

Complex Entropy-Informational Criteria in Ferrous Metallurgy

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ARTICLE INFO	ABSTRACT
Received: 16 Dec 2024	The author has developed an information assessment of technological schemes of producing steel by refining cast iron and by the method of direct production. There are presented methods of entropy-information analysis of technological stages according to the dynamics of increasing the content of Iron from raw materials to the final product, as well as entropy-information analysis of technological stages of producing ferrous metals depending on the method of smelting. The novelty of the research topic lies in the fact that for the first time objective and fundamental information criteria expressed in universal units of information, bits, were applied to analyzing the technology of chemical and metallurgical processes and schemes for producing ferrous metals. The prospect of research extends to any metallurgical and chemical industries.
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INTRODUCTION

Algorithms of calculating the information capacity of a system proposed by Shannon, make it possible to identify the ratio of the amount of deterministic information and the amount of stochastic information (which cannot be predicted in advance), and thereby give a qualitative and quantitative assessment of technological perfection.

Let's consider the use of this formula to quantify the uncertainty of the quality of a product or technological process through the uncertainty of the main element of the system. As the probability of detecting the main element of the technological system, one can take its content in the product, expressed in fractions of a unit. For example, this is the content of the extracted chemical element in the products of the technological stage. The same applies to the process of extracting an element into a particular product, since in this case the extraction indicator is identical to the probability of the transition of this element from one state of the system to another state. To assess the quality of a product or technological conversions, both of these indicators can be equally used: content and extraction. Thus, the amount of information will also serve as a quantitative measure of the certainty of the technological process.

OBJECTIVES

In general characterization of the entropy-information analysis of any objects, there is widely used the Shannon statistical formula to express the uncertainty of any system [1]:

$$H = - \sum_{i=1}^N p_i \log_2 p_i, \quad (1)$$

p_i is the probability of detecting some uniform element of the system in their set N , $\sum_{i=1}^N p_i = 1$, $p_i \geq 0$, $i = 1, 2, \dots, N$.

Before publishing the theory developed by C. Shannon, R. Hartley proposed to determine the maximum amount of information according to the formula [1]:

$$H_{\max} = \log_2 N, \text{ bit/element}, \quad (2)$$

H_{\max} is the information amount; N , is the number of the system elements.

Iron is considered a common element in fact and is in fourth place in terms of content: 4.4% in the earth's crust (oxygen 49.9 %, silicon 26.9 %, aluminium 7.5 % [2-4].

Depending on the characteristics of the mined ore, the following stages of its preparation for smelting are used: crushing and sorting into size classes, enrichment, averaging, agglomeration. In some cases, roasting of ores is also used.

To prepare for the smelting of the mined ore depending on the properties the appropriate periods are used: division and sorting according to size classes, mutual enrichment, averaging, agglomeration. In certain embodiments, in addition, the roasting of ores is used.

The calculations show that for blast-furnace steel production, the best indicator is the iron content in concentrates of approximately 64.5-66.9 %. According to the scientific and technical model, two key stages pre-vail in steelmaking:

– direct production of steel, a blooming process, in this case there is a one-stage industrial production according to the scheme ore ÷ steel;

- obtaining steel by refining pig iron, in this case, a two-stage industrial production according to the scheme ore ÷ pig iron ÷ steel.

METHODS

Let us determine the quality of technological stages and finished products based on a comparative analysis of competing schemes according to a single generalized criterion of complex uncertainty and completeness of the technological scheme of steel production. Since the extraction, as mentioned earlier, of any component is proportional to its content in the original substance and inversely proportional to the content in the product, then, as a first approximation, the extraction of iron from the earth's crust into an ore deposit can be estimated by relation.

$$\beta_0 \cong \frac{\alpha_{3.K.}}{\alpha_{p.M.}} \cdot 100\%.$$

Since for iron $\alpha_{3.K.} = 5,1\%$, and in balance ore $\alpha_{p.M.} \cong 50,0\%$ on average, then the extraction rate at the zero level is determined by the formula proposed by us:

$$\beta_0 \cong \frac{\alpha_{3.K.}}{\alpha_{p.M.}} \cdot 100\% = \frac{5,1}{50,0} \cdot 100\% = 10,2\%,$$

β_0 is the recovery indicator at the zero level of the technological flowsheet; $\alpha_{3.K.}$ is the iron content indicator in the earth crust; $\alpha_{p.M.}$ is the iron content indicator at the ore deposit.

RESULTS

Information Assessment for steel production by iron refining Based on Shannon information formula (1), let's carry out an entropy-information analysis of each technological stage to calculate the complex uncertainty and completeness of the technological scheme as a whole using the example of typical metallurgical industries, steel by refining cast iron (the data in Table 1):

$$H_{\alpha,0} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,5000}{\ln 2} = 1,0000,$$

$$H_{\beta,0} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,1020}{\ln 2} = 3,2934,$$

$$H_{\alpha,1} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,6550}{\ln 2} = 0,6104,$$

$$H_{\beta,1} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,8700}{\ln 2} = 0,2009,$$

$$H_{\alpha,2} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,8830}{\ln 2} = 0,1795,$$

$$H_{\beta,2} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9910}{\ln 2} = 0,0130,$$

$$H_{\alpha,3} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9550}{\ln 2} = 0,0664,$$

$$H_{\beta,3} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9980}{\ln 2} = 0,0029,$$

$$H_{\alpha,4} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9950}{\ln 2} = 0,0072,$$

$$H_{\beta,4} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9990}{\ln 2} = 0,0014,$$

$$H_{\alpha,5} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9999}{\ln 2} = 0,0001,$$

$$H_{\beta,5} = -\frac{\ln p}{\ln 2} = -\frac{\ln 0,9999}{\ln 2} = 0,0001,$$

$$H_{\alpha\beta,0} = H_{\alpha,0} + H_{\beta,0} = 1,0000 + 3,2934 = 4,2934,$$

$$H_{\alpha\beta,1} = H_{\alpha,1} + H_{\beta,1} = 0,6104 + 0,2009 = 0,8113,$$

$$H_{\alpha\beta,2} = H_{\alpha,2} + H_{\beta,2} = 0,1795 + 0,0130 = 0,1925,$$

$$H_{\alpha\beta,3} = H_{\alpha,3} + H_{\beta,3} = 0,0664 + 0,0029 = 0,0693,$$

$$H_{\alpha\beta,4} = H_{\alpha,4} + H_{\beta,4} = 0,0072 + 0,0014 = 0,0086$$

$$H_{\alpha\beta,5} = H_{\alpha,5} + H_{\beta,5} = 0,0001 + 0,0001 = 0,0002$$

Therefore, the complex indicator of uncertainty in general for the technological scheme of steel production by iron refining based on the additivity property is expressed as follows:

$$H_{\alpha\beta,k} = \sum_{i=0}^5 H_{\alpha\beta,i} = 4,2934 + 0,8113 + 0,1925 + 0,0693 + 0,0086 + 0,0002 = 5,3753.$$

Having obtained the characteristic of the complex uncertainty H_k of the technological flow chart by summing the uncertainty over operations, it is possible, using the inverted formula (1), to finally find the corresponding characteristic of the complex certainty of the technological scheme:

$$p_k = \exp(-H_k \ln 2) = 2^{-H_k}, \text{ unit fraction (3)}$$

Then, having obtained the characteristic of the complex uncertainty H_k of the technological flowsheet, using the inverted formula (3), we can find the corresponding characteristic of the complex certainty of the technological scheme [2] of steel production by refining pig iron:

$$p_{\alpha\beta,0} = 2^{-4,2934} = 0,0510,$$

$$p_{\alpha\beta,1} = 2^{-0,8113} = 0,5696,$$

$$p_{\alpha\beta,2} = 2^{-0,1925} = 0,8751,$$

$$p_{\alpha\beta,3} = 2^{-0,0693} = 0,9531,$$

$$p_{\alpha\beta,4} = 2^{-0,0086} = 0,9940,$$

$$p_{\alpha\beta,5} = 2^{-0,0002} = 0,9998.$$

Then the complex indicator of the certainty of the technological scheme as a whole for the production of steel by refining pig iron is equal to:

$$p_{\alpha\beta,k} = \prod_{i=0}^5 p_{\alpha\beta,i} = 0,0510 \cdot 0,5696 \cdot 0,8751 \cdot 0,9531 \cdot 0,9940 \cdot 0,9998 = 2,4087 \cdot 10^{-2}.$$

Thus, the complex certainty of the technological scheme of steel production is expressed through the product of the fractional content and extraction of iron at each level of the scheme. The results of comparative calculations for stages and in general for the technological scheme of steel production by refining cast iron are presented in Table 1. We will illustrate the verification of calculated indicators according to the proposed models with practical data (Table 1)

graphically in accordance with Fig. 1. As a result, the correlation ($R = 0,847942$) of reference indicators for the extraction and content of the target component of the technological organization of steel production by refining iron with an integral model is revealed. The correlation coefficient for the differentiated model d_n is much higher and ($R = 0,991408$). Thus, the formulas obtained by us for the informational assessment of the technological scheme of steel production by refining pig iron are highly correlated with the theoretical hierarchical concept.

Table 1. Information characteristics of the iron content and recovery indicators in the technological flowsheet of steel production by iron refining

Technological conversions	Content indicators α		Recovery indicators β		$H_{\alpha\beta}$	$p_{\alpha\beta}$
	α	H_{α}	β	H_{β}		
Mining	0.5000	1.0000	0.1020	3.2934	4.2934	0.0510
Dressing	0.6550	0.6104	0.8700	0.2009	0.8113	0.5696
Metallization	0.8830	0.1795	0.9910	0.0130	0.1925	0.8751
Cupola smelting	0.9550	0.0664	0.9980	0.0029	0.0693	0.9531
Resmelting	0.9950	0.0072	0.9990	0.0014	0.0086	0.9940

Refining	0.9999	0.0001	0.9999	0.0001	0.0002	0.9998
H_k , bit	-	1.8636	-	3.5117	5.3753	-
p_k , u.f.	0.2748	-	0.0877	-	-	$2.4 \cdot 10^{-2}$

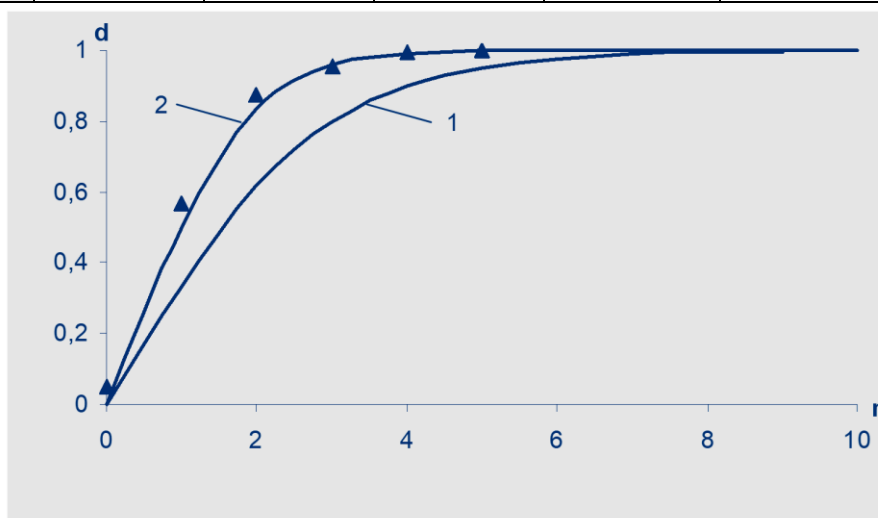


Figure 1. Verification of the calculated data on the proposed models with practical indicators for steel production by iron refining

The closest interdependence with a differentiated modification of the hierarchical concept indicates the most detailed development of any redistribution in the ferrous metallurgy.

DISCUSSION

Based on Shannon's information formula (1), we will carry out an entropy-information analysis of each technological process to calculate the complex uncertainty and completeness of the technological scheme as a whole, using the example of steel production by direct steel production. Taking into account the data for α, β in Table 2, we obtain:

$$H_{\alpha\beta,0} = H_{\alpha,0} + H_{\beta,0} = 1,0000 + 3,2934 = 4,2934,$$

$$H_{\alpha\beta,1} = H_{\alpha,1} + H_{\beta,1} = 0,4860 + 0,1649 = 0,6509,$$

$$H_{\alpha\beta,2} = H_{\alpha,2} + H_{\beta,2} = 0,0291 + 0,0072 = 0,0363,$$

$$H_{\alpha\beta,3} = H_{\alpha,3} + H_{\beta,3} = 0,0130 + 0,0029 = 0,0159,$$

$$H_{\alpha\beta,4} = H_{\alpha,4} + H_{\beta,4} = 0,0072 + 0,0014 = 0,0086,$$

$$H_{\alpha\beta,5} = H_{\alpha,5} + H_{\beta,5} = 0,0001 + 0,0001 = 0,0002.$$

$$H_{\alpha\beta,k} = \sum_{i=0}^5 H_{\alpha\beta,i} = 4,2934 + 0,6509 + 0,0363 + 0,0159 + 0,0086 + 0,0002 = 5,0053.$$

Then the complex certainty indicator of the technological flowsheet in the whole steel direct production will be [1]:

$$p_{\alpha\beta,k} = \prod_{i=0}^5 p_{\alpha\beta,i} = 0,0510 \cdot 0,6368 \cdot 0,9751 \cdot 0,9890 \cdot 0,9940 \cdot 0,9998 = 3,1131 \cdot 10^{-2}.$$

It follows that the complex certainty of the technological scheme for the direct production of steel is expressed through the product of the fractional content and extraction of iron at each level of the scheme.

The results of comparative calculations by stages and in general according to the technological scheme by direct steel production are presented in Table 2.

Comparison of reference data on the extraction and content of the target component of the technological organization of production of steel with an integral model reveals an adequate correlation ($R = 0,733544$) and a higher correlation ($R = 0,96213$) with a differential model.

Table 2. Information characteristics of the iron content and recovery in the technological flowsheet of steel direct production

Technological conversions	Content indicators α		Recovery indicators β		$H_{\alpha\beta}$	$P_{\alpha\beta}$
	α	H_{α}	β	H_{β}		
Mining	0.5000	1.0000	0.1020	3.2934	4.2934	0.0510
Dressing	0.7140	0.4860	0.8920	0.1649	0.6509	0.6368
Metallization	0.9800	0.0291	0.9950	0.0072	0.0363	0.9751
Cupola smelting	0.9910	0.0130	0.9980	0.0029	0.0159	0.9890
Resmelting	0.9950	0.0072	0.9990	0.0014	0.0086	0.9940
Refining	0.9999	0.0001	0.9999	0.0001	0.0002	0.9998
H_k , bit	-	1.5354	-	3.4699	5.0053	-
p_k , u.f.	0.3449	-	0.0902	-	-	$3.11 \cdot 10^{-2}$

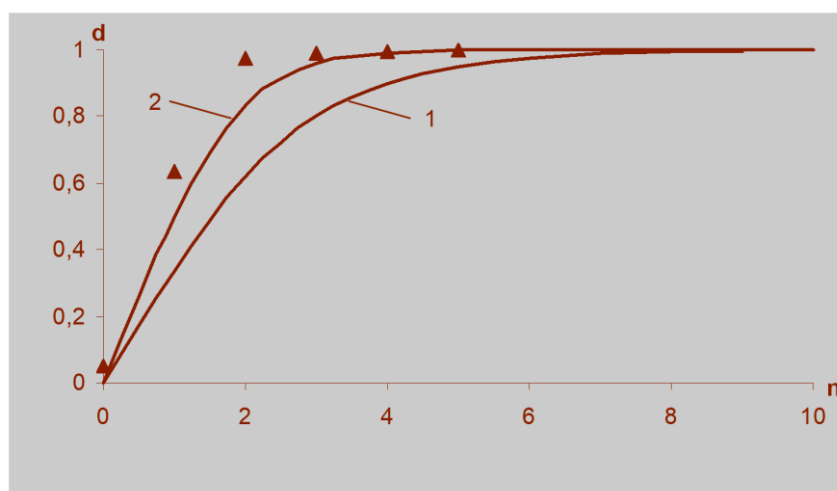


Figure 2. Verification of the calculated data according to the proposed models with practical indicators for steel direct production

Comparison of the calculated data according to the proposed models with practical data in the production technology of steel direct obtaining (Table 2) will be illustrated graphically in coordinates in accordance with Fig. 2.

As in the previous case, we have established a closer correlation with the differentiated model of an ideal hierarchical system, which also indicates the most fundamental study of each conversion of steel direct production [2-6].

CONCLUSION

The calculations proposed for the information analysis of production have become a direct result, already in the first approximation and correlate with the growth dynamics of the deterministic component in an abstract hierarchical system, which justifies the expediency of further entropy-information analysis of such systems.

In order to improve steelmaking production, specialists from many of the world's leading metallurgical companies continue to develop environmentally friendly and cheaper steelmaking technologies. One such process that involves the direct production of iron from ores with a low metal content in electric furnaces, which excludes blast furnace production, was the High Intensity Smelting Process (HISmelt) developed by the Anglo-Australian mining company Rio Tinto. In recent years, the global steel industry has been actively searching for cost-effective technologies that can replace the traditional steel production process through blast furnaces and oxygen converters. But according to our estimates, the production of steel by refining pig iron will still prevail over any other processes for obtaining steel, at least by coke-free metallurgy, as it is more adequate to the ideal hierarchical scheme.

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