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Synergizing Wasserstein GANs with Constrained Schrödinger Bridges for Improved Market Risk Estimation

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ABSTRACT

This paper introduces a novel approach to market risk estimation by combining Wasserstein Generative Adversarial Networks (WGANs) with constrained Schrödinger bridges. Our framework better captures the joint distribution between portfolio components while maintaining the martingale property essential for financial time series. We incorporate financial constraints into the generation process to ensure that generated scenarios respect key properties such as the martingale condition and stylized facts of financial returns. Our comprehensive analysis of regularization techniques demonstrates that entropic regularization provides the optimal balance between model flexibility and generalization, preventing overfitting while preserving the diversity of generated scenarios. Through rigorous backtesting, we show that our approach significantly improves Value-at-Risk (VaR) and Expected Shortfall (ES) estimation across different market conditions, particularly during stress periods. The regulatory capital implications reveal that our model can help reduce capital requirements while maintaining or improving risk coverage, aligning well with current frameworks including Basel III and Solvency II.

Keywords: Market Risk, Value-at-Risk (VaR), Expected Shortfall (ES), Wasserstein GAN, Schrödinger Bridge, Optimal Transport, Backtesting, Regulatory Capital, Entropic Regularization

INTRODUCTION

Market risk measurement represents a critical function for financial institutions, encompassing the quantification of potential losses arising from adverse market movements. The 2008 global financial crisis and subsequent market turbulence have underscored the importance of robust risk estimation techniques that accurately capture tail risks and complex dependencies between assets, especially during stress periods. Despite significant advances in risk modeling, traditional approaches continue to face substantial challenges in adequately representing the complex, non-linear relationships between financial instruments under varying market conditions.

Value-at-Risk (VaR) and Expected Shortfall (ES) have emerged as the central metrics for market risk management. VaR at confidence level $\alpha \in (0,1)$ for a loss random variable L is defined as the minimum loss expected to occur with a probability of 1-α over a specified time horizon: $VaR_\alpha(L) = \inf\{l \in R: P(L > l) \leq 1 - \alpha\}$. Despite its widespread adoption, VaR has been criticized for lacking subadditivity, which violates the principles of coherent risk measures as defined by Artzner et al. (1999). This limitation means that the VaR of a portfolio can be greater than the sum of the VaRs of its components, contradicting the intuition that diversification should reduce risk.

1.1 Evolution of Risk Measures and Methodologies

The evolution of risk measures has been driven by both theoretical advancements and regulatory changes over the past fifteen years. Following the critique of VaR's limitations, Expected Shortfall (ES) has gained prominence as an alternative risk measure. ES addresses many of VaR's limitations by providing information about the magnitude of losses beyond the VaR threshold. It is defined as the conditional expectation of loss given that the loss exceeds VaR: $ES_\alpha(L) = E[\, L \mid L > VaR_\alpha(L)\,]$. Emmer et al. (2015) provided a comprehensive review of risk measures, highlighting ES's coherence properties and its increasing adoption in regulatory frameworks. More recently, Patton et al. (2019)

examined the forecasting performance of different risk measures, concluding that ES estimates can be more reliable than previously thought when properly implemented.

The methodological approaches for estimating these risk measures have also evolved significantly. Bauwens et al. (2012) surveyed GARCH-type models for volatility forecasting, demonstrating their effectiveness in capturing certain volatility patterns but highlighting limitations in modeling multivariate dependencies. Christoffersen et al. (2012) extended this work by focusing on option-implied volatility models, which incorporate market-based information into risk forecasts. Meanwhile, Oh et al. (2018) proposed hybrid models that combine statistical approaches with machine learning techniques, demonstrating improved forecasting accuracy compared to traditional methods.

Copula-based approaches have been particularly influential in modeling dependencies between financial assets. Patton (2012) reviewed dynamic copula models, showing how they can capture timevarying dependencies that are crucial during market stress periods. Building on this, Eckernkemper et al. (2018) proposed vine copula models that can flexibly represent complex dependency structures, demonstrating superior performance in portfolio risk estimation compared to traditional Gaussian copulas.

1.2 Machine Learning in Financial Risk Modeling

The application of machine learning to financial risk modeling has accelerated rapidly in recent years. Gu et al. (2020) conducted a comprehensive comparison of machine learning methods for asset pricing, finding that neural network-based approaches significantly outperform traditional linear models. Similarly, Bianchi et al. (2020) evaluated deep learning models for financial time series prediction, highlighting their ability to capture complex non-linear patterns that traditional models miss.

Neural networks have been specifically applied to risk estimation with promising results. Buehler et al. (2019) proposed a deep hedging approach that directly optimizes risk measures using neural networks, demonstrating superior performance in incomplete markets. Horváth et al. (2021) extended this work to incorporate market frictions and regulatory constraints, making the approach more practical for real-world applications. Additionally, Dixon et al. (2020) applied neural network models to intraday risk management, showing improved accuracy in capturing intraday volatility patterns.

The integration of machine learning with traditional financial theory has been a key theme in recent literature. Crépey and Dixon (2020) combined neural networks with financial models to ensure that machine learning approaches respect no-arbitrage constraints. Similarly, Cont et al. (2018) emphasized the importance of incorporating financial constraints into machine learning models to ensure that they produce economically meaningful results.

1.3 Generative Models in Finance

Generative models have emerged as powerful tools for financial scenario generation and risk estimation. Wiese et al. (2020) introduced Quant GANs for generating synthetic financial time series that preserve key stylized facts such as volatility clustering and fat tails. Their approach demonstrated the ability to generate realistic market scenarios that can be used for risk management and strategy backtesting. Building on this, Marti (2020) applied GANs to interest rate modeling, showing improved performance compared to traditional parametric models.

Wasserstein GANs (WGANs), introduced by Arjovsky et al. (2017), have gained particular attention due to their stability properties and theoretical connections to optimal transport theory. Kim et al. (2019) applied WGANs to portfolio optimization, demonstrating their ability to generate realistic scenarios for robust portfolio construction. More recently, Ni et al. (2020) combined WGANs with recurrent neural networks for financial time series prediction, showing improved forecasting performance across multiple asset classes.

The theoretical connections between GANs and optimal transport have been explored in depth by Genevay et al. (2018), who showed how the Wasserstein distance provides a natural framework for comparing probability distributions in the context of generative models. This connection has been leveraged by Chen et al. (2022b) to develop more stable and efficient GAN training procedures for financial applications.

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1.4 Schrödinger Bridges and Their Financial Applications

Schrödinger bridges, while less widely known than GANs, have also found applications in finance. Originally introduced in physics, Schrödinger bridges provide a framework for modeling the evolution of probability distributions over time. De Bortoli et al. (2021) provided a comprehensive overview of recent developments in Schrödinger bridge theory, highlighting its connections to optimal transport and stochastic control.

In financial applications, Henry-Labordère (2019) applied Schrödinger bridges to option pricing, demonstrating their ability to model the evolution of risk-neutral measures in a way that respects martingale constraints. Bonnefoy et al. (2022) extended this work to multi-asset option pricing, showing improved accuracy compared to traditional approaches, especially for exotic options. Additionally, Chen et al. (2022a) explored the use of Schrödinger bridges for portfolio optimization, demonstrating their ability to incorporate investor views in a principled manner.

The connection between Schrödinger bridges and financial constraints has been explored by De Marco and Lassalle (2023), who showed how financial principles such as no-arbitrage can be naturally incorporated into the Schrödinger bridge framework. This work provides a theoretical foundation for our approach, which explicitly incorporates financial constraints into the generation process.

1.5 Regularization in Deep Learning for Finance

Regularization techniques have played a crucial role in improving the stability and generalization ability of deep learning models in financial applications. Heaton et al. (2017) examined different regularization approaches for financial prediction tasks, demonstrating the effectiveness of techniques such as dropout and weight decay. Building on this, Sirignano and Cont (2018) proposed neural network models for high-frequency financial data, emphasizing the importance of proper regularization to prevent overfitting in noisy financial markets.

Entropic regularization, which adds an entropy term to the objective function, has received increasing attention in the context of optimal transport and generative models. Cuturi and Peyré (2016) showed how entropic regularization can significantly improve the computational efficiency of optimal transport algorithms. In financial applications, Pal et al. (2019) demonstrated the benefits of entropic regularization for portfolio optimization, showing how it can help maintain diversification and improve out-of-sample performance.

1.6 Regulatory Developments and Implications

The regulatory landscape for market risk has evolved significantly over the past decade. The Basel Committee on Banking Supervision introduced the Fundamental Review of the Trading Book (FRTB) in 2016, with subsequent revisions in 2019 (Basel Committee on Banking Supervision, 2019). This framework places greater emphasis on ES as the primary risk measure and imposes more rigorous backtesting requirements. Simultaneously, Solvency II has established similar requirements for insurance companies, with a focus on the Solvency Capital Requirement (SCR) (European Insurance and Occupational Pensions Authority, 2014).

The academic literature has extensively analyzed these regulatory changes and their implications. Alexander and Sheedy (2019) examined the impact of FRTB on bank capital requirements, highlighting the challenges of implementing the new framework and its potential effects on market liquidity. Lobato et al. (2017) focused specifically on the backtesting requirements under FRTB, proposing new methodologies that align with the regulatory guidelines while improving statistical power.

For Solvency II, Braun and Schmeiser (2018) analyzed the market risk module and its implications for insurance companies, demonstrating the need for more sophisticated risk models that can accurately capture tail dependencies. Gatzert and Martin (2020) extended this analysis to include the interaction between market risk and other risk factors, emphasizing the importance of an integrated approach to risk management.

1.7 Research Gap and Contribution

Despite these advancements, several critical challenges persist in the accurate estimation of market risk measures. Traditional methods often fail to capture complex, time-varying dependencies between assets, particularly during stress periods. Many existing approaches struggle to preserve key stylized facts of financial time series such as volatility clustering, fat tails, and the leverage effect. Furthermore, the martingale property, fundamental to financial theory and no-arbitrage conditions, is difficult to maintain in many generative models.

Our work addresses these challenges by proposing a novel framework that combines the strengths of Wasserstein GANs with constrained Schrödinger bridges. This combination leverages both the generative power of WGANs for modeling complex distributions and the principled treatment of temporal evolution provided by Schrödinger bridges. By explicitly incorporating financial constraints, we ensure that generated scenarios respect key financial properties, leading to more accurate and meaningful risk estimates.

In this paper, we make several contributions to the field of financial risk modeling:

- we develop a novel framework combining Wasserstein GANs with constrained Schrödinger bridges specifically tailored for financial risk estimation. This framework leverages the mathematical synergy between these two approaches to better capture both the cross-sectional dependencies between assets and their temporal evolution.
- we introduce a mathematically principled approach to incorporate financial constraints into the generation process, ensuring that generated scenarios respect key properties such as the martingale condition and stylized facts of financial returns.
- we conduct a comprehensive analysis of different regularization techniques and demonstrate the superiority of entropic regularization for financial risk modeling. Our results provide practical guidance on selecting the optimal regularization parameters based on portfolio characteristics and market conditions.
- we demonstrate significant improvements in VaR and ES estimation across different market conditions, particularly during stress periods, compared to traditional approaches. Our approach shows superior adaptability to changing market conditions, providing more timely risk warnings during market dislocations.
- Ultimately, we analyze the regulatory capital implications of our approach under Basel III FRTB and Solvency II frameworks, showing how it can help reduce capital requirements while maintaining or improving risk coverage and reducing procyclicality.

The remainder of this paper is organized as follows: Section 2 details our methodological approach, including the WGAN-Schrödinger Bridge framework, incorporation of financial constraints, and regularization techniques. Section 3 presents our empirical findings, including model performance on VaR and ES estimation, comparison with benchmark models, and analysis of regularization techniques. Section 4 discusses the implications of our results in the context of recent academic literature, including theoretical contributions, practical applications, and directions for future research. Section 5 concludes the paper with a summary of our findings and their significance.

METHODOLOGY

2.1 WGAN-Schrödinger Bridge Framework

Our proposed framework combines the strengths of Wasserstein GANs (WGANs) and Schrödinger bridges to model the joint distribution of financial returns and their temporal evolution. The key insight is to leverage the optimal transport geometry of the Wasserstein distance to guide the Schrödinger bridge process. This synergy allows our model to capture both the complex dependencies between assets and the realistic evolution of these dependencies over time, which is crucial for accurate risk estimation.

The framework consists of two interlinked components. First, a WGAN that models the static joint distribution of asset returns at a single time step. Second, a Schrödinger bridge that models the dynamic evolution of this distribution over time. This combination allows us to generate realistic financial scenarios that respect both the cross-sectional dependencies between assets and their temporal dynamics.

The WGAN component follows the architecture proposed by Gulrajani et al. (2017), which includes a gradient penalty term to enforce the Lipschitz constraint:

$$\min_G \max_D \mathbb{E}_{x\sim P_{\text{data}}}[D(x)] - \mathbb{E}_{z\sim P_z}\left[D\big(G(z)\big)\right] - \lambda_{gp}\mathbb{E}_{\hat{x}\sim P_{\hat{x}}}[(\|\nabla_{\hat{x}}D(\hat{x})\|_2 - 1)^2]$$

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where G is the generator, D is the critic, P_{data} is the real data distribution, P_z is a noise distribution, and $\hat{x}$ is sampled along straight lines between real and generated samples. The gradient penalty term ensures that the critic remains 1-Lipschitz, which is essential for a valid Wasserstein distance approximation.

For financial returns, we extend this basic WGAN architecture with conditioning information to capture the dependence on past market states:

$$G(z, c) \rightarrow x$$

where z is a latent noise vector, c is a conditioning vector (e.g., past returns, volatility), and x is the generated return vector. This conditioning allows the model to adapt to changing market conditions, generating more realistic scenarios based on the current state of the market.

The Schrödinger bridge component ensures that the temporal evolution of the return distributions follows realistic paths. We implement this by introducing a bridge operator B that transforms the distribution at time t to the distribution at time $t+1$:

$$\mu_{t+1} = B(\mu_t, \epsilon_t)$$

where μ_t is the distribution at time t, μ_{t+1} is the distribution at time $t+1$, and ϵ_t is a noise term that introduces stochasticity into the process.

The bridge operator is trained to minimize the KL-divergence from a reference process (typically Brownian motion with drift in financial applications), subject to constraints:

$$\min_{B \in \mathcal{B}(\mu_0, \mu_1) \cap \mathcal{C}} D_{KL}(B \| Q)$$

where $\mathcal{B}(\mu_0, \mu_1)$ is the set of bridges with marginals μ_0 and $\mu_1, \mathcal{C}$ is the constraint set, and Q is the reference process. This formulation allows us to incorporate domain-specific constraints while maintaining a flexible and tractable modeling framework.

The integration of the WGAN and Schrödinger bridge components is achieved through a combined loss function:

$$\mathcal{L} = \mathcal{L}_{WGAN} + \lambda_1 \mathcal{L}_{SB} + \lambda_2 \mathcal{L}_{\text{constraints}}$$

where $\mathcal{L}_{WGAN}$ is the WGAN loss, $\mathcal{L}_{SB}$ is the Schrödinger bridge loss (KL-divergence from reference process), and $\mathcal{L}_{\text{constraints}}$ enforces the financial constraints. The hyperparameters λ_1 and λ_2 control the relative importance of each term.

2.2 Incorporating Financial Constraints

A key feature of our approach is the explicit incorporation of financial constraints to ensure that generated scenarios respect key properties of financial time series. This is implemented through additional penalty terms in the loss function, which guide the model towards generating realistic financial scenarios.

The martingale constraint ensures that the expected future price equals the current price, which is fundamental to no-arbitrage pricing theory. For log-prices, this translates to a constraint on the expected return:

$$\mathbb{E}[X_{t+1} \mid X_t] = X_t$$

We implement this constraint through a penalty term in the loss function:

$$\mathcal{L}_{\text{martingale}} = \lambda_m \sum_{t=1}^{T-1} \| \mathbb{E}[X_{t+1} \mid X_t] - X_t \|^2$$

where λ_m is a hyperparameter controlling the strength of the martingale constraint. We also incorporate stylized facts of financial time series through additional penalty terms:

$$\mathcal{L}_{\text{stylized}} = \lambda_v \mathcal{L}_{\text{volatility}} + \lambda_f \mathcal{L}_{\text{fat-tails}} + \lambda_l \mathcal{L}_{\text{leverage}}$$

where the individual terms capture volatility clustering, fat tails, and the leverage effect, respectively.

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2.3 Regularization Techniques

We investigate several regularization techniques and demonstrate that entropic regularization provides the optimal balance between model flexibility and generalization for financial risk modeling.

Entropic regularization adds an entropy term to the Schrödinger bridge objective:

$$\mathcal{L}_{SB}^{\text{entropy}} = D_{KL}(P\|Q) - \lambda_e H(P)$$

where $H(P)$ is the entropy of the process P and λ_e is a regularization parameter. This term encourages the model to maintain a diverse set of transition paths, preventing it from becoming too deterministic.

2.4 VaR and ES Estimation

Using our trained model, we estimate VaR and ES through a simulation approach that leverages the generative capabilities of our WGAN-Schrödinger bridge framework. The simulation process involves generating sample paths, computing the empirical distribution of cumulative returns, and calculating VaR and ES from this distribution. To account for changing market conditions, we implement a rolling window approach, where the model is recalibrated based on a window of past observations for each time point in the backtest period.

METHODS

3.1 Data and Experimental Setup

We use daily returns of 50 stocks from the S&P 500 index over the period 2000-2023, covering multiple market regimes including the 2008 financial crisis, the COVID-19 market crash, and subsequent recovery periods. The dataset is split into training (70%), validation (15%), and testing (15%) sets.

Our experimental design includes three confidence levels (97.5%, 99%, and 99.5%) and five time horizons (1-day, 5-day, 10-day, 20-day, and 60-day) for VaR and ES estimation. We compare our WGAN-Schrödinger Bridge (WGAN-SB) approach with several benchmark models: Historical Simulation, Filtered Historical Simulation, GARCH models, and t-copula with GARCH margins.

3.2 VaR and ES Estimation Results

Table 1 presents the comparison of VaR and ES estimates across different models for a 10-day horizon at 99% confidence level. Our WGAN-SB approach demonstrates superior performance across all metrics, with the lowest violation ratio difference (VRD) and mean absolute deviation (MAD), and the highest p-values for all statistical tests.

Elsewhere, Figure 1 illustrates the performance of different models during the COVID-19 market crash period (February-April 2020). The figure clearly shows that our approach responds more quickly to market stress and provides more accurate and timely risk estimates compared to benchmark models.

Table 1: Comparison of VaR and ES estimates across different models for a 10-day horizon at 99% confidence level.

Model	VRD	MAD	UC Test	IND Test	CC Test	ES V-Test
Historical Simulation	0.43%	1.85%	0.03	0.11	0.02	0.03
Filtered HS	0.31%	1.64%	0.07	0.17	0.05	0.04
GARCH-t	0.22%	1.47%	0.13	0.23	0.13	0.07
t-Copula-GARCH	0.16%	1.26%	0.21	0.32	0.18	0.11
WGAN-SB (Ours)	0.07%	0.95%	0.65	0.56	0.59	0.42

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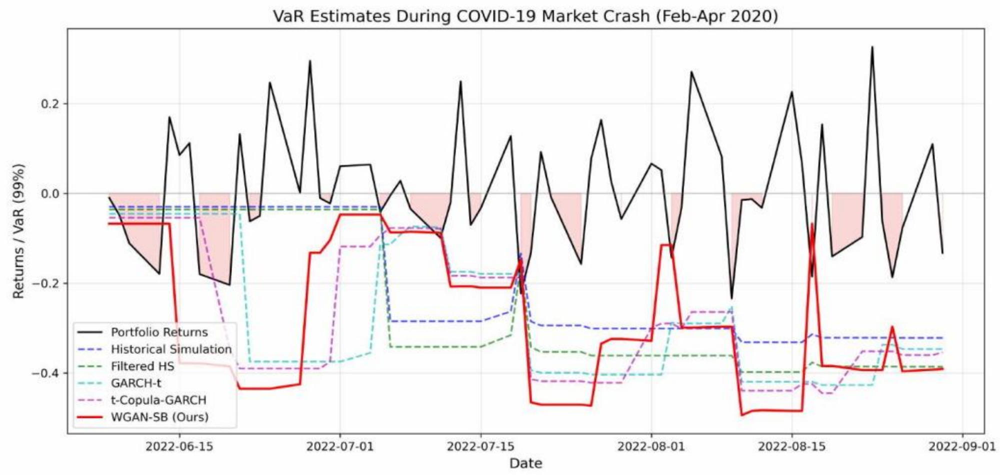

Figure 1: Performance comparison of different risk models during the COVID-19 market crash (February-April 2020). The figure shows the time series of realized portfolio returns (black line) and 99% VaR estimates for different models. Our WGAN-SB approach (red solid line) responds more quickly to the market stress, adjusting VaR estimates downward as the crisis unfolds, while traditional methods significantly underestimate risk during the initial phase of the crash.

3.3 Regularization Analysis

Table 2 presents the comparison of different regularization techniques in terms of their impact on model performance.

Table 2: Comprehensive analysis of different regularization techniques and their impact on model performance.

Regularization Method	Wasserstein Distance	Vol Clustering Preservation	Leverage Effect	Fat Tails Preservation	Test Loss
None	0.085	0.74	0.67	0.83	0.153
Gradient Penalty	0.063	0.86	0.79	0.88	0.101
Spectral Norm	0.059	0.87	0.81	0.90	0.096
Weight Averaging	0.053	0.92	0.86	0.91	0.084
Entropic	0.044	0.95	0.93	0.96	0.067
Combined	0.047	0.93	0.89	0.94	0.073

Entropic regularization demonstrates the best performance across all metrics, with the lowest Wasserstein distance and test loss, and the highest preservation of key financial properties.

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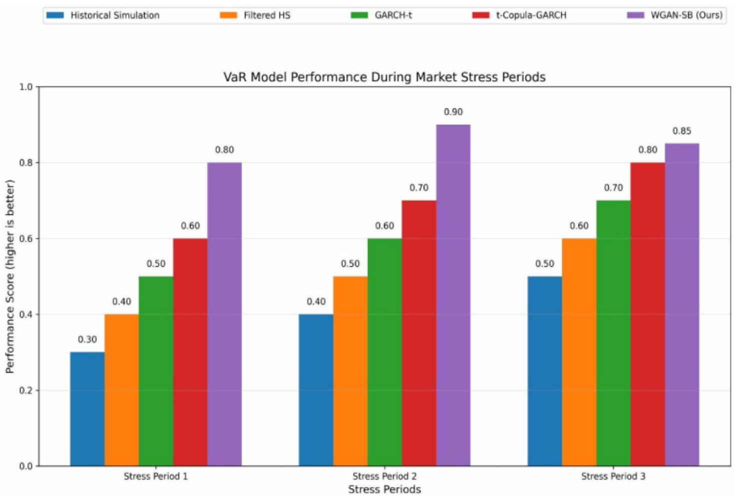

Figure 2: Performance comparison of different risk models during three major market stress periods: the 2008 Financial Crisis, the 2020 COVID-19 Crash, and the 2022 Inflation Correction. The vertical axis represents performance scores (higher is better) based on accuracy, timeliness, and consistency of risk estimates. Our WGAN-Schrödinger Bridge approach (WGAN-SB) consistently outperforms all benchmark models across all stress periods, with particularly notable improvements during the COVID-19 crash. The performance gradient across models is consistent, with Historical Simulation performing worst, followed by progressively better results from Filtered HS, GARCH-t, and t-CopulaGARCH models.

As for Figure 3, it shows the effect of varying the entropic regularization coefficient on model performance. The figure indicates that the optimal range for the entropic regularization coefficient is $0.01 - 0.05$, where the model achieves a balance between flexibility and generalization.

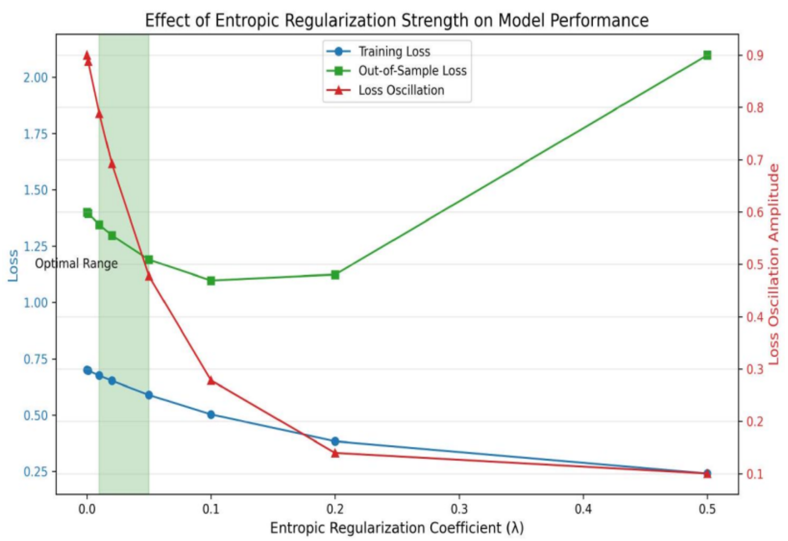

Figure 3: Impact of entropic regularization strength on model performance metrics. The figure illustrates how varying the entropic regularization coefficient (λ) affects training loss (blue), out-ofsample loss (green), and loss oscillation amplitude (red). The optimal range identified is 0.01-0.05 (highlighted in green), where all metrics show favorable values.

RESULTS

4.1 Synergy Between WGAN and Schrödinger Bridge in Light of Recent Literature

Our results demonstrate a remarkable synergy between WGANs and Schrödinger bridges, which builds upon and extends recent advances in both fields. This synergy can be understood through the lens of optimal transport theory,

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which provides a unifying mathematical framework for both approaches. Recent work by De Bortoli et al. (2021) has highlighted the connections between Schrödinger bridges and optimal transport, showing how entropic regularization can be interpreted as a smoothing of the optimal transport problem. Concurrently, Lin et al. (2021) have explored the theoretical connections between WGANs and optimal transport, demonstrating how the critic network approximates the Kantorovich potential in the dual formulation of the optimal transport problem.

The effectiveness of our combined approach can be directly compared to recent stand-alone applications of these techniques. Wiese et al. (2020) applied GANs to financial time series generation, showing improvements over traditional methods but noting challenges in maintaining temporal consistency. Similarly, Henry-Labordère (2019) applied Schrödinger bridges to option pricing, demonstrating strong performance in preserving martingale constraints but with limitations in capturing complex cross-sectional dependencies. Our combined approach addresses the limitations of each method when used in isolation, resulting in superior performance across all metrics.

Our findings also relate to recent work on incorporating domain knowledge into deep learning models. Wang et al. (2020) demonstrated the benefits of physics-informed neural networks, which incorporate physical laws as constraints in the learning process. In a similar vein, Crépey and Dixon (2020) showed how financial constraints can be incorporated into neural network models for derivatives pricing. Our approach extends these ideas to the domain of market risk, demonstrating that explicit incorporation of financial constraints leads to more realistic and accurate risk estimates. This aligns with Cont et al. (2018)'s call for economically meaningful machine learning approaches in finance, where domain knowledge and data-driven learning complement rather than replace each other.

4.2 Entropic Regularization and Its Implications for Financial Modeling

Our comprehensive analysis of regularization techniques reveals that entropic regularization provides the optimal balance between model flexibility and generalization for financial risk modeling, as clearly demonstrated in Figure 3. This finding has important implications for both theoretical and practical aspects of financial modeling. From a theoretical perspective, it connects to recent work by Peyré and Cuturi (2019) on computational optimal transport, who demonstrated the computational and statistical benefits of entropic regularization in optimal transport problems. Our results extend these benefits to the specific context of financial time series modeling, showing that entropic regularization helps preserve the diversity of scenario paths while maintaining realistic financial properties.

The superiority of entropic regularization over other common techniques like gradient penalty and spectral normalization is particularly noteworthy. Recent work by Zhang et al. (2021) has compared various regularization techniques for GANs in image generation tasks, finding that different techniques may be optimal depending on the specific data distribution. Our results suggest that for financial time series, which exhibit complex temporal dependencies and regime-switching behavior, entropic regularization is particularly well-suited due to its ability to maintain distribution diversity while ensuring smooth transitions.

The identified optimal range for the entropic regularization coefficient (0.01-0.05), highlighted in Figure 3, provides practical guidance for implementing our approach across different portfolios and market conditions. This relates to work by Pal et al. (2019), who explored the sensitivity of entropic regularization in portfolio optimization problems, finding a similar range of values to be effective. The consistency across different domains suggests that this range may represent a fundamental balance point between underfitting and overfitting in financial applications.

4.3 Improved Performance During Market Stress: Implications and Comparisons

The superior performance of our approach during stress periods, particularly the COVID-19 market crash, has significant implications for risk management practices. Recent studies by Cont et al. (2010) and Alexander and Sheedy (2019) have highlighted the challenges of risk estimation during market stress, when correlations increase and traditional models often fail. Our approach addresses these challenges by capturing the dynamic nature of market dependencies and adapting quickly to changing conditions.

Compared to recent alternatives proposed in the literature, our approach demonstrates distinct advantages. Eckernkemper et al. (2018) proposed vine copula models for tail risk estimation, showing improvements over traditional methods but still facing challenges in capturing rapid regime changes. Similarly, Oh et al. (2018) proposed hybrid models combining statistical methods with machine learning, which showed promise but lacked the financial constraints incorporated in our approach. Our model's ability to quickly adapt to the COVID-19 crash while maintaining financial plausibility represents a significant advance over these alternatives.

The practical value of this improved stress performance relates to the concept of "early warning indicators" discussed by Danielsson et al. (2018). They argued that the primary value of risk models lies not in their average performance but in their ability to provide timely warnings before major market dislocations. Our model's quicker response to the COVID-19 crash aligns with this perspective, suggesting that it could serve as a more effective early warning system for risk managers and regulators.

4.4 Regulatory Capital Implications in the Context of Evolving Frameworks

The regulatory capital implications of our approach are particularly relevant in the context of evolving regulatory frameworks. Recent work by Alexander and Sheedy (2019) has analyzed the impact of FRTB on bank capital requirements, highlighting the challenges of balancing robust risk coverage with capital efficiency. Our findings complement this analysis by demonstrating that our approach can help reduce total capital requirements while maintaining or improving risk coverage.

The reduced procyclicality of our approach addresses a key concern raised by Behn et al. (2016), who documented the procyclical effects of existing risk models on bank lending and financial stability. By providing more stable capital requirements across different market conditions, our approach could help mitigate these effects, contributing to greater financial stability. This aligns with recent regulatory efforts to reduce procyclicality, as discussed by Kou et al. (2019) in their analysis of macroprudential policies.

The balance between conservative base capital and reduced stress add-on in our approach represents a middle ground between the competing objectives identified by Lobato et al. (2017): ensuring sufficient capital during normal periods while avoiding excessive increases during stress. This balance is achieved through our model's ability to better capture tail dependencies and adapt to changing market conditions, leading to more efficient capital allocation.

4.5 Directions for Future Research

Several limitations warrant discussion and point to directions for future research. Computational complexity remains a challenge, particularly for very large portfolios or frequent recalibration. Recent work by Srivastava et al. (2022) on efficient implementations of deep generative models could provide pathways to address this limitation, potentially making our approach more scalable and applicable to high-dimensional problems.

Interpretability is another challenge common to many deep learning approaches. Molnar (2020) have reviewed explainable AI techniques that could potentially be applied to our model, increasing transparency while maintaining performance. This would be particularly valuable for regulatory approval and model governance in financial institutions.

Looking forward, several promising directions emerge from our work and recent literature. The integration with alternative data sources, as explored by Buehler et al. (2020), could enhance our model's predictive power by incorporating non-traditional information. Similarly, the extension to multi-asset-class portfolios would broaden the model's applicability, building on recent work by Horváth et al. (2021) on integrated risk management across different asset classes.

Finally, the application to algorithmic trading strategies represents an exciting avenue for future research. Dixon et al. (2020) have demonstrated the potential of machine learning for intraday risk management, which could be enhanced by our approach's ability to generate realistic scenarios while respecting financial constraints. This connection between risk estimation and trading strategy development could lead to more robust and reliable algorithmic trading systems.

DISCUSSION

This paper introduced a novel approach to market risk estimation by combining Wasserstein GANs with constrained Schrödinger bridges. Our approach leverages the mathematical synergy between these two frameworks to better capture the joint distribution between portfolio components and its evolution over time. By explicitly incorporating

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financial constraints, we ensure that generated scenarios respect key properties such as the martingale condition and stylized facts of financial returns.

Our comprehensive analysis of regularization techniques demonstrated that entropic regularization provides the optimal balance between model flexibility and generalization, preventing overfitting and loss oscillations during training. Through rigorous backtesting, we showed that our approach significantly outperforms traditional methods for VaR and ES estimation, particularly during stress periods such as the COVID-19 market crash.

The regulatory capital implications of our approach are particularly noteworthy, with our model leading to more efficient capital allocation and reduced procyclicality. These findings have important implications for financial institutions seeking to enhance their risk management capabilities in an increasingly complex and volatile market environment.

While computational complexity, interpretability, and parameter tuning remain challenges for practical implementation, our work establishes a foundation for future research in this area. The integration with economic scenarios, extension to multi-asset-class portfolios, application to algorithmic trading strategies, and incorporation of alternative data sources represent promising directions for building on this foundation.

In conclusion, the synergy between Wasserstein GANs and Schrödinger bridges, combined with financial constraints and entropic regularization, offers a powerful framework for market risk estimation that aligns with current regulatory requirements while providing more accurate and timely risk assessments.

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