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Research Article

Nano (1,2,3) Generalized Pre-Closed and Nano (1,2,3) Pre Generalized Closed Sets in Nano Tri Topological Space

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ABSTRACT

The objective of this paper is to introduced and explored the concepts of Nano (1,2,3) Generalized Pre closed sets and Nano (1,2,3) Pre-Generalized closed sets in Nano tri topological spaces.

Keywords: N(1,2,3)pre-open, N(1,2,3)pre-closed, N(1,2,3)gp closure, N(1,2,3)pg closure, N(1,2,3)gp interior, N(1,2,3)pg interior, N(1,2,3)gp closed, N(1,2,3)pg closed.

1.1 INTRODUCTION

Jackson et al. [5] have introduced and explored a new class of open sets in tri-topological space. Jeyasudha et al.[6] have introduced Tri-closed sets in tri-topological spaces. Recent work in nano tritopology has focused on the introduction and exploration of various new categories of sets, such as Nano (1,2,3) pre-generalized closed sets. Researchers have been analyzing the properties and characteristics of these sets within the framework of Nano Tritopological spaces. This research extends earlier concepts of Nano closed sets and aims to provide a deeper understanding of topological structures in the context of nanotechnology and advanced mathematical theories. Nano tri-topology offers a rich field for theoretical exploration, contributing to the advancement of topology as a discipline and providing new insights into the behavior of topological spaces. In future research ,Nano (1,2,3) pre-generalized closed sets and Nano (1,2,3) generalized pre closed in Nano tri topological space can be applied to the topological analysis of data, helping to identify patterns and structures within complex datasets, which is useful in various scientific and engineering applications.In this paper a new category of sets termed Nano (1,2,3) generalized pre closed set and Nano (1,2,3) pre generalized closed set in Nano Tritopological space is inaugurate and explore some of its effects.

2.NANO (1,2,3) GENERALIZED PRE CLOSED SETS IN NANO TRITOPOLOGICAL SPACE

In this paper, we define a new category of sets termed Nano (1,2,3) generalized preclosed sets in Nano Tritopological space is introduced and their properties are examined.

Definition: 2.1 Let (U, $N_{\tau_{(1,2,3)}}(X)$) be a Nano Tri topological space and P subset of U. Then P is stated be **N(1,2,3)pre open** if P⊆ $N_{\tau_{(1,2,3)}}$Int($N_{\tau_{(1,2,3)}}$Cl(P)). The supplement of N(1,2,3)pre open set in U is called N(1,2,3) pre closed in U.

Definition: 2.2 If (U, $N_{\tau_{(1,2,3)}}(X)$)is a NanoTritopological space with reffering to X where $X⊆U$& if $P⊆U$, Then

i) The **N(1,2,3) pre closure** of P is defined as ∩ of all N(1,2,3)those sets that include P and it is denoted by$N_{\tau_{(1,2,3)}}$*cl(P)*. $N_{\tau_{(1,2,3)}}$*pcl(P)* is the smallest N(1,2,3)pre closed set containing P.

ii) The **N(1,2,3) pre interior** of P is ∪ all N(1,2,3)pre open subsets within P are those sets that lie entirely within Pand it is denoted by $N_{\tau_{(1,2,3)}}$*pInt(P)*. $N_{\tau_{(1,2,3)}}$*pInt(P)* is the largest N(1,2,3)pre open subset of P

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Definition: 2.3 Let (P, $N_{\tau_{(1,2,3)}}(X)$) and (Q, $N_{\tau'_{(1,2,3)}}(X)$) be two Nano Tritopological spaces. Then a mapping f: (P, $N_{\tau_{(1,2,3)}}(X)$) → (Q, $N_{\tau'_{(1,2,3)}}(X)$)is **Nano(1,2,3) generalized pre closed sets (shortly $N_{\tau_{(1,2,3)}}$ *gp closed*)** on U if the reverse image of every N(1,2,3)closed set in (Q, $N_{\tau'_{(1,2,3)}}(X)$) is N (1,2,3)gp closed in (P, $N_{\tau_{(1,2,3)}}(X)$).

Example:2.4U={h,o,w,z}, U/R_1= {{h},{o},{w,z}}, X_1={h,w}, τ_{R_1}={∪, ∅,{h},{w,z},{h,w,z}}, U/R_2={{w},{z},{h,o}}, X_2={o,z}, τ_{R_2}={∪, ∅,{z}},{h,o},{h,o,z}, U/R_3={o}{w}{h,z}}, X_3={h,o}, τ_{R_3}={∪, ∅,{o},{h,z},{h,o,z}}, $N_{\tau_{R_{(1,2,3)}}}(X)$={∪ , ∅,{h},{o},{z},{h,o},{h,z},{w,z},{h,o,z},{h,w,z}}.N(1,2,3)closedsets={∪ , ∅,{o},{w},{h,o},{o,w},{w,z},{h,o,w},{h,w,z},{o,w,z}}.N(1,2,3)preclosedsets={U,∅,{h},{o},{w},{h,o},{h,z},{o,w},{w,z },{h,o,w},{h,w,z},{h,o,z},{o,w,z}}.N(1,2,3)generalizedpreclosedsetsR={U,∅,{h},{o},{w},{z},{h,o},{h,w},{h,z},{o,w},{ o,z},{w,z},{h,o,w},{h,o,z},{h,w,z},{o,w,z}}.

Theorem: 2.5 IfPbe a N(1,2,3) closed set in (U, $N_{\tau_{(1,2,3)}}(X)$), Then P is N(1,2,3) pre closed set.

Proof: Let P be a N(1,2,3) closed set. Then we have $N_{\tau_{(1,2,3)}}Cl(P)) = P$. To prove that $N_{\tau_{(1,2,3)}}Cl(N_{\tau_{(1,2,3)}}Int(P)) \subseteq P$ which implies that P is a N(1,2,3) pre closed set.$N_{\tau_{(1,2,3)}}Cl(N_{\tau_{(1,2,3)}}Int(P)) = N_{\tau_{(1,2,3)}}Int(P) \subseteq P$. Hence P is a N(1,2,3) pre closed set.

Theorem: 2.6 IfP be a N(1,2,3) generalized pre closed subset of (U, $N_{\tau_{(1,2,3)}}(X)$). If P⊆ $Q \subseteq N_{\tau_{(1,2,3)}}pCl(P)$, then Q is also a N(1,2,3) generalized pre closed subset of (U, $N_{\tau_{(1,2,3)'}}(X)$).

Proof: Let (U, $N_{\tau_{(1,2,3)}}(X)$) be a N(1,2,3) open set of a N(1,2,3) generalized pre closed subset of $N_{\tau_{(1,2,3)}}(X)$∀Q⊆U. As P⊆ Q we have P⊆ U. As P is a N(1,2,3) generalized pre closed set. $N_{\tau_{(1,2,3)}}PCl(P) \subseteq U$. Given Q⊆ $N_{\tau_{(1,2,3)}}PCl(P)$ we have $N_{\tau_{(1,2,3)}}PCl(Q) \subseteq N_{\tau_{(1,2,3)}}PCl(P)$. We have $N_{\tau_{(1,2,3)}}PCl(Q) \subseteq U$, whenever Q⊆ U, U is N(1,2,3) open. Hence Q is also a N(1,2,3) generalized pre closed subset of $N_{\tau_{(1,2,3)}}(X)$.

Theorem:2.7 If P and Q are two N(1,2,3)generalized pre closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$) , then PUQis also a N(1,2,3) generalized pre closed set in (U, $N_{\tau_{(1,2,3)}}(X)$)

Proof: Let P and Q be two N(1,2,3) generalized pre closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$). Let V be N (1,2,3) open set in U such that P⊆V and Q⊆V. Then we have $P \cup Q \subseteq V$. P and Q are N (1,2,3) generalized pre closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$) ,$N_{\tau_{(1,2,3)}}pCl(P) \subseteq V$ and $N_{\tau_{(1,2,3)}}pCl(Q)$ ⊆V. Now $N_{\tau_{(1,2,3)}}pCl(P \cup Q)$=$N_{\tau_{(1,2,3)}}pCl(P) \cup N_{\tau_{(1,2,3)}}pCl(Q)$ ⊆V. Thus we have $N_{\tau_{(1,2,3)}}pCl(P \cup Q) \subseteq V$. Whenever $P \cup Q \subseteq V$, V is a N(1,2,3) open set in (U, $N_{\tau_{(1,2,3)}}(X)$) This implies $P \cup Q$ is a N(1,2,3) generalized pre closed set in (U, $N_{\tau_{(1,2,3)}}(X)$)

3.NANO (1,2,3) PRE GENERALIZEDCLOSEDSETSINNANO TRITOPOLOGICAL SPACE

In this paper, we define a current category of sets termed Nano (1,2,3) pre-generalized closed sets in Nano Tritopological space is introduced and their properties are examined.

Definition: 3.1 Let (P, $N_{\tau_{(1,2,3)}}(X)$) and (Q, $N_{\tau'_{(1,2,3)}}(Y)$) be two NanoTritopological spaces. Then a mapping f: (P, $N_{\tau_{(1,2,3)}}(X)$) → (Q, $N_{\tau'_{(1,2,3)}}(Y)$)is **Nano (1,2,3)pre generalized closed sets (shortly $N_{\tau_{(1,2,3)}}$pg closed)** on P if the inverse image of every Nano (1, 2,3)closed set in (Q, $N_{\tau'_{(1,2,3)}}(X)$) is N (1, 2,3) pgclosed in (P, $N_{\tau_{(1,2,3)}}(X)$).

Example:3.2U={c,k,o,s}, U/R_1= {{c},{k},{o,s}}, X_1={c,o}, τ_{R_1}={∪, ∅,{c},{o,s},{c,o,s}}, U/R_2={{o},{s},{c,k}}, X_2={k,s}, τ_{R_2}={∪, ∅,{s}},{c,k},{c,k,s}, U/R_3={k}{o}{c,s}}, X_3={c,k}, τ_{R_3}={∪, ∅,{k},{c,s},{c,k,s}}, $N_{\tau_{R_{(1,2,3)}}}(X)$={∪ , ∅,{c},{k},{s},{c,k},{c,s},{o,s}, {c,k,s}, {c,o,s} }. N (1,2,3) closed sets ={∪ , ∅,{k},{o},{c,k},{k,o},{o,s},{c,k,s},{c,o,s},{k,o,s},}.N(1,2,3)preclosedsets={U,∅,{c},{k},{o},{c,k},{c,s},{k,o},{o,s},{c,k, o},{c,o,s},{c,k,s},{k,o,s}}.N(1,2,3)pregeneralizedclosedsetsR={U,∅,{c},{k},{o},{s},{c,k},{c,s},{k,o},{o,s},{c,k,o},{c,k,s },{c,o,s},{k,o,s}}.

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Theorem: 3.3 IfP is a N(1,2,3) closed set in (U, $N_{\tau_{(1,2,3)}}(X)$), thenP isN(1,2,3) pre generalized closed set.

Proof: Let P be a N(1,2,3) closed set of U and A⊆V, V is N(1,2,3) open in U. Since P is N(1,2,3) closed . $N_{\tau_{(1,2,3)}}Cl(P)) = P \subseteq V$.That is $N_{\tau_{(1,2,3)}} Cl(P) \subseteq N_{\tau_{(1,2,3)}} pCl(P)) \subseteq V$,Where V is N (1,2,3)pre open in U. Therefore P is a N(1,2,3) pre generalized closed set.

Theorem: 3.4 If P is a N(1,2,3) generalized closed set in (U, $N_{\tau_{(1,2,3)}}(X)$), thenP isN(1,2,3) pre generalized closed set.

Proof: Let P be a N(1,2,3) generalized closed set of U and P⊆U, U is N(1,2,3) open in X. Since P is N(1,2,3)closed . $N_{\tau_{(1,2,3)}}Cl(P)) \subseteq U$=P. That is $N_{\tau_{(1,2,3)}} Cl(P) \subseteq N_{\tau_{(1,2,3)}} pCl(P)) \subseteq U$,Where P⊆U ,U is N(1,2,3) pre open in X. Therefore P is a N(1,2,3) pre generalized closed set.

Example:3.5U={3,5,7,9}, U/R_1= {{3},{9},{5,7}}, X_1={3,5}, τ_{R_1}={∪, ∅,{3}{3,5,7}{5,7}}, U/R_2={{5},{7},{3,9}}, X_2={7,9}, τ_{R_2}={∪, ∅,{7},{3,7,9},{3,9}}, U/R_3={{3}{5}{7,9}}, X_3={5,7}, τ_{R_3}={∪, ∅,{5},{5,7,9},{7,9}}, $N_{\tau_{R_{(1,2,3)}}}(X)$={∪ , ∅,{3},{5},{7},{3,9},{5,7},{7,9},{3,5,7},{3,7,9},{5,7,9}}. N(1,2,3) closed sets ={U,ϕ, {3},{5},{9},{3,5},{3,9},{5,7},{3,5,9},{3,7,9}{5,7,9},}. N(1,2,3) pre closed sets ={U,ϕ,{3},{5},{7},{3,7},{3,9},{5,7},{7,9},{3,5,9},{3,5,9},{3,7,9},{5,7,9}}. N(1,2,3) generalized closed sets ={U,∅,{3},{5},{7},{9},{3,5},{3,7},{3,9},{5,7},{5,9},{3,5,9},{3,7,9},{3,5,9},{5,7,9}}.N(1,2,3) pre generalized closed sets R= ={U,∅,{3},{5},{7},{9},{3,5},{3,7},{3,9},{5,7},{5,9},{3,5,7},{3,7,9},{3,5,9},{5,7,9}}. Here N(1,2,3) generalized closed sets are Nano (1,2,3) pre generalized closed set.

Theorem: 3.6 IfP be a N(1,2,3) pre generalized closed subset of (U, $N_{\tau_{(1,2,3)}}(X)$). If P⊆ $Q \subseteq N_{\tau_{(1,2,3)}}pCl(P)$, then Q is also a N(1,2,3) pre generalized closed subset of (U, $N_{\tau_{(1,2,3)}}(X)$)

Proof: Let (U, $N_{\tau_{(1,2,3)}}(X)$) be a N(1,2,3)pre open set of a N (1,2,3)pre generalized closed subset of $N_{\tau_{(1,2,3)}}(X)$suchthat Q⊆U. As P⊆ Q we have P⊆ U. As P is a N(1,2,3) pre generalized closed set. $N_{\tau_{(1,2,3)}}PCl(P) \subseteq U$. Given Q⊆ $N_{\tau_{(1,2,3)}}PCl(P)$ we have $N_{\tau_{(1,2,3)}}PCl(Q) \subseteq N_{\tau_{(1,2,3)}}PCl(P)$. We have $N_{\tau_{(1,2,3)}}PCl(Q) \subseteq U$, whenever Q⊆ U, U is N(1,2,3) pre open. Hence Q is also a N(1,2,3) pre generalized closed subset of $N_{\tau_{(1,2,3)}}(X)$.

Theorem: 3.7 If P and Q are two N(1,2,3)pre generalized closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$) , then PUQis also a N(1,2,3) pre generalized closed set in (U, $N_{\tau_{(1,2,3)}}(X)$)

Proof: Let P and Q be two N(1,2,3) pre generalized closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$). Let V be N(1,2,3)pre open set in U such that P⊆V and Q⊆V. Then we have $P \cup Q \subseteq V$. P and Q are N(1,2,3) pre generalized closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$) , $N_{\tau_{(1,2,3)}}pCl(P) \subseteq V$ and $N_{\tau_{(1,2,3)}}pCl(Q)$ ⊆V. Now $N_{\tau_{(1,2,3)}}pCl(P \cup Q)$=$N_{\tau_{(1,2,3)}}pCl(P) \cup N_{\tau_{(1,2,3)}}pCl(Q)$ ⊆V. Thus we have $N_{\tau_{(1,2,3)}}pCl(P \cup Q) \subseteq V$. Whenever $P \cup Q \subseteq V$, V is a N(1,2,3)pre open set in (U, $N_{\tau_{(1,2,3)}}(X)$) This implies $P \cup Q$ is a N(1,2,3) pre generalized closed set in (U, $N_{\tau_{(1,2,3)}}(X)$).

Example:3.8U={6,7,8,9}, U/R_1= {{6},{9},{7,8}}, X_1={6,7}, τ_{R_1}={∪, ∅,{6}{6,7,8}{7,8}}, U/R_2={{7},{8},{6,9}}, X_2={8,9}, τ_{R_2}={∪, ∅,{8},{6,8,9},{6,9}}, U/R_3={{6}{7}{8,9}}, X_3={7,8},τ_{R_3}={∪ , ∅,{7},{7,8,9},{8,9}},$N_{\tau_{R_{(1,2,3)*}}}(X)$={, ∅,{6},{7},{8},{6,9},{7,8},{8,9},{6,7,8},{6,8,9},{7,8,9}}. N(1,2,3) closed sets ={U,ϕ, {6},{7},{9},{6,7},{6,9},{7,8},{6,7,9},{6,8,9}{7,8,9}}. N(1,2,3) pre closed sets ={U,ϕ,{6},{7},{8},{6,8},{6,9},{7,8},{8,9},{6,7,8},{6,7,9},{6,8,9},{7,8,9}}. N(1,2,3) pre generalized closed sets R={U,∅,{6},{7},{8},{9},{6,7},{6,8},{6,9},{7,8},{7,9},{6,7,8},{6,8,9},{6,7,9},{7,8,9}}.Here{6,7}∩{6,8}∩{6,9}={6},{7, 8}∩{7,9}={7},{7,8}∩ {8,9}={8} which is again a N(1,2,3) pre generalized closed set.

Theorem: 3.9 If P is N(1,2,3) generalized closed set , then P is N(1,2,3) pre generalized closed set.

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Proof: Let P be N(1,2,3) generalized closed set of U and P⊆V, V is N(1,2,3) open in U. Since P is N(1,2,3) open . $N_{\tau_{(1,2,3)}}Cl(P)) = P \subseteq V$.That is $N_{\tau_{(1,2,3)}} \; Cl(P) \subseteq N_{\tau_{(1,2,3)}} \; pCl(P)) \subseteq V$,Where V is N(1,2,3)pre open in U. Therefore P is a N(1,2,3) pre generalized closed set.

Example:3.10U={6,7,8,9}, U/R_1= {{6},{9},{7,8}}, X_1={6,7}, τ_{R_1}={∪, ∅,{6}{6,7,8}{7,8}},U/R_2={{7},{8},{6,9}}, X_2={8,9}, τ_{R_2}={∪, ∅,{8},{6,8,9},{6,9}}, U/R_3={{6}{7}{8,9}}, X_3={7,8},τ_{R_3}={∪ , ∅,{7},{7,8,9},{8,9}},$N_{\tau_{R_{(1,2,3)^*}}}(X)$={, ∅,{6},{7},{8},{6,9},{7,8},{8,9},{6,7,8},{6,8,9},{7,8,9}}. N(1,2,3) closed sets ={U,ϕ, {6},{7},{9},{6,7},{6,9},{7,8},{6,7,9},{6,8,9}{7,8,9}}. N(1,2,3) closed sets ={U,ϕ, {6},{7},{9},{6,7},{6,9},{7,8},{6,7,9},{6,8,9}{7,8,9}}. N(1,2,3) pre closed sets ={U,ϕ,{6}, {7},{8},{6,8},{6,9},{7,8},{8,9},{6,7,8},{6,7,9},{6,8,9},{7,8,9}}. N(1,2,3) generalized pre closed sets R={U,∅,{6},{7},{8},{9},{6,7},{6,8},{6,9},{7,8},{7,9}, {8,9},{6,7,8},{6,8,9},{6,7,9},{7,8,9}}. N(1,2,3) pre generalized closed sets R= ={U,∅,{6},{7},{8},{9},{6,7},{6,8},{6,9},{7,8},{7,9}, {6,7,8},{6,8,9},{6,7,9},{7,8,9}}. Here N(1,2,3) generalized pre closed sets are N(1,2,3) pre generalized closed set.

CONCLUSION

In this paper, certain characteristics properties of Nano (1,2,3) Generalized Pre Closed and Nano (1,2,3) Pre Generalized Closed Set in NanoTritopological space are examined. This exploration will be expanded upon in forthcoming research endeavors, potentially with practical applications in mind.

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