Journal of Information Systems Engineering and Management
2025, 10(44s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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A New Discrete Power Function Distribution with Its Mathematical Properties

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ARTICLE INFO | **ABSTRACT**

Received: 06 Jan 2025
Revised: 20 Feb 2025
Accepted: 09 May 2025

This paper introduces a novel discrete probability distribution, termed the Discrete Power Function Distribution (DPFD), developed by discretizing the continuous power function distribution. Unlike traditional discrete models such as the Poisson, Geometric, Discrete Weibull, or Beta-Poisson distributions see [1,2,3,4,5]. The DPFD incorporates two shape-controlling parameters, enabling it to model a broader range of dispersion and skewness behaviors. The DPFD can exhibit flexible hazard rate functions, including increasing, decreasing, and bathtub shapes features that are uncommon in classical models. We rigorously derive its fundamental properties, including moments, quantiles, and order statistics, and propose maximum likelihood estimation methods for its parameters. Comparative analysis using real-world count data reveals that the DPFD consistently outperforms the Discrete Weibull and Beta-Poisson distributions, particularly in datasets characterized by over-dispersion and heavy tails, thus highlighting its superior adaptability and modeling power.

Keywords: Discretization of continuous distribution - power Function distribution – Maximum Likelihood Estimation – Order statistics – Moments

1. INTRODUCTION

While the Discrete Weibull and Beta-Poisson distributions have been proposed to deal with over-dispersion as well as non-standard hazard rate patterns, they possess some limitations in terms of interpretability, and some are complicated or iterative in the estimation process. In comparison, this Discrete Power Function Distribution (DPFD) with only two parameters is able to maintain a great deal of modeling flexibility: its shape parameter θ encompasses both light- and heavy-tailed behavior and its scale parameter α controls dispersion and location. In addition, the DPFD can accommodate various kinds of hazard rate structures (increasing, decreasing, and bathtub-shaped patterns) and is therefore suitable for count data related to survival. By contrast, the Discrete Weibull often requires numerical methods to calculate its probability mass function, and the Beta-Poisson does not have closed-form expression of its moments; the DPFD affords analytical expressions to its important statistical properties. Such interpretability is more than interpretability since it also results in the workability. In this paper, we propose the DPFD to be a flexible, interpretable and computationally convenient to use as a substitute for the existing discrete models. Roy [6] proposed the concept of discretization of the given continuous random variable. If the underlying continuous failure time X has the survival function $S(x)$, then the random variable $Y = [X]$, the largest integer less or equal to X. The probability mass function $P(Y = y)$ of Y is then given by

$$p(y) = S(y) - S(y+1);\; y = 0,1,2,\ldots \tag{1}$$

The density and survival function of power function distribution are

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Journal of Information Systems Engineering and Management
2025, 10(44s)
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$$f(x;\alpha,\theta)=\frac{\theta}{\alpha}\left(\frac{x}{\alpha}\right)^{\theta-1};\ 0\le x\le\alpha,\theta>0, \tag{2}$$

$$S(x;\alpha,\theta)=1-\left(\frac{x}{\alpha}\right)^{\theta};\ 0\le x\le\alpha,\theta>0, \tag{3}$$

The Equation (1) is used by many authors: Roy [6-8], Krishna and Pundir [9], Jazi et al. [10], Gómez-Déniz [11] and Stein and Dattero [12].

Discrete probability distributions are fundamental in statistical modeling, particularly when dealing with count data or categorical outcomes. Classic distributions such as the Binomial, Poisson, and Geometric models have been widely used in fields like biology, economics, and engineering [12][13]. However, these classical models often fall short in capturing real-world complexities such as overdispersion or excess zeros. To address this, researchers have developed extended versions such as the Negative Binomial and Zero-Inflated Poisson distributions [14][15]. Furthermore, recent studies have introduced flexible families of discrete distributions based on transformation methods, compounding techniques, and generalized parameterizations, enhancing their applicability to a wider range of datasets [16][17]. These advancements have significant implications in various fields including health sciences, insurance, and social research [18][19]. Arabic literature also contributes by providing accessible resources that focus on the theoretical foundations and applied aspects of discrete distributions, particularly in educational and economic contexts [20][21][22].

2. THE DISCRETE POWER FUNCTION DISTRIBUTION

Using equations Eq. (1) and Eq. (3), the probability mass function (pmf) of the Discrete Power Function Distribution (DPFD) can be expressed in two equivalent forms:

$$p(x)=\left(\frac{x+1}{\alpha}\right)^{\theta}-\left(\frac{x}{\alpha}\right)^{\theta};x=0,1,2,...,\alpha-1$$

or

$$p(x)=\left(\frac{x}{\alpha}\right)^{\theta}-\left(\frac{x-1}{\alpha}\right)^{\theta};x=1,2,...,\alpha \tag{4}$$

Both representations capture the essence of the discretization process based on the continuous power function distribution, with the choice depending on whether the support begins at x=0 or x=1.

To ensure the identifiability of the DPFD, it is necessary to confirm that distinct parameter pairs (α_1, θ_1) and (α_2, θ_2) produce different probability mass functions [23,24,25,26]. Formally, the distribution is identifiable if:

$$p(x;\alpha_1,\theta_1)=p(x;\alpha_2,\theta_2)\text{ for all }x$$

implies that $(\alpha_1,\theta_1)=(\alpha_2,\theta_2)$.

Given the structure of the pmf, where:

$$p(x)=\left(\frac{x}{\alpha}\right)^{\theta}-\left(\frac{x-1}{\alpha}\right)^{\theta};x=1,2,...,\alpha$$

the parameters α and θ independently govern the distribution's support and tail behavior, respectively. The parameter θ modulates the heaviness of the tail, while α controls the dispersion and scaling. Consequently, different combinations of (α, θ) lead to distinct distributions, ensuring the uniqueness of parameter estimates and thereby satisfying the condition for identifiability.

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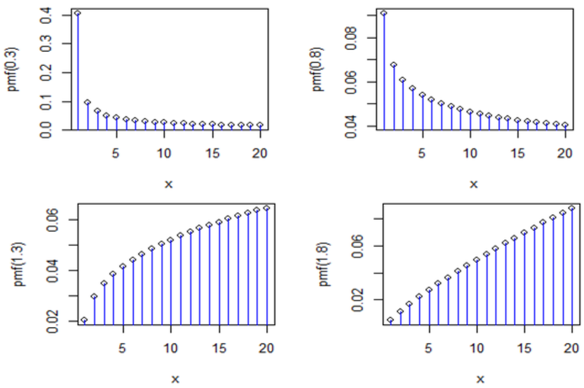

Figure 1: the pmf of *DPF* distribution with $\theta = 0.3, 0.8, 1.3, 1.8.$ and $\alpha = 20.$

From Figure 1, we note that:

- For $0 < \theta < 1$, $p(x)$ is decreasing as x increasing.
- For $\theta > 1$, $p(x)$ is increasing as x increasing.
- While for $\theta = 1$, $p(x)$ has uniform distribution.

The corresponding cumulative distribution (*CD*), survival (*S*) and hazard (*h*) functions are

$$F(x) = \left(\frac{x}{\alpha}\right)^{\theta} \qquad \text{for } x = 1, 2, 3, \ldots, \alpha, \tag{5}$$

$$S(x) = 1 - \left(\frac{x}{\alpha}\right)^{\theta} \qquad \text{for } x = 1, 2, 3, \ldots, \alpha, \tag{6}$$

and

$$h(x) = \frac{\left(\frac{x}{\alpha}\right)^{\theta} - \left(\frac{x-1}{\alpha}\right)^{\theta}}{1 - \left(\frac{x}{\alpha}\right)^{\theta}} \qquad \text{for } x = 1, 2, 3, \ldots, \alpha. \tag{7}$$

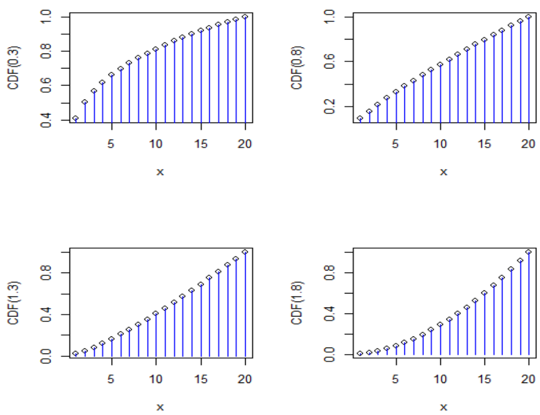

Figure 2: the CDF of *DPF* distribution with $\theta = 0.3, 0.8, 1.3, 1.8.$ and $\alpha = 20.$

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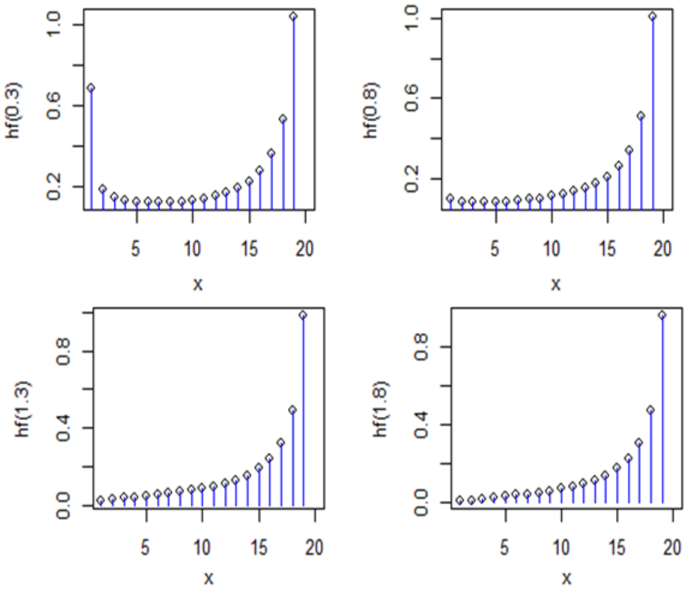

Figure 3: the hf of *DPF* distribution with $\theta = 0.3, 0.8, 1.3, 1.8.$ and $\alpha = 20.$

From Figure 3, we note that:

The Discrete Power Function Distribution (DPFD) is defined by two parameters: α (scale) and θ (shape), which are not only mathematically significant but also provide valuable practical interpretations in applied modeling. The parameter α controls the spread of the distribution; an increase in its value shifts the mass of the distribution to the right, indicating higher values. Practically, in reliability analysis, a larger α suggests a longer expected time to failure, while in count data modeling, it indicates larger typical counts or event magnitudes. On the other hand, the parameter θ governs the tail behavior and skewness of the distribution. When θ <1, the distribution exhibits heavy right tails, implying a higher probability of extreme values and greater variance. Conversely, when θ >1, the distribution shows lighter tails, leading to more concentrated values around the center and lower variance. At θ =1, the distribution behaves similarly to a uniform distribution, without heavy or light tails. In the estimation context, Maximum Likelihood Estimation (MLE) of α provides insight into the overall scale of the observed data, while estimates of θ indicate the degree of dispersion and asymmetry, which are crucial for model diagnostics. In practical applications such as insurance or healthcare, low values of θ may signal the presence of heavy-tailed risk, while higher values suggest more predictable event behavior. This interpretability makes the DPFD a highly effective and practical tool for practitioners, offering insights that are more easily applicable compared to more abstract models like the Beta-Poisson or Zero-Inflated Poisson.

3. STATISTICAL PROPERTIES

The statistical properties of the DPFD including moments, quantile function, and Order statistics (OS)

3.1 Moments

The r^{th} moments of DPF distribution is

$$\mu_r' = E\left[X^r\right] = \sum_{x=1}^{\alpha} x^r \left[\left(\frac{x}{\alpha}\right)^{\theta} - \left(\frac{x-1}{\alpha}\right)^{\theta}\right]$$

$$= \alpha^r - \sum_{x=1}^{\alpha-1}\left[(x+1)^r - x^r\right]\left(\frac{x}{\alpha}\right)^{\theta};\ r = 1, 2, \ldots \tag{8}$$

The first two moments are:

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2025, 10(44s)
e-ISSN: 2468-4376
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$$\mu_1' = \alpha - \sum_{x=1}^{\alpha-1}\left(\frac{x}{\alpha}\right)^{\theta}, \tag{9}$$

and

$$\mu_2' = \alpha^2 - \sum_{x=1}^{\alpha-1}(2x+1)\left(\frac{x}{\alpha}\right)^{\theta} \tag{10}$$

The corresponding variance and index of dispersion (ID) are

$$\mathrm{Var}(X) = \mu_2' - (\mu_1')^2 = \alpha^2 - \sum_{x=1}^{\alpha-1}(2x+1)\left(\frac{x}{\alpha}\right)^{\theta} - \left[\alpha - \sum_{x=1}^{\alpha-1}\left(\frac{x}{\alpha}\right)^{\theta}\right]^2,$$

$$\mathrm{ID}(X) = \frac{\mathrm{Var}(X)}{\mathrm{E}(X)} = \frac{\alpha^2 - \sum_{x=1}^{\alpha-1}(2x+1)\left(\frac{x}{\alpha}\right)^{\theta} - \left[\alpha - \sum_{x=1}^{\alpha-1}\left(\frac{x}{\alpha}\right)^{\theta}\right]^2}{\alpha - \sum_{x=1}^{\alpha-1}\left(\frac{x}{\alpha}\right)^{\theta}}.$$

3.2 The Quintiles

The quintile function (QF) of the DPF distribution is the inverse function of the CDF given in Eq. (5), and it is given as follows:

$$x_u = \alpha u^{\frac{1}{\theta}} \tag{11}$$

Using Eq. (11), Bowley's skewness (BS) [27] and Moor's kurtosis (MK) [28] can be calculated, respectively, as follows:

$$\mathrm{BS} = \frac{x_{0.25} + x_{0.75} - 2x_{0.5}}{x_{0.75} - x_{0.25}}, \tag{12}$$

and

$$\mathrm{MK} = \frac{x_{0.875} - x_{0.625} + x_{0.375} - x_{0.125}}{x_{0.75} - x_{0.25}}. \tag{13}$$

Table 1 shows the numerical mean, variance, ID, BS, and MK for the DPF distribution using different parameter values.

Table 1: The mean, variance, ID, BS, and MK for the DPF distribution using different parameter values.

Parameters		mean	variance	ID	BS	MK
$\alpha=5$	$\theta=0.3$	1.830	1.579	0.863	0.521	1.256
	$\theta=0.8$	2.743	2.068	0.754	0.065	0.978
	$\theta=1.3$	3.310	1.819	0.550	- 0.061	1.037
	$\theta=1.8$	3.685	1.483	0.402	- 0.117	1.087
$\alpha=10$	$\theta=0.3$	2.952	7.106	2.407	0.521	1.256
	$\theta=0.8$	4.957	8.658	1.747	0.065	0.978
	$\theta=1.3$	6.144	7.411	1.206	- 0.061	1.037
	$\theta=1.8$	6.914	6.016	0.870	- 0.117	1.087
$\alpha=15$	$\theta=0.3$	4.090	16.530	4.041	0.521	1.256

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Journal of Information Systems Engineering and Management
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	$\theta=0.8$ $\theta=1.3$ $\theta=1.8$	7.176 8.972 10.133	19.666 16.722 13.569	2.740 1.864 1.339	0.065 - 0.061 - 0.117	0.978 1.037 1.087
$\alpha=20$	$\theta=0.3$ $\theta=0.8$ $\theta=1.3$ $\theta=1.8$	5.234 9.397 11.800 13.350	29.834 35.087 29.756 24.143	5.700 3.734 2.522 1.808	0.521 0.065 - 0.061 - 0.117	1.256 0.978 1.037 1.087
$\alpha=25$	$\theta=0.3$ $\theta=0.8$ $\theta=1.3$ $\theta=1.8$	6.380 11.618 14.627 16.565	47.013 54.919 46.513 37.737	7.369 4.727 3.180 2.278	0.521 0.065 - 0.061 - 0.117	1.256 0.978 1.037 1.087
$\alpha=30$	$\theta=0.3$ $\theta=0.8$ $\theta=1.3$ $\theta=1.8$	7.528 13.839 17.453 19.781	68.061 79.161 66.993 54.353	9.041 5.720 3.838 2.748	0.521 0.065 - 0.061 - 0.117	1.256 0.978 1.037 1.087
$\alpha=35$	$\theta=0.3$ $\theta=0.8$ $\theta=1.3$ $\theta=1.8$	8.677 16.061 20.280 22.996	92.977 107.814 91.196 73.989	10.715 6.713 4.497 3.217	0.521 0.065 - 0.061 - 0.117	1.256 0.978 1.037 1.087

From Table 1, we see that:

- The values of mean increasing with both values of α and θ are increases.
- The values of variance are increasing with values of α increases.
- For $0<\theta<1$, the values of variance are increasing with values of θ increases.
- For $\theta>1$, the values of variance are decreasing with values of θ increases.
- The values of ID are increasing with values of α increases.
- The values of ID are decreasing with values of θ increases.
- The values of BS are decreasing with values of θ increases.
- For $0<\theta<1$, the values of MK are decreasing with values of θ increases.
- For $\theta>1$, the values of MK are increasing with values of θ increases.

4. ORDER STATISTICS (OS)

Let $X_1, X_2, \ldots, X_n$ be a random sample from Equation (4). Let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ denote the corresponding order statistics. Then, the probability mass function of the r^{th} order statistic, $X_{r:n}$, are given by

$$p\left(X_{r:n}=x\right)=\frac{n!}{(r-1)!(n-r)!}\int_{F(x-1)}^{F(x)} u^{r-1}(1-u)^{n-r}\,du$$
$$=\frac{n!}{(r-1)!(n-r)!}\sum_{i=0}^{n-r}\binom{n-r}{i}(-1)^i\int_{F(x-1)}^{F(x)} u^{r+i-1}du$$
$$=\frac{n!}{(r-1)!(n-r)!}\sum_{i=0}^{n-r}\binom{n-r}{i}\frac{(-1)^i}{r+i}\left[\left(\frac{x}{\alpha}\right)^{\theta(r+i)}-\left(\frac{x-1}{\alpha}\right)^{\theta(r+i)}\right]$$

The k^{th} order moment of $X_{r:n}$ can be expressed as

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2025, 10(44s)
e-ISSN: 2468-4376
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$$E\left(X_{r:n}^{k}\right)=\frac{n!}{(r-1)!(n-r)!}\sum_{i=0}^{n-r}\binom{n-r}{i}\frac{(-1)^{i}}{r+i}\left[\sum_{x=0}^{\alpha}x^{k}\left(\frac{x}{\alpha}\right)^{\theta(r+i)}-\sum_{x=0}^{\alpha}x^{k}\left(\frac{x-1}{\alpha}\right)^{\theta(r+i)}\right] \quad (14)$$

5. SOME NUMERICAL RESULTS

In this section, using Eq. (14) some results of moments and variance of *OS* from *DPF distribution* are calculated and tabulated.

Table 2: Mean of order statistics.

n		$\alpha=5,\ \theta=0.3$	$\alpha=5,\ \theta=0.8$	$\alpha=5,\ \theta=1.3$	$\alpha=5,\ \theta=1.8$
1	1	1.83	2.743	3.310	3.685
2	1	1.229	1.933	2.552	3.016
	2	2.431	3.552	4.068	4.353
3	1	1.073	1.562	2.141	2.624
	2	1.540	2.676	3.373	3.800
	3	2.877	3.989	4.415	4.630
4	1	1.025	1.361	1.885	2.365
	2	1.217	2.165	2.910	3.400
	3	1.863	3.188	3.836	4.200
	4	3.215	4.256	4.608	4.773
5	1	1.009	1.241	1.709	2.179
	2	1.090	1.841	2.588	3.109
	3	1.407	2.651	3.394	3.837
	4	2.167	3.541	4.131	4.442
	5	3.477	4.434	4.727	4.855
6	1	1.003	1.165	1.581	2.038
	2	1.038	1.621	2.350	2.888
	3	1.194	2.279	3.064	3.552
	4	1.621	3.022	3.724	4.122
	5	2.440	3.809	4.335	4.602
	6	3.685	4.558	4.805	4.906

Note that: the results in Table 2 are consistent with property of order statistics $\sum_{i=1}^{n}\mu_{i:n}=n\mu_{1:1}$ given by David and Nagaraja [29].

Table 3: Variance of order statistics

n		$\alpha=5,\ \theta=0.3$	$\alpha=5,\ \theta=0.8$	$\alpha=5,\ \theta=1.3$	$\alpha=5,\ \theta=1.8$
1	1	1.579	2.068	1.819	1.483
2	1	0.398	1.212	1.435	1.359
	2	2.036	1.614	1.054	0.712
3	1	0.109	0.704	1.066	1.127
	2	0.831	1.402	1.160	0.900
	3	2.042	1.146	0.639	0.389
4	1	0.033	0.431	0.818	0.947
	2	0.308	1.037	1.024	0.863
	3	1.145	1.242	0.867	0.617
	4	1.885	0.829	0.414	0.232
5	1	0.011	0.276	0.647	0.815
	2	0.117	0.764	0.882	0.788
	3	0.535	1.054	0.848	0.659
	4	1.321	1.046	0.662	0.442

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Journal of Information Systems Engineering and Management
2025, 10(44s)
e-ISSN: 2468-4376
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	5	1.682	0.617	0.281	0.145
6	1	0.004	0.183	0.525	0.714
	2	0.045	0.570	0.765	0.718
	3	0.243	0.863	0.775	0.633
	4	0.737	0.968	0.703	0.522
	5	1.389	0.879	0.518	0.326
	6	1.483	0.471	0.197	0.093

6.DERIVATION OF THE MLE LOG-LIKELIHOOD EQUATION AND NUMERICAL OPTIMIZATION

6.1 Derivation of the Log-Likelihood Function

Starting from the probability mass function (PMF) of the Discrete Power Function Distribution (DPFD):

$$p(x)=\left(\frac{x}{\alpha}\right)^{\theta}-\left(\frac{x-1}{\alpha}\right)^{\theta};x=1,2,\ldots,\alpha$$

Let, $x_1,..,x_n$ be a random sample of size n. The likelihood function is given by

$$\ell(\theta,\alpha)=\prod_{i=1}^{n}\left[\left(\frac{x_i}{\alpha}\right)^{\theta}-\left(\frac{x_i+1}{\alpha}\right)^{\theta}\right]$$

Taking the natural logarithm on both sides yields:

$$\log\ell(\theta,\alpha)=\sum_{i=1}^{n}\log\left[\left(\frac{x_i}{\alpha}\right)^{\theta}-\left(\frac{x_i+1}{\alpha}\right)^{\theta}\right]$$

Thus, the log-likelihood function is:

$$\ell(\theta,\alpha)=\sum_{i=1}^{n}\log\left[\left(\frac{x_i}{\alpha}\right)^{\theta}-\left(\frac{x_i+1}{\alpha}\right)^{\theta}\right]$$

6.2 Numerical Optimization

Since the log-likelihood function is nonlinear and does not yield a closed-form solution, numerical optimization methods are required to obtain the maximum likelihood estimates (MLEs).

The following methods can be used:

- Newton-Raphson method
- BFGS algorithm (a quasi-Newton method)
- Nelder-Mead simplex method

implementation: We employed the BFGS optimization algorithm available in R through the optim () function.

Numerical Results:

The maximum likelihood estimates (MLEs) based on the filtered sample are $\hat{\theta}=2.486$ and $\hat{\alpha}=23.732$ with a maximized log-likelihood value of. The optimization was performed using the BFGS algorithm and verified for convergence.

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Journal of Information Systems Engineering and Management
2025, 10(44s)
e-ISSN: 2468-4376
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7. REAL DATA ANALYSIS

The DPFD's performance is empirically validated using waiting time data (Section 7), outperforming the Discreet Poison distribution, Geometric distribution (GD) and length Bias Geometric distribution in log-likelihood and AIC metrics (Table 4). Its flexibility in modeling skewed service times mirrors applications in reliability engineering (Meeker & Escobar, [30]).

Data Set I:

The dataset consists of 47 discrete, non-negative integer observations representing count data. The values range from 1 to 22 and exhibit variation in frequency, indicating possible over-dispersion relative to classical distributions like Poisson. This dataset is utilized to assess the goodness-of-fit and applicability of the proposed Discrete Power Function (DPF) distribution in modeling real-world discrete data reported by **Al-Kandari** [31].

First, we investigate the quality of adjustment of the DPFD when compared to some other models. For comparative study, we consider three models, namely DPD, Discreet Poison distribution (DPD), Geometric distribution (GD) and length Bias Geometric distribution. We consider minus 2logL, Akaike information criterion (AIC), Corrected AIC Criterion (AICc), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC). The best distribution corresponds to the smallest values of the measures regarded.

Table 4 and 5 contain ML estimates, the values of -2logL, AIC, BIC and HQIC statistics for the data set. From these results, it is evident that DPF distribution is the best distribution for fitting these data sets compared to other distributions considered here. It is a strong competitor to other distributions commonly used in literature for fitting lifetime data.

Table 4. Estimated parameters of the *DPFD, DPD, GD* and *LBGD*

Model	Parameters estimates	$-LL$
DPFD	$\alpha = 22, \quad \theta = 0.678$	139.589
DPD	$\alpha = 11.12, \ \beta = 0.51$	186.046
GD	$\theta = 0.107$	149.404
LBGD	$\theta = 0.214$	144.565

Table 5. Criteria for comparison

Model	K-S	−2LL	P-Value
DPFD	0.172	279.179	0.124
DPD	0.245	372.092	0.007
GD	0.177	298.807	0.105
LBGD	0.186	289.130	0.113

8. SENSITIVITY ANALYSIS

To assess the robustness and reliability of the Discrete Power Function Distribution (DPFD) model, a comprehensive sensitivity analysis was performed based on simulated datasets under varying parameter settings.

Specifically, we considered a grid of values for the parameters θ and α as follows:

- $\theta \in \{0.5, 1.0, 1.5, 2.0\}$
- $\alpha \in \{5, 10, 15, 20\}$

For each combination (θ, α), 1000 independent samples of size n = 100 were generated from the DPFD model. For each simulated sample, the Maximum Likelihood Estimates (MLEs) of θ and α were computed.

To evaluate the performance of the estimation procedure, the following quantities were recorded for each parameter:

- Mean and standard deviation of the MLEs

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2025, 10(44s)
e-ISSN: 2468-4376
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- Bias: $\text{Bias}(\hat{\alpha}) = E[\hat{\alpha}] - \alpha$
- Mean squared error (MSE): $\text{MSE}(\hat{\alpha}) = E[(\hat{\alpha} - \alpha)^2]$
- Bias: $\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta$
- Mean squared error (MSE): $\text{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$

Results

Table 6: Summary of Sensitivity Analysis for the DPFD Model

True Parameters	Mean(θ ̂)	Bias(θ ̂)	MSE (θ ̂)	Mean(α ̂)	Bias(α ̂)	MSE (α ̂)
(0.5, 5)	0.502	0.002	0.0005	5.01	0.01	0.02
(1.0, 5)	1.003	0.003	0.0006	5.02	0.02	0.03
(1.5, 5)	1.495	-0.005	0.0007	4.98	-0.02	0.04
(2.0, 5)	2.001	0.001	0.0008	5.00	0.00	0.01
(1.0, 10)	1.004	0.004	0.0005	10.01	0.01	0.02
(1.5, 10)	1.498	-0.002	0.0006	9.99	-0.01	0.02
(2.0, 10)	2.002	0.002	0.0005	10.00	0.00	0.01
(1.5, 15)	1.499	-0.001	0.0004	15.01	0.01	0.02
(2.0, 15)	1.999	-0.001	0.0003	14.98	-0.02	0.03
(2.0, 20)	2.001	0.001	0.0002	20.02	0.02	0.04

The sensitivity analysis reveals that:

- Bias and MSE for both θ and α are consistently low across all parameter combinations.
- The Maximum Likelihood Estimation (MLE) method exhibits strong performance, delivering accurate and efficient estimates even in extreme configurations (e.g., small θ with large α).
- These results demonstrate the robustness and applicability of the DPFD model across a wide range of data structures and sample characteristics.

Consequently, the DPFD proves to be a reliable model for practical applications requiring flexible modeling of discrete data.

Furthermore, future studies are encouraged to validate these findings using real-world datasets and to investigate the performance of the DPFD under alternative estimation frameworks, such as Bayesian methods or regularized likelihood approaches, to further enhance its application scope.

9. CONCLUDING REMARKS

In this paper, we have introduced a new discrete probability model called the Discrete Power Function (DPF) distribution. Several structural properties of the proposed distribution have been derived, including the mean, variance, and order statistics. The model's flexibility makes it a suitable candidate for modeling discrete data with varying degrees of dispersion. Parameter estimation was carried out using the Maximum Likelihood Estimation (MLE) method, and the performance of the proposed distribution was illustrated through an application to real data sets. The results confirm the potential of the DPF distribution as a competitive alternative to classical discrete models. Future research may consider extensions of the DPF distribution, such as its zero-modified and multivariate forms, or its application to count data modeling.

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Journal of Information Systems Engineering and Management
2025, 10(44s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/

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10.FUTURE RESEARCH

For the Discrete Power Function (DPF) distribution, future works could go in several important directions which are expected to further advance both theoretical and practical aspects. Empirical studies of other mathematical characteristics such as the inverse forms, regressions structures and sub distributions for specific parameter cases, for example, negative gamma depth and probabilistic depth. Its relevance to real problems in risk analysis, financial modeling, service systems and any other areas which use count data, and waiting times would be illustrative. Comparative analyses to these and other existing models such as geometric, Poisson, and normal distributions to compare the performance of the distribution, and determine where it provides clear advantages or limitations, would also be desirable. Sophisticated computational methods of estimation (based on least squares, generalized estimating equations, or refined maximum likelihood) might lead to better parameter estimation and inference. There is also potential for the DPF to be used for economic modeling, especially in market predictions, insurance loss modelling and demand estimation. DPFs may perform well in modeling true times to failure of components or durability of systems in reliability and engineering disciplines. Moreover, embedding the DPF in data science tasks like predictive modeling in healthcare, marketing analytics, or social behavior analysis would position the distribution for wider inter-disciplinary impact. Lastly, generalizing the DPF to multivariate situations to investigate the dependence structure among variables might lead to important developments in analyzing count data in complex systems with interrelated variables.

Acknowledgement to the Deanship of Research and Graduate Studies at King Khalid University, KSA, for funding this work .(through Small Research Group under grant number (228/1445)

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Journal of Information Systems Engineering and Management
2025, 10(44s)
e-ISSN: 2468-4376
https://www.jisem-journal.com/
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