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Research Article

Combinatorial Properties of Anti T-Subalgebra and Ideals on Fuzzy BP-Algebra with ŧ -Norms

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ABSTRACT

Introduction: We propose the ŧ-norm of the Anti-fuzzy T-subalgebra and the T-ideal of the BP-algebra, and investigate some of their properties in this work. In addition, we define properties of Cartesian products of Anti fuzzy T subalgebras and T-ideals of BP algebras. These are treated in detail along with other algebraic properties.

Objectives: The primary objective of this study is to investigate the combinatorial properties of Anti T-subalgebras and T-ideals within the framework of fuzzy BP-algebras using ŧ-norms. The research aims to explore and characterize their algebraic structures, including operations, interconnections, intersections, and Cartesian products.

Methods: The concepts of t-norms can be applied to explore the algebraic properties of anti-fuzzy T-ideals and T-subalgebras within BP-algebras. These concepts are particularly relevant in formulating and proving theorems, lemmas, and illustrative examples related to the structure and behavior of such algebraic systems.

Conclusions: In this study, we investigated the ŧ-norm of Anti-fuzzy T-subalgebras and T-ideals within the structure of BP-algebras and established several fundamental properties. We further introduced the concept of Cartesian products of Anti-fuzzy T-subalgebras and T-ideals, demonstrating key structural characteristics and interactions. The interconnection and intersection of these algebraic structures were also analyzed in depth, offering new insights into their theoretical behavior. The findings presented in this work provide a solid foundation for future extensions and generalizations. In particular, the notions developed here can be further explored in the context of intuitionistic Q-fuzzy sets, interval-valued Q-fuzzy sets, and Q-bipolar fuzzy sets, opening avenues for advanced research in generalized fuzzy algebraic systems.

Keywords: Anti Fuzzy Sets (AFS), Anti Fuzzy Subset (AFSb), Anti Fuzzy T-Ideal (AFTI), Anti Fuzzy T-Subalgebra (AFTSA).
Mathematics Subject Classification: MSC2020-zbMATH-03B52

INTRODUCTION

In 2014, Abu[1] first discussed the idea of T-Fuzzy -subalgebras of -algebras. Jefferson[2] and [3] invented L-Fuzzy BP-Algebras in 2016 and also established the notation of FTIs in BP-Algebras in same year. In 1991, M. M. Gupta and

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Research Article

Qi. J[4] investigated the idea of the theory of T-norms and fuzzy inference techniques. The concept for the 2015 concert L-FTIs in β −Algebras was developed by K. Raja[5]. Fuzzy d-Algberas under T-norms were developed in 2022 after R. Rasuli [6] and [7] developed the notation for fuzzy congruence on product lattices in 2021. FSA and FTIs in TM-Algebras were defined by A. Tamilarasi and K. Megalai[8] in 2011. L.A. Zadeh and I introduced the notation for fuzzy sets in 1965[9].

The notation on Characteristics of ŧ-Norms under the AFTSA and Ideals in BP-Algebra is introduced in this work. It also defines the characteristics of the cartesian product of the AFTSA and T-Ideals in BP-Algebra, and it investigates several qualities.

BASIC CONCEPTS

Definition: 2.1 [9]

Let,X be a non-empty set. A FSb of the set,Ĝ is $\check{V}$: $X\rightarrow$ [0, 1].

Definition: 2.2[3]

A FS,$\check{V}$ in a BP-algebra, Ĝ is named a FTI of,Ĝ

(i) $\check{V}(0) \geq \check{V}(ɭ)$
(ii) $\check{V}(ɭ * ɉ) \geq min\{\check{V}((ɭ * ɳ) * ɉ), \check{V}(ɳ)\}, \forall\, ɭ, ɳ, ɉ \in$ Ĝ.

Result: 2.3

A ŧ-norm, Ŧ is Ŧ: $[0,1] \times [0,1] \rightarrow [0,1]$

(i) $Ŧ(ɭ, 1) = ɭ$
(ii) $Ŧ(ɭ, ɳ) \leq Ŧ(ɭ, ɉ)$
(iii) $Ŧ(ɭ, ɳ) = Ŧ(ɳ, ɭ)$
(iv) $Ŧ(ɭ, Ŧ(ɳ, ɉ)) = Ŧ(Ŧ(ɭ, ɳ), ɉ), \forall ɭ, ɳ, ɉ \in [0,1]$.

ON CHARACTERISTICS OF ŧ -NORMS UNDER THE ANTI FUZZY T-SUB ALGEBRA AND IDEALS IN BP-ALGEBRA

Definition: 3.1

Let, $\tilde{\omega}$ be the $AFSb$ of the T-Algebra Ĝ. Then, $\tilde{\omega}$ is christened $AFTSA$ of, Ĝ concealed by ŧ −norm, $Ŧ(AFTSA\ Ŧ(Ĝ))$ iff $\tilde{\omega}(ɭ * ɳ) \leq Ŧ\{max\{\tilde{\omega}(ɭ), \tilde{\omega}(ɳ)\}\}, \forall\, ɭ, ɳ \in$ Ĝ.

Example: 3.1.1

Let, Ĝ = {0,1,2,3} be a set given by

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then, (Ĝ,∗ ,0) is a T-Algebra.

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Research Article

Define $AFSb$ $\tilde{\omega}: (\hat{G}, *, 0) \to [0,1]$ as

$$\tilde{\omega}(ƀ) = \begin{cases} 0.45, & if\ ƀ = 0 \\ 0.50, & if\ ƀ \neq 0 \end{cases}$$

$Ŧ(a,b) = Ŧ_p(a,b) = ab, \forall\ a,b \in [0,1]$ then $\tilde{\omega}$ is$FTSA$ $Ŧ(\hat{G})$.

Definition: 3.2

Let,$\tilde{\omega}$ be the AFS of a T-Algebra,$\hat{G}$ and call $\tilde{\omega} \in [0,1]$ a $AFTI$ of,$\hat{G}$concealed bythe ŧ −norm, $Ŧ(AFTI\ Ŧ(\hat{G}))$.

(i) $\tilde{\omega}(0) \leq \tilde{\omega}(ƀ)$

(ii) $\tilde{\omega}(ƀ * ɉ) \leq Ŧ\left\{max\{\tilde{\omega}((ƀ * ƞ) * ɉ), \tilde{\omega}(ƞ)\}\right\}, \ \forall\ ƀ, ƞ, ɉ \in \hat{G}$

Definition: 3.3

Let, $\tilde{\omega}$ be a $AFSb$ in T-Algebra of, $\hat{G}$ and, $\tilde{\omega}$ as $AFTI$ $Ŧ(\hat{G})$then

(i) $\tilde{\omega}$, is $AFTSA$ $Ŧ(\hat{G})$.

(ii) $\tilde{\omega}(0) \leq \tilde{\omega}(ƀ)\ and \tilde{\omega}(ƀ * ɉ) \leq Ŧ\left\{max\{\ Ŧmax\{\tilde{\omega}((ƀ * ƞ) * ɉ), \tilde{\omega}(ƞ)\}, \tilde{\omega}(ƞ)\}\right\}, \forall\ ƀ, ƞ, ɉ \in \hat{G}$

Example: 3.3.1

Let, $\hat{G} = \{0,1,2,3\}$ be a set given by

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	0
3	3	3	3	0

$(\hat{G}, *, 0)$ is a T-Algebra.

AFS $\tilde{\omega}: (\hat{G}, *, 0) \to [0,1]$ as

$$\tilde{\omega}(ƀ) = \begin{cases} 0.85, & if\ ƀ = 0 \\ 0.25, & if\ ƀ \neq 0 \end{cases}$$

$Ŧ(a,b) = ab = T_p(a,b), \forall\ a,b \in [0,1]$, then $\tilde{\omega}$, is $AFTI$ $Ŧ(\hat{G})$.

Definition: 3.4

Let, $\tilde{\omega} \in [0,1]$ and ß$\in [0,1]$. The Cartesian product of, $\tilde{\omega}$ and ß is represented by, $\tilde{\omega} \times ß: \hat{G} \times Ĩ \to [0,1]$ and is represented by

$$(\tilde{\omega} \times ß)(ƀ, ƞ) = Ŧ\left\{max\{\tilde{\omega}(ƀ), ß(ƞ)\}\right\}, \forall\ ƀ, ƞ \in \hat{G} \times Ĩ.$$

Proposition: 3.5

Let, $\tilde{\omega} \in [0,1]$ and, Ŧ be idempotent . Then, $\tilde{\omega}$ is $AFTSA$ $Ŧ(\hat{G})$ iff the highest tier, $\tilde{\omega}_ŧ$ is a sub algebra of, $\hat{G}$ $\forall$ Ŧ $\in$ [0,1].

Proof:

Let, $\tilde{\omega}$ is $AFTSA$ $Ŧ(\hat{G})$, $\forall\ ƀ, ƞ \in \tilde{\omega}_ŧ$.

Then

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Research Article

$$\tilde{\omega}(ƚ, ƞ) \leq Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\} \geq Ŧ(ŧ, ŧ) = ŧ$$

Thus, $ƚ * ƞ \in \tilde{\omega}_ŧ$, and so, $\tilde{\omega}_ŧ$ ill be a SA of, $\hat{G}$, $\forall$ $Ŧ \in [0,1]$.

Converse

Let, $\tilde{\omega}_ŧ$ is a SA of $\hat{G}, \forall ŧ \in [0,1]$

Let, $ŧ = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$ and $ƚ, ƞ \in \tilde{\omega}_ŧ$.

As $\tilde{\omega}_ŧ$ is a SA of $\hat{G}$, so $ƚ * ƞ \in \tilde{\omega}_ŧ$ and thus, $\tilde{\omega}(ƚ, ƞ) \leq ŧ = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$

Then, $\tilde{\omega}$ is *AFTSA* $Ŧ(\hat{G})$.

Proposition: 3.6

Let, $\tilde{\omega}$ be a algebra of a T-Algebra $\hat{G}$, and $\mathring{S} \in [0,1]$ such that $\mathring{S}(ƚ) = \begin{cases} ŧ, if\ ƚ \in \mathring{S} \\ 0, if\ ƚ \neq \mathring{S} \end{cases}$ since, $ŧ \in [0,1]$. If, $Ŧ$ be idem then, $\mathring{S}$ is *AFTSA* $Ŧ(\hat{G})$.

Proof:

W.K.T, $\tilde{\omega} = \mathring{S}_ŧ$.

Let, $ƚ, ƞ \in \hat{G}$

(i) Let, $ƚ, ƞ \in \tilde{\omega}$ then, $ƚ * ƞ \in \mathring{S}$ and so
$$\tilde{\omega}(ƚ * ƞ) = ŧ \leq ŧ = Ŧ(ŧ, ŧ) = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$$

(ii) Let, $ƚ \in \mathring{S}$ *and* $ƞ \notin \mathring{S}$ then, $\tilde{\omega}(ƚ) = ŧ$ and, $\tilde{\omega}(ƞ) = 0$ and so
$$\tilde{\omega}(ƚ * ƞ) \leq 0 = Ŧ(ŧ, 0) = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$$

(iii) If, $ƚ \notin \mathring{S}$ and $ƞ \in \mathring{S}$ then, $\tilde{\omega}(ƚ) = 0\ and\ \tilde{\omega}(ƞ) = ŧ$ and so, $\tilde{\omega}(ƚ * ƞ) \leq 0 = Ŧ(0, ŧ) = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$

(iv) Let, $ƚ \notin \mathring{S}$ and $ƞ \notin \mathring{S}$ then, $\tilde{\omega}(ƚ) = 0\ and\ \tilde{\omega}(ƞ) = 0$ and so, $\tilde{\omega}(ƚ * ƞ) \leq 0 = Ŧ(0,0) = Ŧ\left\{max\{\tilde{\omega}(ƚ), \tilde{\omega}(ƞ)\}\right\}$
Thus (i) and (ii) we get, $\tilde{\omega}$ is *FTSA* $Ŧ(\hat{G})$.

Proposition: 3.7

Let, $\breve{\varepsilon}$ *be a AFTSA* $Ŧ(\hat{G})$ *and* $\tilde{\eta}$ be a *AFTSA* $Ŧ(\tilde{I})$. Then, $\breve{\varepsilon} \times \tilde{\eta}$ in *AFTSA* $\hat{G}$ under $ŧ$ −norm, $Ŧ(AFTSA\ Ŧ(\hat{G} \times \tilde{I}))$.

Proof:

Let, $(ƚ_1, ƞ_1), (ƚ_2, ƞ_2) \in \hat{G} \times \tilde{I}$.

Then, $(\breve{\varepsilon} \times \tilde{\eta})((ƚ_1, ƞ_1) * (ƚ_2, ƞ_2)) = \breve{\varepsilon} \times \tilde{\eta}(ƚ_1, ƞ_1, ƚ_2, ƞ_2)$

$$= Ŧ\left\{max\{ʄ(ƚ_1 * ƚ_2), \tilde{\eta}(ƞ_1 * ƞ_2)\}\right\}$$

$$\leq Ŧ\left\{ Ŧ\{max(\breve{\varepsilon}(ƚ_1), \breve{\varepsilon}(ƚ_2))\},\ Ŧ\{max(\tilde{\eta}(ƞ_1), \tilde{\eta}(ƞ_2))\}\right\}$$

$$= Ŧ\left\{ Ŧ\left\{max\left(\breve{\varepsilon}(ƚ_1), \tilde{\eta}(ƞ_1)\right)\right\},\ Ŧ\{max(\breve{\varepsilon}(ƚ_2), \tilde{\eta}(ƞ_2))\}\right\}$$

$$= Ŧ\left\{max\{(\breve{\varepsilon} \times \tilde{\eta})(ƚ_1, ƞ_1), (\breve{\varepsilon} \times \tilde{\eta})(ƚ_2, ƞ_2)\}\right\}$$

$\Rightarrow (\breve{\varepsilon} \times \tilde{\eta})((ƚ_1, ƞ_1) * (ƚ_2, ƞ_2)) \leq Ŧ\left\{max\{(\breve{\varepsilon} \times \tilde{\eta})(ƚ_1, ƞ_1), (\breve{\varepsilon} \times \tilde{\eta})(ƚ_2, ƞ_2)\}\right\}$ and so, $\breve{\varepsilon} \times \tilde{\eta}$ in *FTSA* $Ŧ(\hat{G} \times \tilde{I})$.

Proposition: 3.8

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Research Article

Let, $\breve{\varepsilon}$ be a $AFTI$ $Ŧ(\hat{G})$ and $\tilde{\eta}$be $AFTI$ $Ŧ(\tilde{I})$. Then, $(\breve{\varepsilon}\times\tilde{\eta})$be a AFTI, $\hat{G}$ under ŧ −norm , $Ŧ(AFTI\ Ŧ(\hat{G}\times\tilde{I}))$.

Proof:

(i) Let, $(l, n) \in \hat{G}\times\tilde{I}$

Then, $(\breve{\varepsilon}\times\tilde{\eta})(0,0) = Ŧ\left\{max\{\breve{\varepsilon}(0),\tilde{\eta}(0)\}\right\}$

$$\leq Ŧ\left\{max\{\breve{\varepsilon}(l),\tilde{\eta}(l)\}\right\}.$$

(ii) $$(\breve{\varepsilon}\times\tilde{\eta})\big((l_1, j_1) * (l_1, j_1)\big) = (\breve{\varepsilon}\times\tilde{\eta})(l_1 * l_2, j_1 * j_2)$$
$$\leq Ŧ\{max(\{\breve{\varepsilon}((l_1 * n_1) * j_1), \tilde{\eta}((l_2 * n_2) * j_2)\}), (\{\breve{\varepsilon}(n_1), \tilde{\eta}(n_2)\})\}$$
$$= Ŧ\left\{max\left(\{(\breve{\varepsilon}\times\tilde{\eta})((l_1 * n_1) * j_1, (l_2 * n_2) * j_2)\}, (\breve{\varepsilon}\times\tilde{\eta})(n_1, n_2)\right)\right\}$$
$$(\breve{\varepsilon}\times\tilde{\eta})\big((l_1, j_1) * (l_1, j_1)\big) \leq Ŧ\left\{max\left(\{(\breve{\varepsilon}\times\tilde{\eta})((l_1 * n_1) * j_1, (l_2 * n_2) * j_2)\}, (\breve{\varepsilon}\times\tilde{\eta})(n_1, n_2)\right)\right\}$$

Therefore (i) and (ii), $(\breve{\varepsilon}\times\tilde{\eta})$ be a FTI $Ŧ(\hat{G}\times\tilde{I})$.

Proposition: 3.9

Let, $\breve{\varepsilon} \in [0,1]^{\hat{G}}$ and $\tilde{\eta} \in [0,1]^{\tilde{I}}$. If, $(\breve{\varepsilon}\times\tilde{\eta})$ be a $AFTI$ $Ŧ(\hat{G}\times\tilde{I})$ then at least one of the following two statements must be satisfied

(i) $\breve{\varepsilon}(0) \leq \breve{\varepsilon}(l)$ then either,$\tilde{\eta}(0) \leq \breve{\varepsilon}(l)$ or $\tilde{\eta}(0) \leq \tilde{\eta}(n)$, for all $l \in \hat{G}$ and $n \in \tilde{I}$.

(ii) $\tilde{\eta}(0) \leq \tilde{\eta}(n)$then either,$\breve{\varepsilon}(0) \leq \tilde{\eta}(n)$ or $\breve{\varepsilon}(0) \leq \breve{\varepsilon}(l)$, for all $l \in \hat{G}$ and $n \in \tilde{I}$.

Proof:

(i) Let none of the statement (i) and (ii) holds, then, $(l, n) \in \hat{G}\times\tilde{I}$st,$\breve{\varepsilon}(0) < \breve{\varepsilon}(l)$ and $\tilde{\eta}(0) < \tilde{\eta}(n)$.

Thus

$$(\breve{\varepsilon}\times\tilde{\eta})(l, n) = Ŧ\left\{max\left(\breve{\varepsilon}(l),\tilde{\eta}(n)\right)\right\}$$
$$< Ŧ\left\{max\left(\breve{\varepsilon}(0),\tilde{\eta}(0)\right)\right\}$$
$$= (\breve{\varepsilon}\times\tilde{\eta})(0,0)$$

and its contradiction with, $(\breve{\varepsilon}\times\tilde{\eta})$ be a FTI $Ŧ(\hat{G}\times\tilde{I})$.

(ii) Let, $\tilde{\eta}(0) > \tilde{\eta}(n)$ such that, $(l, n) \in \hat{G}\times\tilde{I}$

We have, $\breve{\varepsilon}(0) > \tilde{\eta}(n)$ and $\breve{\varepsilon}(0) > \tilde{\eta}(l)$.

So, $\tilde{\eta}(0) \leq \tilde{\eta}(n) < \breve{\varepsilon}(0)$ and, $\breve{\varepsilon}(0) = Ŧ\left\{max\left(f(l),\tilde{\eta}(n)\right)\right\}$

Thus

$$(\breve{\varepsilon}\times\tilde{\eta})(l, n) = Ŧ\left\{max\left(\breve{\varepsilon}(l),\tilde{\eta}(n)\right)\right\}$$
$$< Ŧ\left\{max\left(\breve{\varepsilon}(0),\tilde{\eta}(0)\right)\right\}$$
$$= f(0)$$
$$= Ŧ\left\{max\left(\breve{\varepsilon}(0),\tilde{\eta}(0)\right)\right\}$$
$$= (\breve{\varepsilon}\times\tilde{\eta})(0,0)$$

and it is contradiction with, $(\breve{\varepsilon}\times\tilde{\eta})$ be a $AFTI$ $Ŧ(\hat{G}\times\tilde{I})$.

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Research Article

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