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Research Article

Statistical Probability Distribution for the Covid-19 Mortality Rate.

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ABSTRACT

In the study, a discrete distribution is proposed by considering the Poisson distribution as a base distribution and the Lindley distribution as a parametric distribution which is named as Poisson extended Lindley distribution (PELD). The new distribution is known as the Poisson extended Lindley distribution. Its density function, as well as its survival and hazard rate curves are shown graphically. The study presented the Moment generating function and its related measures such as moments about the origin and mean, coefficient of variation, skewness, and kurtosis. To calculate the distribution parameters, distinct estimating strategies are employed. In order to compare and draw conclusions about the performance of the different estimators, a thorough numerical analysis was done. Here, a real data set on the COVID-19 mortality rate was examined to show how adaptable and practical the suggested distribution is.

Keywords: Poisson extended Lindley distribution (PELD), Survival analysis, Maximum likelihood, Method of Moments, COVID-19, Mortality.

1.1 INTRODUCTION

Probability distributions are a very practical and helpful tool for explaining and forecasting a variety of real-world occurrences that are documented in a variety of applied disciplines. On the other hand, a substantial amount of research and studies have been done to develop distributions that are more flexible and able to pull out all of the information from the data.

In this respect, there have been substantial efforts made to build the classical distributions, which rely on various operations such as the addition of parameters, transformation, composition, and compounding of random variables. Generalized probability distributions have emerged as a result of the widespread availability of the method for adding parameters. This approach improves the quality of expressing data relating to natural events and also increases the accuracy of characterizing the tail shape of distributions. Numerous areas, including medical, finance, bioengineering, and statistics, may benefit from the employment of contemporary numerical methods. Lastly, numbers play a vital part in our everyday lives. A great deal of statistical analysis is premised on the existence of a probability model or distribution.

The application of statistical models and probability distributions to analyze mortality data has seen growing prominence in public health and epidemiology.

Age-at-death distributions have emerged as a significant focus area. Aliverti[1] et al. developed a dynamic model leveraging mixtures of skewed distributions to analyse age-at-death data, advocating for more sophisticated approaches to mortality analysis (Aliverti[1] et al, 2022). Similarly, Mazzuco[2] et al. proposed a model combining half-normal and generalized skew-normal distributions to handle diverse mortality patterns effectively (Mazzuco[2] et al., 2018). These approaches underscore the necessity of adaptable statistical frameworks that can account for the intricate nature of mortality trends.

The COVID-19 pandemic spurred the development of innovative statistical methods tailored for mortality analysis. Muse et al. introduced a modified log-logistic tangent (LLT) distribution to model COVID-19 mortality in Somalia, highlighting its advantageous statistical attributes (Lukas[3] et al., 2021). In a related effort, Nagy[4] et al. designed a novel discrete distribution to analyze COVID-19 mortality rates, showcasing how statistical tools can address emerging health crises (Nagy[4] et al., 2021). These advancements demonstrate the importance of customized models in responding to novel public health challenges.

Regression-based methods are another widely explored area in mortality modeling. Prescott[5] et al. examined regression-based assessments in the Veterans Affairs Healthcare System, emphasizing the trade-off between model complexity and interpretability (Prescott[5] et al., 2022). Similarly, Dashti[6] et al. developed simplified models for predicting hospitalization and mortality risks in COVID-19 patients, achieving predictive accuracy comparable to more complex methods (F). These studies highlight the importance of balancing sophistication and usability in public health applications.

The **Gompertz distribution**, a longstanding tool in mortality modeling, remains widely used. Dey[7] et al. examined its statistical properties and application in survival analysis, demonstrating its capacity to model exponentially increasing failure rates (Dey[7] et al., 2018). This distribution's utility is further supported by studies such as Baldi's[8] analysis of the gender gap in youth mortality using an Age-Period-Cohort framework (Baldi's[8], 2023). The robustness of the Gompertz model ensures its relevance across various demographic studies.

Spatial analysis has also become integral to mortality research. Andrade[9] et al. utilized a Poisson probability distribution model alongside spatial techniques to examine COVID-19 mortality patterns across Brazil (Andrade[9] et al., 2020). Hallisey[10] et al. further emphasized the significance of geography in mortality research, using interpolation methods to analyze mortality counts across regions (Hallisey[10] et al., 2017). These spatial approaches highlight the importance of integrating geographic factors to enhance the understanding of mortality trends.

Beyond these methodologies, innovative models have been proposed for demographic forecasting. Aliverti[1] et al. emphasized the importance of age-at-death distributions in mortality modeling, presenting skewed distribution functions as powerful tools for analyzing demographic shifts (Aliverti[1] et al., 2021; 2022). Li[11] proposed the Poisson common factor model, which builds on the Lee-Carter method by incorporating sex-specific factors, enhancing mortality projections for different genders (Li[11], 2012). This model's ability to handle count data, such as death numbers, underscores its utility in public health.

Coherent ensemble averaging methods have been explored to improve the reliability of mortality rate forecasts. Chang[12] & Shi[12] demonstrated how combining multiple mortality models can produce age-coherent forecasts, minimizing the risks of overfitting while ensuring accuracy over extended timeframes (Chang[12] & Shi[12] 2022). This method is particularly relevant for applications in life insurance and pension planning.

Cairns[13] et al. introduced a two-factor model for stochastic mortality, addressing parameter uncertainty and randomness in mortality data (Cairns[13] et al., 2008). Meanwhile, Basellini and Camarda proposed a relational model linking observed adult age-at-death distributions to a standard reference, enhancing the predictive power of mortality models (Basellini[14] & Camarda[14], 2019; 2020). Their three-component framework provides insights into mortality patterns across different life stages.

Finally, maximum entropy models have emerged as an innovative tool in mortality forecasting. Pascariu[15] et al. used statistical moments to predict mortality, demonstrating the adaptability of these models in scenarios where traditional parametric approaches may falter (Pascariu[15] et al., 2019). This, the diverse range of statistical models and distributions developed for mortality analysis reflects their importance in both theoretical and applied research. From traditional methods like the Gompertz distribution to novel approaches such as maximum entropy models, these tools are invaluable for improving demographic forecasts, informing public health strategies, and supporting actuarial science.

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This article focuses on the introduction of a new statistical distribution, the Poisson Extended Lindley Distribution (PELD), as an effective tool for modelling over-dispersed mortality data. Derived through a compound distribution framework, PELD integrates the Poisson distribution with the Extended Lindley distribution, enabling a more precise representation of mortality patterns.

1.2 Objectives

- To develop Poisson Extended Lindley distribution (PELD) and its properties.
- To find out the application of Poisson Extended Lindley distribution (PELD).

1.3 Materials & Method

In this study, the Poisson distribution with parameter λ as base distribution and Lindley distribution with parameter θ combine these parameter together then obtain a distribution called Poisson Extended Lindley Distribution (PELD).

1.4 Proposed Distribution

1.4.1 Poisson Extended Lindley Distribution (PELD)

Let X follows Poisson distribution with parameter λ again the parameter λ follows Extended Lindley distribution with parameter θ having probability density function (pdf)

$$f(\lambda,\theta)=\frac{\theta^2}{1+\theta+\theta^2}(1+\lambda+\theta)e^{-\theta\lambda} \quad ;\lambda>0,\theta>0 \qquad (1.1)$$

Then the Poisson Extended Lindley distribution (PELD) can be obtained by the method of compound distribution as

$$P(X=x)=\int_0^\infty P(x,\lambda)\,f(\lambda,\theta)\,d\lambda \qquad (1.2)$$

$$P(X=x)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\frac{\theta^2}{1+\theta+\theta^2}(1+\lambda+\theta)e^{-\theta\lambda}\,d\lambda$$

$$=\frac{\theta^2}{1+\theta+\theta^2}\int_0^\infty \frac{\lambda^x}{x!}(1+\lambda+\theta)e^{-(\theta+1)\lambda}\,d\lambda$$

$$=\frac{\theta^2}{1+\theta+\theta^2}\int_0^\infty \left(\frac{\lambda^x}{x!}+\frac{\lambda^{x+1}}{x!}+\theta\frac{\lambda^x}{x!}\right)e^{-(\theta+1)\lambda}\,d\lambda$$

$$=\frac{\theta^2}{1+\theta+\theta^2}\left[\frac{1}{(\theta+1)^{x+1}}+\frac{(\theta+1)}{(\theta+1)^{x+2}}+\theta\frac{1}{(\theta+1)^{x+1}}\right]$$

Hence

$$P(X=x)=\frac{\theta^2}{1+\theta+\theta^2}\left[\frac{(\theta+1)^2+x+1}{(\theta+1)^{x+2}}\right];\ x=0,1,2,\ldots;\theta>0 \qquad (1.3)$$

cumulative distribution function (CDF) of PELD can be obtained as

$$F(X\le x)=\sum_{t=0}^{x}P(X=t) \qquad (1.4)$$

$$F(X\le x)=\frac{\theta^2}{1+\theta+\theta^2}\sum_{t=0}^{x}\frac{(\theta+1)^2+t+1}{(\theta+1)^{t+2}}=1-\frac{\theta^3+2\theta^2+3\theta+\theta x+1}{(1+\theta+\theta^2)(\theta+1)^{x+2}} \qquad (1.5)$$

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The PELD is formulated by combining the Poisson distribution (parameter λ) with the Extended Lindley distribution (parameter θ). The derived expressions for the PMF, CDF, and other distributional properties establish PELD as a flexible model for count data.

1.5 Distributional Properties

1.5.1 Behaviour of PMF and CDF

The behaviour and nature of PMF of PELD for the different values of θ , is depicted in Fig-1. 1. It can be seen that the value of PMF is decreasing as the value of variable increases for the fixed values of parameter θ while PMF decreases for the increasing values of parameter θ at fixed values of random variable X. The behaviour of CDF of PELD for varying value of parameter θ has been shown graphically in Fig-1.2.

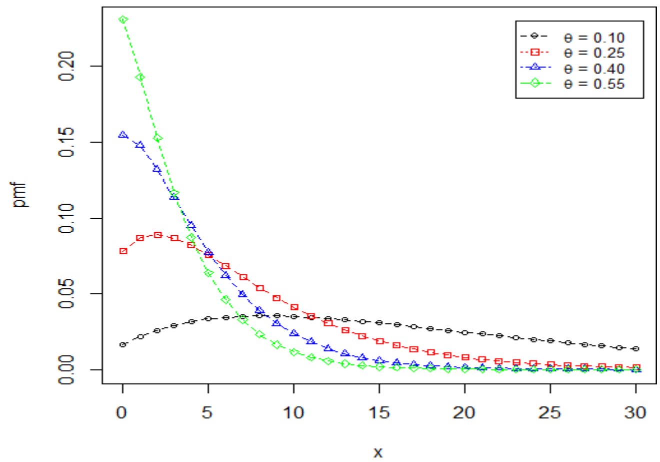

Fig-1.1: -Behaviour of probability mass function (PMF) of Poisson Extended Lindley distribution (PELD)

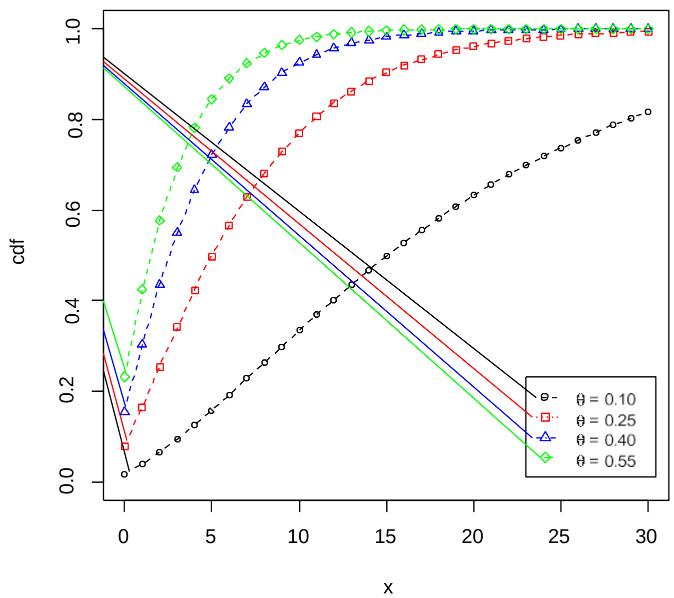

Fig-1.2: Behaviour1of cumulative density function (CDF) of Poisson Extended Lindley distribution (PELD)

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1.6 Mathematical Properties

Moments of PELD

1.6.1 Raw Moments of PELD

The r^{th} raw moment about the origin of PELD can be obtained as

$$\mu_r' = E\left[E\left(X \mid \lambda\right)\right] \tag{1.6}$$

Form (1.2)

$$\mu_r' = \int_0^\infty \left[\left\{\sum_0^\infty x^r \frac{e^{-\lambda}\lambda^x}{x!}\right\} \frac{\theta^2}{1+\theta+\theta^2}\left(1+\lambda+\theta\right)e^{-\theta\lambda}\right] d\lambda \tag{1.7}$$

Putting r = 1, 2, 3, 4, one can obtain the raw moments of PELD and further can be used to obtain the central moments. Hence

$$\begin{aligned}
\mu_1' &= \int_0^\infty \left[\left\{\sum_0^\infty x \frac{e^{-\lambda}\lambda^x}{x!}\right\} \frac{\theta^2}{1+\theta+\theta^2}\left(1+\lambda+\theta\right)e^{-\theta\lambda}\right] d\lambda \\
&= \int_0^\infty \left[\lambda \frac{\theta^2}{1+\theta+\theta^2}\left(1+\lambda+\theta\right)e^{-\theta\lambda}\right] d\lambda \\
&= \frac{\theta^2}{1+\theta+\theta^2}\int_0^\infty \left(\lambda+\lambda^2+\theta\lambda\right)e^{-\theta\lambda}\, d\lambda \\
&= \frac{\theta^2}{1+\theta+\theta^2}\left[\frac{1}{\theta^2}+\frac{2}{\theta^3}+\frac{1}{\theta}\right] \\
&= \frac{2+\theta+\theta^2}{\theta\left(1+\theta+\theta^2\right)}
\end{aligned} \tag{1.8}$$

Hence

$$\mu_1' = E\left(X\right) = \frac{2+\theta+\theta^2}{\theta\left(1+\theta+\theta^2\right)}$$

Similarly

$$\begin{aligned}
\mu_2' &= \int_0^\infty \left[\left\{\sum_0^\infty x^2 \frac{e^{-\lambda}\lambda^x}{x!}\right\} \frac{\theta^2}{1+\theta+\theta^2}\left(1+\lambda+\theta\right)e^{-\theta\lambda}\right] d\lambda \\
&= \frac{\theta^2}{1+\theta+\theta^2}\int_0^\infty \left[\left\{\lambda+\lambda^2\right\}\left(1+\lambda+\theta\right)e^{-\theta\lambda}\right] d\lambda \\
&= \frac{\theta^2}{1+\theta+\theta^2}\left[\frac{6}{\theta^4}+\frac{4}{\theta^3}+\frac{3}{\theta^2}+\frac{1}{\theta}\right] \\
&= \frac{6+4\theta+3\theta^2+\theta^3}{\theta^2\left(1+\theta+\theta^2\right)}
\end{aligned} \tag{1.9}$$

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$$\mu_3' = \int_0^\infty \left[\left\{ \sum_0^\infty x^3 \frac{e^{-\lambda}\lambda^x}{x!} \right\} \frac{\theta^2}{1+\theta+\theta^2} (1+\lambda+\theta) e^{-\theta\lambda} \right] d\lambda$$
$$= \frac{\theta^2}{1+\theta+\theta^2} \int_0^\infty \left[\left\{ \lambda^3 + 3\lambda^2 + \lambda \right\} (1+\lambda+\theta) e^{-\theta\lambda} \right] d\lambda$$
$$= \frac{\theta^2}{1+\theta+\theta^2} \left[\frac{24}{\theta^5} + \frac{24}{\theta^4} + \frac{14}{\theta^3} + \frac{7}{\theta^2} + \frac{1}{\theta} \right] \tag{1.10}$$
$$= \frac{24 + 24\theta + 14\theta^2 + 7\theta^3 + \theta^4}{\theta^3 \left(1+\theta+\theta^2\right)}$$

and

$$\mu_4' = \int_0^\infty \left[\left\{ \sum_0^\infty x^4 \frac{e^{-\lambda}\lambda^x}{x!} \right\} \frac{\theta^2}{1+\theta+\theta^2} (1+\lambda+\theta) e^{-\theta\lambda} \right] d\lambda$$
$$= \frac{\theta^2}{1+\theta+\theta^2} \int_0^\infty \left[\left\{ \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda \right\} (1+\lambda+\theta) e^{-\theta\lambda} \right] d\lambda$$
$$= \frac{\theta^2}{1+\theta+\theta^2} \left[\frac{120}{\theta^6} + \frac{168}{\theta^5} + \frac{102}{\theta^4} + \frac{36}{\theta^3} + \frac{31}{\theta^2} + \frac{1}{\theta} \right] \tag{1.11}$$
$$= \frac{120 + 168\theta + 102\theta^2 + 36\theta^3 + 31\theta^4 + \theta^5}{\theta^4 \left(1+\theta+\theta^2\right)}$$

Using these raw moments one can obtain the central moments as

$$\mu_2 = \mu_2' - \mu_1'^2$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 \tag{1.12}$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

1.6.2 Coefficient of Dispersion, Skewness and Kurtosis

The coefficient of variation (CV) of the distribution can be obtained as the ration of standard deviation by its mean

$$CV = \frac{\sqrt{\mu_2}}{mean} \tag{1.13}$$

The Pearson's coefficients have been calculated to obtain the expression of skewness as kurtosis

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \tag{1.14}$$

and

$$\gamma_2 = \beta_2 - 3 = \frac{\mu_4}{\mu_2^2} - 3 \tag{1.15}$$

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Using (1.14) and (1.15), one can calculate the values of skewness and kurtosis. From the Table 1.1, it can be seen that the PELD distribution is over dispersive as its variance is greater than mean. The value of mean and variance are decreasing for increasing value of parameter θ , while coefficient of variation (CV) is increasing in nature as the value of parameter increases. The skewness of PELD increases as the value of parameter increases, it a positive skewed distribution as the value of skewness is positive for all the values of parameter. As well as, the values of kurtosis increases as the value of parameter increases but all the values are positive hence it can be said that is leptokurtic distribution and suitable for peaked over dispersive datasets.

1.7 Moments and Descriptive Statistics

The moments about the origin and the mean, variance, skewness, and kurtosis of the PELD are derived. The model is characterized by positive skewness and leptokurtic behaviour, making it suitable for peaked, over-dispersed datasets. As shown in Table 1, increasing values of the parameter θ lead to decreasing mean and variance but increasing skewness and kurtosis, indicating an adaptable model for diverse mortality data.

Table 1: - Values of descriptive statistics of Poisson Extended Lindley distribution (PELD) for various values of parameter θ

Value of θ	Mean	Variance	CV	Skewness	Kurtosis
0.1	19.009	218.027	0.77678	1.43325	3.05591
0.2	9.03226	58.0957	0.84387	1.47786	3.19537
0.3	5.73141	27.0789	0.90793	1.5349	3.38794
0.4	4.10256	15.7972	0.9688	1.59652	3.61355
0.5	3.14286	10.4082	1.02651	1.65818	3.85915
0.6	2.51701	7.40617	1.08122	1.71741	4.11677
0.7	2.08089	5.55995	1.13315	1.77301	4.38203
0.8	1.7623	4.34309	1.18255	1.82457	4.65295
0.9	1.52112	3.4987	1.22968	1.87211	4.92894
1.0	1.33333	2.88889	1.27475	1.9159	5.21006

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1.8. Reliability Characteristics

1.8.1 Survival Function

The survival function $S(x)$ of random variable T at any point x defined as

$$S(x) = P(T > x); x = 0, 1, 2, ... \tag{1.16}$$

can be calculated as

$$S(x) = P(T > x) + P(T = x) - P(T = x)$$
$$= P(T \geq x) - P(T = x)$$

Hence

$$S(x) = \sum_{t=x}^{\infty} \frac{\theta^2}{1+\theta+\theta^2} \left[\frac{(\theta+1)^2 + t + 1}{(\theta+1)^{t+2}} \right] - P(T = x)$$
$$= \frac{\theta^2 + 2\theta^2 + 2\theta + x\theta + 1}{(1+\theta+\theta^2)(\theta+1)^{x+1}} - \frac{\theta^2}{1+\theta+\theta^2} \left[\frac{(\theta+1)^2 + x + 1}{(\theta+1)^{x+2}} \right] \tag{1.17}$$
$$= \frac{\theta^3 + 2\theta^2 + 3\theta + \theta x + 1}{(1+\theta+\theta^2)(\theta+1)^{x+2}}; \theta > 0, x = 0, 1, 2, ...$$

From the Fig 1.3 it is seen seen that the survival of Poisson Extended Lindley distribution (PELD) rapidly decreases as the value of parameter θ increases also it is decreases in same manner as the value of random variable increases.

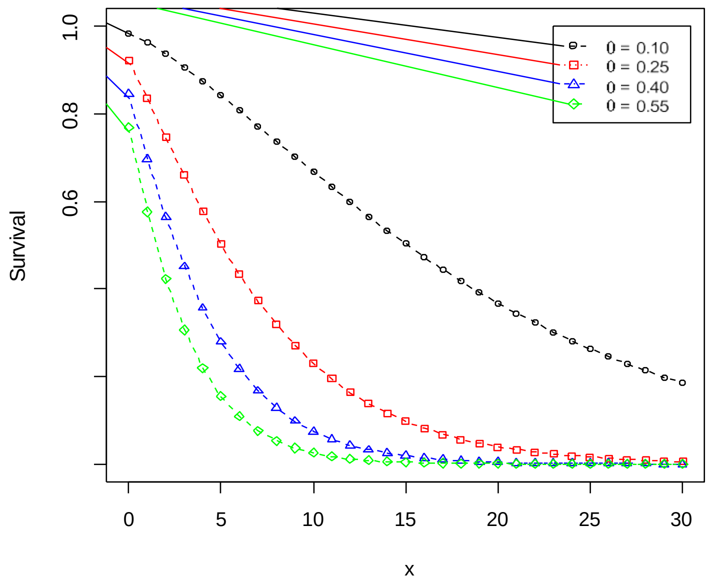

Fig 1.3: - Behavior of survival function of Poisson Extended Lindley distribution (PELD)

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1.8.2 Hazard Rate or Mortality rate

The hazard rate is the measure of failure or mortality in indefinite small time interval and defined as the conditional probability that the system may fail in a very small time interval given that it was at surviving stage at the starting time point of that time interval. It can be calculated as

$$h(x) = \frac{P(X=x)}{S(x)} \quad ; x = 0,1,2,...$$

$$= \frac{\dfrac{\theta^2}{1+\theta+\theta^2}\left[\dfrac{(\theta+1)^2+x+1}{(\theta+1)^{x+2}}\right]}{\dfrac{\theta^3+2\theta^2+3\theta+\theta x+1}{(1+\theta+\theta^2)(\theta+1)^{x+2}}} \qquad (1.18)$$

hence

$$h(x) = \frac{\theta^2\left[(\theta+1)^2+x+1\right]}{\theta^3+2\theta^2+3\theta+\theta x+1} \quad ;\theta>0, x=0,1,2,... \qquad (1.19)$$

From the Fig 1.4 is can be observed that Poisson Extended Lindley distribution (PELD) has increasing hazard rate or mortality rate for any value of parameter. As the value of parameter increases, the hazard rate increases rapidly

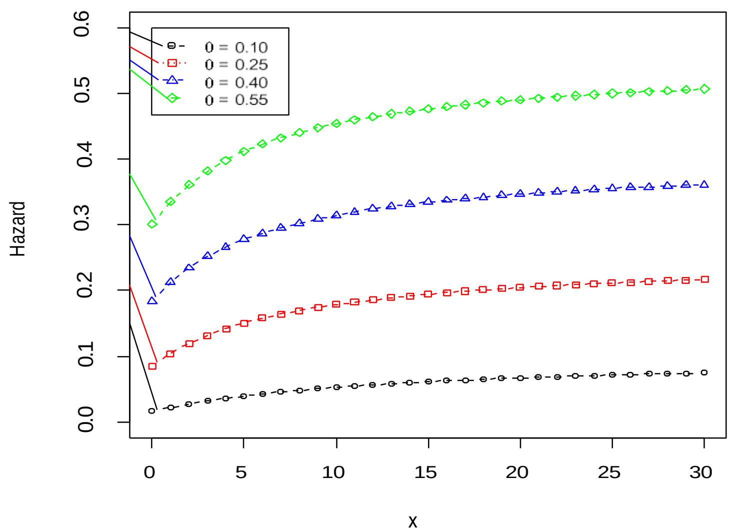

Fig 1.4:- Behavior of hazard rate of Poisson Extended Lindley distribution (PELD).

1.8.3 Second Rate of Failure

The hazard rate defined in (1.19) is bounded and hence cannot be convex also it is not additive for series system. The second rate of failure (SRF) defined as $SRF(x) = \log\left(S(x)/S(x+1)\right)$ was introduced by Roy (2002) (also used by Xie et al, 2002) to overcome these inherent problems of the failure rate. For discrete Poisson Extended Lindley distribution (PELD)

$$SRF(x) = \log\left(\frac{S(x)}{S(x+1)}\right) = \log\left(\frac{(x+\theta x)(e^{\theta}-1)+e^{\theta}(\theta+2)-1}{\left\{(x+\theta x+\theta+1)(e^{\theta}-1)+e^{\theta}(\theta+2)-1\right\}e^{-\theta}}\right) \qquad (1.20)$$

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1.9 Parameter Estimation

1.9.1 Method of Moments

In this method, one makes the equation of sample moments and population moments and solve for the estimates of parameters. For Poisson Extended Lindley distribution (PELD), equating the sample mean equals to population mean to get the moments estimator, $\tilde{\theta}$ of parameter θ,

$$E(X)=\bar{x}=\frac{2+\theta+\theta^2}{\theta\left(1+\theta+\theta^2\right)} \tag{1.21}$$

$$\bar{x}\left(\theta+\theta^2+\theta^3\right)=2+\theta+\theta^2$$

$$\theta^3\bar{x}+(\bar{x}-1)\theta^2+(\bar{x}-1)\theta+2=0 \tag{1.22}$$

2

Solving the equation (1.22), one can get the moment estimator $\tilde{\theta}$ of parameter θ.

1.9.2 Method of Maximum Likelihood

The method of maximum likelihood consists of maximizing the likelihood function to get the estimator of parameter. For the given sample $x_1, x_2, ..., x_n$ of size n, the Likelihood function of the parameter of Poisson Extended Lindley distribution (PELD) is as follows

$$L=\prod_{i=1}^{n}\mathrm{P}(x_i,\theta)$$
$$=\prod_{i=1}^{n}\frac{\theta^2}{1+\theta+\theta^2}\left[\frac{(\theta+1)^2+x_i+1}{(\theta+1)^{x_i+2}}\right] \tag{1.23}$$

Taking log both side for log likelihood function

$$\log L=n\log\left(\frac{\theta^2}{1+\theta+\theta^2}\right)+\sum_{i=1}^{n}\log\left((\theta+1)^2+x_i+1\right)-\sum_{i=1}^{n}(x_i+2)\log(\theta+1) \tag{1.24}$$

Differentiating equation (1.24) w.r.t θ and equating equals to zero can get the log likelihood equation

$$\frac{2n}{\theta}+\frac{n(1+2\theta)}{1+\theta+\theta^2}+\sum_{i=1}^{n}\left(\frac{2+2\theta}{(\theta+1)^2+x_i+1}\right)-\frac{\sum_{i=1}(x_i+2)}{\theta+1}=0 \tag{1.25}$$

Solving the non-linear equation (1.25), one can get the ML estimate of the parameter θ.

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2.0 APPLICATION OF THE MODEL

As Poisson Extended Lindley distribution (PELD) is an over-dispersed model, so it can be used for the modelling of over-dispersed data. The applicability of Poisson Extended Lindley distribution (PELD) has been compared with Poisson-Exponential distribution (PED) which is a well-known over-dispersed distribution. The chi-square distribution has been used to test the significance of goodness of fit. From Table 1.2 it can be observed that PELD is better fit than PED, hence it can be used for the modelling of over-dispersed data.

Table 1.2- Observed and expected distribution of families according to the number of child deaths in Northeast Libya.

Number of Child Deaths	Observed Frequency	Expected frequency	
		Poisson-Exponential	Poisson-ELD
0	805	807.2	806.12
1	306	300.73	308.19
2	93	95.4	91.9
3	36	33.55	35.52
4	7	10.56	8.42
5	2	3.98	3.87
6	1	1.17	1.12
7	2	0.73	1.44
ML estimate		1.9027	2.1467
Chi-square		4.76	1.41
p-value		0.313	0.842

3.0 DISCUSSION

PELD's ability to model over-dispersed mortality data is evident from its performance against PED. The positive skewness and leptokurtic nature of the distribution align with the characteristics of the dataset. Additionally, the model's flexibility in capturing varying mortality patterns exponentiates its potential for broader applications in demographic and public health research.

4.0 CONCLUSION

This study presents the Poisson Extended Lindley Distribution (PELD) as a robust model for over-dispersed mortality data. The derived distributional properties, combined with practical parameter estimation methods, highlight its utility in analyzing complex datasets. The application to child mortality data in North-East Libya demonstrates its effectiveness compared to existing models. PELD's adaptability and accuracy make it a valuable tool for mortality analysis, public health planning, and demographic forecasting.

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