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Optimal Inventory Control with Discount-Driven Special Orders: A Three-Phase Approach

Babitha. N[1], T.P.M. Pakkala[2], Ashwini Kumari[3], Mohankumari C[4],
[1]*Department of Statistics, St. Joseph's University, Bangalore-560027*
Karnataka. India
babitha@sju.edu.in
[2]*Department of Statistics, Mangalore University, Mangalagangothri-574199. India tpm.pakkala@gmail.com*
[3]*Assistant Professor, Department of Mathematics, Nitte (Deemed to be University), NMAM Institute of Technology, Nitte- 574110, Udupi Dist., Karnataka, India*
[4]*Assistant Professor, Department of Statistics and Analytics, Mount Carmel College, Autonomous, Bengaluru, Karnataka, India.*
Mail Id: mohankumari@mccblr.edu.in

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ABSTRACT

Instead of regular orders, special orders are placed whenever price change occurs in the inventory policies. When price reduction is available to the retailer, the objective of the retailer is to get a maximum benefit from the orders placed under discount. In order to get rid of excess stock, wholesaler offers discount to the retailers. Quite often the discount will give a minimum duration. Beyond this duration, in order to sell the excess stock the discount will be given until the "Supplies last". This leads to the division of the planning horizon into three parts. Optimal policies have been discussed under deterministic, stochastic and no discount durations. This method has a stronger economic interpretation in terms of optimality as the nature of optimal orders will be different in three different situations. Since the determination of the orders placed during the stochastic period of discount is not straight forward, attention need to be given to determine the optimal policies, unlike earlier methods. A sensitivity analysis is carried out with respect to various parameters to determine optimal order levels and reorder levels.

Keywords: Temporary sales, Special order, Minimum guaranteed period, Discount price, Optimal ordering policy.

1. INTRODUCTION

Many variations of special offers of purchase prices have been modelled under inventory policies. It is quite common that the wholesaler offers discounts on purchase price. Due to unavoidable reasons either excess stock is produced or stock produced is unsold for certain reason. Recently this has happened due to lockdown on account of COVID 19. In order to get rid of excess stock, wholesaler offers discount to the retailers, further, to have good supply chain relation between the wholesaler and the retailer, wholesaler will give a minimum duration of discount. Beyond this guaranteed minimum duration excess stock may still remain. Wholesaler decides to offer the discount till the "Supplies last". It is not possible to determine the exact duration of the discount. Similar to this, many special offers provided do not have fixed duration of length, however it is certain that the offer will last for a certain minimum duration. In this work, the discount duration has two components, namely, guaranteed minimum duration and the remaining duration is which is random. The second segment of the duration of the discount, based on a judgment or estimation, could be modeled through a probability distribution function.

Modelling of the similar situation is found in Arcelus F.J. et al. (2003). However, during second segment of random discount duration the policy given was with equal order sizes. Babitha and Pakkala T.P.M (2014) have established that equal order sizes during this period are suboptimal. The first segment of discount duration which is guaranteed minimum duration is called as deterministic duration and the next segment of random duration is called as stochastic duration. When price reduction is given to the retailer, the objective is to get maximum benefit from special orders that is different from regular orders. Optimal policies have been discussed under deterministic and stochastic discount durations. Many articles have discussed regarding temporary sales. To mention a few, initially price change problem was studied by Naddor in 1966. Later on various solutions are given by Lev and Weiss (1990),Goyal (1990),Goyal and Gupta (1990) and Datta T.K and Pal. A.K (1991). Arcelus F.J and Srinivasan G. (1995, and 1998), presented a paper on a profit maximizing retailer's decision model when a vendor offers temporary sale at a reduced price. Random duration of discount is discussed by Arcelus F.J et al. (2001, 2003, 2006 and 2008) and developed a profit maximizing retailer's decision model when a vendor offers temporary sale at a reduced price incorporating random duration of discount. Pakkala T.P.M and Babitha (2013) considered a common practical situation of unknown ending date of the discount in their paper. The optimal order sizes and reorder sizes are determined for a general probability distribution for the discount duration, which was varying as the probability of residual duration of discount are different. Optimal orders are based on the combination of first single period and it is embedded with remaining discount period. A dynamic programming solution to the model is suggested.

Babitha and Pakkala T.P.M (2014) developed optimal inventory policies under temporary discount based on a functional relationship between order levels and hazard rate function and also mean residual life function. Arcelus. F.J (2003) et al., examined the response of a retailer when vendor offers temporary price discount, here the length of discount is divided into two parts. They developed profit maximizing policies by considering both deterministic period and stochastic period of price discount when the total duration of discount is uncertain. However effect of changing residual probability is not taken into consideration in this work. Reviews of most of the papers on price reduction can be seen in ArcelusF.J et al. (2003).

Dynamic pricing and inventory management are inherently interlinked, with price-sensitive demand influencing optimal ordering policies. Several studies have explored inventory replenishment strategies under fluctuating prices. Chen et al. (2021) analyzed how retailers adjust their order quantities in response to temporary discounts and identified strategies for maximizing profit margins. Tang and Yin (2018) examined the impact of dynamic pricing on perishable goods inventory and found that incorporating price elasticity into order decision models enhances profitability. Zhao et al. (2020) developed an optimal inventory control model under random price discounts, demonstrating that a mixed strategy of regular and special orders minimizes costs. Huang et al. (2021) conducted a sensitivity analysis on order quantities based on price fluctuations, showing that minor price changes significantly impact profit margins. Zhang and Luo (2022) explored the role of supplier reliability in special order decisions and found that inconsistent discount offerings lead to increased inventory risks. Choi and Park (2023) studied how lead time variability affects ordering policies under discount pricing, concluding that shorter lead times enhance the benefits of special orders.

In this paper, an optimal inventory policy is determined by considering deterministic, stochastic and regular durations. Since the deterministic period will have identical structure for all orders within this period, equal orders are optimal. However, in the stochastic period as the residual probability of closure of the discount differs among orders, equal orders are not optimal. During the regular period orders are placed based on the known results.

2. MODEL ASSUMPTION AND FORMULATION

Initially at the starting point of time, a special offer is announced by the vendor for unknown termination date, which includes a minimum guaranteed duration of fixed length (deterministic) and extended for some random duration. An infinite planning horizon is considered and it is divided into three parts. The first is deterministic duration, in which discount will remain surely, the next one is the uncertain duration (stochastic) of discount and the last one is the regular price period. The objective is to determine the number of optimal orders during the planning horizon. Since the procedure of obtaining the optimal order levels in these three periods is different because of economic implications, the planning horizon is divided into three parts.

Finding the number of orders in the deterministic case is straight forward by Schwarz (1972). Hence the optimal order sizes during the deterministic period can be obtained using this method. The problem faced during stochastic period is two-fold. The first one being order levels need not remain same and the second is the number of decision variable is random. Lev and Weiss(1990) gave a procedure to determine optimal order levels when there is a change

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in price. During the regular price period EOQ can be determined easily. Combination of all these is to be worked out in order to give a complete policy for the planning horizon.

Assumptions:

a) Deterministic(constant) demand
b) Lead time is zero
c) Shortages are not allowed

Reason for considering deterministic demand is justified in Arcelus F.J.et. al., (2003). If the planning horizon is long enough so as to have sufficient large number of orders during regular price period then EOQ is applicable during this period. Otherwise we can obtain optimal order sizes using method mentioned in Schwarz (1972).

2.1. Notations

The nomenclature in this paper is as follows:

R- Annual demand rate.

c- Purchasing price per unit.

d- Unit price discount.

(c-d)- Discounted price.

K- Fixed ordering or setup cost per replenishment.

P- Selling price per unit.

F- Holding cost per unit price.

r_i- Re order points (i=0, 1,2...) in units.

Q_m^*- Optimal order quantity under reduced price for deterministic(minimum guaranteed) period.

T_1- Minimum guaranteed duration.

X- Random discount period.

T_2- Point at which discount closes, T_1+X.

Q_i- Optimal order quantity (i= 1, 2,3, . . .)under reduced price for stochastic period.

G(x)- Cumulative distribution function of X.

g(x)- Density function of X.

Q_0- Order quantity under regular price.

H- Yearly profit when discount is off.

L- Planning horizon.

φ_i- Expected profit with discount in deterministic case (i=d) and stochastic case (i=s_1, s_2, s_3...).

Ψ_i- Total conditional profit obtained by combining discount duration and post discount duration (i=0,1, 2, 3...).

2.2 Ordering Strategy

A special offer starts and it is assumed that at this time point the inventory is zero. The retailer have a sure chance to place orders under discount price till minimum guaranteed period T_1. The period from [0, T_1] is the deterministic period, T_1 onwards until discount is on the period is called stochastic period, that is [T_1, T_2].

The optimal decision variables are chosen based on the following two policies that gives maximum profit.

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Policy 1:

1.a) During the deterministic period equal order sizes are placed whenever inventory level reaches zero so that at the end of this period, T_1, inventory depletes. At T_1 an order up to Q_1 is placed.

1.b) In the stochastic period order up to Q_{i+1} is placed whenever inventory level reaches $r_i \geq 0$, i=1,2,....

1.c) During discount is off, order levels of size Q_0 are placed whenever inventory level reaches zero.

Policy 2:

2.a) During the deterministic period equal order sizes are placed whenever inventory level reaches zero and the inventory level at T_1 is positive.

2.b) In the stochastic period order up to Q_{i+1} is placed whenever inventory level reaches $r_i \geq 0$, i=0, 1, 2, ...

2.c) same as 1.c)

The retailers ordering strategy is explained in the figure given below.

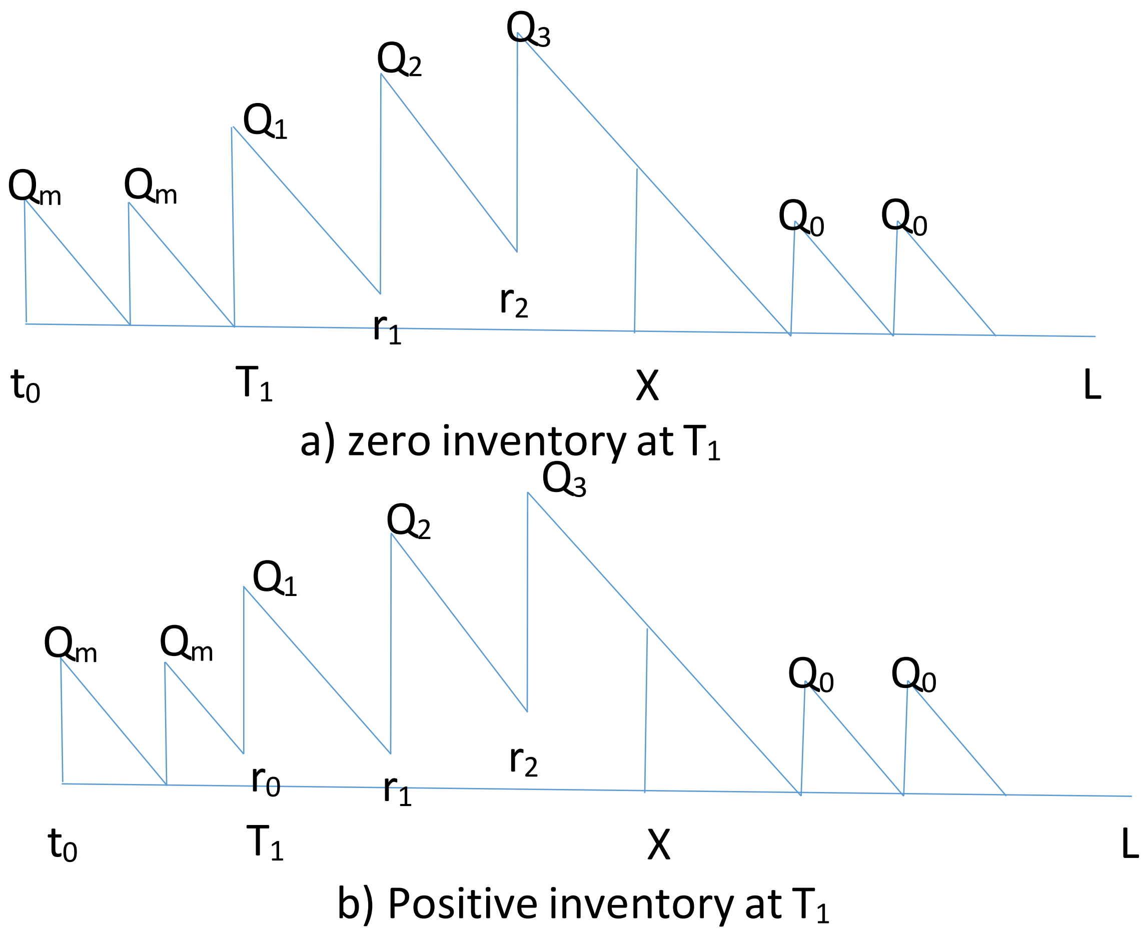

Fig 1: Ordering strategy

3. RETAILER'S OBJECTIVE

The main objective is to determine optimal order quantities and re order quantities to maximize the profit.

3.1 Model formulation for Policy1

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At the starting point of time, the inventory level is assumed to zero. In this policy, discount is on, from starting point to until T1 surely. The number of special orders placed in this period is based on the following property.

Property 1: Equal orders of same size need to be placed during 0 to T1.

Optimal number of orders n of same sizes are placed during minimum guaranteed period, the expression for n is given by Schwarz (1972). That is

$$n = \sqrt{\frac{RcT_1^{\ 2}}{2K} + \frac{1}{4}} - \frac{1}{2} \qquad (1)$$

Once the number of orders are obtained then the optimal quantity are determined using the following relation,

$$Q_m^{\ *} = \frac{RT_1}{n} \qquad (2)$$

Using (1) and (2), the profit φ_d for single period in deterministic period is calculated. That is,

$$\varphi_d = (P - c + d)Q_m - \frac{(c-d)FQ_m^{\ 2}}{2R} - K \qquad (3)$$

Since the orders placed are of same size, the expected profit over the minimum guaranteed duration is obtained just by multiplying n to the above profit function. In this policy the inventory level is zero at the end of minimum guaranteed duration, hence at T1 an order up to level Q1 is placed which is determined optimally.

After T1, the discount may close at any time and assumed to be at random point and the length of the period is uncertain. Based on past experience, retailer can estimate trend or probability distribution of the remaining duration of discount hence a probability distribution for the period of discount beyond T_1 is assumed to be known. One an order is already placed at T_1, further such special orders may be placed in the stochastic period. Stochastic period starts from $T_1$and ends at T_2. After T_1 next order under discount is placed when inventory level reaches r_1, if the discount is on at this point. Profit over the interval starting from T_1 with zero initial inventory (r_0=0) until inventory level reaches r_1 is given by,

$$\varphi_{s1} = P(Q_1 - r_1) - (c-d)(Q_1 - r_0) - \frac{(c-d)F(Q_1 - r_1)^2}{2R} - K \qquad (4)$$

If the discount is off when the inventory reaches r1 then wait till the inventory depletes to zero. If discount is on at any reorder point ri-1 in the stochastic period, orders of size Qi are placed at this point i=2,3,... Profit over such order cycles are given by,

$$\varphi_{si} = P(Q_i - r_i) - (c-d)(Q_i - r_{i-1}) - \frac{(c-d)F(Q_i - r_i)^2}{2R} - K \quad i = 2,3,... \qquad (5)$$

If discount closes during ith replenishment cycle in the stochastic period, that is, after placing i orders (including the one that is placed at T1) discount ends before the next planned order then the following expression gives incremental profit from the last reorder point till inventory level reaches zero, which is not considered in the regular terms.

$$\xi_i = \Pr_i - \frac{r_i^{\ 2}(c-d)F}{2R} \quad \text{for } i = 1,2,3,...... \qquad (6)$$

Once the discount is over we assume zero reorder level and EOQ are ordered. During this regular price period the optimal inventory level is determined according to Wilson's EOQ model. However, if the planning horizon is short then we can make use of equal order sizes and order sizes can be obtained by finding optimal number of orders over the planning horizon based on the relation given by Schwarz (1972).

The EOQ Q0 is ordered by paying fixed charge K per order, purchase price is c and maintenance cost is F. The demand

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R is satisfied when the units are sold at price P. The yearly profit is calculated using the following expression

$$H = (P-c)R - \frac{cFQ_0}{2} - \frac{KR}{Q_0}$$

$$= (P-c)R - cFQ_0 \tag{7}$$

$$where \quad Q_0 = \sqrt{\frac{2RK}{cF}}$$

Expected profit:

The conditional profit for placing an ith order, before discount is closed in stochastic period is,

$$\psi_i = n\varphi_d + \sum_{j=1}^{i}\varphi_j + \xi_i + H\left(L - \frac{nQ_m}{R} - \left\{\sum_{j=1}^{i}\frac{Q_i - r_i}{R}\right\} - \frac{r_i}{R}\right)$$

$$where \quad \xi_i = \Pr_i - \frac{r_i^2(c-d)F}{2R}$$

Therefore the profit according to Policy 1 is obtained using the following expression. That is,

$$\psi = n\varphi_d + \sum_{i=1}^{\infty}P(N=i)\left[\sum_{j=1}^{i}\varphi_j + \xi_i + H\left(L - \frac{nQ_m}{R} - \left\{\sum_{j=1}^{i}\frac{Q_i - r_i}{R}\right\} - \frac{r_i}{R}\right)\right] \tag{8}$$

$$with \quad P(N=i) = \int_{t_{i-1}}^{t_i} g(x)dx \quad , \quad t_i = \left[\frac{Q_1 - r_1 + Q_2 - r_2 + ...Q_i - r_i}{R}\right]$$

Though the range of the distribution of discount is infinity, but practically the number is very small. It is because, as time passes the probabilities becomes negligible. In the several numerical study carried out it is observed that a maximum number of orders under discount is 4.

3.2. Model formulation for Policy 2

In Policy 2, we consider the situation that at T1 the inventory level is positive, and it is Ib. No order is placed at T1. It is placed when inventory level is r0, if discount is on at r0, where r0 is a decision variable. If discount is off at r0, no special order is placed in stochastic period.

In order to determine number of orders in the deterministic period, we consider the interval 0 to (T1+Ib)/R for deterministic period and number of optimal orders is determined by the relation given by Schwarz (1972) that is,

$$n = \sqrt{\frac{Rc\left(T_1 + \frac{I_b}{R}\right)^2}{2K} + \frac{1}{4}} - \frac{1}{2}$$

The optimal order size is computed using the relation

$$Q_m = \frac{R\left(T_1 + \frac{I_b}{R}\right)}{n}.$$

Hence the decision variable Ib will determine optimal order sizes during the deterministic period. The decision variable r0 in Policy 2 will determine the time point at which first order is placed under discount price during stochastic period.

Since the length of discount duration is random it may get over before placing an order under discount in the stochastic period. If the discount is closed before the first order after T1, then wait till the complete depletion of inventory and place regular orders then onwards. If discount is on until inventory level reaches ri, then order size of Qi-ri is placed in the ith cycle of stochastic period. Again if discount is off when inventory reaches ri next order will be placed under regular price whenever inventory reaches zero.

The profit component from deterministic period is $n\varphi_d$. If there are orders placed during stochastic period then while combining profit of combining two period requires an adjustment term in the profit, ξ0,is required because stochastic period starts before closing of items purchased during previous period, where ξ0 is as given in (6) with i = 0. Further, φj is the profit component corresponding to jth ordering cycle during stochastic period. After the closure of stochastic period, the adjustment factor ξi is incorporated followed by profit component from the regular period. Hence expected profit for policy 2 is given by,

$$\psi = n\varphi_d - \xi_0 + P(N=0)\left\{\xi_0 + H\left(L - \frac{nQ_m}{R}\right)\right\}$$
$$+\sum_{i=1}^{\infty} P(N=i)\left[\sum_{j=1}^{i}\varphi_j + \xi_i + H\left(L - \frac{nQ_m}{R} + \frac{r_0}{R} - \left\{\sum_{j=1}^{i}\frac{Q_i - r_i}{R}\right\} - \frac{r_i}{R}\right)\right]$$

$$where \quad \xi_i = \Pr_i - \frac{r_i^2(c-d)F}{2R}, \quad i = 0, 1, 2, \ldots$$

$$P(N=i) = \int_{t_{i-1}-T_1}^{t_i - T_1} g(x)\,dx$$

g(x) refers to the density function of the discount duration after T1. For density function the probabilities can be computed. Normally, based on the past experience of such discount scenario the density function is estimated and used.

The profit function have decision variables Q1, r1, Q2, r2,…and regular order quantities for post discount are determined similarly as discussed in Policy 1 to obtain maximum profit but the difference is, here, the first order Q1 is placed during stochastic period at reorder point r0, which is positive. The profit is obtained using the same expression which is discussed in Policy 1.

4. SOLUTION PROCEDURE

In Policy 1, the optimal number of orders, n, of equal sizes are determined first for deterministic period according to Schwarz (1972) till minimum guaranteed period. A different procedure is adopted for stochastic period. Profit is computed for different number of decision variables which is kept on increasing until the condition $\psi_{i-1} < \psi_i \text{ and } \psi_i > \psi_{i+1}$ is satisfied. Normally this condition is satisfied whenever number of orders is a small positive number, hence a comfortable number of decision variables are chosen. Profit is calculated by increasing the number of decision variables and the procedure is terminated when profit is gradually decreasing. Hence decision variables that gives maximum profit are finally selected. Profit which is calculated includes, the profit under deterministic period, stochastic period and regular price period. In Policy 2, same procedure is used to compute profit but here we get 2 more additional decision variables because of positive inventory at T1.

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The illustration is given by taking an example of Weibull distribution for additional discount duration. Optimal solutions are obtained for different parameters of the distribution.

5. NUMERICAL ILLUSTRATION

Since the expression of profit function involves many variables it is not possible to obtain closed form solutions, so a program is developed to get the optimal solutions for both policies and hence the final policy is obtained. Here, Weibull distribution is considered for the discount period after deterministic period for illustration.

Weibull distribution for stochastic period:

The random variable X follows the following density function with a shape parameter β and a scale parameter λ is,

$$g(x)=\lambda\beta x^{\beta-1}e^{-\lambda x^{\beta}} \qquad for\ x>0;\ \beta>0\ and\ \lambda>0$$

which have the following mean and variance

$$E(X)=\frac{\Gamma\left(\frac{1}{\beta}+1\right)}{\lambda^{\frac{1}{\beta}}}=m \ and \quad V(X)=\frac{\Gamma\left(\frac{2}{\beta}+1\right)-\Gamma\left(\frac{1}{\beta}+1\right)^{2}}{\lambda^{\frac{2}{\beta}}}=v$$

5.1 Optimal solutions in case of basic parameters for Policy 1

The Basic parameters are (K=50, c=10.5, F=0.3, d=3, R=1560, P=13, λ=0.8,β =3.2, T1=0.25, m=0.96, v=0.11)

Optimal solutions for deterministic period: n=2, Qm=195, Q1*=2391.56, φ_d =995.08, H=3199

Optimal solutions for stochastic period: Q1=985.37, r1=0, Q2=2388.76, r2=10.0, Q3=2391.55, r3=19.9and combined profit (i.e profit for whole planning horizon) =6813.80.

5.2 Optimal solutions in case of basic parameters for Policy 2

Optimal solutions for deterministic period: n=2, Qm=205, Q1*=2391.56, φ_d =1047.19, H=3199

Optimal solutions for stochastic period: Q1=971.48, r1=0, Q2=2388.89, r2= 10, Q3=2391.6, r3=19.0, Ib=19.9, r0=0, and combined profit (i.e profit for whole planning horizon) =6824.97.

Table 1: Table represents optimal solutions for different values of the parameters of Weibull distribution.

Parameters (λ, β)		Qm	n	Ib, r0	Order quantity (stochastic period)	Re order level	Profit by considering all the three periods
(0.8,2.5)	Policy 1	195	2	-	950.17 2293.24 2391.55	0 10.9 19.9	6809.67
	Policy 2	205	2	19.1 0	940.30 2297.19 2391.55	0 9.98 19.2	6821.36
	Policy 1	195	2	-	950.09 1340.54 2391.42	0.001 0.03 0.04	6603.13

(0.3,2.5)	Policy 2	204.9	2	19.9 0.011	942.39 1336.87 2391.5	0.002 0.01 0.09	6613.49
(0.8,5.9)	Policy 1	195	2	-	1083.92 2391.55	0 6.05	6802.24
	Policy 2	204.9	2	19 0	562.46 587.08 2391.55	0 0 3.09	6972.84
(3.8,5.9)	Policy1	195	2	-	817.52 2391.55	0 5.6	6777.46
	Policy 2	204	2	20 0	410.09 429.78 2391.56	0 0 7.04	6896.14
(0.8,3.5)	Policy 1	312	1	-	995.15 2391.28 2391.55	0 19.9 19.9	3906.63
	Policy 2	204	2	4.9 0	991.50 2391.28 2391.56	0 19 19.5	3905.75
(3.8,7.9)	Policy 1	195	2	-	483.49 488.82 2391.55	0 0 1.26	6944.63
	Policy 2	202.5	2	14.9 0	476.02 481.86 2391.55	0 0 12.26	6947.32

The table shows i) set of parameters ii) two policies iii) order levels under deterministic period iv) number of optimal orders v) the decision variable, that will determine optimal order sizes during the deterministic period and the time point at which first order is placed under discount price during stochastic period vi) optimal order levels vii) reorder levels during stochastic period and viii) profit for complete policy. It is observed that in the optimal policy two or three optimal orders are placed at different re order levels during stochastic period. Profit is calculated for entire planning horizon. Though most of the cases Policy 2 is better than Policy 1 as can be seen in case of the parameter (0.8, 3.5), Policy 1 is better than Policy 2. It can be observed that as residual random variable of discount becoming shorter then the optimal order and re order levels are higher. Similar property has been established in Babitha and Pakkala T.P.M (2014). Number of orders during deterministic period is not much affected by the random variable of discount duration.

6. CONCLUDING COMMENTS

The profit function over the planning horizon is considered under two prices, that is, discount over a random duration and regular price in the remaining duration. Here we discuss a special case of discount period which is definitely known to last for certain period and also surely extends for a random duration. Based on the economic impact of the prices on profit the planning horizon is divided into three regions. First, guaranteed discount duration period, second random period for the discount and the rest is the regular price period. These are respectively called as deterministic period, stochastic period, and regular price period. Profit function is appropriately modified according to the periods. It may be noted that a positive reorder levels are feasible in the beginning of the stochastic period. The optimal solution is obtained through comparison of two types of policies and the best one is chosen. Through the numerical study, it is found that Policy 2 is more often optimal however in some cases Policy 1 is optimal. Unlike earlier work carried out, the policy considered here generalizes and obtains varying order levels over different periods and further varying order levels within stochastic period. The impact of randomness of discount duration is very much reflected in the numerical study carried out, as the variations in order level and reorder levels are observed during the stochastic period.

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