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Research Article

Properties Of Bipolar Valued Vague Normal Ideals

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ABSTRACT

The main objective of this paper is to present the notation of BVVI. As a consequence, properties of BVVNIs of a semiring is introduced. Also, Translation of BVVNIs of a semiring is examined. Later, BVVI was extended the homomorphism in BVVI.

Keywords: Fuzzy Set , Vague Subset , BVFS, BVVS, BVVSS, BVVI, BVVNI , ?(Ť), !(Ť),Q ̃_(〖([1]〗 _-^+, 〖1]〗 _+^+],[〖ϱ]〗 _-^-, 〖ϱ]〗 _+^-])) (Ť), P _(〖([1]〗 _-^+, 〖1]〗 _+^+],[〖ϱ]〗 _-^-, 〖ϱ]〗 _+^-])) (Ť), G ̃_(〖([1]〗 _-^+, 〖1]〗 _+^+],[〖ϱ]〗 _-^-, 〖ϱ]〗 _+^-])) (Ť),

Ť, Ť, Ť.

1 INTRODUCTION

The concept of (BVVI) of a semiring is used here. In 1965, Zadeh [16] introduced the notion of a fuzzy subset of a set, *Fuzzy Sets* are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as *Intuitionistic Fuzzy Sets*, *Interval Valued Fuzzy Sets*, *Fuzzy Sets*, *vague sets*, Soft Sets etc. Grattan-Guiness [9] discussed about fuzzy membership mapped onto interval and many quantities. *vague set* is an extension of *Fuzzy Set* and it is appeared as a unique case of context dependent *Fuzzy Sets*. The VS was introduced by W.L.Gau and D.J.Buehrer [8]. Lee [10] introduced the notion of BVFSs. BVFSs are an extension of *Fuzzy Sets* whose membership degree range is enlarged from enlarged from the interval $[0,1]$ to $[-1,1]$. In a BVFSs, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree $(0,1]$ indicates that elements somewhat satisfy the property and the membership degree $[-1,0)$ indicates that elements somewhat satisfy the implicit implicit counter property. BVFSss and intuitionistic *Fuzzy Sets* look similar each other. However, they are different each other [10, 11]. fuzzy subgroup was was introduced by Azriel Rosenfeld [5]. RanjitBiswas [13] introduced the *vague Groups*. Cicily Flora. S and Arockiarani.I [7] have introduced a new class of generalized BVss. Anitha.M.S., et.al.[1] defined as BVFSs of a group and Balasubramanian.A et.al [6] have defined the BIVG. K.Murugalingam and K.Arjunan [12] have discussed about IVFSS and then BVMFSSs of a semiring have been introduced by Yasodara.B and KE.Sathappan [14]. Anitha.K. et al.[2, 3, 4] defined as BVVSS. Here, the concept of BVVNIs of a

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semiring is introduced and estaiblished some results. Particularly, some properties of BVVNI of a semiring are introduced in this paper.

2 PRELIMINARIES

In this phase, we recall a few number of the important standards and definitions,which are probably vital for this paper.

Definition 2.1 [5]

The vague value of ù in Ť is described as the range [$t_{\check{S}}$(ù) ,1- $f_{\check{S}}$(ù)] The vague value of ù in Ť is represented by the range [$t_{\check{S}}$(ù) ,1- $f_{\check{S}}$(ù)] and is written by $\check{\mathcal{G}}_{\check{T}}$(ù) , i.e., $\check{\mathcal{G}}_{\check{T}}$(ù) = [$t_{\check{S}}$(ù) ,1- $f_{\check{S}}$(ù)].

Example 2.2 A BVVSS Š = {ǩ, ℓ, ḿ} is [Ť] = {< ǩ, [0.4,0.6] ,[−0.5, −0.2]>, < ℓ, [0.2,0.4] ,[−0.6, −0.3], < ḿ , [0.1,0.6] ,[−0.6, −0.2]> }.

Definition 2.3. A set Š has two BVVSS Ť = ⟨ $\check{\mathcal{G}}^{+}_{\check{T}}$, $\check{\mathcal{G}}^{-}_{\check{T}}$⟩ and $\breve{\mathcal{D}}$ = ⟨ $\check{\mathcal{G}}^{+}_{\mathcal{D}}$, $\check{\mathcal{G}}^{-}_{\mathcal{D}}$⟩. These relationships and operations are defined:

(i) [Ť] ⊂ [$\breve{\mathcal{D}}$] if and only if $\check{\mathcal{G}}^{+}_{\check{T}}$(ù) ≤ $\check{\mathcal{G}}^{+}_{\check{T}}$(ù) and $\check{\mathcal{G}}^{-}_{\check{T}}$(ù) ≥ $\check{\mathcal{G}}^{-}_{\check{T}}$(ù) , ∀ ù∈ $\check{\mathcal{H}}$.

(ii) [Ť] = [$\breve{\mathcal{D}}$] if and only if $\check{\mathcal{G}}^{+}_{\check{T}}$(ù) = $\check{\mathcal{G}}^{+}_{\check{T}}$(ù) and $\check{\mathcal{G}}^{-}_{\check{T}}$(ù) = $\check{\mathcal{G}}^{-}_{\check{T}}$(ù), ∀ ù∈ $\check{\mathcal{H}}$.

(iii) [Ť] ∩ [$\breve{\mathcal{D}}$] = { ⟨ ù, *rmin* ($\check{\mathcal{G}}^{+}_{\check{T}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)), *rmax* ($\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}$(ù)) ⟩ / ù∈ $\check{\mathcal{H}}$ }.

(iv) [Ť] ∪ [$\breve{\mathcal{D}}$] = { ⟨ ù, *rmax* ($\check{\mathcal{G}}^{+}_{\check{T}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)), *rmin* ($\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}^{-}_{\check{T}}$(ù))⟩ / ù∈ $\check{\mathcal{H}}$ }.

Here *rmin* ($\check{\mathcal{G}}^{+}_{\check{T}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)) = [*min* { $t^{+}_{\check{T}}$(ù), $t^{+}_{\mathcal{D}}$(ù) }, *min*{1−$f^{+}_{\check{T}}$(ù), 1−$f^{+}_{\mathcal{D}}$(ù)}],

rmax ($\check{\mathcal{G}}^{+}_{\check{T}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)) = [*max* { $t^{+}_{\check{T}}$(ù), $t^{+}_{\mathcal{D}}$(ù)}, *max* {1−$f^{+}_{\check{T}}$(ù), 1−$f^{+}_{\mathcal{D}}$(ù) }],

rmin ($\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}^{-}_{\check{T}}$(ù)) = [*min* {−1−$f^{-}_{\check{T}}$(ù) , −1−$f^{-}_{\mathcal{D}}$(ù) }, *min* { $t^{-}_{\check{T}}$(ù), $t^{-}_{\mathcal{D}}$(ù)}],

rmax ($\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}^{-}_{\check{T}}$(ù)) = [*max* {{−1−$f^{-}_{\check{T}}$(ù) , − 1−$f^{-}_{\mathcal{D}}$(ù}, *max* { $t^{-}_{\check{T}}$(ù), $t^{-}_{\mathcal{D}}$(ù)}].

Definition 2.4. Consider $\check{\mathcal{G}}$ to be a semiring. If the following requirement are satisfied BVVSS Ť of $\check{\mathcal{G}}$ is defined as a BVVSS of BVVSS

(i) $\check{\mathcal{G}}^{+}_{\check{T}}$(ù + $\grave{v}$) ≥ *rmin*{ $\check{\mathcal{G}}^{+}_{\check{T}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$($\grave{v}$)},

(ii) $\check{\mathcal{G}}$(ù$\grave{v}$) ≥ *rmin*{$\check{\mathcal{G}}$(ù), $\check{\mathcal{G}}^{+}_{\check{T}}$($\grave{v}$)},

(iii) $\check{\mathcal{G}}^{-}_{\check{T}}$(ù + $\grave{v}$) ≤ *rmax* { $\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}^{-}_{\check{T}}$($\grave{v}$)},

(iv) $\check{\mathcal{G}}^{-}_{\check{T}}$(ù$\grave{v}$)≤ *rmax* { $\check{\mathcal{G}}^{-}_{\check{T}}$(ù), $\check{\mathcal{G}}^{-}_{\check{T}}$($\grave{v}$)}, *for all* ù *and* $\grave{v}$ ∈ $\check{\mathcal{G}}$.

Definition 2.5. Let Ť = ⟨ $\check{\mathcal{G}}^{+}_{\check{T}}$, $\check{\mathcal{G}}^{-}_{\check{T}}$⟩ be a BVVSS of U. Then the following translations defined as

$(i)?(\check{T}) = <$ ù, rmin {{[1/2,1/2] $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)}, rmax {[−1/2, −1/2] $\check{\mathcal{G}}^{-}_{\check{T}}$(ù)} >/*for all* ù ∈ *U*}.

$(ii)!(\check{T}) = <$ ù, rmax {{[1/2,1/2] $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)}, rmin {[−1/2, −1/2] $\check{\mathcal{G}}^{-}_{\check{T}}$(ù)} >/*for all* ù ∈ *U*}.

$(iii)\tilde{\mathcal{Q}}_{([\iota^{\pm}_{-},\iota^{\pm}_{+}],[\varrho^{-}_{-},\varrho^{-}_{+}])}(\check{T})$ = < ù, rmin{[$\iota^{\pm}_{-}$, ι^{+}_{+}] $\check{\mathcal{G}}^{+}_{\check{T}}$(ù)}, rmax {[ϱ^{-}_{-}, ϱ^{-}_{+}] $\check{\mathcal{G}}^{-}_{\check{T}}$(ù)} > /*for all* ù ∈ *U*} and [$\iota^{\pm}_{-}$, ι^{+}_{+}] ∈ [0,1] and [ϱ^{-}_{-}, ϱ^{-}_{+}] ∈ [−1,0].

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$(iv)\tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\pm}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathrm{T}}) =<$ ù, rmax$\{[\iota_-^{\pm},\ \iota_+^{\pm}]\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù})\}$, rmin $\{[\ \varrho_-^{-},\ \varrho_+^{-}]\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù})\}$ >

$/for\ all$ ù $\in U\}$ and $[\iota_-^{\pm},\ \iota_+^{\pm}] \in [0,1]$ and $[\ \varrho_-^{-},\ \varrho_+^{-}] \in [-1,0]$.

$(v)\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\pm}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathrm{T}}) = \{ <$ ù, $[\iota_-^{\pm},\ \iota_+^{\pm}]\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}), -[\ \varrho_-^{-},\ \varrho_+^{-}]\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}) >\}$

$/for\ all$ ù $\in U\}$ and $[\iota_-^{\pm},\ \iota_+^{\pm}] \in [0,1]$ and $[\ \varrho_-^{-},\ \varrho_+^{-}] \in [-1,0]$.

Example 2.6.

Let U $= \{\acute{k}, \acute{\ell}, \acute{m}\}$ be a set. Then $\check{\mathrm{T}} = \{<\acute{k},\ [0.5,0.6]\ ,[-0.4,-0.3]>, <\acute{\ell}, [0.1,0.2]\ ,$ $[-0.5,-0.3], <\acute{m}\ , [0.1,0.6]\ ,[-0.6,-0.2]>\}$ is a BVVSS of U. Let $[\iota_-^{\pm},\ \iota_+^{\pm}] =$ $[0.3,0.3]$ and $[\ \varrho_-^{-},\ \varrho_+^{-}]\ =\ [-0.4,-0.4]$.Then

$(i)?\,(\check{\mathrm{T}}) = <\acute{k},\ [0.5,0.5], [-0.4,-0.3]>, <\acute{\ell}, [0.1,0.2]\ ,[-0.5,-0.5], <\acute{m}\ , [0.5,0.5]\ ,$

$[-0.2,-0.1]>\}$.

$(ii)!\,(\check{\mathrm{T}}) =\ <\acute{k},\ [0.5,0.6], [-0.5,-0.5]>, <\acute{\ell}, [0.5,0.5]\ ,[-0.7,-0.6], <\acute{m}\ , [0.6,0.7]\ ,$

$[-0.5,-0.5]>\}$.

$(iii)\,\widetilde{\mathcal{Q}}_{([0.3,0.3]),([-0.2,-0.1])}(\check{\mathrm{T}}) = <\acute{k},\ [0.3,0.3], [-0.4,-0.3]>, <\acute{\ell}, [0.1,0.2]\ ,[-0.4,-0.4],$ $<\acute{m}\ , [0.3,0.3]\ ,[-0.2,-0.1]>\}$.

$(iv)\tilde{\mathcal{P}}_{([0.3,0.3]),([-0.4,-0.4])}(\check{\mathrm{T}})\ = <\acute{k},\ [0.5,0.6], [-0.4,-0.4]>, <\acute{\ell}, [0.3,0.3]\ ,[-0.7,-0.6],$ $<\acute{m}\ , [0.6,0.7]\ ,[-0.4,-0.4]>\}$

$(v)\,\widetilde{\mathcal{G}}_{[0.3,-0.4]}(\check{\mathrm{T}}) =\ <\acute{k},\ [015,0.18], [-0.16,-0.12]>, <\acute{\ell}, [0.03,0.06]\ ,[-0.28,-0.24],$ $<\acute{m}\ , [0.18,0.21]\ ,[-0.08,-0.04]>\}$.

Definition 2.7.

Let $\check{\mathrm{T}} = \langle\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}\rangle$ be a BVVSS of U .Then $^{\circ}\,\check{\mathrm{T}}\ =\ <{}^{\circ}\,\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ {}^{\circ}\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}>$ is defined as $^{\circ}\,\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}) = <$ $\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}),\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+})>$ for all ù in U and $^{\circ}\,\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}) = <\ -\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}),\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-})>$ for all ù in U.

Definition 2.8.

Let $\check{\mathrm{T}} = \langle\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}\rangle$ be a BVVSS of U .Then $^{\Delta}\,\check{\mathrm{T}}\ =\ <\ ^{\Delta}\,\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \ ^{\Delta}\,\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}>$ is defined as $^{\Delta}\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}) = <$ $\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù})/\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+})>$ for all ù in U and $^{\Delta}\ \ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}) = <\ -\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù})/\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-})>$ for all ù in U.

Definition 2.9.

Let $\check{\mathrm{T}} = \langle\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}\rangle$ be a BVVSS of U .Then $^{\oplus}\check{\mathrm{T}}\ \ =\ <\ ^{\oplus}\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \ ^{\oplus}\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}>$ is defined as $^{\oplus}\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}) =\ <$ $\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+}(\text{ù}) + [1] -\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{+})>$ for all ù in U and $^{\oplus}\ \ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}) = <\ -\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}(\text{ù}) + [1] -\ \mathcal{H}(\check{\mathcal{G}}_{\check{\mathrm{T}}}^{-})>$ for all ù in U.

3 BIPOLAR VALUED VAGUE IDEALS:

In this section introduced BVVnis of a semiring with translation and studied their properties.

Definition 3.1.

Let $\mathfrak{P}$ be a semiring. A BVVSS $\check{\check{T}} = \langle\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{+},\ \check{\mathcal{G}}_{\check{\mathrm{T}}}^{-}\rangle$ of $\mathfrak{P}$ is said to be a BVVI of $\mathfrak{P}$ if the following conditions are satisfied,

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(i) $\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v}) \geq rmin\{\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$,

(ii) $\check{\mathcal{G}}(\grave{u}\grave{v}) \geq rmax\{\check{\mathcal{G}}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$,

(iii) $\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v}) \leq rmax\{\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}$,

(iv) $\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}\grave{v}) \leq rmin\{\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}$, *for all* ù *and* $\grave{v} \in \mathfrak{P}$

Definition 3.2.

Let $\mathfrak{P}$ be a semiring. A BVVI $\check{\check{T}} = \langle \check{\mathcal{G}}_{\Upsilon}^{+}, \check{\mathcal{G}}_{\Upsilon}^{-}\rangle$ of $\mathfrak{P}$ is said to be a BVVNI of $\boldsymbol{\mathcal{R}}$ if the following conditions are satisfied,

(i) $\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v}) = \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v}\grave{u})$

(ii) $\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}\grave{v}) = \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v}\grave{u})$ for all ù and $\grave{v}$ in $\mathfrak{P}$

Theorem 3.3

If $\check{\Upsilon} = \langle \check{\mathcal{G}}_{\Upsilon}^{+}, \check{\mathcal{G}}_{\Upsilon}^{-}\rangle$ is a BVVI of a semiring $\mathfrak{P}$, then

(i) $?(\check{\Upsilon}) = \langle ?\check{\mathcal{G}}_{\Upsilon}^{+}, ?\check{\mathcal{G}}_{\Upsilon}^{-}\rangle$ is a BVVI of $\mathfrak{P}$.

(ii) $!(\check{\Upsilon}) = \langle !\check{\mathcal{G}}_{\Upsilon}^{+}, !\check{\mathcal{G}}_{\Upsilon}^{-}\rangle$ is a BVVI of $\mathfrak{P}$.

(iii) $\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm}, \iota_{+}^{\pm}],[\varrho_{-}^{-}, \varrho_{+}^{-}])}(\check{\Upsilon})$ is a BVVI of $\mathfrak{P}$.

(iv) $\tilde{\mathcal{P}}_{([\iota_{-}^{\pm}, \iota_{+}^{\pm}],[\varrho_{-}^{-}, \varrho_{+}^{-}])}(\check{\Upsilon})$ is a BVVI of $\mathfrak{P}$.

(v) $\tilde{\mathcal{G}}_{([\iota_{-}^{\pm}, \iota_{+}^{\pm}],[\varrho_{-}^{-}, \varrho_{+}^{-}])}(\check{\Upsilon})$ is a BVVI of $\mathfrak{P}$.

(vi) $!(?(\check{\Upsilon})) = ?(!(\check{\Upsilon})) =< [1/2,1/2], [-1/2,-1/2] >$ is also a BVVI of $\mathfrak{P}$.

Proof.

(i) For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v}) = \text{rmin}\{[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v})\} \geq rmin\{[1/2,1/2], \text{rmin}\{\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmin\{rmin\,[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u})\}, \text{rmin}\{[1/2,1/2]\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmin\{?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\} \Rightarrow ?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v}) \geq rmin\{?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For

every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v}) = \text{rmin}\{[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v})\} \geq rmin\{[1/2,1/2], \text{rmax}\{\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = \{rmax\{rmin\,[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u})\}, \text{rmin}\{[1/2,1/2]\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmax\{?\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), ?\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\} \Rightarrow ?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v}) \geq rmax\{?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v}) = \text{rmax}\{[-1/2,-1/2], \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v})\} \leq rmax\{[-1/2,-1/2], \text{rmax}\{\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}\} = rmax\{rmax\,[-1/2,-1/2], \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u})\}, \text{rmax}\{[-1/2,-1/2]\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}\} = rmax\{?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\} \Rightarrow ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v}) \geq rmax\{?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}\grave{v}) = \text{rmax}\{[-1/2,-1/2], \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}\grave{v})\} \leq rmax\{[-1/2,-1/2], \text{rmin}\{\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}\} = \{rmin\{rmax\,[-1/2,-1/2], \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u})\}, \text{rmax}\{[-1/2,-1/2]\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}\} = rmin\{?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\} \Rightarrow ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}\grave{v}) \geq rmin\{?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}), ?\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$.

(ii) For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v}) = \text{rmax}\{[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v})\} \geq rmax\{[1/2,1/2], \text{rmax}\{\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmin\{rmax\,[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u})\}, \text{rmax}\{[1/2,1/2]\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmin\{!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\} \Rightarrow !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}+\grave{v}) \geq rmax\{!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v}) = \text{rmax}\{[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v})\} \geq rmax\{[1/2,1/2], \text{rmax}\{\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = \{rmax\{rmax\,[1/2,1/2], \check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u})\}, \text{rmax}\{[1/2,1/2]\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}\} = rmax\{!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\} \Rightarrow !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}\grave{v}) \geq rmax\{!\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{u}), !\,\check{\mathcal{G}}_{\Upsilon}^{+}(\grave{v})\}$, $\forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$, $!\,\check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v}) = \text{rmin}\{[-1/2,-1/2], \check{\mathcal{G}}_{\Upsilon}^{-}(\grave{u}+\grave{v})\} \leq$

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$rmin\{[-1/2,-1/2], \text{rmax}\{\check{\mathcal{G}}_{Ŧ}^{-}(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = rmax\{rmin[-1/2,-1/2], \check{\mathcal{G}}_{Ŧ}^{-}(ù)\}, \text{rmin}\{[-1/2,-1/2]\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = rmax\{!\,\check{\mathcal{G}}_{Ŧ}^{-}(ù), !\,\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\} \Rightarrow !\,\check{\mathcal{G}}_{Ŧ}^{-}(ù+\grave{v}) \leq rmax\{!\,\check{\mathcal{G}}_{Ŧ}^{-}(ù), !\,\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. For every $ù, \grave{v}$ *in* $\mathfrak{P}$, $!\,\check{\mathcal{G}}_{Ŧ}^{-}(ù\grave{v}) = \text{rmin}\{[-1/2,-1/2], \check{\mathcal{G}}_{Ŧ}^{-}(ù\grave{v})\} \leq rmin\{[-1/2,-1/2], \text{rmin}\{\check{\mathcal{G}}_{Ŧ}^{-}(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = \{rmin\{rmin[-1/2,-1/2], \check{\mathcal{G}}_{Ŧ}^{-}(ù)\}, \text{rmin}\{[-1/2,-1/2]\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = rmin\{!\,\check{\mathcal{G}}_{Ŧ}^{-}(ù), !\,\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\} \Rightarrow !\,\check{\mathcal{G}}_{Ŧ}^{-}(ù\grave{v}) \leq rmin\{!\,\check{\mathcal{G}}_{Ŧ}^{-}(ù), !\,\check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$.

(iii) For every $ù, \grave{v}$ *in* $\mathfrak{P}$, $\iota_{-}^{\pm}, \iota_{+}^{\ddagger}$ in [0,1] and $\varrho_{-}^{-}, \varrho_{+}^{\mp}$ in [-1,0], $\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) = \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù+\grave{v})\} \geq \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \text{rmin}\{\check{\mathcal{G}}_{Ŧ}^{+}(ù), \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} = \text{rmin}\{\text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} \geq rmin\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v}) \Rightarrow \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) \geq rmin\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù\grave{v}) = \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù\grave{v})\} \geq \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \text{rmax}\{\check{\mathcal{G}}_{Ŧ}^{+}(ù), \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} = \text{rmax}\{\text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} \leq rmax\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\} \Rightarrow \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) \geq rmax\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. For every $ù, \grave{v}$ *in* $\mathfrak{P}$, $\iota_{-}^{\pm}, \iota_{+}^{\ddagger}$ in [0,1] and $\varrho_{-}^{-}, \varrho_{+}^{\mp}$ in [-1,0], $\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v}) = \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(ù+\grave{v})\} \leq \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \text{rmax}\{\check{\mathcal{G}}_{Ŧ}^{-}(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = \text{rmax}\{\text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(ù)\}, \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} \leq rmax\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v}) \leq rmax\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù\grave{v}) = \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(ù\grave{v})\} \leq \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \text{rmin}\{\check{\mathcal{G}}_{Ŧ}^{-}(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = \text{rmin}\{\text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(ù)\}, \text{rmax}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} \leq rmin\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v}) \leq rmin\{\tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{Q}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$.

(iv) For every $ù, \grave{v}$ *in* $\mathfrak{P}$. $\iota_{-}^{\pm}, \iota_{+}^{\ddagger}$ in [0,1] and $\varrho_{-}^{-}, \varrho_{+}^{\mp}$ in [-1,0], $\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) = \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}]\,(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v})\} \geq \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \text{rmin}\,(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} = \text{rmin}\{\text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} \geq rmin\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\} \Rightarrow \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) \geq rmin\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù\grave{v}) = \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}]\,(\check{\mathcal{G}}_{Ŧ}^{+})(ù\grave{v})\} \geq \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \text{rmax}\,(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} = \text{rmax}\{\text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, \text{rmax}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}\} \geq rmax\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\} \Rightarrow \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù\grave{v}) \geq rmax\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. For every $\iota_{-}^{\pm}, \iota_{+}^{\ddagger}$ in [0,1] and $\varrho_{-}^{-}, \varrho_{+}^{\mp}$ in [-1,0], $\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v}) = \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}]\,(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v})\} \leq \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \text{rmax}\,(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = \text{rmax}\{\text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} \leq rmax\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù+\grave{v}) \leq rmin\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù\grave{v}) = \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}]\,(\check{\mathcal{G}}_{Ŧ}^{-})(ù\grave{v})\} \leq \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \text{rmin}\,(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} = \text{rmin}\{\text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(ù)\}, \text{rmin}\{[\varrho_{-}^{-}, \varrho_{+}^{\mp}], \check{\mathcal{G}}_{Ŧ}^{-}(\grave{v})\}\} \leq rmin\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù\grave{v}) \leq rmin\{\tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(ù), \tilde{\mathcal{P}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{-})(\grave{v})\}, \forall\, ù, \grave{v} \in \mathfrak{P}$.

(v) For every $ù, \grave{v}$ *in* $\mathfrak{P}$, $\iota_{-}^{\pm}, \iota_{+}^{\ddagger}$ in [0,1] and $\varrho_{-}^{-}, \varrho_{+}^{\mp}$ in [-1,0], $\tilde{\mathcal{G}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) = [\iota_{-}^{\pm}, \iota_{+}^{\ddagger}]\,(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) \geq [\iota_{-}^{\pm}, \iota_{+}^{\ddagger}](\text{rmin}\{\check{\mathcal{G}}_{Ŧ}^{+}(ù), \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\}) = \text{rmin}\{[\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(ù)\}, [\iota_{-}^{\pm}, \iota_{+}^{\ddagger}], \check{\mathcal{G}}_{Ŧ}^{+}(\grave{v})\} \geq rmin\{\tilde{\mathcal{G}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù), \tilde{\mathcal{G}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(\grave{v})\} \Rightarrow \tilde{\mathcal{G}}_{([\iota_{-}^{\pm},\iota_{+}^{\ddagger}],[\varrho_{-}^{-},\varrho_{+}^{\mp}])}(\check{\mathcal{G}}_{Ŧ}^{+})(ù+\grave{v}) \geq$

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$rmin\,\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{v})\}, \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}\,\grave{v}) = [\iota_-^{\pm}, \iota_+^{\ddagger}]\;(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}\,\grave{v}) \geq [\iota_-^{\pm}, \iota_+^{\ddagger}](rmax\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}), \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}) = rmax\{[\iota_-^{\pm}, \iota_+^{\ddagger}], \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\},[\iota_-^{\pm}, \iota_+^{\ddagger}], \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} \geq rmax\,\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{v})\} \Rightarrow \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}\,\grave{v}) \geq rmax\,\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{+})(\grave{v})\}, \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. For every $\grave{u}, \grave{v}$ *in* $\mathfrak{P}$ $\iota_-^{\pm}, \iota_+^{\ddagger}$ in [0,1] and $\varrho_-^{-}, \varrho_+^{-}$ in [-1,0], $\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u} + \grave{v}) = -[\varrho_-^{-}, \varrho_+^{-}]\;(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u} + \grave{v}) \leq -[\varrho_-^{-}, \varrho_+^{-}]\;(rmax\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}), \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}) = rmax\{-[\varrho_-^{-}, \varrho_+^{-}], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\},[\iota_-^{\pm}, \iota_+^{\ddagger}], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} \leq rmax\,\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u} + \grave{v}) \leq rmax\,\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{v})\}, \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. And $\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}\,\grave{v}) = -[\varrho_-^{-}, \varrho_+^{-}]\;(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}\,\grave{v}) - [\varrho_-^{-}, \varrho_+^{-}]\;(rmin = \{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}), \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\})\; rmin\{-[\varrho_-^{-}, \varrho_+^{-}], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\}, -[\varrho_-^{-}, \varrho_+^{-}], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} \leq rmin\;\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{v})\} \Rightarrow \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}\,\grave{v}) \leq rmin\;\{\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{u}), \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{\mathcal{G}}_{\check{T}}^{-})(\grave{v})\}, \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$.

(vi) For every $\grave{u}$ *in* $\mathfrak{P}$, $?\,\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) = rmin\{[1/2,1/2], \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\} \leq [1/2,1/2]$ and $!\,\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) = rmax\{[1/2,1/2], \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\} \geq [1/2,1/2]$ so $!(?\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})) = ?(!\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})) = [1/2,1/2]$ And $?\,\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}) = rmax\{[1/2,1/2], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\} \geq [1/2,1/2]$ and $!\,\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) = rmin\{[1/2,1/2], \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\} = [1/2,1/2]$ so $!(?\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})) = ?(!\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})) \leq [1/2,1/2]$. $!(?(\check{T})) = ?(!(\check{T})) = <[1/2,1/2],[-1/2,-1/2]>$ is also a BVVI of $\mathfrak{P}$.

Theorem 3.4

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+}, \check{\mathcal{G}}_{\check{T}}^{-}\rangle$ and $\check{S} = \langle \check{\mathcal{G}}_{\check{S}}^{+}, \check{\mathcal{G}}_{\check{S}}^{-}\rangle$ are BVVI s of a semiring $\mathfrak{P}$, then

(i) $?(\check{T} \cap \check{S}) = ?(\check{T}) \cap ?(\check{S})$ is also a BVVI of $\mathfrak{P}$;

(ii) $!(\check{T} \cap \check{S}) = !(\check{T}) \cap !(\check{S})$ is also a BVVI of $\mathfrak{P}$;

(iii) $\tilde{\mathcal{Q}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T} \cap \check{S}) = \tilde{\mathcal{Q}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T}) \cap \tilde{\mathcal{Q}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{S})$ is a BVVI of $\mathfrak{P}$. $?\,\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u} + \grave{v})$ is also a BVVI of $\mathfrak{P}$;

(iv) $\tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T} \cap \check{S}) = \tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T}) \cap \tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{S})$ is a BVVI of $\mathfrak{P}$;

(v) $\tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T} \cap \check{S}) = \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T}) \cap \tilde{\mathcal{G}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{S})$ is a BVVI of $\mathfrak{P}$;

(vi) $\tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\tilde{\mathcal{Q}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T}) = \tilde{\mathcal{Q}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}\;(\tilde{\mathcal{P}}_{([\iota_-^{\pm},\iota_+^{\ddagger}],[\varrho_-^{-},\varrho_+^{-}])}(\check{T}))$ is also a BVVI of $\mathfrak{P}$.

Proof.

The proof follows from the Theorems 3.2 and 3.3.

Theorem 3.5

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+}, \check{\mathcal{G}}_{\check{T}}^{-}\rangle$ is a BVVI of a semiring $\check{T}$, then ${}^{\oplus}\check{T} = <{}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}, {}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{-}>$ is a BVVI of the semiring $\mathfrak{P}$.

Proof.

Let $\grave{u}$ *and* $\grave{v}$ *in* $\mathfrak{P}$. Now ${}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u} + \grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u} + \grave{v}) + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq rmin\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}), \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) = rmin\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}), \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} = rmin\left\{{}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}), {}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\right\} \Rightarrow {}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u} + \grave{v}) \geq rmin\{{}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}), {}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\; \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. And ${}^{\oplus}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u} + \grave{v}) + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq rmax\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}), \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) = rmax\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}), \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} =$

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rmax$\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} \Longrightarrow {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) \geq$ rmax$\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. Also ${}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) = \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) + [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) =$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}) - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} =$ rmax$\left\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\right\} \Longrightarrow {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) \leq$ rmax $\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. And ${}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}\,\grave{v}) = \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) =$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}) - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} - [1] - \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} =$ rmin$\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. $\Longrightarrow {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}\,\grave{v}) \leq$ rmin$\{{}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\oplus}\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$.

Theorem 3.6

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+},\ \check{\mathcal{G}}_{\check{T}}^{-} \rangle$ is a BVVI of a semiring $\grave{\mathbb{S}}$, then ${}^{\circ}\check{T} = < {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+},\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-} >$ is a BVVI of the semiring $\grave{\mathbb{S}}$.

Proof.

Let $\grave{u}$ *and* $\grave{v}$ *in* $\acute{\mathbb{S}}$. Now ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+} =$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} =$ rmin$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} \Longrightarrow {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v}) \geq \{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. And ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq$ rmax$\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) =$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} =$ rmax$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\} \Longrightarrow {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) \geq$ rmax$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. Also ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) = -\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq -$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) =$ rmax $\{-\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}),-\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} =$ rmax$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} \Longrightarrow {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v}) \leq$ rmax$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. And ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}\,\grave{v}) = -\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq -$rmin$\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) =$ rmin $\{-\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}),-\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\,\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} =$ rmin$\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\} \Longrightarrow {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}\,\grave{v}) \leq$ rmin $\{{}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. Hence ${}^{\circ}\check{T}$ is a BVVI of a $\grave{\mathbb{S}}$.

Theorem 3.7

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+},\ \check{\mathcal{G}}_{\check{T}}^{-} \rangle$ is a BVVI of a semiring $\grave{\mathbb{S}}$, then ${}^{\circ}\check{T} = < {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+},\ {}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-} >$ is a BVVI of the semiring $\grave{\mathbb{S}}$.

(i) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) < [1]$, then ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{+} < \check{\mathcal{G}}_{\check{T}}^{+}$;

(ii) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) > [1]$, then ${}^{\circ}\check{\mathcal{G}}_{\check{T}}^{-} > \check{\mathcal{G}}_{\check{T}}^{-}$;

(iii) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) < [1]$ and $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) > [1]$, then ${}^{\circ}\check{T} < \check{T}$.

Proof.

(i), *(ii)* and (iii) are trivial.

Theorem 3.8

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+},\ \check{\mathcal{G}}_{\check{T}}^{-} \rangle$ is a BVVI of a semiring $\grave{\mathbb{S}}$, then ${}^{\Delta}\check{T} = < {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{+},\ {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-} >$ is a BVVI of the semiring $\grave{\mathbb{S}}$.

Proof.

Let $\grave{u}$ *and* $\grave{v}$ *in* $\acute{\mathbb{S}}$. Now ${}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ /\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) =$ rmin $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} =$ rmin$\left\{{}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\right\} \Longrightarrow {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}\ (\grave{u}+\ \grave{v}) \geq$ rmin$\{{}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. *And* ${}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) = \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) \geq$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ /\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) =$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}),\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+})\} =$ rmax $\left\{{}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\right\}\ {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}+\ \grave{v}) \geq$ rmax$\{{}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}),\ {}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\}\ \forall\ \grave{u},\grave{v} \in \mathfrak{P}$. Also ${}^{\Delta}\ \check{\mathcal{G}}_{\check{T}}^{-}\ (\grave{u}+\ \grave{v}) = -\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\ \grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq$ rmax $\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}$

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$/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) = \text{rmax}\,\{-\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}) \quad /\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}), -\check{\mathcal{G}}_{\check{T}}^{+}(\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} = \text{rmax}\left\{{}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\right\} \Rightarrow {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\grave{v}) \leq$ $\text{rmax}\{{}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. *And* ${}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{+}(\grave{u}\,\grave{v}) = -\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}\,\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) \leq \text{rmin}\,\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}$ $/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) = \text{rmin}\,\{\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}),\ \check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\,/\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-})\} = \text{rmin}\left\{{}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\right\} \Rightarrow {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}+\grave{v}) \leq$ $\text{rmin}\{{}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{u}),\ {}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-}(\grave{v})\}\ \forall\, \grave{u}, \grave{v} \in \mathfrak{P}$. ${}^{\Delta}\check{T}$ is a BVVI of $\check{\mathbb{S}}$.

Theorem 3.9

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+}, \check{\mathcal{G}}_{\check{T}}^{-}\rangle$ is a BVVI of a semiring $\check{\mathbb{S}}$, then

(i) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) < [1]$, then ${}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{+} < \check{\mathcal{G}}_{\check{T}}^{+}$;

(ii) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) > [1]$, then ${}^{\Delta}\check{\mathcal{G}}_{\check{T}}^{-} < \check{\mathcal{G}}_{\check{T}}^{-}$;

(iv) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) < [1]$ and $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) > [1]$, then ${}^{\Delta}\check{T} > \check{T}$;

(v) If $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{+}) < [1]$ and $\grave{\mathcal{H}}(\check{\mathcal{G}}_{\check{T}}^{-}) > [1]$, then ${}^{\Delta}\check{T}$ is a normal BVVI of $\check{T}$.

Proof.

(i), (*ii*), (iii) and (iv) are trivial.

Theorem 3.10

If $\check{T} = \langle \check{\mathcal{G}}_{\check{T}}^{+}, \check{\mathcal{G}}_{\check{T}}^{-}\rangle$ be a BVVI of a semiring $\check{\mathbb{S}}$, then

(*i*) $\check{T} = \check{T}$, (*ii*) ${}^{\Delta}\check{T} = \check{T}$.

Proof.

It can be easily proved.

4 CONCLUSION

The concept Properties of BVVNIs of a semiring is discussed in this section and BVVNIs of a semiring with translation have been introduced. These ideas are applied to further research in the creation of BVVSSs . As a result, our upcoming research will examine some of the qualities based on the idea of homomorphism in BVVI.

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