

Sustainable Supply Chain Model for Agile Manufacturing and Advertisement-Driven Demand Under Uncertainty

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ABSTRACT

Supply chain management (SCM) is of great importance to ensure efficiency, sustainability, and response in today's competitive and environmentally sensitive industry. This study combines agile manufacturing with advertisement-dependent demand to enhance supply chain operations in the automotive industry, where changing preferences need adaptable production tactics. An Interval -Valued Triangular Intuitionistic Fuzzy Number (IVTIFN)- based decision-making model is suggested to address uncertainty in production scheduling, inventory control, and transportation management. The model incorporates carbon emission estimations to guarantee satisfaction with environmental requirements increasing overall efficiency. A Simulated Annealing (SA) technique optimizes inventory management and lowers total costs, resulting in an effective inventory solution. Sensitivity analysis and numerical example demonstrates the effect of important parameters on optimal cost and other decision variables. The results shows that combining agile manufacturing with demand forecasting considerably improves supply chain flexibility, cost-effectiveness, and sustainability.

Keywords: supply chain management, carbon emission, advertisement dependent demand, interval valued triangular intuitionistic fuzzy number, agile manufacturing

1.INTRODUCTION

Supply chain management involves coordination and processes involved in the manufacture, sale, and delivering of products or services from starting point to the end customer. This includes everything from searching for the basic materials for the product to sending the finished good directly to the customer. Managing the supply chain requires five components: planning, sourcing, organizing, delivering, and managing. Every link in the chain is carefully planned before any supplier contracts are finalized or the product is released into the market. It is essential to ensure a smooth transition between phases and that each stakeholder is aware of their respective responsibilities. The sourcing process is finding suppliers to buy products and services to fulfill the plan's specifications. There are quality and quantity requirements that must be fulfilled. Quantity values are dynamic and subject to vary over time. This study includes the impact of carbon dioxide emissions on supply chain management. Without the concept of carbon emissions, supply chain management (SCM) is defined by adding environmental (emissions) concerns to the conventional economic (cost) emphasis.

The greenhouse effect increases by human-generated greenhouse gas (GHG) emissions. This adds to the warming of the planet. When fossil fuels like coal, oil, and natural gas burns, they release carbon dioxide (CO₂), which is seen as a major factor behind climate change. The United States and China are the two biggest polluters. Per capita emissions are greater in the US. It is the major oil and gas firms that are driving emissions worldwide. The carbon dioxide percentage in the environment has grown by around 50% since pre-industrial times due to emissions from human activity. Although they have fluctuated, the rising amounts of emissions for all greenhouse gases have remained constant while the emissions of methane (CH₄) have almost the same immediate effects. In contrast, fluorinated gases (F-gases) and nitrous oxide (N₂O) have a smaller function. Methane, nitrous oxide, and carbon dioxide emissions reached at high level in 2023. The primary greenhouse gases released and eliminated are methane, carbon dioxide, and nitrogen dioxide. Investment in green technology is a top priority for any business to cut emissions and fight global warming.

Advertising is one of the best methods for spreading the product's popularity among all customers. Since product advertising increases brand recognition and assesses the brand's accessibility to customers in different marketplaces, and product advertising is essential for supply chain management. Advertisements by manufacturers and suppliers frequently include details on their products especially when launching new or upgrade variants of their current models. Customers understand the way to utilize the product. With the use of modern technologies or well-known people, manufacturers and suppliers wish to advertise in renowned print and electronic media, among other channels, to attract more and more customers to purchase their goods. In this study the demand is considered as advertisement dependent. Manufacturers, suppliers, and retailers may give promotions or market their products to attract customers. They employ popular media platforms, including social media, television, newspapers, movies, and posters.

Agile manufacturing is a production methodology emphasizing responsiveness, flexibility, and adaptation to supply chain delays, customer needs and market shifts. It is designed to help businesses quickly, effectively and economically produce high-quality goods in a changing and uncertain environment. This manufacturing focuses on the client, ensuring production procedures are adapted to quickly and precisely satisfy changing customer demands. Integration with the supply chain is essential to guarantee seamless operations, encouraging cooperation and real-time communication between distributors, suppliers, and partners. The capacity of agile manufacturing to increase demand flexibility, shorten lead times, and boost overall supply chain resilience makes it crucial for supply chain management. Moreover, by cutting waste, maximizing resources, and closely coordinating production with market demands, agile manufacturing helps to save costs. It guarantees that businesses maintain their adaptability in a competitive environment, producing high-quality goods more quickly and sustainably. Adopting agile manufacturing has become crucial for sectors like fashion, electronics, and automotive to successfully navigate the hurdles of globalization and consumer-driven marketplaces. While traditional manufacturing uses established, frequently subtractive techniques such as machining, casting, and molding to convert raw materials into final goods, with a focus on high-volume, low-cost mass production. Traditional techniques are ideal for creating big quantities of things. These manufacturers may struggle to produce customized goods. Tooling and mold production can be costly, making it unsuitable for small batches. Subtractive techniques can result in substantial material waste.

Fuzzy refers to the things that are not clear or vague. A fuzzy number is generally an extended version of an ordinary number containing a group of possible values, with each possible value weighing 0 and 1. Fuzzy number enables the inclusion of uncertainty in parameters, characteristics, geometry, beginning conditions, and other areas. In the present study, an interval-valued triangular intuitionistic fuzzy number (IVTIFN) is employed instead of a traditional one to understand the uncertainty better. Triangular fuzzy numbers (TFN) and intuitionistic fuzzy numbers (IFN) are used to create an interval-valued triangular intuitionistic fuzzy number (IVTIFN), which permits ambiguity and uncertainty.

The current paper extends the earlier model by considering the advertisement-reliable demand rate. This study uses the producer's production rate as a decision variable. Fuzzy numbers are usually used to express uncertainty. Interval value-Fuzzy numbers have membership intervals, fuzzy numbers have

individual memberships. In this work, the idea of an IVTIFN is introduced. A mathematical model is built to obtain the optimal cost by using these factors. Lastly, a numerical example have demonstrated model's feasibility.

Research motivation and research contribution of this work

Environmental legislation and consumer demand for sustainable practices are increasing pressure on the global automobile industry to manage carbon emissions. In this regard, supply chain management is essential, particularly in sectors like the automotive industry that have intricate networks. A revolutionary strategy for improving responsiveness and adaptation in uncertain situations is agile manufacturing. Agile supply chain models that use triangular interval-valued fuzzy numbers provide a viable means of addressing uncertainty and maximizing sustainability goals. Despite its promise, not much study is currently done that integrates carbon emission issues, advertisement-dependent demand, and agile production comprehensively. These difficulties motivate the study of a comprehensive strategy that incorporates environmental factors, flexible manufacturing, and demand management led by advertisements into supply chain optimization tactics.

The following noteworthy contributions in the area of supply chain are made by this study:

- A novel framework is suggested that merges agile manufacturing concepts with supply chain operations, addressing the complexity of advertisement-dependent demand and carbon emission constraints while maintaining adaptability and sustainability.
- This article proposes the use of interval valued triangular intuitionistic fuzzy number to describe uncertainty in demand forecasting, production scheduling, and inventory management. This method improves decision-making accuracy in uncertain and dynamic contexts.
- It develops strategies to lower carbon emissions across the supply chain, bringing operational procedures into accordance with the objectives of global sustainability. As part of this, supply chain planning and decision-making must consider emission in effect.
- The study explicitly considers the influence of advertising campaigns on demand patterns. It outlines how to modify production and logistics strategies to successfully respond to such variances.
- The study contributes to the scholarly conversation by providing a thorough method that incorporates supply chain management sustainability concepts, agile manufacturing, and fuzzy logic.

Through these contributions, our study advances the state of knowledge in the field by enhancing the academic discussion on supply chain management, advertising-dependent demand, triangular interval-valued fuzzy numbers, and agile manufacturing. Our study presents an innovative strategy that addresses the complexity of demand-dependent advertising and carbon emission limits while preserving agility and sustainability by fusing agile manufacturing principles with supply chain operations.

1.2 Organization

The remaining paper consist of the following section: the literature background of several authors is discussed in Section 2, while Table 1 presents the comparison analysis. Section 3 contains the necessary preliminaries and definitions. The notations, assumptions, and mathematical formulations is represented in Section 4 whereas Section 5 provides a thorough analysis of the optimization technique. Finally, a numerical example is described in Section 6, and a sensitivity analysis is carried out across several parameters. Managerial insights is presented and a conclusion is drawn in Section 7 while section 8 proposes possible directions for future research and limitations.

2. LITERATURE BACKGROUND

This section outlines the current literature of the proposed model including supply chain management, carbon emission, interval-valued triangular intuitionistic fuzzy number (IVTIFN), advertisement dependent demand, agile manufacturing.

2.1 sustainability in supply chain management

Supply chain management (SCM) has a big environmental effect since it comprises many activities that use a lot of energy and emit carbon dioxide. These effects results from operations that rely significantly on fossil fuels and inefficient energy practices, such as manufacturing, logistics, transportation, and warehousing. Considering carbon emissions, Ahmed and Sarkar (2018) created an integrated model of sustainable supply chain management (SSCM). Asadkhani et al. (2022) include carbon taxes and cap-and-trade legislation. For instance, Govindan et al. (2014) examined ecologically conscious supply chain methods and emphasized the value of life cycle analysis. Wu et al. (2017) examined green supply chain models that incorporate carbon-taxing strategies. Esfahbodi et al. (2016) highlighted the importance of sustainable practices. Ghadge et al. (2019) on the integration of sustainability and risk management. Ramanathan et al. (2014) address some of the fundamental elements of supply chain collaboration with an emphasis on retailers, logistics and suppliers to make the companies more eco friendly. Xu et al. (2024) presents a systematic literature review of supply chain management under cap and trade regulation. Zhang et al. (2025) focuses in creating and refining carbon neutral logistics networks as way to minimize the carbon footprint of distribution and transportation operations. They suggest using innovative modelling approaches to create supply chain networks that are cost effective and operationally efficient while operating carbon neutrally. According to Roberts et al. (2025), stakeholder collaboration is crucial to accomplish sustainability goals in a variety of businesses. Singh et al. (2024) investigate the way AI (artificial intelligence) may be used to evaluate big datasets and offer practical advice for manufacturing, supply chains, and other corporate activities. Poswal et al. (2022) presented a study of a three-layer fuzzy sustainable manufacturing model has been established, with the producer, remanufacturer, and retailer as contributors, and also the effects of economic and environmental factors were addressed

2.2 Carbon emission in supply chain management

Incorporating carbon emissions into optimization models has become a crucial topic of study as businesses strive to balance environmental responsibility and financial success. According to Xie et al. (2021), improving carbon efficiency (CEE) is the process of producing the most economic output while reducing carbon emissions. According to Chhabra et al. (2023), knowledge spillover is an effective approach for reducing carbon emissions. Sharing and transferring knowledge across several companies and industries may lead to adopting innovative and sustainable practices. Singh et al. (2010) created a plan for deteriorating items that took trade credit programs and the price of carbon emissions under effect. A smart manufacturing system based on production consistency was presented by Kugele et al. (2022) to improve overall productivity, carbon ejection control, and production process quality. Chen and Wang (2017) include cap-and-trade laws ingreen product mix selection models using game theory. In a two-tier supply chain, the study investigates possible behavioural shifts under cap-and-trade. Benjaafar et al. (2013) demonstrate how standard models may be altered by linking carbon emission characteristics to different choice factors. Mashud et al. (2022) focused on reducing CO₂ in greenhouse farming. Thomas et al. (2024) built an intelligent supply chain model for electric car batteries, which includes emission control devices. Kumari et al. (2024) studied the impact of carbon emissions on supply chain coordination. In Zhang et al. (2018) study, involves social responsibility, internal and external green supply chain management. Aggarwal (2024) presents a third strategy by proposing a carbon offset market according to the source of the emissions, this strategy seeks to stop them before they are released. Wang et al. (2025) discovered that blockchain technology innovation could contribute to the energy transition by strengthening supply chain management channels. Afgharid et al. (2025) combine recycled coal as concrete and Build Integrated Photovoltaic (BIPV) facades to assess the energy

efficiency and evaluate progressive approaches to decarbonizing Positive Energy Building (PEB) warehouse in Central Taiwan. Jiang et al. (2019) put forward the challenge of reducing carbon emissions in last-mile deliveries. The challenge is to allocate parcel storage spaces while planning delivery routes to lower the overall expenses and carbon emissions of last-mile delivery.

2.3 Interval-valued triangular intuitionistic fuzzy number (IVTIFN)

Dubois and Prade (2008) introduced a concept of a gradual element in fuzzy set theory. Gorzafczany (1983) and Turksen (1986) introduced interval-valued fuzzy set theory. A gradual element is comparable to a fuzzy set in terms of being an element of a set. The interval-valued triangular fuzzy numbers were first introduced by Rabiei et al. (2014). In supply chain management (SCM) research, interval-valued triangular fuzzy numbers (IVTFNs) have emerged as a key tool, especially for handling uncertainty. Unlike, classic crisp numbers or single-valued fuzzy numbers, they can more flexibly describe imprecise and incomplete information, which primarily encourages their use. Deschrijver (2007) presented arithmetic operations for IVTFNs. Hong and Lee (2002) introduced a distance metric and talked about some of its algebraic characteristics. For supply chain success, Singh et al. (2010) attempt to provide an example of supplier selection and quota distribution in a situation of uncertainty. Deng and Zhang (2022) examined the class of uncertain nonlinear multi-agent systems in which the control gains of each subsystem in which the control gains of each subsystem are unknown, and the control directions and dead zone defect are unknown. Shaikh and Gite (2022) creates a fuzzy inventory model that identifies the most optimal inventory cycle time and the lowest average total costs by taking time value of money and inflation in factor. An innovative behavior three-way decision model under IVTFN is proposed by Xie et al. (2024) to resolve the 3D printing composites selection problem. Qudaimi (2021) developed a complete interval-valued triangular fuzzy regression model (IVTFRM) and suggests an Ishita approach to optimize it. Rajput et al. (2022) suggested to avoid the effect of non-random uncertainties of demand rate in the production, the cloudy fuzzy model has been used and shown a profitable business under several circumstances. Arora et al. (2022) offers an explicit condition to control the carbon emission of the manufacturer and reduce the optimum cost. The findings indicate that the collection of used goods that can be remanufactured must be increased.

2.4 Advertisement-selling price dependent demand in supply chain

Using advertising strategies, Hazari et al. (2015) constructed an inventory model in a bi-fuzzy environment. An important part of managing the product's market demand is advertising. Cho (1996) thus created the best manufacturing and advertising policies in the crisp environment. An EPQ model was created by Manna et al. (2017) in which production and demand rates are based on defective rates and advertising, respectively. Shah and Vaghela (2017) investigated the decline in inventory systems with time-sensitive, ad-controlled demand rates. Shekhar et al. (2024) develop comprehensive inventory models that take advantage of advertising demand. Sanjose et al. (2021) explored products whose demand patterns were impacted by timing, price, and frequency of advertising. For a perishable good, Sebatjane (2024) proposes four sustainable inventory models. The product's selling price, expiration date, advertising, inspection errors, and poor quality all influence demand. Limi et al. (2024) investigate the way demand rates are affected by product selling prices and advertising. They additionally consider the slow deterioration in consumer patience that causes a partial backlog of shortages. Rad et al. (2018) create an inventory model in which demand is considered to be dependent by both selling price and advertisement. New retailing techniques have emerged as a result of the quick expansion of e-commerce and consumer tendency to spend the majority of their time online. This trend is carried out by Sarkar et al. (2024) by taking factors such as buy-online-pick-up-in-store, offline, and online for non-permanent objects with ambiguous demand. Goyal et al. (2015) presented an economic ordering policy in which retailer's demand is a function of stock level and price. Chauhan & Singh (2015) introduce the inventory model for deteriorating items with Verhulst's model type demand rate, that is, the integrated demand pattern for growth and maturity stage to the final decay stage for single-period products. Chauhan and & Singh (2014) developed a model to reflect the real situation of market for time

varying deterioration items and varying demand with time, discounted cash flow (DCF) approach to investigate optimal replenishments and ordering policy.

2.5 agile manufacturing

Hemlatha et al. (2021) examined the way lean and agile manufacturing concepts are used in boiler component fabrication to identify the variables influencing WIP (Work In Progress) inventory levels to satisfy the necessary demand for each product. An industry 5.0-driven strategy called additive digital molding is examined by Miguel et al. (2024) as a means of achieving supply chain sustainability and corporate agility. Khan et al. (2024) came up with a plan to help managers to add agile and long-lasting criteria to their sourcing processes by figuring out the key factors that make this possible at the strategic, operational, and performance levels. Tavana et al. (2025) provide a thorough, four-step, integrated process with a final score to evaluate lean and agile supply chain management statistically. Using bibliometric analysis, Susitha et al. (2024) explore the supply chain competitiveness, emphasizing the importance of supply chain agility and the effects of quickly developing digital technology. For volume agility, Sangal and Gupta (2016) suggested a vendor-supplier model with two warehouses to handle the declining inventory. Singh and Gupta (2016) proposed a supply chain model that integrates agility, incorporates quality inspection errors, and considers a demand rate dependent on price. Gautam et al. (2020) investigated a model of sustainable strategies in relation to volume agility, preservation technologies, and price-reliant demand. Singh et al. (2010) developed a supply chain model for a flexible manufacturing system with variable costs.

Table 1. Summary of literature Review

Authors	Demand Pattern	Fuzzy number	Carbon emission	Model type	Manufacturing type
Benjaafar et al. (2013)	deterministic	No	Yes	EPQ	Traditional
Mashud et al. (2022)	constant	No	Yes	EPQ	Traditional
Kumari et al. (2024)	Stock-price-time dependent	No	Yes	SCM	Traditional
Shaikh et al. (2022)	Selling price dependent	Yes	No	EPQ	Traditional
Xie et al. (2024)	Fuzzy demand	Yes	No	EPQ	Traditional
Manna et al. (2017)	Advertisement dependent	No	No	EPQ	Traditional
Shah and Vaghela (2017)	Time and advertisement dependent	No	No	EOQ	Traditional
Shekhar et al. (2024)	Time, price-discount, advertisement dependent	No	No	EOQ	Traditional
San-jose et al. (2021)	Time-price-advertisement dependent	No	No	EPQ	Traditional

Sebatjane (2024)	Advertisement-expiration date-price dependent	No	Yes	Integrated	Traditional
Sangal and Gupta (2016)	Parabolic demand	No	No	Integrated	Agile
Singh and Gupta (2016)	Selling price dependent	No	No	Integrated	Agile
Gautam et al. (2020)	Price dependent	No	Yes	EPQ	Agile
Singh et al. (2010)	Selling price dependent	No	No	Integrated	Flexible
This paper	Advertisement and selling price dependent	Yes	Yes	GSCM	Agile

3. PRELIMINARIES, NOTATIONS AND ASSUMPTIONS

This section provides the basic concept of the interval-valued fuzzy numbers (IVFN) and interval-valued triangular intuitionistic fuzzy numbers (IVTIFN) and also provides notations and assumptions and the list of the abbreviations used, which are essential in developing and understanding the model.

3.1 Preliminaries

Definition 1. Interval-Valued Fuzzy Numbers (IVFN)

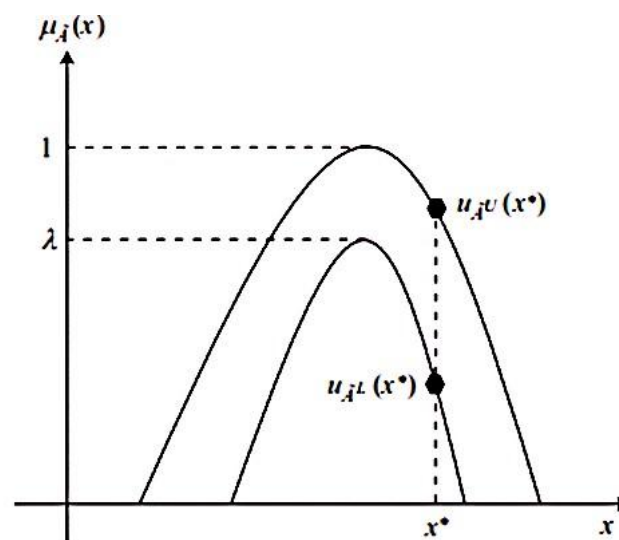


Fig. 1 Interval-valued fuzzy set \tilde{A} .

According to Gorzalczany (1987), an interval-valued fuzzy number (IVFN) is described as

$$\tilde{A} = \{x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]\}, x \in (-\infty, \infty),$$

$$\mu_{\tilde{A}^L}, \mu_{\tilde{A}^U}: (-\infty, \infty) \rightarrow [0, 1],$$

$$\mu_{\tilde{A}}^L \leq \mu_{\tilde{A}}^U, \forall x \in (-\infty, \infty),$$

$$\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x)], x \in (-\infty, \infty)$$

An interval-valued fuzzy number is represented in figure 1 showing that the degree of membership at x^* is in the interval $[\mu_{\tilde{A}}^L(x^*), \mu_{\tilde{A}}^U(x^*)]$,

Definition 2. Interval Valued Triangular Intuitionistic Fuzzy Number (IVTIFN)

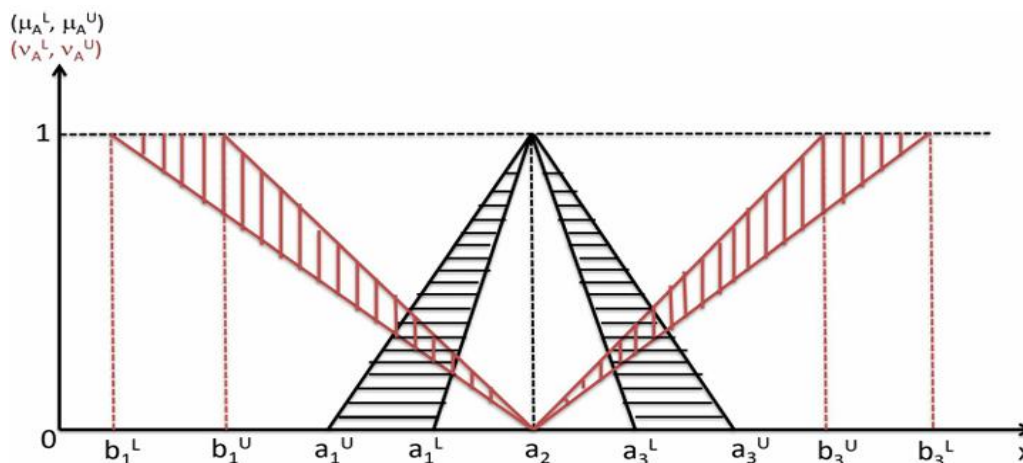


Figure 2 Interval-valued triangular intuitionistic fuzzy number

An IVTIFN is denoted by $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$, and it is defined as

Lower and upper membership functions, respectively, are defined as:

$$\mu_{\tilde{A}}^L(x) = \begin{cases} 1, & \text{if } x = a_2 \\ \frac{x - a_1^L}{a_2 - a_1^L}, & \text{if } a_1^L < x < a_2 \\ \frac{a_3^L - x}{a_3^L - a_2}, & \text{if } a_2 < x < a_3^L \\ 0, & \text{otherwise,} \end{cases}$$

$$\mu_{\tilde{A}}^U(x) = \begin{cases} 1, & \text{if } x = a_2 \\ \frac{x - a_1^U}{a_2 - a_1^U}, & \text{if } a_1^U < x < a_2 \\ \frac{a_3^U - x}{a_3^U - a_2}, & \text{if } a_2 < x < a_3^U \\ 0, & \text{otherwise,} \end{cases}$$

- Lower and upper non-membership functions, respectively, are given by:

$$v_{\tilde{A}}^L(x) = \begin{cases} 0, & \text{if } x = a_2 \\ \frac{a_2 - x}{a_2 - b_1^L}, & \text{if } b_1^L < x < a_2 \\ \frac{a_2 - x}{a_2 - b_3^L}, & \text{if } a_2 < x < b_3^L \\ 1, & \text{otherwise,} \end{cases}$$

$$v_{\tilde{A}}^U(x) = \begin{cases} 0, & \text{if } x = a_2 \\ \frac{x - a_2}{b_1^U - a_2}, & \text{if } b_1^U < x < a_2 \\ \frac{x - a_2}{b_3^U - a_2}, & \text{if } a_2 < x < b_3^U \\ 1, & \text{otherwise,} \end{cases}$$

Where $b_1^L \leq b_1^U \leq a_1^U \leq a_1^L \leq a_2 \leq a_3^L \leq a_3^U \leq b_3^U \leq b_3^L$.

Definition 3 Let $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ and $\tilde{B} = \{(c_1^U, c_1^L, c_2, c_3^L, c_3^U), (d_1^L, d_1^U, c_2, d_3^U, d_3^L)\}$ be two IVTIFNs. Then

- $\tilde{A} \oplus \tilde{B} = \{(a_1^U + c_1^U, a_1^L + c_1^L, a_2 + c_2, a_3^L + c_3^L, a_3^U + c_3^U), (b_1^L + d_1^L, b_1^U + d_1^U, a_2 + c_2, b_3^U + d_3^U, b_3^L + d_3^L)\}$.
- $k\tilde{A} = \begin{cases} \{(ka_1^U, ka_1^L, ka_2, ka_3^L, ka_3^U), (kb_1^L, kb_1^U, ka_2, kb_3^U, kb_3^L)\}, & \text{if } k \geq 0, \\ \{(ka_1^U, ka_1^L, ka_2, ka_1^L, ka_1^U), (kb_1^L, kb_3^U, ka_2, kb_1^U, kb_1^L)\}, & \text{if } k < 0. \end{cases}$
- $\tilde{A} \ominus \tilde{B} = \{(a_1^U - c_3^U, a_1^L - c_3^L, a_2 - c_2, a_3^L - c_1^L, a_3^U - c_1^U), (b_1^L - d_3^L, b_1^U - d_3^U, a_2 - c_2, b_3^U - d_1^U, b_3^L - d_1^L)\}$.
- $\tilde{A} \otimes \tilde{B} = \{(e_1^U, e_1^L, e_2, e_3^L, e_3^U), (f_1^L, f_1^U, e_2, f_3^U, f_3^L)\}$
where

$$e_1^U = \min\{a_1^U c_1^U, a_1^U c_3^U, a_3^U c_1^U, a_3^U c_3^U\},$$

$$e_3^U = \max\{a_1^U c_1^U, a_1^U c_3^U, a_3^U c_1^U, a_3^U c_3^U\},$$

$$e_1^L = \min\{a_1^L c_1^L, a_1^L c_3^L, a_3^L c_1^L, a_3^L c_3^L\},$$

$$e_3^L = \max\{a_1^L c_1^L, a_1^L c_3^L, a_3^L c_1^L, a_3^L c_3^L\},$$

$$f_1^L = \min\{b_1^L d_1^L, b_1^L d_3^L, b_3^L d_1^L, b_3^L d_3^L\},$$

$$f_3^L = \max\{b_1^L d_1^L, b_1^L d_3^L, b_3^L d_1^L, b_3^L d_3^L\},$$

$$f_1^U = \min\{b_1^U d_1^U, b_1^U d_3^U, b_3^U d_1^U, b_3^U d_3^U\},$$

$$f_3^U = \max\{b_1^U d_1^U, b_1^U d_3^U, b_3^U d_1^U, b_3^U d_3^U\},$$

$$e_2 = a_2 c_2.$$

Definition 4 An IVTIFN $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$ is said to be a non-negative IVTIFN iff $b_1^L \geq 0$.

Definition 5 Two IVTIFNs $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$, and $\tilde{B} = \{(c_1^U, c_1^L, c_2, c_3^L, c_3^U), (d_1^L, d_1^U, c_2, d_3^U, d_3^L)\}$, are said to be equal, i.e. $\tilde{A} \cong \tilde{B}$ iff $a_1^U = c_1^U, a_1^L = c_1^L, a_2 = c_2, a_3^L = c_3^L, a_3^U = c_3^U, b_1^L = d_1^L, b_1^U = d_1^U, b_3^U = d_3^U$ and $b_3^L = d_3^L$.

Definition 6 If $\tilde{A} = \{(a_1^U, a_1^L, a_2, a_3^L, a_3^U), (b_1^L, b_1^U, a_2, b_3^U, b_3^L)\}$, be a IVTIFN, then its expected value is given by:

$$EV(\tilde{A}) = \frac{a_1^U + a_1^L + b_1^L + b_1^U + 8a_2 + a_3^L + a_3^U + b_3^U + b_3^L}{16}.$$

3.2 Notations

The notations listed below, grouped into decision variables, supplier parameters, producer parameters, and fuzzy parameters, are consistently used throughout this work to develop our inventory model.

Producer's Parameters

- H_p – holding cost of items for producer including carbon emission(\$/unit/week)
- D_p – deterioration cost of item for producer including carbon emission (\$/unit/week)
- S_p – setup cost of item for producer including carbon emission(\$/unit/week)
- $\lambda(P)$ – cost of production

Supplier's Parameters

- H_s – holding cost of items for supplier including carbon emission (\$/unit/week)
- O_s – ordering cost of items for supplier including carbon emission (\$/unit/week)
- D_s – deterioration cost of item for supplier including carbon emission (\$/unit/week)
- R – rate of inflation
- N – number of cycles
- t_2 – cycle length per delivery; $t_2 = T/N$
- S_{Max} – maximum selling price
- S_{Min} – minimum selling price
- C_A – Fixed advertisement cost

Fuzzy Parameters

- \tilde{H}_p – fuzzy holding cost of stock for producer(\$/unit/week)
- \tilde{P}_{ic} – fuzzy item cost for deterioration for producer (\$/unit/week)
- \tilde{S}_p – fuzzy setup cost for producer (\$/unit/week)
- \tilde{H}_s – fuzzy holding cost of supplier(\$/unit/week)
- \tilde{S}_{ic} – fuzzy item cost for supplier (\$/unit)
- \tilde{O}_s – fuzzy supplier's ordering cost

Decision Variables

- T – cycle length of producer
- S – selling price

3.3 Abbreviations used

- **IVFN**- Interval-Valued Fuzzy Number
- **IVTFN**- Interval-Valued Triangular Fuzzy Number
- **IVTIFN**- Interval-Valued Triangular Intuitionistic Fuzzy Number
- **IVTFRM**- Interval-Valued Triangular Fuzzy Regression Model
- **CEE**- Carbon Emission Efficiency
- **SSCM**- Sustainable Supply Chain Management
- **GHG**- Green House Gas
- **CO₂**- Carbon Dioxide
- **BIPV**- Building Integrated Photovoltaic
- **PEB**- Positive Energy Building
- **WIP**- Work In Progress
- **TFN**- Triangular Fuzzy Number

3.3 Assumptions

The following are the assumptions for the proposed model.

- In terms of environmental criteria, this article makes the assumption that holding costs, item costs, and supplier and retailer setup/ordering costs will result in carbon emission expenses
- Agile manufacturing controls excess holding or shortages at any stage of the cycle by maintaining a controlled production rate. There were no shortages since the production rate is seen as a deciding factor.
- Here, the selling price and the advertising ξ function determine the demand rate. Hence, $d = a_1 \left(\frac{S_{\text{Max}} - S}{S - S_{\text{Min}}} \right) + a_2 \xi^{\theta_2}$, where $\xi = a_1 + a_2 t$, i.e. ξ is the function of time t and a_1 is the selling price scaling parameter, a_2 is the scaling parameter of the advertisement variable, and θ_2 represents parameter of shape.
- The rate of deterioration ($0 \leq \delta \leq 1$) is considered very small.
- Within the inflationary environment, a complete model is developed.
- Agile manufacturing cannot produce at a constant rate. The production cost for this purpose is provided below :

$$v(P) = \left(\gamma + \left(\frac{K}{P} \right) + SP + \psi(P - p_c) \cdot f(P - p_c) \right), \text{ where } f(P - p_c) = \begin{cases} 1, & P > p_c \\ 0, & P \leq p_c \end{cases}$$

Here

γ shows the material cost, $\left(\frac{K}{P} \right)$ is the labor cost and production rate (P) increases, the value of $\left(\frac{K}{P} \right)$ decreases, the term (SP) is the tool or die costs is represented by the term SP , and the term (p_c) denotes the critical value of the machine's manufacturing rate. For a very high manufacturing rate, the formed stock must be defective ($P > p_c$).

4. MATERIALS AND METHODS

The problem description and mathematical formulation for producer and supplier along with the solution methodology is elucidated in this section.

4.1 Model definition

The study focuses on the supply chain model incorporates agile manufacturing processes intended to minimize the cost of carbon emissions. Production rate is regarded as an important factor in agile manufacturing, which offers flexibility by allowing the production cost to vary per unit based on the type of material used. The model considers a dynamic demand rate that changes with the selling price, carbon emission cost are analyzed across multiple stages of the supply chain, including setup/ordering costs, item production costs, and holding costs. The primary objective is to develop an integrated strategy that effectively reduces total costs while managing and controlling carbon emissions. To address uncertainties within the model, an IVTIFN is employed

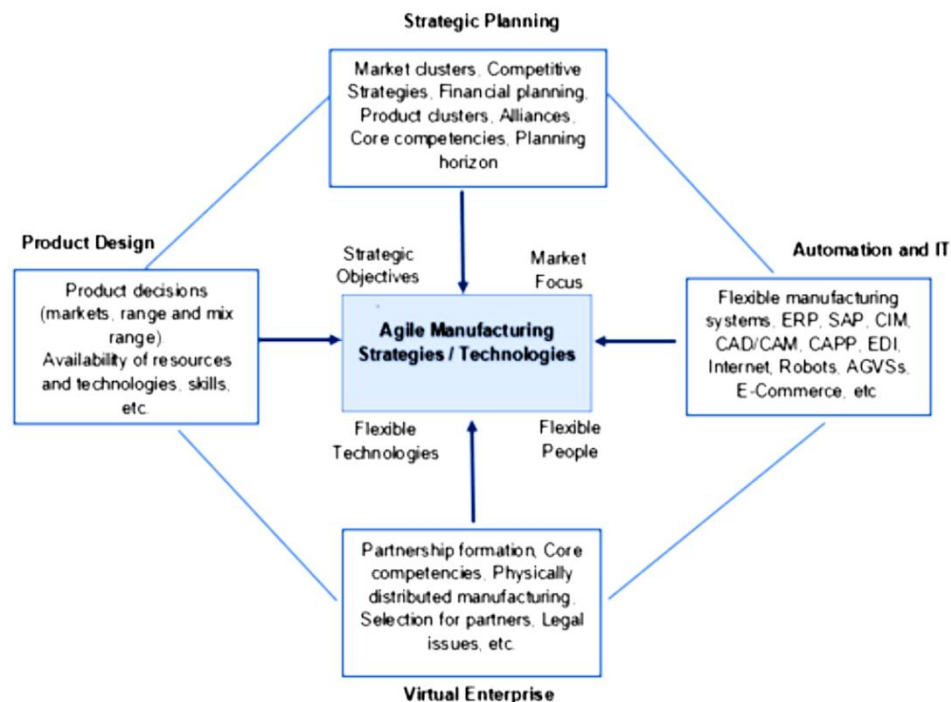


Figure 3 represents the agile manufacturing

4.2 Mathematical formulation of the proposed model

This section proposes a two-echelon supplier and producer model, with the producer's model falling under the agile manufacturing framework. ' P ' symbolizes the producer's rate of production. Demand is seen as dependent on selling price and advertising, and the producer ships the manufactured items to the supplier in multiple shipments.

4.2.1 Producer's model

Agile manufacturing is used to formulate the producer's production model. The manufacturing process begins at $t = 0$ and continues until $t = t_1$, at which point the stock level reaches its maximum. Currently, production leads to the inventory level to rise, while demand and deterioration lead it downwards. Because of demand and the rate of deterioration, manufacturing at times stops and inventory levels drop. At $t = T$, the stock's level decreases to zero. The behaviour of the stock system is shown in Figure 1 for the period $[0, T]$.

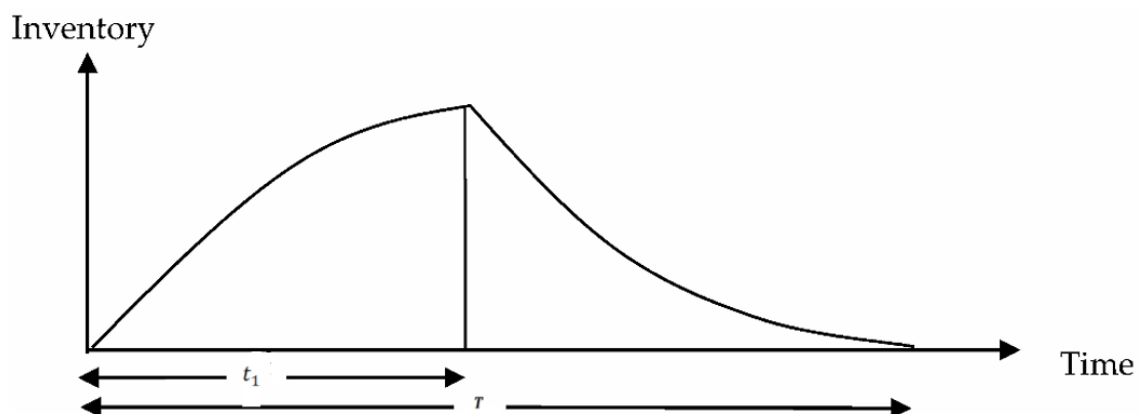


Figure 4 representing change in inventory

Inventory levels are shown to increase in association with production rates and fall in accordance to demand and deterioration rates. The following is a mathematical representation of the inventory position.

$$\frac{dI_{P_1}(t)}{dt} = -\delta I_{P_1}(t) - d + P, 0 \leq t \leq t_1, \quad (1)$$

We see that the demand and rate of deterioration contributes to the inventory level rate decrease throughout time periods $t_1 \leq t \leq T$. The following is a mathematical representation of the inventory position.

$$\frac{dI_{P_2}(t)}{dt} = -\delta I_{P_2}(t) - d, t_1 \leq t \leq T, \quad (2)$$

The inventory level for production time t is obtained by solving the above equations with the conditions $I_{P_1}(0) = 0$ and $I_{P_2}(T) = 0$.

$$I_{P_1}(t) = \left(\frac{P-d}{\delta}\right)(1 - e^{-\delta t}), \quad (3)$$

At any given time t , the inventory level during the non-production period is as follows:

$$I_{P_2}(t) = \left(\frac{d}{\delta}\right)(e^{\delta(T-t)} - 1), \quad (4)$$

After production the maximum level of inventory with initial condition $I_{P_1}(t_1) = I_m$

$$I_m = \left(\frac{P-d}{\delta}\right)(1 - e^{-\delta t_1}), \quad (5)$$

Using the condition of continuity in inventory $I_{P_1}(t_1) = I_{P_2}(t_1)$, we have

$$\left(\frac{P-d}{\delta}\right)(1 - e^{-\delta t_1}) = \left(\frac{d}{\delta}\right)(e^{\delta(T-t_1)} - 1), \quad (6)$$

This give the production run time as

$$t_1 = \frac{1}{\delta} \log\left(\frac{de^{\delta T} + P - d}{P}\right), \quad (7)$$

The costs of material setup, holding, item, and manufacturing are contributed by the producer. carbon emission expenditures are included in the setup, holding, and item costs.

Holding Cost:

The holding cost contains two components: one is holding cost of the products as \tilde{H}_p , and the other is related to carbon emission cost as H'_p total holding cost after accounting for the environmental impact is.

$$\begin{aligned} HC &= \frac{(\tilde{H}_p \oplus H'_p)}{t} \left[\int_0^{t_1} I_{P_1}(t) e^{-Rt} dt + \int_{t_1}^T I_{P_2}(t) e^{-Rt} dt \right], \\ &= \frac{(\tilde{H}_p \oplus H'_p)}{t} \left[\left(\frac{P-d}{\delta}\right) \left\{ \frac{\delta - (\delta+R)e^{-Rt_1} + Re^{-(\delta+R)t_1}}{R(\delta+R)} \right\} + \frac{d}{\delta} \left(\frac{e^{-RT} - e^{-Rt_1}}{R} + \frac{e^{\delta t - (\delta+R)t_1} - e^{-Rt}}{\delta+R} \right) \right] \end{aligned} \quad (8)$$

Setup Cost:

There are two components to the producer's setup cost: the carbon emissions as S'_p and the product setup as \tilde{S}_p . In the end, the overall setup cost when the environment is taken into consideration is.

$$SC = \frac{(\tilde{S}_p \oplus S'_p)}{T}, \quad (9)$$

Item Cost:

Item cost of producer contains two components. One is linked to the deterioration and the sale of \tilde{P}_{ic} and other components P'_{ic} as a result of the carbon emissions. In the end, the total cost of the item under environmental effects is.

$$IC = \frac{(\tilde{P}_{ic} \oplus P'_{ic})}{T} \left[\int_0^{t_1} P e^{-Rt} dt \right] = \frac{(\tilde{P}_{ic} \oplus P'_{ic})}{T} P (1 - e^{-Rt_1}), \quad (10)$$

Production Cost:

The cost of labour, materials, tools and dies, and some extra labour costs are all included in the producer's production costs.

$$PC = v(P) \left[\int_0^{t_1} P e^{-Rt} dt \right] = \left(\gamma + \left(\frac{k}{P} \right) + SP + \psi(P - P_c) \cdot f(P - P_c) \right) \left(\frac{P(1 - e^{-Rt_1})}{R} \right), \quad (11)$$

Producer's Total Cost:

The total cost of producer is formulated as;

$$TC_p = HC \oplus SC \oplus IC \oplus PC$$

$$TC_p = \frac{(\tilde{H}_p \oplus H'_p)}{t} \left\{ \left(\frac{P-d}{\delta} \right) \left\{ \frac{\delta - (\delta+R)e^{-Rt_1} + Re^{-(\delta+R)t_1}}{R(\delta+R)} \right\} \oplus \frac{d}{\delta} \left(\frac{e^{-RT} - e^{-Rt_1}}{R} + \frac{e^{\delta t - (\delta+R)t_1} - e^{-Rt}}{\delta+R} \right) \right\} \oplus \frac{(\tilde{S}_p \oplus S'_p)}{T} \oplus \frac{(\tilde{P}_{ic} \oplus P'_{ic})}{T} P (1 - e^{-Rt_1}) \oplus \left(\gamma + \left(\frac{k}{P} \right) + SP + \psi(P - P_c) \cdot f(P - P_c) \right) \left(\frac{P(1 - e^{-Rt_1})}{R} \right) \quad (12)$$

4.2.2 Supplier's Model:

The figure shown below depicts the behavior of inventory concerning the time in which the inventory decreases due to deterioration and demand

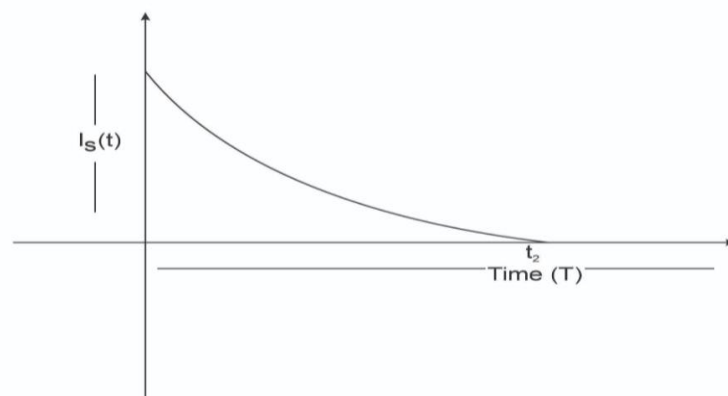


Figure 5. Behaviour of inventory for supplier

Figure 5 illustrates the pattern of demand and inventory storage for the supplier. $I_s(t)$ is the stock that was received from the producer at the beginning of the period. Due to the deterioration and demand, the level of stock $I_s(t)$ continually decreases to fulfil the demand; at $t = t_2$, the inventory level becomes zero. In this case, the supplier provides its clients with inventory in n different orders.

The demand and rate of deterioration leads to the inventory level dropping throughout the time period, $0 \leq t \leq t_2$. Thus, the following differential equation shows the rate of inventory:

$$\frac{dI_s(t)}{dt} = -\delta I_s(t) - D, 0 \leq t \leq t_2, \quad (13)$$

The present stock at any given time t over the period $0 \leq t \leq t_2$ is as follows, with inventory status $I_s(t) = 0$.

$$I_s(t) = \left(\frac{d}{\delta}\right)(e^{\delta(t_2-t)} - 1), \quad (14)$$

The ordering, holding, and item costs, all involves the carbon emission cost which are all carried by the supplier.

The total cost function is provided by the following components:

Holding Cost:

The two elements of holding cost : the products holding cost as \tilde{H}_s , and another one is carbon emission cost as H'_s , so the total holding cost under the environmental impact is.

$$HC = \left(\frac{\tilde{H}_s \oplus H'_s}{T}\right) \left[\int_0^{t_2} I_s(t) e^{-Rt} dt\right] = \frac{(\tilde{H}_s \oplus H'_s)d}{T\delta} \left(\frac{e^{\delta t_2} - e^{-Rt_2}}{\delta + R} + \frac{e^{-Rt_2} - 1}{R}\right), \quad (15)$$

Ordering Cost:

Supplier's ordering cost also contains two types of costs: One is related to the items ordering as \tilde{O}_s , while the other is related to the energy use and carbon emissions as O'_s . In the end, the overall ordering cost when the environmental factors are taken into consideration is.

$$OC = \left(\frac{\tilde{O}_s \oplus O'_s}{T}\right), \quad (16)$$

Item Cost:

The producer's item cost also involves two parts, the deterioration and the sale of \tilde{S}_{ic} and other components S'_{ic} as a result of the carbon emissions. Thus, the total cost of the item under environmental effects is.

$$IC = \frac{(\tilde{S}_{ic} \oplus S'_{ic})}{T} I_s(0) = \frac{(\tilde{S}_{ic} \oplus S'_{ic})}{T} \left(\frac{d}{\delta}\right)(e^{\delta t_2} - 1), \quad (17)$$

Supplier's Total Cost:

The producer's total cost formulated as;

$$TC_s = HC \oplus OC \oplus IC$$

$$TC_s = \frac{(\tilde{H}_s + H_s)d}{T\delta} \left(\frac{e^{\delta t_2 - e^{-Rt_2}}}{\delta + R} + \frac{e^{-Rt_2 - 1}}{R} \right) \oplus \left(\frac{\tilde{O}_s + O'_s}{T} \right) \oplus \frac{(\tilde{S}_{ic} + S'_{ic})}{T} \left(\frac{d}{\delta} \right) (e^{\delta t_2} - 1), \quad (18)$$

Total cost of the model can be represented as;

$$TC = TC_p \oplus TC_s$$

$$TC = \left[\frac{(\tilde{H}_p \oplus H'_p)}{t} \left\{ \left(\frac{P-d}{\delta} \right) \left\{ \frac{\delta - (\delta + R)e^{-Rt_1} + Re^{-(\delta + R)t_1}}{R(\delta + R)} \right\} \oplus \frac{d}{\delta} \left(\frac{e^{-RT} - e^{-Rt_1}}{R} + \frac{e^{\delta t - (\delta + R)t_1} - e^{-Rt_1}}{\delta + R} \right) \right\} \oplus \frac{(\tilde{S}_p \oplus S'_p)}{T} \oplus \frac{(\tilde{P}_{ic} \oplus P'_{ic})}{T} P(1 - e^{-Rt_1}) \oplus \left(\gamma + \left(\frac{k}{P} \right) + SP + \psi(P - P_c) \cdot f(P - P_c) \right) \left(\frac{P(1 - e^{-Rt_1})}{R} \right) \oplus C_A \oplus \frac{(\tilde{H}_s \oplus H_s)d}{T\delta} \left(\frac{e^{\delta t_2 - e^{-Rt_2}}}{\delta + R} + \frac{e^{-Rt_2 - 1}}{R} \right) \oplus \left(\frac{\tilde{O}_s \oplus O'_s}{T} \right) \oplus \frac{(\tilde{S}_{ic} \oplus S'_{ic})}{T} \left(\frac{d}{\delta} \right) (e^{\delta t_2} - 1) \right], \quad (19)$$

5. SOLUTION METHODOLOGY

The method use for the optimization of the total cost of the model is SA (Simulated Annealing)

The simulated annealing approach is used to optimize the total cost and decision variables. Physical annealing is the foundation of the simulated annealing (SA) technique. SA is a probabilistic method for figuring out a function's global optimum. To convert the material into a corresponding structure, physical annealing involves heating it to the proper annealing temperature and then progressively cooling it down. When heated, the molecular structure of the substance becomes less stiff and more malleable.

This metaheuristic was developed to approximate global optimization of optimization problems through a wide search space. For a significant percentage of local optima, global optima can be found by Simulated Annealing in distinct search spaces. Simulated Annealing is a better alternative than precise approaches in instances when discovering an estimated global optimum in a short length of time is more critical than locating an actual exact local optimum.

The following steps are used to get the optimal value:

Step 1. Initialize: choose a random place to get started.

Step 2. Move: move in a certain way to alter the positioning . i.e. generate initial value of $F(S, T) = F(S_0, T_0)$

Step 3. Determine how the technique influence the result or the outcome itself.

Step 4. Depending on how the outcome changes, decide whether to accept or reject the move.

Step 5. Revise and repeat: to update a perfect value, decrease the respected parameters. Now return to step2 and until the "optimal point" is achieved, the procedure is repeated.

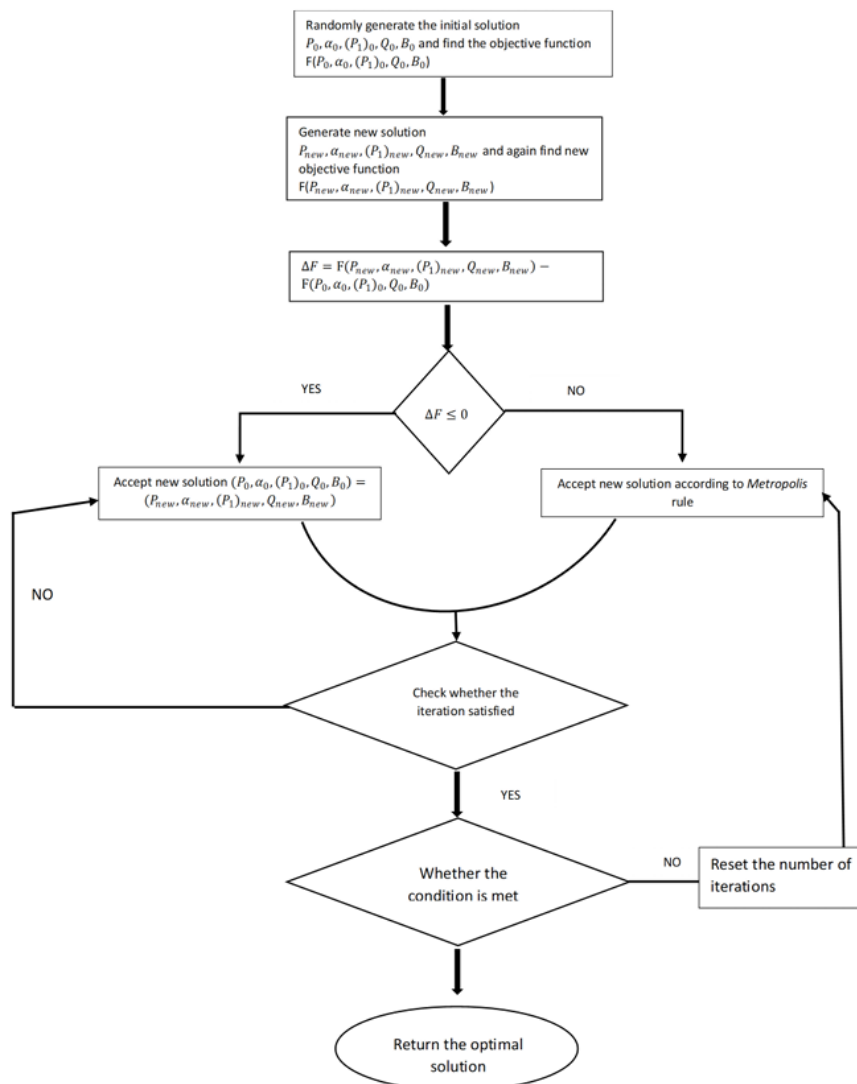


Figure 6 Flow chart of SA

6. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

This section includes a numerical experiment to validate the current investigation. The model's importance is then demonstrated through a sensitivity analysis. The data has been taken from the Vandana et al. [58].

6.1 Numerical example

The optimality of the suggested model is demonstrated in detail by the numerical example. The model considers three decision variables for optimizing the system's overall performance. Table presents the suitable values of the input parameters taken into consideration to give a thorough picture of the model's performance. All things considered, this example demonstrate that the suggested approach is successful in maximizing system performance and producing the optimal cost while accounting for related costs.

Table 2 represents the values given to the parameters

Input parameters	values
Item cost of for producer (\tilde{P}_{ic})	$\{((6.5,6.7,6.9), (6.4,6.7,7.0))\}$ /unit/week
Holding cost of stock for producer(\tilde{H}_p)	$\{(2.6,2.8,3.0),(2.5,2.8,3.1)\}$ /unit/week
Producer's setup cost (\tilde{S}_p)	$\{(890,990,1090),(790,990,1190)\}$ /setup
Item cost of for supplier (\tilde{S}_{ic})	$\{(5.3,5.5,5.7), (5.2,5.5,5.8)\}$ /unit/week
Holding cost of stock for supplier(\tilde{H}_s)	$\{(2.6,2.7,2.8),(2.5,2.7,2.9)\}$ /unit/week
Supplier's ordering cost (\tilde{O}_s)	$\{(198,298,398), (98,298,498)\}$ /unit/week
Carbon emission cost of items for producer (P'_{ic})	\$0.3/unit/week
Carbon emission cost of holding items for producer (H'_p)	\$0.2/unit/week
Carbon emission cost for set up for producer (S'_p)	\$10/setup
Carbon emission cost of items for supplier (S'_{ic})	\$0.5/unit/week,
Carbon emission cost of holding items for supplier(H'_s)	\$0.3/unit/week
Carbon emission cost of ordering items for supplier (O'_s)	\$2/order
Production rate (P)	182 units/week
critical value of the machine's manufacturing rate(p_c)	220units/week,
Rate of inflation(R)	0.01
Deterioration rate (δ)	0.02
scaling parameter for selling price(a_1)	1
scaling parameter of the advertisement(a_2)	0.10
shape parameter.(θ_2)	1
maximum selling price (S_{Max})	\$250
miniimum selling price (S_{Min})	\$100
material cost (γ)	\$90/week
(K)	\$3500/week
(ψ)	\$0.04/week
(f)	20 units/week
advertisement cos(C_A)	\$4/item

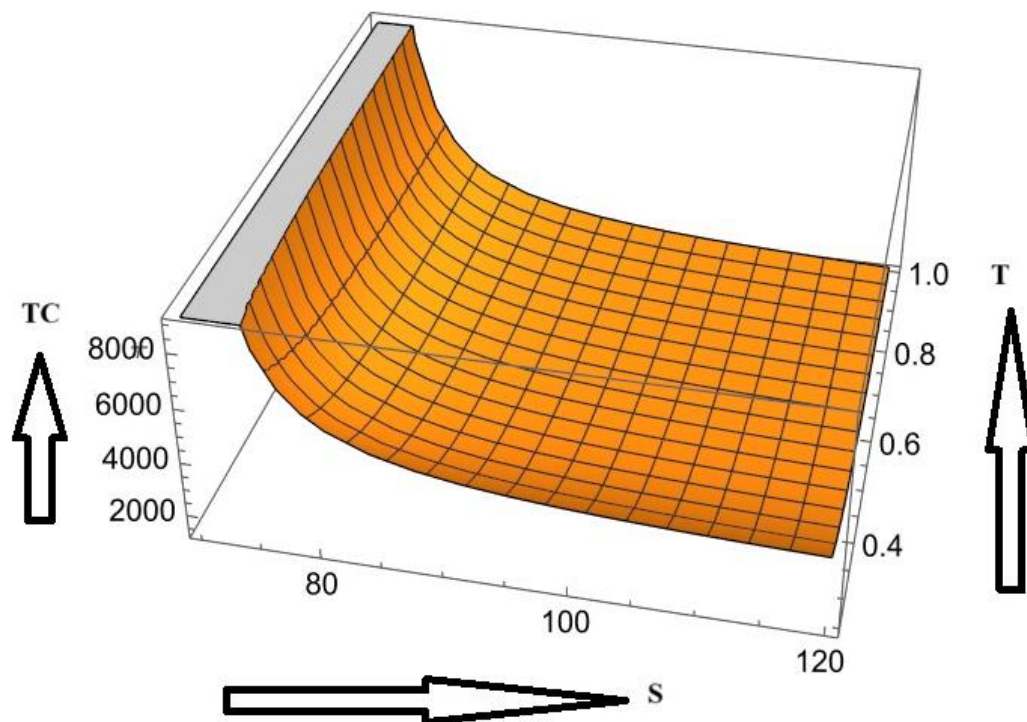


Figure 7 shows that the relationship between Selling price, Time cycle and Total cost

Result:

Selling price	Time cycle	Total Cost
\$108	0.50	\$3279.29

Discussion

The result evaluates the impact of selling price (S) and time cycle (T) on the total cost (TC) within the given supply chain system

- Impact of selling price (S) on total cost (TC)
The figure 7 indicates that the selling price and total cost have an inverse connection. When the selling price (S) rises, the total cost (TC) falls off considerably. This pattern suggests that raising the selling price could increase profitability while lowering costs. But demand may be impacted by prices that are too high; thus, an ideal balance is required.
- Impact of Time Cycle (T) on Total Cost (TC)
The decrease in total cost (TC) with increasing time cycle (T) is related to less frequent production setups and improved inventory management. On the other hand, an excessively long cycle might result in higher holding costs, but reduced profit.

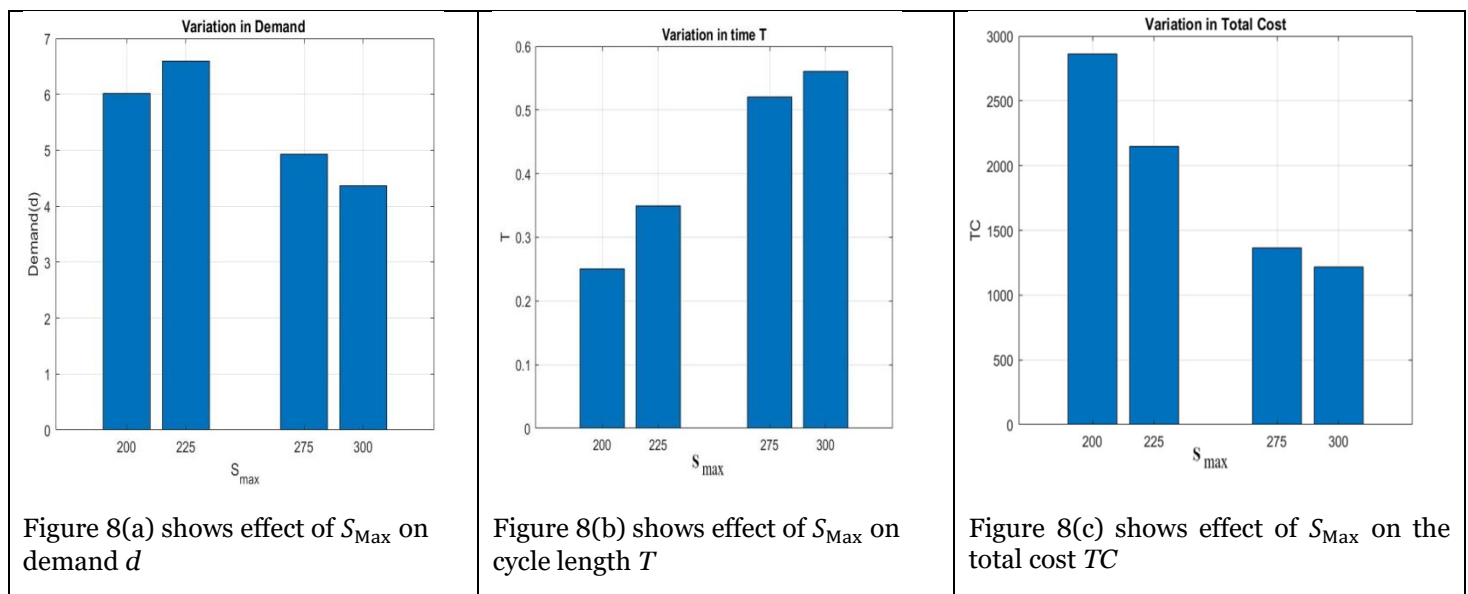
The findings imply that keeping selling price (S) unchanged may help reduce total cost (TC). By choosing the optimum value of time cycle (T), excessive inventory buildup may be avoided, and operating costs can be decreased.

6.2 Sensitivity Analysis

Sensitivity analysis examines the effects of some changes in input parameters decision variables and objective function of the model.

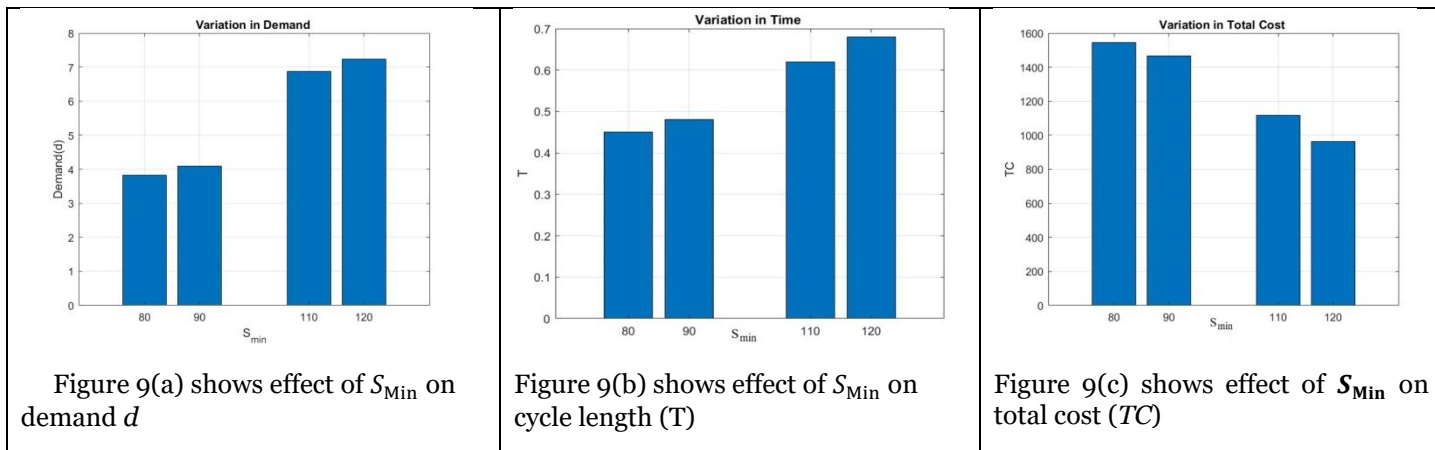
Table 2. Sensitivity analysis of parameter S_{Max} (maximum selling price), S_{Min} (minimum selling price), a_1 (scaling parameter of selling price) with respect to demand (d), cycle length (T) and total cost (TC)

Parameter	%change		Demand (d)	Cycle length (T)	Total cost (TC)
	-20	200	6.02	0.25	2861.46
	-10	225	6.60	0.35	2150.55
S_{Max}	10	275	4.93	0.52	1366.69
	20	300	4.36	0.56	1217.48
	-20	80	3.82	0.45	1545.39
	-10	90	4.10	0.48	1465.62
S_{Min}	10	110	6.88	0.62	1117.26
	20	120	7.23	0.68	964.11
	-20	0.5	8.93	0.15	4429.9
a_1	-10	0.75	13.39	0.38	2050.44
	10	1.25	22.31	0.88	818.92
	20	1.5	26.78	1.2	490.40



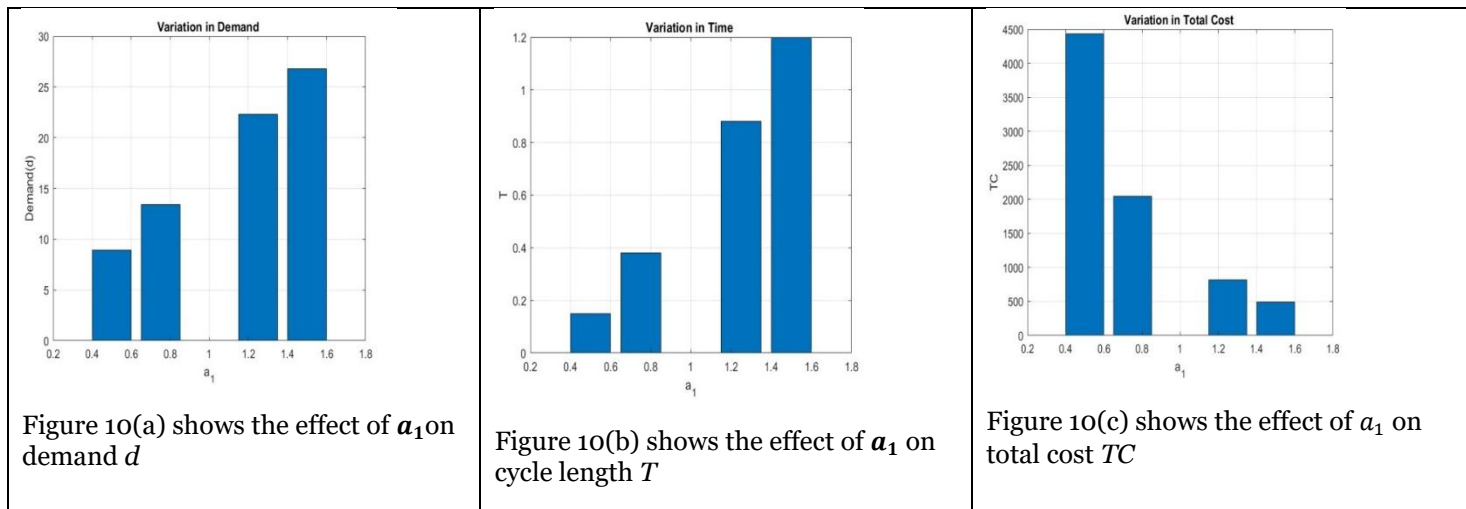
From table 2 and figure 8(a), 8(b), 8(c), the observations are drawn as follows.

- From table 2 and figure 8(a), 8(b), 8(c), it is observed that as S_{Max} (maximum selling price) grows, demand (d) increases slowly until it reaches its highest point. After this point, demand steadily decreases indicating that higher S_{Max} selling price may not be beneficial for maintaining high demand.
- The cycle length (T) increases with S_{Max} showing a direct variation. Higher S_{Max} values may lead to longer cycles, indicating improved operational or inventory management efficiency.
- The value of S_{Max} varies inversely with the total cost (TC) which means that higher the S_{Max} , lesser will be the total inventory cost.



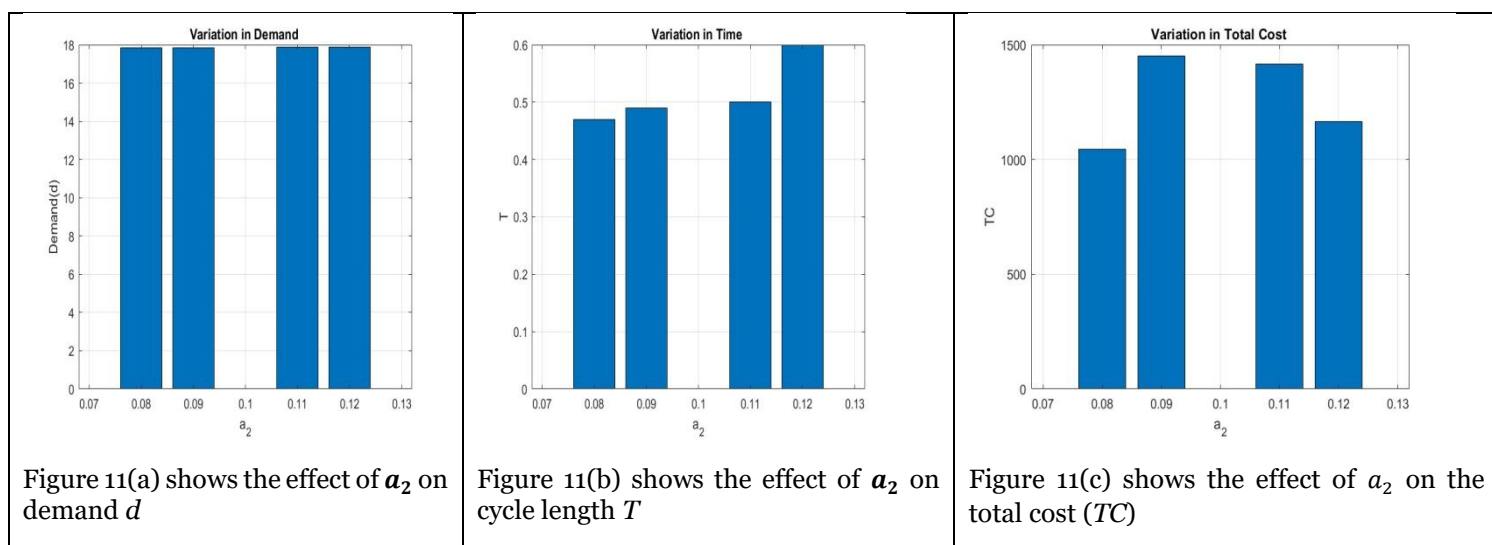
From table 2 and figure 9(a), 9(b), 9(c), the following observation are drawn

- It is clear that as the value of S_{Min} (minimum selling price) increases, the demand d increases significantly i.e., when the value of S_{Min} is 80, the demand d is 3.82, but when S_{Min} reaches to 120, the demand increases to 7.23. This indicates that higher the value of S_{Min} higher will be the value of demand.
- As S_{Min} rises, the cycle length T increases steadily, i.e., when S_{Min} is 80, the cycle length T is 0.45 and when S_{Min} rises to 120 then T rises to 0.68. This demonstrates that larger S_{Min} requires larger cycle length to manage resources and meet demand.
- The total cost TC decreases as S_{Min} increases. When the value of S_{Min} is 80, the total cost is 1545.39, but when S_{Min} is 120 then the total cost drops to 964.11. Thus, it is seen that S_{Min} varies inversely with regard to the total cost.



From table 2 and figure 10(a), 10(b), 10(c), the following observation are drawn

- Demand (d) rises steadily as a_1 rises. When a_1 (scaling parameter) is 0.5 the value demand is 8.93 while the value to demand reaches to its maximum 26.78 when a_1 is 1.5. This shows that demand rises proportionally to the greater values of a_1 .
- With an increase in a_1 , the cycle length T also increases. The cycle length T is 0.15 at a_1 0.5. the cycle length T achieves a maximum value of 1.2 when a_1 is 1.5 and value of cycle length T increases to 0.88 when a_1 is 1.25. The increase in cycle length T shows a nonlinear trend, with larger increment observed at higher values of a_1 .
- The total cost (TC) drops significantly as a_1 increases. The total cost (TC) reaches its maximum of 4429.9 when a_1 is 0.5. It falls to its lowest value 490.92 when a_1 is 1.5. The graph shows an exponential decrease in cost, indicating that increasing a_1 results in significant cost reductions, particularly at initial values of a_1 .



The following observations are found in table 2 and Figures 11(a), 11(b), and 11(c).

- Despite changes in a_2 (scaling parameter of advertisement), the demand (d) stays mostly constant. It begins at 17.83 when a_2 is reduced by 20%, and it only slightly 17.87 when a_2 is raised by 20%. This shows that changes in a_2 have minimal impact on demand, which stays relatively steady.
- There is a moderate positive relationship between a_2 and cycle length (T), which is demonstrated by the fact that the cycle length increases slightly as a_2 climbs i.e., cycle length reaches to 0.47 at a 20% drop in a_2 and reaches to 0.6 at a 20% increase.
- Changes in a_2 cause variation in the total cost (TC). The entire cost comes to 1045.94 when a_2 is reduced by 20%. The total cost increases to 1451.4 with a 10% decrease and then decreases slightly to 1415.86 at a 10% increase and then the total cost drops to 1164.8 when a_2 rises by 20%. This implies that the overall cost exhibits inconsistent sensitivity to changes and does not follow a linear trend.

7. MANAGERIAL INSIGHT, CONCLUSION

The section involves the managerial insight found and the conclusion.

7.1 Managerial insight

The research produced analytical and numerical data, which is used to develop useful insights.

- The study indicates the relevance of containing carbon emission factors in supply chain choices. Managers can reduce costs while adhering to environmental rules, increasing corporate social responsibility, and fulfilling stakeholder's expectations.
- Managers can improve product demand by using advertisement-dependent demand models to customize the marketing strategy. Investment in successful advertising may have a substantial impact on production scheduling, inventory management and overall profitability.
- Agile manufacturing provides the flexibility to cut lead times, adjust to changing market conditions, and lessen the risk of supply chain interruptions. By using agile frameworks, managers in sectors like the automobile industry may improve their response to changing customer expectations.
- Interval Valued Triangular Fuzzy Number (IVTFN) is used to help manage uncertainty in production scheduling and demand forecasting. To increase the correctness of their decisions, managers should spend money on tools and training that make the adoption of fuzzy logic model easier.
- Sensitivity analysis reveals a substantial correlation between selling prices and demand. Managers must carefully balance maximum and minimum price points to maximize demands, cycle length and total costs without sacrificing customer satisfaction.
- An effective method for maximizing supply chain costs is the Social Group Optimization (SGO) algorithm. Similar meta heuristic optimization approaches may be used or explored by managers to address complicated, multidimensional problems in the supply chain.
- Carbon emission and adaptive manufacturing techniques help organizations achieve long-term sustainability. Managers should prioritize investments in green technology and practices that cut emissions while being economically viable.
- Reaching cost and environmental targets requires efficient collaboration between manufacturers, retailers, and suppliers. Managers should encourage cooperation, openness, and instantaneous communication amongst supply chain participants.
- The model highlights the importance of accommodating inflation, fluctuating demand, and other external variables in decision making. Managers should remain vigilant about external market forces and adopt adaptive planning strategies to sustain operations.
- Since the study reveals advanced methodology, their practical use in real-world situations might require specific approaches, rigorous training, and clear alignment with company goals. Managers may evaluate feasibility and allocate necessary resources to ensure seamless transitions.

7.2 Conclusion

This research emphasizes the revolutionary potential of incorporating sustainability, agile manufacturing, and novel optimization methodologies into supply chain management. We tackled uncertainty in production scheduling and inventory management by using triangular interval-valued fuzzy numbers and advertisement-dependent demand models, focusing on carbon emission reduction. Simulated Annealing is used for cost optimization and obtaining optimal solution.

Simulated Annealing (SA) optimizes total costs, revealing key insights: higher S_{Max} reduces costs and extends cycle time, while S_{Min} boosts demand but requires longer cycles. The selling price scaling parameter (a_1) influences demand and cost effectiveness, while advertising (a_2) has a nonlinear effect on total cost.

Base on these findings, the study emphasizes the vital role of strategic parameter optimization in managing the complex interaction of cost efficiency, resource management, and demand responsiveness. By carefully examining and altering critical criteria such as selling prices, advertising methods, and manufacturing rates, industries may strike a harmonic equilibrium that reduces costs while increasing productivity. This strategic approach is especially important in industries such as automotive, where changing customer demands and rigorous environmental regulations require agile and adaptable supply chain solutions.

Finally this study reinforces the importance of employing data-driven approaches to encourage durability, adaptation and sustainability in supply chains, ensuring that industries are agile to both market changes and environmental imperatives.

8. FUTURE SCOPE AND LIMITATIONS

Although the present model has proven its benefit, operational efficiency, and optimization, there are certain limits and proposals for the future scope:

8.1 Future scopes

Possible future implications for the present research are as follows.

- **Integration of Advanced Technologies** improves decision-making and transparency in supply chain operations, future research can integrate innovative technologies like blockchain, artificial intelligence, and the Internet of Things (IoT). Demand forecasting, carbon monitoring and optimization procedures may all be further enhanced by these technologies.
- **Multi-Objective Optimization-** The existing model may be expanded to handle multi-objective optimization issues, including simultaneously balancing carbon emissions, profit, and consumer satisfaction, to offer a more thorough framework for decision making.
- **Industry Specific Applications-** while this research focuses on the automotive industry, the model may be expanded and tailored to other industries with specific issues and requirements such as healthcare, electronics or food supply chains which have unique challenges and requirements.
- **Dynamic Consumer Behaviour-** The agility of demand forecasting that is dependent on advertisements may be increased by integrating real time consumer behaviour data and market trends into the model..

8.2 limitations of the proposed model

Below are the limitations of the proposed model after the analysis:

- Although the study's primary focus is the automotive sector, its conclusion might not apply to other industries with distinct supply chain frameworks or operating processes.

- The suggested model is based on several assumptions, such as low degradation rates and settings with constant inflation, which may not accurately reflect the problems that arise in the real world.
- An IVTIFN improves the representation of uncertainty but add difficulty to decision-making, which can be difficult for the sectors unfamiliar with advanced fuzzy logic concepts.
- Including carbon emissions in the cost calculation is beneficial for sustainability but may oversimplify the true environmental impact due to the limited scope or exclusion of other greenhouse gases.
- The study considers advertisement-dependent demand a significant element, which may overlook other important demand variables such as competitors or consumer behavior adjustments.
- While Social Group Optimization (SGO) is marketed as a reliable approach, further research is necessary to determine exactly how it performs in comparison to more recent or sector-specific optimization strategies.
- Agile manufacturing is effectively handled but the model may fail to reflect significant changes or interruptions in global supply networks, such as those induced by geopolitical crises or pandemics.

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