frac{1}{q} \sum_{m=1}^q k_{ijm}^1, k_{ij}^2 = \frac{1}{q} \sum_{m=1}^q k_{ijm}^2, k_{ij}^3 = \frac{1}{q} \sum_{m=1}^q k_{ijm}^3, k_{ij}^4 = \frac{1}{q} \sum_{m=1}^q k_{ijm}^4 \\ m_{ij}^1 &= \frac{1}{q} \sum_{m=1}^q m_{ijm}^1, m_{ij}^2 = \frac{1}{q} \sum_{m=1}^q m_{ijm}^2, m_{ij}^3 = \frac{1}{q} \sum_{m=1}^q m_{ijm}^3, m_{ij}^4 = \frac{1}{q} \sum_{m=1}^q m_{ijm}^4. \end{aligned} \quad (11)$$

Similarly, the aggregated importance weights  $w_j$  can be calculated as:

$$w_j = \frac{1}{q} \sum_{m=1}^q w_{jm} \quad (12)$$

**Step 3 :** The weighted bipolar fuzzy decision matrix is given as:

$$H = [h_{ij}] \quad (13)$$

Where  $h_{ij} = y_{ij}w_j = \langle (\delta_{ij}^1, \delta_{ij}^2, \delta_{ij}^3, \delta_{ij}^4), (\varepsilon_{ij}^1, \varepsilon_{ij}^2, \varepsilon_{ij}^3, \varepsilon_{ij}^4) \rangle$

**Step 4 :** The bipolar fuzzy positive ideal solution (BFPIS)  $A^*$  and bipolar fuzzy negative ideal solution (BFNIS)  $A^-$  are identified as:



$$\begin{aligned} B^+ &= \{h_1^*, h_2^*, \dots, h_t^*\} \\ &= \{(\max h_{ij} \mid j \in A), (\min h_{ij} \mid j \in D) \mid i = 1, 2, \dots, s\}, \\ B^- &= \{h_1^-, h_2^-, \dots, h_n^-\} \\ &= \{(\min h_{ij} \mid j \in A), (\max h_{ij} \mid j \in D) \mid i = 1, 2, \dots, s\}. \end{aligned} \quad (14)$$

**Step 5 :** The distance  $d_i^+$  of each alternative from BFPIS solution is given as:

$$\sqrt{\frac{1}{2} \sum_{j=1}^t \left\{ (\delta_{ij}^1 - \delta_{j^*}^1)^2 + (\delta_{ij}^2 - \delta_{j^*}^2)^2 + (\delta_{ij}^3 - \delta_{j^*}^3)^2 + (\delta_{ij}^4 - \delta_{j^*}^4)^2 \right.} \quad (15)$$

$$\left. + (\varepsilon_{ij}^1 - \varepsilon_{j^*}^1)^2 + (\varepsilon_{ij}^2 - \varepsilon_{j^*}^2)^2 + (\varepsilon_{ij}^3 - \varepsilon_{j^*}^3)^2 + (\varepsilon_{ij}^4 - \varepsilon_{j^*}^4)^2 \right\}$$

Similarly, the distance  $d_i^-$  of each alternative from BFNIS is given as:

$$\sqrt{\frac{1}{2} \sum_{j=1}^t \left\{ (\delta_{ij}^1 - \delta_j^1)^2 + (\delta_{ij}^2 - \delta_j^2)^2 + (\delta_{ij}^3 - \delta_j^3)^2 + (\delta_{ij}^4 - \delta_j^4)^2 \right\}} \quad (16)$$

$$\left. + (\varepsilon_{ij}^1 - \varepsilon_j^1)^2 + (\varepsilon_{ij}^2 - \varepsilon_j^2)^2 + (\varepsilon_{ij}^3 - \varepsilon_j^3)^2 + (\varepsilon_{ij}^4 - \varepsilon_j^4)^2 \right\}$$

**Step 6 :** The closeness coefficient of alternative  $A_i$  from BFPIS is given as:

$$D_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad \text{for } i = 1, \dots, s \quad (17)$$

$A_j$  is closer to  $A^*$  as  $D_i$  approaches 1. A preference order can be determined by the descending order of  $C_i$ , ( $i = 1, \dots, s$ )

## 6. Defuzzification using the Bhattacharya and Cosine Operator by including the TOPSIS

To convert the fuzzy weight into a crisp value, we use the defuzzification formula:

$$D(W_i) = \frac{k_1 + k_2 + k_3 + k_4}{4} - \frac{m_1 + m_2 + m_3 + m_4}{4} \quad (18)$$

Where The first term represents the average positive weight, The second term represents the average negative weight, The difference provides the final crisp weight.

Compute the Normalized Weight using Delta Based Aggregation.

The final normalized weight is computed as:

$$W_i^* = \frac{D(W_i)}{\sum_{i=1}^n D(W_i)} \quad (19)$$

Where  $W_i^*$  represents the final priority weight for each criterion.

The Trapezoidal Bipolar Fuzzy AHP method effectively handles uncertainty by integrating positive and negative membership values. The final defuzzified and normalized weights guide decision-making, ensuring accuracy in ranking alternatives.

## 7. Ranking of Bipolar Fuzzy Numbers

Let  $G = \{g_1, g_2, \dots, g_n\}$  be the set of BFNs then for any distinct  $g_i, g_j \in G$ , the ranking function  $Re$  from  $G$  to real line  $R$  is mapping satisfying the following characteristics.

(a) If  $Re(g_i) < Re(g_j)$ , then  $g_i < g_j$ ,

(b) If  $Re(g_i) = Re(g_j)$ , then  $g_i = g_j$ ,

(c) If  $Re(g_i) > Re(g_j)$ , then  $g_i > g_j$ .

We define here the ranking function for a BFN  $g_i = \langle (K_i, M_i) \rangle = \langle (k_{i1}, k_{i2}, k_{i3}, k_{i4}), (m_{i1}, m_{i2}, m_{i3}, m_{i4}) \rangle$  as:

$$[p(K_{im}) + \gamma(K_i)] - [|p(M_{im})| + \gamma(M_i)], m = 1, 2, 3, 4. \quad (20)$$

Where,  $p(K_{im})$  and  $p(M_{im})$  represent the mean of  $K_{im}$  and mean of  $M_{ik}$  respectively,  $\gamma(K_i)$  denotes the area of  $K_i$  and  $\gamma(M_i)$  denotes the area of  $M_i$ . The areas can be calculated by taking the mode value of integration of left and right membership functions separately and then adding it into  $\int_0^1 dx$  or  $\left| \int_{-1}^0 (-1) dx \right|$

Therefore, for any BFNs  $g_i$  and  $g_j$ ,

$$\text{If } [p(K_{im}) + \gamma(K_i)] - [|p(M_{im})| + \gamma(M_i)] < [p(K_{jm}) + \gamma(K_j)] - [|p(M_{jm})| + \gamma(M_j)], \text{ then } g_i < g_j; \quad (21)$$

$$\text{If } [p(K_{im}) + \gamma(K_i)] - [|p(M_{im})| + \gamma(M_i)] = [p(K_{jm}) + \gamma(K_j)] - [|p(M_{jm})| + \gamma(M_j)], \text{ then } g_i = g_j; \quad (22)$$

and

$$\text{if } [p(K_{im}) + \gamma(K_i)] - [|p(M_{im})| + \gamma(M_i)] > [p(K_{jm}) + \gamma(K_j)] - [|p(M_{jm})| + \gamma(M_j)], \text{ then } g_i > g_j. \quad (23)$$

If  $\gamma(K_i), \gamma(M_i) \geq 1$ , for each  $i$ , then ranking function for BFNs can also be defined as

$$p(K_{im})\gamma(K_i) - |p(M_{im})|\gamma(M_i), m = 1, 2, 3, 4. \quad (24)$$

## 8. Sensitivity Analysis Process in Relative Closeness (RC) Ranking

**Step 1:** Define the Decision Matrix

The decision matrix consists of alternatives  $A_1$  to  $A_9$  and criteria  $C_1$  to  $C_6$ . Each alternative is assigned a value for each criterion based on performance.

**Step 2:** Normalize the Decision Matrix

Normalization ensures all values are within the same scale (typically between 0 and 1). The most common normalization method used in MCDM (Multi-Criteria Decision Making) is:

$$Y_{ij}^{norm} = \frac{Y_{ij}}{\sqrt{\sum_{i=1}^t Y_{ij}^2}} \quad (25)$$

Where  $Y_{ij}^{norm}$  norm is normalized value,  $Y_{ij}$  is an original value in decision matrix,  $t$  is total number of alternatives

**Step 3:** Apply Weighting to Criteria

Different weight distributions ( $\Omega$ ) are considered to analyze their impact. The weighted normalized decision matrix is created by multiplying each normalized value by its appropriate weight:

$$U_{ij} = Y_{ij}^{norm} \times w_j \quad (26)$$

Where  $U_{ij}$  the weighted normalized value,  $w_j$  is the weight allocated to criterion  $j$

$$p(K_{im})\gamma(K_i) - |p(M_{im})|\gamma(M_i), m = 1, 2, 3, 4.$$

**Step 4:** Compute the Ideal and Negative-Ideal Solutions

Two reference points are determined:

Ideal Solution ( $B^+$ ): The best value for each criterion

Negative-Ideal Solution ( $B^-$ ): The worst value for each criterion

$$B^+ = (\max U_{ij} | j \in D$$

$$B^- = (\min U_{ij} | j \in D \quad (27)$$

**Step 5:** Calculate the Separation Measures

For each alternative, compute its distance from an ideal and negative-ideal solutions using Euclidean distance:

$$T_i^+ = \sqrt{\sum_{j=1}^s (U_{ij} - B_j^+)^2}$$

$$T_i^- = \sqrt{\sum_{j=1}^s (U_{ij} - B_j^-)^2} \quad (28)$$

Where  $T_i^+$  is the distance from an ideal solution,  $T_i^-$  is the distance from the negative-ideal solution

**Step 6:** Compute Relative Closeness ( $CC_i$ ) for Each Alternative

The RC coefficient for each alternative is computed as:

$$CC_i = \frac{T_i^-}{T_i^+ + T_i^-} \quad (29)$$

A higher  $CC_i$  value means the alternative is closer to an ideal solution, making it preferable.

**Step 7:** Rank the Alternatives

Alternatives are ranked descending based on their RC ( $CC_i$ ) scores. The option with the highest  $CC_i$  is the most suitable.

**Step 8:** Perform Sensitivity Analysis

## 9. Illustrative Example:

Let's assume we are evaluating nine alternatives ( $A_1$  to  $A_9$ ) based on six criteria ( $C_1$  to  $C_6$ ) using the Trapezoidal Bipolar Fuzzy AHP approach. Each alternative is assigned trapezoidal fuzzy numbers representing both positive and negative membership values, which help in decision-making under uncertainty.

**Table 1:** Number of Alternatives and Criteria

Alternatives		Criteria	
$A_1$	Zoho CRM	$C_1$	Cost
$A_2$	Google Workspace	$C_2$	Ease of Use
$A_3$	Microsoft 365	$C_3$	Features and Functionalities
$A_4$	Slack	$C_4$	Security
$A_5$	Asana	$C_5$	Scalability

A <sub>6</sub>	Dropbox	C <sub>6</sub>	Customer Support
A <sub>7</sub>	Shopify		
A <sub>8</sub>	Zoom		
A <sub>9</sub>	AWS		

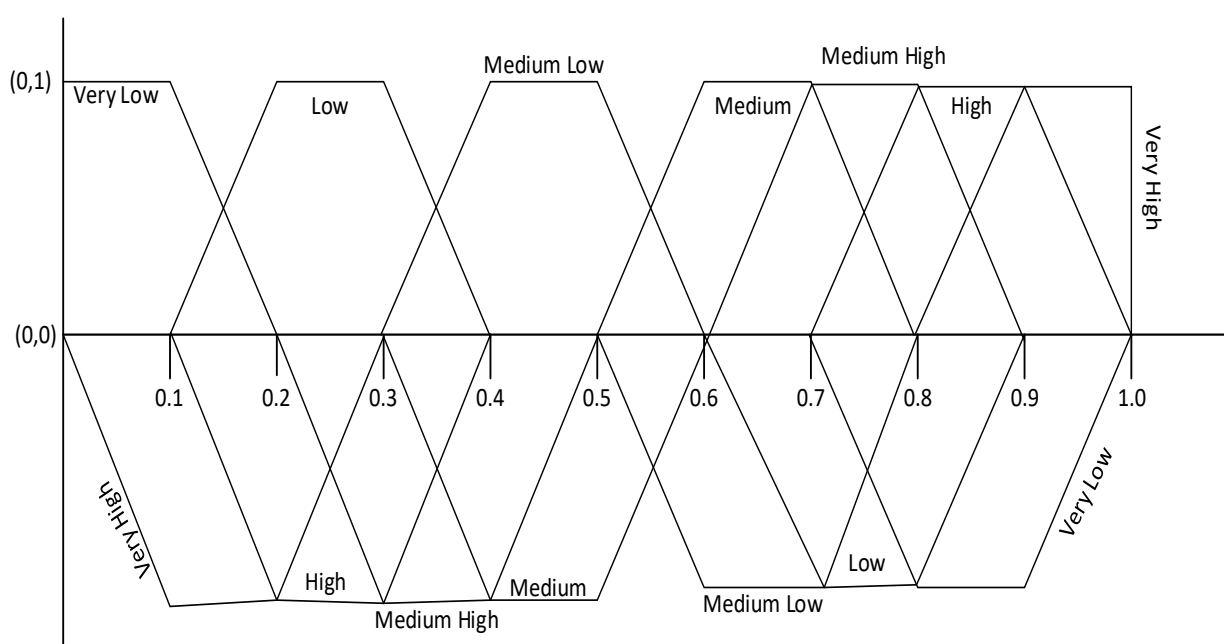
Three decision makers  $D_1$ ,  $D_2$  and  $D_3$  are responsible for the selection. Let us consider the Bipolar fuzzy linguistic rating set of SaaS,

$S_1 = \{\text{Very Low} = \text{VL}, \text{Low} = \text{L}, \text{Medium Low} = \text{ML}, \text{Medium} = \text{M}, \text{Medium High} = \text{MH}, \text{High} = \text{H}, \text{Very High} = \text{VH}\}$  for all the six Criteria. Figure 4 shows the trapezoidal bipolar fuzzy numbers that represent these linguistic values. The numerical domain of linguistic values considered here is the closed interval  $[0, 1]$ .

**Table 2:** Bipolar Linguistic Variables

Linguistic Term	Abbreviations	
Very Low	VL	$(0, 0, 0.1, 0.2), (0.7, 0.8, 0.9, 1.0)$
Low	L	$(0.1, 0.2, 0.3, 0.4), (0.6, 0.7, 0.8, 0.9)$
Medium Low	ML	$(0.3, 0.4, 0.5, 0.6), (0.5, 0.6, 0.7, 0.8)$
Medium	M	$(0.5, 0.6, 0.7, 0.8), (0.3, 0.4, 0.5, 0.6)$
Medium High	MH	$(0.6, 0.7, 0.8, 0.9), (0.2, 0.3, 0.4, 0.5)$
High	H	$(0.7, 0.8, 0.9, 1.0), (0.1, 0.2, 0.3, 0.4)$
Very High	VH	$(0.8, 0.9, 1.0, 1.0), (0, 0.1, 0.2, 0.3)$

Tables 3 and 4 display the linguistic values and their related trapezoidal bipolar fuzzy numbers, as illustrated in Figure 4, which indicate the performance rate of the alternatives (proposals). Using Eq. 11, the aggregated weights of the criteria across three decision-makers are obtained. Similarly, by Eq. 12, the aggregated performance ratings of alternatives for given conflicting criteria across the three decision-makers, are obtained and are also presented in Tables 3 and 4. Table 5 provides the weighted bipolar fuzzy decision matrix for criteria  $C_1$ ,  $C_2$  and  $C_3$ , while Table 6 provides results for criteria  $C_4$ ,  $C_5$  and  $C_6$



**Figure 4:** Satisfaction and dissatisfaction degree for linguistic values**Table 3 :** Performance rating of alternatives and Weighted Decision Matrix for  $C_1$ ,  $C_2$ , and  $C_3$ 

Decision Makers	Alternatives	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>	A <sub>9</sub>
<b>D<sub>1</sub></b>	<b>C<sub>1</sub></b>	M	ML	MH	H	MH	ML	M	H	MH
	<b>C<sub>2</sub></b>	ML	M	M	MH	M	MH	VH	M	ML
	<b>C<sub>3</sub></b>	MH	M	M	H	MH	M	ML	MH	L
<b>D<sub>2</sub></b>	<b>C<sub>1</sub></b>	M	L	MH	H	MH	L	MH	H	MH
	<b>C<sub>2</sub></b>	L	M	ML	MH	M	MH	M	MH	M
	<b>C<sub>3</sub></b>	MH	ML	M	H	MH	M	M	H	MH
<b>D<sub>3</sub></b>	<b>C<sub>1</sub></b>	M	ML	H	VH	H	ML	L	MH	H
	<b>C<sub>2</sub></b>	ML	M	MH	H	M	MH	M	ML	MH
	<b>C<sub>3</sub></b>	H	MH	M	VH	MH	M	ML	M	H

**Table 4:** Performance rating of alternatives and Weighted Decision Matrix for  $C_4$ ,  $C_5$ , and  $C_6$ 

Decision Makers	Alternatives	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>	A <sub>9</sub>
<b>D<sub>1</sub></b>	<b>C<sub>4</sub></b>	H	MH	H	M	H	MH	M	L	L
	<b>C<sub>5</sub></b>	MH	M	MH	H	M	ML	M	MH	M
	<b>C<sub>6</sub></b>	ML	MH	M	MH	ML	M	M	VH	MH
<b>D<sub>2</sub></b>	<b>C<sub>4</sub></b>	H	MH	H	M	H	MH	H	M	H
	<b>C<sub>5</sub></b>	MH	M	MH	H	M	L	MH	H	M
	<b>C<sub>6</sub></b>	L	MH	M	MH	L	M	M	MH	ML
<b>D<sub>3</sub></b>	<b>C<sub>4</sub></b>	VH	H	VH	M	H	MH	MH	H	M
	<b>C<sub>5</sub></b>	H	M	MH	H	M	ML	M	MH	H
	<b>C<sub>6</sub></b>	ML	MH	M	MH	ML	M	MH	M	MH

**Table 5:** Weighted Decision Matrix for  $C_1$ ,  $C_2$ , and  $C_3$ 

	<b>C<sub>1</sub></b>	<b>C<sub>2</sub></b>	<b>C<sub>3</sub></b>
<b>A<sub>1</sub></b>	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)

<b>A<sub>2</sub></b>	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.16,0.19,0.22,0.26),(0.11,0.14,0.18,0.21)
<b>A<sub>3</sub></b>	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.16,0.19,0.22,0.26),(0.11,0.14,0.18,0.21)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)
<b>A<sub>4</sub></b>	(0.24,0.28,0.31,0.33),(0.02,0.06,0.09,0.12)	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.24,0.28,0.31,0.33),(0.02,0.06,0.09,0.12)
<b>A<sub>5</sub></b>	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)
<b>A<sub>6</sub></b>	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)
<b>A<sub>7</sub></b>	(0.13,0.17,0.2,0.23),(0.12,0.16,0.19,0.22)	(0.2,0.23,0.27,0.29),(0.07,0.1,0.13,0.17)	(0.12,0.15,0.19,0.22),(0.15,0.18,0.21,0.25)
<b>A<sub>8</sub></b>	(0.22,0.26,0.29,0.32),(0.04,0.08,0.11,0.14)	(0.15,0.19,0.22,0.25),(0.11,0.15,0.18,0.21)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)
<b>A<sub>9</sub></b>	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.15,0.19,0.22,0.25),(0.11,0.15,0.18,0.21)	(0.15,0.19,0.22,0.25),(0.1,0.13,0.17,0.2)

**Table 6:** Weighted Decision Matrix for  $C_4$ ,  $C_5$ , and  $C_6$ 

	<b>C<sub>4</sub></b>	<b>C<sub>5</sub></b>	<b>C<sub>6</sub></b>
<b>A<sub>1</sub></b>	(0.24,0.28,0.31,0.33),(0.02,0.06,0.09,0.12)	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)
<b>A<sub>2</sub></b>	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)
<b>A<sub>3</sub></b>	(0.24,0.28,0.31,0.33),(0.02,0.06,0.09,0.12)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)
<b>A<sub>4</sub></b>	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.23,0.27,0.3,0.33),(0.03,0.07,0.1,0.13)	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)
<b>A<sub>5</sub></b>	(0.23,0.27,0.3,0.33),(0.03,0.07,0.1,0.13)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)
<b>A<sub>6</sub></b>	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)	(0.08,0.11,0.15,0.18),(0.18,0.21,0.24,0.28)	(0.17,0.2,0.23,0.27),(0.1,0.13,0.17,0.2)

A <sub>7</sub>	(0.2,0.23,0.27,0.3),(0.07,0.1,0.13,0.17)	(0.18,0.21,0.24,0.28),(0.09,0.12,0.16,0.19)	(0.18,0.21,0.24,0.28),(0.09,0.12,0.16,0.19)
A <sub>8</sub>	(0.14,0.18,0.21,0.24),(0.11,0.15,0.18,0.21)	(0.21,0.24,0.28,0.31),(0.06,0.09,0.12,0.16)	(0.21,0.24,0.28,0.3) (0.05,0.09,0.12,0.15)
A <sub>9</sub>	(0.14,0.17,0.21,0.24),(0.11,0.15,0.18,0.21)	(0.19,0.22,0.26,0.29),(0.08,0.11,0.14,0.18)	(0.17,0.2,0.24,0.27),(0.1,0.13,0.16,0.2)

Since decision-making requires crisp values, we apply the defuzzification formula:

**Table 7:** Defuzzification for  $C_1$ ,  $C_2$ , and  $C_3$

	$C_1$	$C_2$	$C_3$
A <sub>1</sub>	(0.535, 0.535, 0.53 , 0.535)	(0.45 , 0.45 , 0.455, 0.45)	(0.575, 0.575, 0.58 , 0.575)
A <sub>2</sub>	(0.45 , 0.45 , 0.455, 0.45)	(0.535, 0.535, 0.53 , 0.535)	(0.525, 0.525, 0.52 , 0.525)
A <sub>3</sub>	(0.575, 0.575, 0.58 , 0.575)	(0.525, 0.525, 0.52 , 0.525)	(0.535, 0.535, 0.53 , 0.535)
A <sub>4</sub>	(0.61 , 0.61 , 0.61 , 0.605)	(0.575, 0.575, 0.58 , 0.575)	(0.61 , 0.61 , 0.61 , 0.605)
A <sub>5</sub>	(0.575, 0.575, 0.58 , 0.575)	(0.535, 0.535, 0.53 , 0.535)	(0.565, 0.565, 0.57 , 0.565)
A <sub>6</sub>	(0.45 , 0.45 , 0.455, 0.45)	(0.565, 0.565, 0.57 , 0.565)	(0.535, 0.535, 0.53 , 0.535)
A <sub>7</sub>	(0.505, 0.505, 0.505, 0.505)	(0.565, 0.565, 0.57 , 0.56)	(0.485, 0.485, 0.49 , 0.485)
A <sub>8</sub>	(0.59 , 0.59 , 0.59 , 0.59 )	(0.52 , 0.52 , 0.52 , 0.52)	(0.565, 0.565, 0.57 , 0.565)
A <sub>9</sub>	(0.575, 0.575, 0.58 , 0.575)	(0.52 , 0.52 , 0.52 , 0.52)	(0.525, 0.53 , 0.525, 0.525)

**Table 8:** Defuzzification for  $C_4$ ,  $C_5$ , and  $C_6$

	$C_4$	$C_5$	$C_6$
A <sub>1</sub>	(0.61 , 0.61 , 0.61 , 0.605)	(0.575, 0.575, 0.58 , 0.575)	(0.45 , 0.45 , 0.455, 0.45)
A <sub>2</sub>	(0.575, 0.575, 0.58 , 0.575)	(0.535, 0.535, 0.53 , 0.535)	(0.565, 0.565, 0.57 , 0.565)
A <sub>3</sub>	(0.61 , 0.61 , 0.61 , 0.605)	(0.565, 0.565, 0.57 , 0.565)	(0.535, 0.535, 0.53 , 0.535)
A <sub>4</sub>	(0.535, 0.535, 0.53 , 0.535)	(0.6 , 0.6 , 0.6 , 0.6)	(0.565, 0.565, 0.57 , 0.565)
A <sub>5</sub>	(0.6 , 0.6 , 0.6 , 0.6)	(0.535, 0.535, 0.53 , 0.535)	(0.45 , 0.45 , 0.455, 0.45)
A <sub>6</sub>	(0.565, 0.565, 0.57 , 0.565)	(0.45 , 0.45 , 0.455, 0.45)	(0.535, 0.535, 0.53 , 0.535)
A <sub>7</sub>	(0.565, 0.565, 0.57 , 0.565)	(0.545, 0.545, 0.54 , 0.545)	(0.545, 0.545, 0.54 , 0.545)
A <sub>8</sub>	(0.515, 0.515, 0.515, 0.515)	(0.575, 0.575, 0.58 , 0.575)	(0.58 , 0.575, 0.58 , 0.575)
A <sub>9</sub>	(0.515, 0.51 , 0.515, 0.515)	(0.555, 0.555, 0.56 , 0.555)	(0.535, 0.535, 0.54 , 0.535)

Delta-based aggregation is used in MCDM to regulate the best alternative by considering normalized decision values and a weighting factor. This approach follows defuzzification, meaning it works with crisp numerical values derived from fuzzy data.



The Delta values (0.2099, 0.1852, 0.1605, 0.1358, 0.1111, 0.0864, 0.061, 0.0370, 0.0123) represent differences between alternatives based on their weighted normalized scores. A smaller Delta value, such as 0.0123, indicates higher preference, while a larger Delta value, like 0.2099, suggests lower preference in decision-making.

This matrix represents the final weights assigned to each criterion after normalization and weighting. The values for each criterion  $C_1$  to  $C_6$  are given as:

**Table 9:** Delta Based Aggregation (Weighted Normalized Decision Matrix)

$C_1$	0.57117284	0.57117284	0.57302469	0.57012346
$C_2$	0.55037037	0.55037037	0.55154321	0.5495679
$C_3$	0.5654321	0.56574074	0.5667284	0.56438272
$C_4$	0.5854321	0.58537037	0.58679012	0.58345679
$C_5$	0.56858025	0.56858025	0.57067901	0.56858025
$C_6$	0.55197531	0.55092593	0.55308642	0.55092593

**Table 10:** Final Criteria Weight

	Final Criteria Weight
$C_1$	0.5714
$C_2$	0.5505
$C_3$	0.5656
$C_4$	0.5853
$C_5$	0.5691
$C_6$	0.5517

### Relative Closeness ranking

In MCDM, RC ranking is used to evaluate and prioritize alternatives in relation to an ideal solution. Weighted scores are produced, the option matrix is normalized, and each alternative is compared to the ideal and anti-ideal solutions.

In the given data, each alternative  $A_1$  to  $A_9$  is assessed across six criteria ( $C_1 - C_6$ ) with different performance values. The RC value indicates how close an alternative is to an ideal solution. A higher RC value signifies a better and more preferred alternative, while a lower RC value indicates a less favorable option.

**Table 11:** Sample Data

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
$A_1$	0.7	0.8	0.6	0.7	0.8	0.7
$A_2$	0.8	0.7	0.7	0.8	0.7	0.8
$A_3$	0.7	0.7	0.8	0.7	0.7	0.7
$A_4$	0.8	0.8	0.7	0.7	0.8	0.8
$A_5$	0.7	0.7	0.7	0.8	0.7	0.7
$A_6$	0.6	0.7	0.6	0.6	0.7	0.7
$A_7$	0.7	0.7	0.7	0.8	0.7	0.8
$A_8$	0.7	0.6	0.7	0.7	0.7	0.6
$A_9$	0.6	0.7	0.6	0.7	0.6	0.7

**Table 12:** Relative Closeness Rank

	Relative Closeness	Rank
A <sub>1</sub>	0.5542	6
A <sub>2</sub>	0.5561	5
A <sub>3</sub>	0.5729	4
A <sub>4</sub>	0.5989	2
A <sub>5</sub>	0.5775	3
A <sub>6</sub>	0.4415	8
A <sub>7</sub>	0.6352	1
A <sub>8</sub>	0.372	9
A <sub>9</sub>	0.4444	7

The omega value provided is (0.35, 0.31, 0.34)

**Table 13:** Sensitivity Analysis

Omega	Relative Closeness	Rank
(0.3, 0.3, 0.4)	0.554	A <sub>7</sub> > A <sub>4</sub> > A <sub>5</sub> > A <sub>3</sub> > A <sub>2</sub> > A <sub>1</sub> > A <sub>9</sub> > A <sub>6</sub> > A <sub>8</sub>
	0.556	
	0.5731	
	0.5987	
	0.5777	
	0.4411	
	0.6351	
	0.3723	
	0.4443	
(0.1, 0.5, 0.4)	0.553	A <sub>7</sub> > A <sub>4</sub> > A <sub>5</sub> > A <sub>3</sub> > A <sub>2</sub> > A <sub>1</sub> > A <sub>9</sub> > A <sub>6</sub> > A <sub>8</sub>
	0.5559	
	0.5738	
	0.5978	
	0.5785	
	0.4398	
	0.6349	
	0.3738	
	0.4433	
(0.6, 0.2, 0.2)	0.5552	A <sub>7</sub> > A <sub>4</sub> > A <sub>5</sub> > A <sub>3</sub> > A <sub>2</sub> > A <sub>1</sub> > A <sub>9</sub> > A <sub>6</sub> > A <sub>8</sub>
	0.5561	
	0.5723	
	0.6	
	0.5764	
	0.4428	
	0.6353	
	0.37	
	0.4453	
(0.1, 0.7, 0.2)	0.553	A <sub>7</sub> > A <sub>4</sub> > A <sub>5</sub> > A <sub>3</sub> > A <sub>2</sub> > A <sub>1</sub> > A <sub>9</sub> > A <sub>6</sub> > A <sub>8</sub>
	0.5557	

	0.5739	
	0.5979	
	0.5783	
	0.4399	
	0.6346	
	0.3738	
	0.4432	

#### 4. CONCLUSION

The study successfully applied the Trapezoidal Bipolar Fuzzy AHP and TOPSIS to assess and rank leading SaaS cloud service providers, including Zoho CRM, Google Workspace, Microsoft 365, Slack, Asana, Dropbox, Shopify, Zoom, and AWS. By prioritizing key MCDM aspects such as cost, ease of use, features, security, scalability, and customer support, the study provided a systematic decision-making framework for selecting the best SaaS provider. The Bhattacharya and Cosine operator-based enhancement improved decision-making accuracy under uncertainty. A Python-based experimental tool validated the approach, and sensitivity analysis confirmed its robustness and stability. The findings indicate that the proposed model effectively minimizes complexity in uncertain SaaS selection scenarios and enhances service consistency and accuracy. This study contributes to optimized cloud computing adoption, assisting businesses in making data-driven, reliable, and cost-effective SaaS selection decisions.

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