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Research Article

A Hybrid Parallel-Sequential Service Model for Tandem Communication Networks with Load-Dependent and Time-Variant Behaviour

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ABSTRACT

Queueing models are vital analytical tools for examining a wide range of real-world scenarios, such as those found in communication networks, transportation systems, machine maintenance, and production lines. These systems often exhibit non-stationary arrival and service processes, reflecting their dynamic nature over time. In this study, we develop and evaluate a queueing model that integrates both sequential and parallel service mechanisms, characterized by non-stationary Poisson arrivals and time-dependent service rates. The model assumes the presence of two parallel queues, each serviced independently, which subsequently converge into a single queue connected to a downstream service station. Such configurations are commonly observed in manufacturing systems and network infrastructures. By employing differential equations, we derive the joint probability generating function for the queue lengths. Additionally, we obtain explicit expressions for key performance indicators, including the average no. of customers in the queue, the average waiting time, and the variation in queue size in response to changes in service station throughput. The results demonstrate that non-stationary arrival and service patterns have a significant effect on overall system performance. This investigation draws inspiration from earlier foundational models while extending their applicability to more complex and realistic scenarios.

Keywords: Non-stationary Poisson process, parallel and sequential queuing model, load-dependent service, performance metrics.

INTRODUCTION

Over the past few decades, extensive research in queuing theory, initiated by A.K. Erlang in 1902, has inspired numerous scholars to develop mathematical models for congestion control. Numerous writers devised diverse models with distinct assumptions to analyze waiting lines with greater precision and efficacy. Cappe et al. and Abry et al. (2002) have established that the traffic of computer networks cannot be adequately analyzed using Poisson processes for traffic prediction, as these techniques are suited for smoother and less bursty traffic. Currently, most communication systems in networks must manage bursty and diverse traffic. Leland et al. (1994) asserted that traffic on Ethernet LANs lacks reduced burstiness and exhibits behaviour and similar scope. Rakesh Singhai et al. (2007) noted in their research of MAN and WAN that these networks exhibit burstiness due to time-dependent arrival and service procedures. Numerous studies indicate that many contemporary communication networks experience significant load circumstances characterized by time-dependent arrival and service operations. Nonetheless, scant research has been documented concerning non-stationary queueing models featuring arrival and service processes based on time, with the exception of the contributions by Massey and Whitt (1994), Misses (1993, 1996), Duffield et al. (2001), and Ward Whitt (2016), who have formulated certain queueing models utilizing diffusion approximation or simulation studies.Recently, Durga Aparajitha and Raj Kumar et al. (2014), Rao et al. (2017, 2019), and Durga Aparajitha et al. (2017, 2024) have examined queuing models featuring non-stationary service processes, while the arrival processes adhere to Poisson distributions. Srilatha et al. (2019) created a non-

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stationary queuing model featuring both the processes of arrival and services are time-dependent. In numerous communication networks, both the processes of arrival and services exhibit temporal dependence, and the configuration comprises parallel and series connections. Specifically, two queues operate in parallel, each linked to independent service stations. Upon completion of service at these stations, customers may proceed to a third queue, which is arranged in tandem with a third service station. After receiving service from the third station, customers exit the system. Such systems are prevalent and beneficial for analyzing and predicting delays in MAN, WAN, and LAN environments, where arrival and service operations are time-dependent, and the service duration for each customer is contingent upon the number of size in the buffer linked to the service station. This form of network analysis is essential for modeling multi-node communication networks. In this research, we design and study a tandem queueing model featuring both parallel and sequential connections, characterized by non-stationary (time-dependent) arrival and service processes, as well as dynamic bandwidth allocation (load-dependent service). It is presumed that two queues operate in parallel, and upon receiving service from the service station, they merge into another queue linked to an additional service station. The queue order adheres to a FIFO and the queue size is infinite.

The following sections of this paper are structured as follows: Section 2 presents the development of the proposed queueing model, founded on a set of postulates and formulated using differential-difference equations. Section 3 details the derivation of key system characteristics and performance metrics. Section 4 provides a numerical illustration of the model along with an extensive sensitivity analysis. Section 5 offers a comparative evaluation between the proposed model and existing models, particularly those based on homogeneous Poisson processes. Section 6 concludes the study with key insights, summarizing the contributions and outlining potential avenues for future research in this area.

2. QUEUEING MODEL

The development of a queuing model is briefly described in this section. Look at three service stations and three corresponding queues set up in both parallel and sequential arrangements. Following their experience at the first or second service station, customers move on to the third line, which is connected to the service stations in a sequential manner. After receiving assistance from the first or second service stations, customers proceed to the third line. With average arrival rates of $\lambda_1(t)$ for the first queue and $\lambda_2(t)$ for the second, the arrival process follows a NHPP.

The formula for calculating the arrival rate of customers in the line is $\lambda_1(t) = (\lambda_1 + \gamma_1 t)$ *and* $\lambda_2(t) = (\lambda_2 + \gamma_2 t)$. It is implicit that the three service stations' service processes follow NHPP, with service rates for the first, second, and third service stations being μ1(t), μ2(t), and μ3(t), respectively. Additionally, it is proposed that the no. of customers in the linked queue determines the service rate at the service stations, specifically: $\mu_1(n_1,t) = n_1(\alpha_1 + \beta_1 t), \mu_2(n_2,t) = n_2(\alpha_2 + \beta_2 t)$ *and* $\mu_3(n_3,t) = n_3(\alpha_3 + \beta_3 t)$ After receiving service at the first or second service station, customers move on to the third line, which is connected to the service stations in a sequential manner. After being served by the first or second service stations, customers move on to the third line. The queue capacity is endless, and the discipline adheres to the first-in, first-out concept. A schematic diagram illustrating the structure of the proposed queueing model is presented in **Figure 1**.

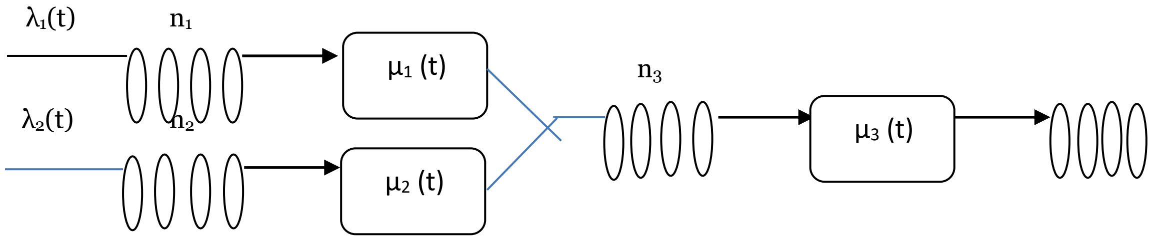

Figure 1 presents a schematic representation of the queueing model.

Here n_1, n_2 and n_3 represent the quantity of customers in the first, second, and third queues, respectively. Let $P_{n_1,n_2,n_3}(t)$ denote the probability of having n_1 customers in the first line up, n_2 customers in the second line up,

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and n_3 customers in the third line up during a time period of length t, where n_1, n_2 and $n_3$are integers. Similarly $P_{n_1,n_2,n_3,}(t+h)$ denote the probability of n_1 customers in the first line up, n_2 customers in the second line up, and n_3 customers in the third line up arriving within a time period of length t+h.

The differential equations that delineate the model are as follows:

For diminutive values of h,

$$\frac{\partial P_{n_1,n_2,n_3}(t)}{\partial t} = -((1-\lambda_1(t)h)(1-\lambda_2(t)h)(1-n_1\mu_1(t)h)(1-n_2\mu_2(t)h)(1-n_3\mu_3(t)h))P_{n_1,n_2,n_3}(t) + \lambda_1(t)h\, P_{n_1-1,n_2,n_3}(t)$$

$$+\lambda_2(t)hP_{n_1,n_2-1,n_3}(t) + (n_1+1)h\mu_1(t)P_{n_1+1,n_2,n_3-1}(t)$$

$$+(n_2+1)h\mu_2(t)P_{n_1,n_2+1,n_3-1}(t) + (n_3+1)h\mu_3(t)P_{n_1,n_2,n_3+1}(t); \quad \forall n_1,n_2,n_3 > 0$$

As h→0

$$\frac{\partial P_{n_1,n_2,n_3}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)+n_2\mu_2(t)+n_3\mu_3(t)\big)P_{n_1,n_2,n_3}(t) + \lambda_1(t)P_{n_1-1,n_2,n_3}(t)$$

$$+\lambda_2(t)P_{n_1,n_2-1,n_3}(t) + (n_1+1)\mu_1(t)P_{n_1+1,n_2,n_3-1}(t)$$

$$+(n_2+1)\mu_2(t)P_{n_1,n_2+1,n_3-1}(t) + (n_3+1)\mu_3(t)P_{n_1,n_2,n_3+1}(t); \quad \forall n_1,n_2,n_3 > 0$$

$$\frac{\partial P_{0,n_2,n_3}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)+n_2\mu_2(t)+n_3\mu_3(t)\big)P_{0,n_2,n_3}(t) + \lambda_2(t)P_{0,n_2-1,n_3}(t)$$

$$+\mu_1(t)P_{1,n_2,n_3-1}(t) + (n_2+1)\mu_2(t)P_{0,n_2+1,n_3-1}(t)$$

$$+(n_3+1)\mu_3(t)P_{0,n_2,n_3+1}(t); \qquad \forall\, n_2,n_3 > 0$$

$$\frac{\partial P_{n_1,0,n_3}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)+n_3\mu_3(t)\big)P_{n_1,0,n_3}(t) + \lambda_1(t)P_{n_1-1,n_2,n_3}(t)$$

$$+(n_1+1)\mu_1(t)P_{n_1+1,0,n_3-1}(t) + \mu_2(t)P_{n_1,1,n_3}(t) + (n_3+1)\mu_3(t)P_{n_1,0,n_3+1}(t);$$

$\forall n_1,n_3 > 0$

$$\frac{\partial P_{n_1,n_2,0}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)+n_2\mu_2(t)\big)P_{n_1,n_2,0}(t) + \lambda_1(t)P_{n_1-1,n_2,0}(t)$$

$$+\lambda_2(t)P_{n_1,n_2-1,0}(t) + (n_1+1)\mu_1(t)P_{n_1+1,n_2,n_3-1}(t) + (n_3+1)\mu_3(t)P_{n_1,n_2,1}(t);$$

$\forall n_1,n_2 > 0$

$$\frac{\partial P_{0,0,n_3}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_3\mu_3(t)\big)P_{0,0,n_3}(t) + \mu_1(t)P_{1,0,n_3-1}(t)$$

$$+\mu_2(t)P_{0,1,n_3-1}(t) + (n_3+1)\mu_3(t)P_{0,0,n_3+1}(t); \qquad \forall\, n_3 > 0$$

$$\frac{\partial P_{n_1 0,0}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)\big)P_{n_1,0,0}(t) + \lambda_1(t)P_{n_1-1,0,0}(t) + \mu_3(t)P_{n_1,0,1}(t); \quad \forall n_1 > 0$$

$$\frac{\partial P_{0,n_2,0}(t)}{\partial t} = -\big(\lambda_1(t)+\lambda_2(t)+n_2\mu_2(t)\big)P_{0,n_2,0}(t) + \lambda_2(t)P_{0,n_2-1,0}(t)+\mu_3(t)P_{0,n_2,1}(t); \forall\ n_2 > 0$$

$$\frac{\partial P_{0,0,0}(t)}{\partial t} = -(\lambda_1(t)+\lambda_2(t))P_{0,0,0}(t) + \mu_3(t)(1)$$

Let the PGF of $P_{n_1,n_2,n_3}(t)$ be

$$P(S_1,S_2,S_3,t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} P_{n\,1,n_2,n_3}(t){s_1}^{n_1}{s_2}^{n_2}\,{s_3}^{n_3} \qquad (2)$$

By multiplying equation (1) with $s_1^{n_1}, s_2^{n_2}, s_3^{n_3}$ and summing over alln_1, n_2,n_3 obtain

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$$\frac{\partial P(t)}{\partial t} = -\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\left(\lambda_1(t)+\lambda_2(t)+n_1\mu_1(t)+n_2\mu_2(t)+n_3\mu_3(t)\right)P_{n\,1,n_2,n_3}(t)s_1^{\,n_1}s_2^{\,n_2}s_3^{\,n_3}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\lambda_1(t)P_{n_1-1,n_2,n_3}(t)\,s_1^{\,n_1}s_2^{\,n_2}s_3^{\,n_3}+\sum_{n_1=0}^{\infty}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\lambda_2(t)P_{n_1,n_2-1,n_3}(t)\,s_1^{\,n_1}s_2^{\,n_2}s_3^{\,n_3}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}(n_1+1)\mu_1(t)P_{n_1+1,n_2,n_3-1}(t)\,s_1^{\,n_1}s_2^{\,n_2}s_3^{\,n_3}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}(n_2+1)\mu_2(t)P_{n_1,n_2+1,n_3-1}(t)\,s_1^{\,n_1}s_2^{\,n_2}s_3^{\,n_3}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}(n_3+1)\mu_3(t)P_{n_1,n_2,n_3+1}(t)\,;\quad \forall n_1,n_2,n_3>0 \qquad (3)$$

After simplifying, we get

$$\frac{\partial P(s_1,s_2,s_3,t)}{\partial t} = -\mu_1(t)(s_3-s_1)\frac{\partial P(s_1,s_2,s_3,t)}{\partial s_1}-\mu_2(t)(s_3-s_2)\frac{\partial P(s_1,s_2,s_3,t)}{\partial s_2}$$
$$-\mu_3(t)(1-s_3)\frac{\partial P(s_1,s_2,s_3,t)}{\partial s_3}+P(S_1,S_2,t)[\lambda(t)(s_1-1)+\lambda_2(t)(s_2-1)] \quad (4)$$

Utilizing Lagrangian's approach to solve the model (4), the auxiliary model become

$$\frac{dt}{1}=\frac{ds_1}{-\mu_1(t)(s_3-s_1)}=\frac{ds_2}{-\mu_2(t)(s_3-s_2)}=\frac{ds_3}{-\mu_3(t)(1-s_3)}$$
$$=\frac{dP}{P(s_1,s_2,t)[\lambda_1(t)(s_1-1)+\lambda_2(t)(s_2-1)]} \qquad (5)$$

Given that both the arrival and service rates exhibit linearity and time dependent, they can be expressed in the following form:

$$\lambda_1(t)=\lambda_1+\gamma_1 t;\; 0\le\gamma_1\le 1,\;\; \lambda_2(t)=\lambda_2+\gamma_2 t; 0\le\gamma_2\le 1$$
$$\mu_1(t)=\alpha_1+\beta_1 t;\; 0\le\beta_1\le 1, \mu_2(t)=\alpha_2+\beta_2 t; 0\le\beta_2\le 1 and\;\; \mu_3(t)=\alpha_3+\beta_3 t;$$
$$0\le\beta_3\le 1,$$

By calculating the initial and fourth conditions in the model (5), obtain

$$a=(s_3-1)e^{-\int\mu_3(t)dt}$$

By calculating the initial and third terms in the model (5), obtain

$$b=s_2e^{-\int\mu_2(t)dt}+(s_3-1)e^{-\int\mu_3(t)dt}\left(\int\mu_2(t)e^{\int[\mu_3(t)-\mu_2(t)]dt}dt\right)+\int\mu_2(t)\,e^{-\int\mu_2(t)dt}dt$$

By calculating the initial and second terms in the model (5), obtain

$$c=s_1e^{-\int\mu_1(t)dt}+(s_3-1)e^{-\int\mu_3(t)dt}\left(\int\mu_1(t)e^{\int[\mu_3(t)-\mu_1(t)]dt}dt\right)+\int\mu_1(t)\,e^{-\int\mu_1(t)dt}dt$$

Consider the arrival rates $\lambda_1(t)$ *and* $\lambda_2(t)$ and service rates $\mu_1(t),\mu_2(t)$ *and* $\mu_3(t)$ are linear function of time$\lambda_1(t)=\lambda_1+\gamma_1 t$, $\lambda_2(t)=\lambda_2+\gamma_2 t$, $\mu_1(t)=\alpha_1+\beta_1(t)$, $\mu_2(t)=\alpha_2+\beta_2(t) and$ $\mu_3(t)=\alpha_3+\beta_3(t)$. By calculating the initial and fifth terms in the model (5), obtain some arbitrary constants. Using the initial circumstances $P_{000}(0)=1, P_{000}(t)=0\;\;\forall t>0$

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$$d = P(s_1,s_2,t)exp\left(-\int\left(\int \lambda_1(t)e^{-\int \mu_1(t)dt}dt\right)\left([s_1e^{-\int \mu_1(t)dt} + (s_3-1)e^{-\int \mu_3(t)dt}\left(\int \mu_1(t)e^{\int[\mu_3(t)-\mu_1(t)]dt}dt\right)\right.\right.$$

$$+\int \mu_1(t)\, e^{-\int \mu_1(t)dt}dt\Big]\int e^{\int \mu_1(t)dt}dt - (s_3-1)e^{-\int \mu_3(t)dt}$$

$$\left(\int e^{\int \mu_1(t)dt}\left(\int \mu_1(t)e^{\int[\mu_3(t)-\mu_1(t)]dt}dt\right)dt\right) + \int -e^{-\int \mu_1(t)dt}\left(\int \mu_1(t)\, e^{-\int \mu_1(t)dt}dt\right)dt - t\Big)$$

$$-(\int \lambda_2(t)e^{-\int \mu_1(t)dt}dt)([s_2e^{-\int \mu_2(t)dt} + (s_3-1)e^{-\int \mu_3(t)dt}\Big)\left(\int \mu_2(t)e^{\int[\mu_3(t)-\mu_2(t)]dt}dt\right)$$

$$+\int \mu_2(t)\, e^{-\int \mu_2(t)dt}dt\Big]\int e^{\int \mu_2(t)dt}dt - (s_3-1)e^{-\int \mu_3(t)dt}$$

$$\left(\int e^{\int \mu_2(t)dt}\left(\int \mu_2(t)e^{\int[\mu_3(t)-\mu_2(t)]dt}dt\right)dt\right) + \int -e^{-\int \mu_2(t)dt}\left(\int \mu_2(t)\, e^{-\int \mu_2(t)dt}dt\right)dt - t\Big)$$

The comprehensive explanation of (5) provides the PGF for the no. of customers in the first line up, second line up and third line ups at time 't' as

$$P(s_1,s_2,s_3,t) = exp\left(\lambda_1\left((s_1-1)e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)\, e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv}{\lambda_1} - \frac{1}{\alpha_1}\right)\right.\right.$$

$$+(s_3-1)e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1} - \frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)$$

$$+(s_3-1)\, e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1} - \frac{1}{\alpha_3}\right)$$

$$+\ \lambda_2\left((s_2-1)e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\frac{\int_0^t(\lambda_2+\gamma_2 v)\, e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2} - \frac{1}{\alpha_2}\right.$$

$$+(s_3-1)e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2} - \frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)$$

$$+(s_3-1)\, e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$\left.-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2} - \frac{1}{\alpha_3}\right) \quad (6)$$

Hence,

Theorem 1:

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The PGF for the no. of customers in each line up of parallel and sequential tandem queueing model with non-stationary arrival and series processes having load dependent service governed by the differential equations presented in equation (1) with the initial circumstances $P_{000}(0) = 1, P_{000}(t) = 0\ \forall t > 0$ This function is denoted as $P(s_1, s_2, s_3, t)$ in given equation (6).

3. FUNDAMENTAL PROPERTIES OF THE QUEUING MODEL:

We get the probability of the queue being empty by extending $P(s1, s2, t)$ as given in equation (6) and grouping the constant terms.

$$P_{0\,0\,0}(t) = e(s_1, s_2, t) = exp\left(-\lambda_1\left(e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)\, e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv}{\lambda_1}-\frac{1}{\alpha_1}\right)\right.\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)$$

$$+\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\right)$$

$$-\,\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\frac{\int_0^t(\lambda_2+\gamma_2 v)\, e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)$$

$$+\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$\left.-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\right) \qquad (7)$$

By substituting $s_2=1$, $s_3=1$ in $P(s_1, s_2, s_2 t)$, we get the PGF for the initial queue size as follows

$$P(s_{1,}t) = exp\left(\lambda_1(s_1-1)e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)\, e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv}{\lambda_1}-\frac{1}{\alpha_1}\right)\right)\ ;\ \lambda_1 < \alpha_1, \beta_1 \ (8)$$

By increasing $P(s_1, t)$ and consolidating the constant terms, we ascertain the likelihood that the initial queue is devoid of elements as

$$P_{0\,.\,.}(t) = exp\left(-\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)\, e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv}{\lambda_1}-\frac{1}{\alpha_1}\right)\right) \qquad (9)$$

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In the initial queue, the average no. of customers as

$$L_1(t) = \lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \tag{10}$$

In the initial service station, the utilization as

$$U_1(t) = 1 - exp\left(-\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \right) \tag{11}$$

In the initial service station, the throughput as

$$ThP_1(t) = (\alpha_1 + \beta_1 t)\left[1 - exp\left(-\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \right) \right] \tag{12}$$

In the initial queue, the average waiting time of a customers as

$$W_1(t) = \frac{\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right)}{(\alpha_1 + \beta_1 t)\left[1 - exp\left(-\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \right) \right]} \tag{13}$$

In the initial queue, the variance of the no. of customers as

$$V_1(t) = \lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \tag{14}$$

In the initial queue, the CV of the no. of customers as

$$CV_1(t) = \left(\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_1 + \gamma_1 v)\, e^{\alpha_1 v + \beta_1 \frac{v^2}{2}} dv}{\lambda_1} - \frac{1}{\alpha_1} \right) \right)^{-1/2} * 100 \tag{15}$$

By substituting s_1=1, s_3=1 in $P(s_1, s_2, s_2, t)$ we get the PGF for the no. of customers in the second queue size as follows

$$P(s_2, t) = exp\left(\lambda_2 \left((s_2 - 1) e^{-\left(\alpha_2 t+\beta_2 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_2 + \gamma_2 v)\, e^{\alpha_2 v + \beta_2 \frac{v^2}{2}} dv}{\lambda_2} - \frac{1}{\alpha_2} \right) \right) \right) \;;\quad \lambda_2 < \alpha_2, \beta_2 \tag{16}$$

By increasing P (s_2, t) and consolidating the constant terms, we ascertain the likelihood that the second queue is devoid of elements as

$$P_{.0.}(t) = exp\left(-\lambda_2 \left(e^{-\left(\alpha_2 t+\beta_2 \frac{t^2}{2}\right)} \left(\frac{\int_0^t (\lambda_2 + \gamma_2 v)\, e^{\alpha_2 v + \beta_2 \frac{v^2}{2}} dv}{\lambda_2} - \frac{1}{\alpha_2} \right) \right) \right) \tag{17}$$

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In the second queue, the average no. of customers as

$$L_2(t) = \left(\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right) \qquad (18)$$

In the second service station, the utilization as

$$U_2(t) = 1-exp\left(-\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right) \qquad (19)$$

In the second service station, the throughput as

$$ThP_2(t) = (\alpha_2+\beta_2 t)\left(1-exp\left(-\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right)\right) \qquad (20)$$

In the second queue, the average waiting time of a customers as

$$W_2(t) = \frac{\left(\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right)}{(\alpha_2+\beta_2 t)\left(1-exp\left(-\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right)\right)} \qquad 21$$

In the second queue, the variance of the no. of customers as

$$V_2(t) = \left(\lambda_2\left(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)\right) \qquad (22)$$

In the second queue, the CV of the no. of customers as

$$CV_2(t) = \left(\lambda_2 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)\right)^{-1/2} * 100 \qquad (23)$$

By substituting $s_1=1$, $s_2=1$ in $P(s_1,s_2,s_2,t)$ we get the PGF for the no. of customers in the third queue size as follows

$$P(s_3,t) = exp\left(\lambda_1\left((s_3-1)e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

$$+(s_3-1)\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\right)$$

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$$+\ \lambda_2\left((s_3-1)e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+(s_3-1)\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\right)\right)$$

$$\lambda_1,\lambda_2<min(\alpha_1+\beta_1 t),(\alpha_2+\beta_2 t)\,,(\alpha_3+\beta_3 t)\quad(24)$$

By increasing P (s_3, t) and consolidating the constant terms, we ascertain the likelihood that the third queue is devoid of elements as

$$P_{..0}(t)=exp\left(-\lambda_1\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

$$+\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\right)\right)$$

$$-\ \lambda_2\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$\left.\left.\left.-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\right)\right)\right)\qquad(25)$$

In the third queue, the average no. of customers as

$$L_3(t)=\left(\lambda_1\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

$$+\,e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

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$$-\frac{\int_0^t (\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\Bigg)\Bigg)$$

$$+\ \lambda_2\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$-\frac{\int_0^t (\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\Bigg)\Bigg)\Bigg)\qquad(26)$$

In the third service station, the utilization as

$$U_3(t)=1-exp\left(-\lambda_1\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$-\frac{\int_0^t (\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\Bigg)\Bigg)$$

$$-\ \lambda_2\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$-\frac{\int_0^t (\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\Bigg)\Bigg)\Bigg)\qquad(27)$$

In the third service station, the throughput as

$$ThP_3(t)=(\alpha_3+\beta_3 t)1-exp\left(-\lambda_1\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

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$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\right)\right)$$

$$-\lambda_2\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$\left.\left.\left.-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\right)\right)\right) \quad (28)$$

In the third queue, the average waiting time of a customers as

$W_3(t)=\frac{L_3(t)}{Thp_3(t)}$

L_3 (t) and Thp_3 (t) are defined in equations (26) and (28) correspondingly. (29)

In the third queue, the Variance of the no. of customers as

$$V_3(t)=\left(\lambda_1\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)\right.\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\right)\right)$$

$$+\lambda_2\left(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)\right.$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}\right.$$

$$-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\Bigg)\Bigg)\Bigg)\Bigg) \quad (30)$$

We derive the CV of the no. of customers in the second queue as

$$CV_3(t)=\left(V_3(t)\right)^{-1/2}*100 \quad (31)$$

We derive the average no. of customers in the queueing system at time t as

$L(t) = L_1(t) + L_2(t) + L_3(t)$

$$L(t)=\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\gamma_1 v)\,e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv}{\lambda_1}-\frac{1}{\alpha_1}\right)+\Bigg(\Bigg(\lambda_2\Bigg(e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_2+\gamma_2 v)\,e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv}{\lambda_2}-\frac{1}{\alpha_2}\right)$$

$$+\Bigg(\lambda_1\Bigg(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}dv}{\alpha_1}\right)$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\Bigg(\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}dv\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v}{\lambda_1}$$

$$-\frac{\int_0^t(\lambda_1+\gamma_1 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_1+\beta_1 v)e^{(\alpha_3-\alpha_1)v+(\beta_3-\beta_1)\frac{v^2}{2}}d\,v\right)dv}{\lambda_1}-\frac{1}{\alpha_3}\Bigg)\Bigg)$$

$$+\lambda_2\Bigg(e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_3-\alpha_2}-\frac{\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}dv}{\alpha_2}\right)$$

$$+e^{-\left(\alpha_3 t+\beta_3\frac{t^2}{2}\right)}\Bigg(\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_2 v+\beta_2\frac{v^2}{2}}dv\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v}{\lambda_2}$$

$$-\frac{\int_0^t(\lambda_2+\gamma_2 v)e^{\alpha_1 v+\beta_1\frac{v^2}{2}}\left(\int_0^t(\alpha_2+\beta_2 v)e^{(\alpha_3-\alpha_2)v+(\beta_3-\beta_2)\frac{v^2}{2}}d\,v\right)dv}{\lambda_2}-\frac{1}{\alpha_3}\Bigg)\Bigg)\Bigg)\Bigg) \quad (32)$$

4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

This part discusses a numerical analysis to assess the performance of the proposed queueing model. Different values for arrival and service rates are considered. Customers first enter two separate queues, receive service from the corresponding servers, and then proceed to a third queue that is allied in sequence to the first two. The arrival of customers is modeled by a NHPP, and the service processes at all three stations also follow non-homogeneous Poisson distributions. The average arrival rates are defined as $\lambda_1(t)=\lambda_1+\gamma_1 t$, $\lambda_2(t)=\lambda_2+\gamma_2 t$, while the service rates are given by $\mu_1(t) = \alpha_1 + \beta_1 t$, $\mu_2(t) = \alpha_2 + \beta_2 t$, and $\mu_3(t) = \alpha_3 + \beta_3 t$. The model's transient behavior is explored by computing performance measures using selected parameter values, highlighting the system's sensitivity to time-

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dependent variations. These values are calculated using the grid search method through LINGO Computer Package 20.0. and Mathcad 15.0 (The Lingo code for computing the performance values is developed and provided in Appendix - B).

t =0.08, 0.09, 0.10, 0.11; λ_1 = 4, 5, 6, 7; γ_1= 6,7,8,9; λ_2 =6,7, 8, 9;γ_2= 10,11,12,13; α_1 = 9.4, 9.8, 10.2,10.6; β_1 = 16,20, 24, 28; α_2= 10.2,10.4 10.6,10.8;β_2 =17,21,25,29; α_3= 11.5, 12, 12.5,13; β_3=20, 25,30,35.

Table 1 displays the calculated values for several performance metrics under different parameter settings, including the probability of queue emptiness, the average no. of customers in each queue, service station utilization, service station throughput, the variance in the no. of customers per queue, and the CV. These results are shown for various values of the parameters t, λ_1, γ_1, λ_2, γ_2, α_1, β_1, α_2, β_2, α_3, and β_3.

Table 1

$P_{000}(t)$, $P_{0..}(t)$, $P_{.0.}(t)$, $P_{..0}(t)$, $L_1(t)$, $L_2(t)$ and $L_3(t)$ for various standards of parameters

t	λ1	ϒ1	λ2	ϒ2	α1	β1	α2	β2	α3	β3	$P_{000}(t)$	$P_{0..}(t)$	$P_{.0.}(t)$	$P_{0..}(t)$	$L_1(t)$	L_2
0.08	3	5	5	9	9	12	10	13	11	16	0.12349	0.97671	0.92765	0.1363	0.02357	0.
0.09	3	5	5	9	9	12	10	13	11	16	0.14933	0.94752	0.88535	0.17801	0.05391	0.1
0.1	3	5	5	9	9	12	10	13	11	16	0.17609	0.9215	0.84856	0.22519	0.08176	0.1
0.11	3	5	5	9	9	12	10	13	11	16	0.20306	0.89827	0.81647	0.27687	0.10729	0.2
0.08	**4**	5	5	9	9	12	10	13	11	16	0.10858	0.97313	0.92765	0.12028	0.02724	0.
0.08	**5**	5	5	9	9	12	10	13	11	16	0.09547	0.96956	0.92765	0.10615	0.03091	0.
0.08	**6**	5	5	9	9	12	10	13	11	16	0.08395	0.966	0.92765	0.09368	0.03459	0.
0.08	**7**	5	5	9	9	12	10	13	11	16	0.07381	0.96246	0.92765	0.08267	0.03826	0.
0.08	3	**6**	5	9	9	12	10	13	11	16	0.12318	0.97426	0.92765	0.1363	0.02607	0.
0.08	3	**7**	5	9	9	12	10	13	11	16	0.12287	0.97182	0.92765	0.1363	0.02858	0.
0.08	3	**8**	5	9	9	12	10	13	11	16	0.12257	0.96939	0.92765	0.1363	0.03109	0.
0.08	3	**9**	5	9	9	12	10	13	11	16	0.12226	0.96696	0.92765	0.1363	0.0336	0.
0.08	3	5	**6**	9	9	12	10	13	11	16	0.08841	0.97671	0.91786	0.09862	0.02357	0.0
0.08	3	5	**7**	9	9	12	10	13	11	16	0.06329	0.97671	0.90817	0.07136	0.02357	0.0
0.08	3	5	**8**	9	9	12	10	13	11	16	0.04531	0.97671	0.89858	0.05163	0.02357	0.1
0.08	3	5	**9**	9	9	12	10	13	11	16	0.03244	0.97671	0.88909	0.03736	0.02357	0.1
0.08	3	5	5	**10**	9	12	10	13	11	16	0.12319	0.97671	0.92539	0.1363	0.02357	0.0
0.08	3	5	5	**11**	9	12	10	13	11	16	0.12289	0.97671	0.92312	0.1363	0.02357	0.0
0.08	3	5	5	**12**	9	12	10	13	11	16	0.12259	0.97671	0.92087	0.1363	0.02357	0.0
0.08	3	5	5	**13**	9	12	10	13	11	16	0.12229	0.97671	0.91862	0.1363	0.02357	0.0
0.08	3	5	5	9	**9.4**	12	10	13	11	16	0.10537	0.96804	0.92765	0.11734	0.03248	0.
0.08	3	5	5	9	**9.8**	12	10	13	11	16	0.08156	0.96042	0.92765	0.09154	0.04038	0.
0.08	3	5	5	9	**10.2**	12	10	13	11	16	0.04939	0.95372	0.92765	0.05583	0.04739	0.

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0.08	3	5	5	9	**10.6**	12	10	13	11	16	0.1118	0.9478	0.92765	0.01271	0.05361	0.
0.08	3	5	5	9	9	**16**	10	13	11	16	0.12359	0.97609	0.92765	0.13649	0.0242	0.
0.08	3	5	5	9	9	**20**	10	13	11	16	0.12368	0.97549	0.92765	0.13667	0.02482	0.
0.08	3	5	5	9	9	**24**	10	13	11	16	0.12377	0.9749	0.92765	0.13685	0.02542	0.
0.08	3	5	5	9	9	**28**	10	13	11	16	0.12386	0.97432	0.92765	0.13704	0.02602	0.
0.08	3	5	5	9	9	12	**10.2**	13	11	16	0.07492	0.97671	0.92244	0.08315	0.02357	0.08
0.08	3	5	5	9	9	12	**10.4**	13	11	16	0.03274	0.97671	0.91755	0.03653	0.02357	0.08
0.08	3	5	5	9	9	12	**10.6**	13	11	16	0.00629	0.97671	0.91297	0.00706	0.02357	0.0
0.08	3	5	5	9	9	12	**10.8**	13	11	16	0.00453	0.97671	0.90867	0.005115	0.02357	0.0
0.08	3	5	5	9	9	12	10	**17**	11	16	0.12365	0.97671	0.92711	0.13656	0.02357	0.07
0.08	3	5	5	9	9	12	10	**21**	11	16	0.12381	0.97671	0.92659	0.13681	0.02357	0.07
0.08	3	5	5	9	9	12	10	**25**	11	16	0.12397	0.97671	0.92608	0.13706	0.02357	0.0
0.08	3	5	5	9	9	12	10	**29**	11	16	0.12413	0.97671	0.92558	0.13731	0.02357	0.0
0.08	3	5	5	9	9	12	10	13	**11.5**	16	0.27941	0.97671	0.92765	0.30839	0.02357	0.
0.08	3	5	5	9	9	12	10	13	**12**	16	0.42403	0.97671	0.92765	0.4681	0.02357	0.
0.08	3	5	5	9	9	12	10	13	**12.5**	16	0.54484	0.97671	0.92765	0.60134	0.02357	0.
0.08	3	5	5	9	9	12	10	13	**13**	16	0.64263	0.97671	0.92765	0.70927	0.02357	0.
0.08	3	5	5	9	9	12	10	13	11	**20**	0.12682	0.97671	0.92765	0.13997	0.02357	0.
0.08	3	5	5	9	9	12	10	13	11	**25**	0.13105	0.97671	0.92765	0.14464	0.02357	0.
0.08	3	5	5	9	9	12	10	13	11	**30**	0.13534	0.97671	0.92765	0.14938	0.02357	0.
0.08	3	5	5	9	9	12	10	13	11	**35**	0.1397	0.97671	0.92765	0.15418	0.02357	0.

The information from Table 1 is presented below:-

- **Effect of Time t:**
  As time (t) ranges from 0.08 to 0.11, the probability that the system and the third queue are empty increases, while The probability that the queue is idle in the first and second queues decreases. Simultaneously, the average no. of customers in the first and second queues rises, whereas it declines in the overall system and the third queue, assuming all other parameters remain constant. Notably, the system's probability of emptiness is highly sensitive to changes in time.
- **Effect of Arrival Rate λ_1:**
  As λ1 increases from 4 to 7, The probability that the queue is idle in the first and third queues decreases, while it remains unchanged in the second queue. Simultaneously, the average no. of customers increases in the first and third queues but stays constant in the second queue, assuming all other parameters remain fixed.
- **Effect of Time-Dependent Arrival Parameter γ_1:**
  When γ_1 ranges from 6 to 9, The probability that the queue is idle decreases in the overall system and the first queue, with no change observed in the second and third queues. The average no. of customers increases in both the system and the first queue, while remaining constant in the second and third queues.
- **Effect of Arrival Rate λ_2:**

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As λ_2 increases from 6 to 9, The probability that the queue is idle decreases in the system, as well as in the second and third queues, while the first queue remains unaffected. Correspondingly, the average no. of customers increases in the system and the second and third queues, with the first queue showing no change.

- **Effect of Time-Dependent Arrival Parameter γ_2:**
  When γ_2 rises from 10 to 13, The probability that the queue is idle decreases in the second queue, while it remains constant in the first and third queues. At the same time, the average no. of customers increases in the second queue and the system, with no change in the first and third queues.
- **Effect of Service Rate Parameter α_1:**
  As α_1 varies between 9.4 and 10.6, The probability that the queue is idle decreases in the first and third queues but remains unchanged in the second queue. The average no. of customers increases in the first and third queues, with the second queue remaining constant.
- **Effect of Time-Dependent Service Parameter β_1:**
  As β_1 increases from 16 to 28, The probability that the queue is idle rises in the system and the third queue, decreases in the first queue, and remains unchanged in the second queue. In parallel, the average no. of customers declines in the system and third queue, increases in the first queue, and remains constant in the second queue.
- **Effect of Service Rate Parameter α_2:**
  An increase in α_2 from 10.2 to 10.8 leads to a decrease in The probability that the queue is idle in the system, as well as in the second and third queues, while the first queue remains unaffected. The average no. of customers rises in the system and the second and third queues, with no change in the first queue.
- **Effect of Time-Dependent Service Parameter β_2:**
  When β_2 increases from 17 to 29, The probability that the queue is idle decreases in the second queue, while the first and third queues remain unchanged. The average no. of customers increases in the second queue, decreases in the third queue and the system, and stays the same in the first queue.
- **Effect of Service Rate Parameter α_3:**
  As α_3 increases from 11.5 to 13, The probability that the queue is idle increases in the system and third queue, with the first and second queues remaining unchanged. Correspondingly, the average no. of customers decreases in the system and the third queue, while remaining constant in the first and second queues.
- **Effect of Time-Dependent Service Parameter β_3:**
  As β_3 rises from 20 to 35, The probability that the queue is idle increases in the system and the third queue, with no effect on the first and second queues. The average no. of customers similarly decreases in the system and third queue and remains unchanged in the first and second queues.

Table2

U_1 (t), U_2 (t), U_3 (t), Thp_1 (t), Thp_2 (t), Thp_3(t), W_1(t), W_2(t) and W_3(t) for various values of parameters

t	λ_1	Υ_1	λ_2	Υ_2	α1	β_1	α2	β_2	α3	β_3	$U_1(t)$	$U_2(t)$	$U_3(t)$	Thp_1 (t)	Thp_2 (t)	Thp_3 (t)	$W_1(t)$	$W_2(t)$	$W_3(t)$
0.08	3	5	5	9	9	12	10	13	11	16	0.02329	0.07235	0.8637	0.23196	0.79872	10.60625	0.10159	0.09402	0.1879
0.09	3	5	5	9	9	12	10	13	11	16	0.05248	0.11465	0.82199	0.52901	1.28063	10.22558	0.1019	0.09509	0.16879
0.1	3	5	5	9	9	12	10	13	11	16	0.0785	0.15144	0.77481	0.80075	1.71126	9.76255	0.1021	0.09596	0.15271
0.11	3	5	5	9	9	12	10	13	11	16	0.10173	0.18353	0.72313	1.04988	2.09779	9.22714	0.10219	0.09666	0.13918
0.	**4**	5	5	9	9	1	10	1	11	1	0.02	0.07	0.87	0.26	0.79	10.8	0.10	0.09	0.19

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08						2		3		6	687	235	972	764	872	0291	178	402	605
0.08	**5**	5	5	9	9	12	10	13	11	16	0.03044	0.07235	0.89385	0.30318	0.79872	10.97646	0.10196	0.09402	0.20434
0.08	**6**	5	5	9	9	12	10	13	11	16	0.034	0.07235	0.90632	0.3386	0.79872	11.12962	0.10215	0.09402	0.21275
0.08	**7**	5	5	9	9	12	10	13	11	16	0.03754	0.07235	0.91733	0.37388	0.79872	11.26478	0.10233	0.09402	0.2213
0.08	3	**6**	5	9	9	12	10	13	11	16	0.02574	0.07235	0.8637	0.25634	0.79872	10.60625	0.10172	0.09402	0.1879
0.08	3	**7**	5	9	9	12	10	13	11	16	0.02818	0.07235	0.8637	0.28065	0.79872	10.60625	0.10184	0.09402	0.1879
0.08	3	**8**	5	9	9	12	10	13	11	16	0.03061	0.07235	0.8637	0.3049	0.79872	10.60625	0.10197	0.09402	0.1879
0.08	3	**9**	5	9	9	12	10	13	11	16	0.03304	0.07235	0.8637	0.32909	0.79872	10.60625	0.1021	0.09402	0.1879
0.08	3	5	**6**	9	9	12	10	13	11	16	0.02329	0.08214	0.90138	0.23196	0.90685	11.06896	0.10159	0.09452	0.20928
0.08	3	5	**7**	9	9	12	10	13	11	16	0.02329	0.09183	0.92864	0.23196	1.01383	11.40376	0.10159	0.09501	0.23151
0.08	3	5	**8**	9	9	12	10	13	11	16	0.02329	0.016142	0.94837	0.23196	1.11964	11.646	0.10159	0.09551	0.25448
0.08	3	5	**9**	9	9	12	10	13	11	16	0.02329	0.11091	0.96264	0.23196	1.22443	11.82127	0.10159	0.09601	0.27808
0.08	3	5	5	**10**	9	12	10	13	11	16	0.02329	0.07461	0.8637	0.23196	0.82375	10.60625	0.10159	0.09414	0.1879
0.08	3	5	5	**11**	9	12	10	13	11	16	0.02329	0.07688	0.8637	0.23196	0.84872	10.60625	0.10159	0.09425	0.1879
0.08	3	5	5	**12**	9	12	10	13	11	16	0.02329	0.07913	0.8637	0.23196	0.87363	10.60625	0.10159	0.09436	0.1879
0.08	3	5	5	**13**	9	12	10	13	11	16	0.02329	0.08138	0.8637	0.23196	0.89848	10.60625	0.10159	0.09448	0.1879
0.08	3	5	5	9	**9.4**	12	10	13	11	16	0.03196	0.07235	0.88266	0.33114	0.79872	10.83913	0.0981	0.09402	0.19768
0.08	3	5	5	9	**9.8**	12	10	13	11	16	0.03958	0.07235	0.90848	0.42586	0.79872	11.1559	0.09483	0.09402	0.21433
0.08	3	5	5	9	**10.2**	12	10	13	11	16	0.04628	0.07235	0.94417	0.51652	0.79872	11.5944	0.09175	0.09402	0.24887
0.08	3	5	5	9	**10.6**	12	10	13	11	16	0.0522	0.07235	0.98729	0.60342	0.79872	12.12389	0.08884	0.09402	0.36004
0.	3	5	5	9	9	**1**	10	1	11	1	0.02	0.07	0.86	0.24	0.79	10.6	0.09	0.09	0.18

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08						**6**		3		6	391	235	351	576	872	0395	846	402	781
0.08	3	5	5	9	9	**20**	10	13	11	16	0.02451	0.07235	0.86333	0.25981	0.79872	10.60167	0.09552	0.09402	0.18772
0.08	3	5	5	9	9	**24**	10	13	11	16	0.0251	0.07235	0.86315	0.2743	0.79872	10.59942	0.09274	0.09402	0.18764
0.08	3	5	5	9	9	**28**	10	13	11	16	0.02568	0.07235	0.86296	0.28868	0.79872	10.5972	0.09013	0.09402	0.18755
0.08	3	5	5	9	9	12	**10.2**	13	11	16	0.02329	0.07756	0.91685	0.23196	0.8718	11.25891	0.10159	0.09261	0.2209
0.08	3	5	5	9	9	12	**10.4**	13	11	16	0.02329	0.08245	0.96347	0.23196	0.94322	11.83143	0.10159	0.09123	0.27974
0.08	3	5	5	9	9	12	**10.6**	13	11	16	0.02329	0.08703	0.99294	0.23196	1.01305	12.19329	0.10159	0.08988	0.40622
0.08	3	5	5	9	9	12	**10.8**	13	11	16	0.02329	0.09133	0.99995	0.23196	1.08137	12.27937	0.10159	0.08857	0.80466
0.08	3	5	5	9	9	12	10	**17**	11	16	0.02329	0.07289	0.86344	0.23196	0.82798	10.6031	0.10159	0.0914	0.18778
0.08	3	5	5	9	9	12	10	**21**	11	16	0.02329	0.07341	0.86319	0.23196	0.85743	10.59998	0.10159	0.08892	0.18766
0.08	3	5	5	9	9	12	10	**25**	11	16	0.02329	0.07392	0.86294	0.23196	0.88706	10.59691	0.10159	0.08657	0.18754
0.08	3	5	5	9	9	12	10	**29**	11	16	0.02329	0.07442	0.86269	0.23196	0.91685	10.59387	0.10159	0.08435	0.18742
0.08	3	5	5	9	9	12	10	13	**11.5**	16	0.02329	0.07235	0.069161	0.23196	0.79872	8.83884	0.10159	0.09402	0.1331
0.08	3	5	5	9	9	12	10	13	**12**	16	0.02329	0.07235	0.0532	0.23196	0.79872	7.06498	0.10159	0.09402	0.10747
0.08	3	5	5	9	9	12	10	13	**12.5**	16	0.02329	0.07235	0.03986	0.23196	0.79872	5.49354	0.10159	0.09402	0.09258
0.08	3	5	5	9	9	12	10	13	**13**	16	0.02329	0.07235	0.02907	0.23196	0.79872	4.15161	0.10159	0.09402	0.08274
0.08	3	5	5	9	9	12	10	13	11	**20**	0.02324	0.07235	0.86003	0.23196	0.79872	10.83633	0.10159	0.09402	0.18145
0.08	3	5	5	9	9	12	10	13	11	**25**	0.02324	0.07235	0.85536	0.23196	0.79872	11.11972	0.10159	0.09402	0.17388
0.08	3	5	5	9	9	12	10	13	11	**30**	0.02324	0.07235	0.85062	0.23196	0.79872	11.39835	0.10159	0.09402	0.1668
0.08	3	5	5	9	9	12	10	13	11	**35**	0.02324	0.07235	0.84581	0.23196	0.79872	11.67216	0.10159	0.09402	0.16017

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Table 2 highlights the strong time sensitivity of key performance indicators, including service station utilization, throughput, and customer waiting times. As time t increases from 0.08 to 0.11:

Time Sensitivity of Performance Metrics (Based on Table 2)

Table 2 highlights the strong time sensitivity of key performance indicators, including service station utilization, throughput, and customer waiting times. As time t increases from 0.08 to 0.11:

- **Utilization** of the first and second service stations increases from 0.02329 to 0.10173 and from 0.07235 to 0.18353, respectively. In contrast, the third queue's utilization declines from 0.8637 to 0.72313.
- **Throughput** at the first and second service stations rises from 0.23196 to 1.04988 and from 0.79872 to 2.09779, respectively, while throughput in the third queue decreases from 10.60625 to 9.22714.
- **Average waiting time** increases slightly in the first and second queues (from 0.10159 to 0.10219 and from 0.09402 to 0.09666, respectively), but decreases significantly in the third queue (from 0.1879 to 0.13918), assuming all other parameters remain fixed.

Impact of Arrival Parameters

- **Increasing λ_1:** Leads to increased utilization, throughput, and average waiting time in the first and third service stations. Metrics for the second queue remain unchanged.
- **Increasing γ_1:** Increases utilization, throughput, and waiting time at the first service station, with no change in the second and third queues.
- **Increasing λ_2:** Results in higher utilization and throughput for the second and third service stations, along with increased waiting times. The first queue remains unaffected.
- **Increasing γ_2:** Raises utilization, throughput, and waiting time at the second service station, while the first and third stations show no change.

Impact of Service Rate Parameters

- **Increasing α_1:** Boosts utilization and throughput in the first and third queues. Waiting time decreases in the first queue, increases in the third, and remains unchanged in the second.
- **Increasing β_1:** Utilization and throughput increase in the first queue and decrease in the third, while remaining constant in the second. Waiting time decreases in the first and third queues and remains unchanged in the second.
- **Increasing α_2:** Enhances utilization and throughput in the second and third queues. Waiting time decreases in the second queue, increases in the third, and remains constant in the first.
- **Increasing β_2:** Utilization and throughput rise in the second queue but decline in the third. Waiting times decrease in both the second and third queues and remain unchanged in the first.
- **Increasing α_3:** Reduces utilization, throughput, and waiting time in the third queue, with no changes observed in the first and second queues.
- **Increasing β_3:**Leads to a decline in utilization of the third queue, an increase in its throughput, and a reduction in waiting time. The first and second queues remain unaffected.

Analysis of Variance and Coefficient of Variation (Based on Table 3)

Table 3 demonstrates that both the **Variance** and the **Coefficient of Variation (CV)** of the no. of customers in each queue are notably influenced by time and system parameters.

Time-Dependent Behaviour:

- **As time t increases:**

Variance in the first and second queues increases, while it decreases in the third queue. CV declines in the first and second queues, but increases in the third queue, assuming all other variables remain constant.

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Impact of Arrival Parameters:

- **Increasing λ_1:** Variance rises in the first and third queues; the second queue remains unaffected. CV decreases in the first and third queues; remains unchanged in the second queue.
- **Increasing γ_1 :** Variance increases in the first queue, with no changes in the second and third queues. CV decreases in the first queue, while remaining constant in the others.
- **Increasing λ_2:** Variance increases in the second and third queues; the first queue remains unaffected. CV decreases in the second and third queues; the first queue remains unchanged.
- **Increasing γ_2:** Variance increases in the second queue only. CV decreases in the second queue**,** with no changes in the first and third queues.

Impact of Service Rate Parameters

- **Increasing** α_1**:** Variance increases in the first and third queues. CV decreases in those queues, assuming other parameters are fixed.
- **Increasing** β_1**:** Variance increases in the first queue, decreases in the third, and stays constant in the second. CV drops in the first queue, rises in the third, and remains unchanged in the second.
- **Increasing** α_2**:** Variance rises in the second and third queues. CV declines in those queues, while remaining unchanged in the first.
- **Increasing** β_2**:** Variance decreases in the third queue, increases in the second, and remains constant in the first. CV decreases in the second, increases in the third, and stays the same in the first.
- **Increasing** α_3**:** Variance decreases in the system and third queue. CV increases in the third queue, with no impact on the others.
- **Increasing** β_3**:** Variance decreases in the system and third queue. CV rises in the third queue, and remains constant in the first and second queues**.**

Table 3 **$V_1(t)$, $V_2(t)$, $V_3(t)$, $CV_1(t)$, $CV_2(t)$, $CV_3(t)$ for different values of parameters**

t	λ1	ϒ1	λ2	ϒ2	α1	β1	α2	β2	α3	β3	$V_1(t)$	$V_2(t)$	$V_3(t)$	V(t)	$CV_1(t)$	$CV_2(t)$	$CV_3(t)$
0.08	3	5	5	9	9	12	10	13	11	16	0.02357	0.0751	1.99291	2.09158	6.51427	3.64909	0.70836
0.09	3	5	5	9	9	12	10	13	11	16	0.05391	0.12177	1.72593	1.90161	4.30694	2.86568	0.76118
0.10	3	5	5	9	9	12	10	13	11	16	0.08176	0.16421	1.49079	1.73676	3.49732	2.46771	0.81901
0.11	3	5	5	9	9	12	10	13	11	16	0.10729	0.20277	1.28421	1.59427	3.05298	2.22074	0.88243
0.08	**4**	5	5	9	9	12	10	13	11	16	0.02724	0.0751	2.1179	2.22024	6.05903	3.6491	0.68714
0.08	**5**	5	5	9	9	12	10	13	11	16	0.03091	0.0751	2.24289	2.3489	5.68758	3.6491	0.66772
0.08	**6**	5	5	9	9	12	10	13	11	16	0.03459	0.0751	2.36788	2.47757	5.37702	3.6491	0.64986

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0.08	**7**	5	5	9	9	12	10	13	11	16	0.03826	0.0751	2.49287	2.60623	3.11234	3.6491	0.63335
0.08	3	**6**	5	9	9	12	10	13	11	16	0.02607	0.0751	1.99291	2.09408	6.19298	3.6491	0.708364
0.08	3	7	5	9	9	12	10	13	11	16	0.02858	0.0751	1.99291	2.09659	5.99497	3.6491	0.708364
0.08	3	**8**	5	9	9	12	10	13	11	16	0.03109	0.0751	1.99291	2.0991	5.67132	3.6491	0.708364
0.08	3	**9**	5	9	9	12	10	13	11	16	0.0336	0.0751	1.99291	2.10161	5.4555	3.6491	0.708364
0.08	3	5	**6**	9	9	12	10	13	11	16	0.02357	0.08521	2.3165	2.42528	6.51427	3.41569	0.65703
0.08	3	5	7	9	9	12	10	13	11	16	0.02357	0.09633	2.64005	2.75995	6.51427	3.22201	0.61544
0.08	3	5	**8**	9	9	12	10	13	11	16	0.02357	0.10694	2.96367	3.09418	6.51427	3.05800	0.58088
0.08	3	5	**9**	9	9	12	10	13	11	16	0.02357	0.11756	3.28726	3.42839	6.51427	2.91661	0.55155
0.08	3	5	5	**10**	9	12	10	13	11	16	0.02357	0.07755	1.99291	2.09403	6.51427	3.59105	0.70836
0.08	3	5	5	**11**	9	12	10	13	11	16	0.02357	0.07999	1.99291	2.09647	6.51427	3.53569	0.70836
0.08	3	5	5	**12**	9	12	10	13	11	16	0.02357	0.08244	1.99291	2.09892	6.51427	3.48218	0.70836
0.08	3	5	5	**13**	9	12	10	13	11	16	0.02357	0.08489	1.99291	2.10137	6.51427	3.43224	0.70836
0.08	3	5	5	9	**9.4**	12	10	13	11	16	0.03248	0.0751	2.14272	2.2503	5.5483	3.6491	0.68315
0.08	3	5	5	9	**9.8**	12	10	13	11	16	0.04038	0.0751	2.39099	2.50647	4.97623	3.6491	0.64671
0.08	3	5	5	9	**10.2**	12	10	13	11	16	0.04739	0.0751	2.88549	3.00798	4.59371	3.6491	0.5887
0.08	3	5	5	9	**10.6**	12	10	13	11	16	0.05361	0.0751	4.36514	4.49385	4.31891	3.6491	0.47863
0.08	3	5	5	9	9	**16**	10	13	11	16	0.0242	0.0751	1.99153	2.09083	6.42867	3.6491	0.70861
0.08	3	5	5	9	9	**20**	10	13	11	16	0.02482	0.0751	1.99017	2.09009	6.34793	3.6491	0.70885
0.08	3	5	5	9	9	**24**	10	13	11	16	0.02542	0.0751	1.98883	2.08935	6.27165	3.6491	0.70909

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0.08	3	5	5	9	9	**28**	10	13	11	16	0.02602	0.0751	1.98752	2.08864	6.19948	3.6491	0.70932
0.08	3	5	5	9	9	12	**10.2**	13	11	16	0.02357	0.08074	2.4871	2.59141	6.51427	3.5194	0.63409
0.08	3	5	5	9	9	12	**10.4**	13	11	16	0.02357	0.08605	3.30967	3.41929	6.51427	3.40903	0.54968
0.08	3	5	5	9	9	12	**10.6**	13	11	16	0.02357	0.09105	4.95321	5.06783	6.51427	3.31397	0.44932
0.08	3	5	5	9	9	12	**10.8**	13	11	16	0.02357	0.09578	9.88073	10.00008	6.51427	3.23126	0.31813
0.08	3	5	5	9	9	12	10	**17**	11	16	0.02357	0.07568	1.99102	2.09027	6.51427	3.63508	0.7087
0.08	3	5	5	9	9	12	10	**21**	11	16	0.02357	0.07624	1.98917	2.08898	6.51427	3.62157	0.70903
0.08	3	5	5	9	9	12	10	**25**	11	16	0.02357	0.0768	1.98734	2.08771	6.51427	3.68853	0.70936
0.08	3	5	5	9	9	12	10	**29**	11	16	0.02357	0.07733	1.98554	2.08644	6.51427	3.59596	0.70968
0.08	3	5	5	9	9	12	10	13	**11.5**	16	0.02357	0.0751	1.17641	1.27508	6.51427	3.6491	0.92198
0.08	3	5	5	9	9	12	10	13	**12**	16	0.02357	0.0751	0.75929	0.85796	6.51427	3.6491	1.14761
0.08	3	5	5	9	9	12	10	13	**12.5**	16	0.02357	0.0751	0.5086	0.60727	6.51427	3.6491	1.40221
0.08	3	5	5	9	9	12	10	13	**13**	16	0.02357	0.0751	0.34352	0.44219	6.51427	3.6491	1.70618
0.08	3	5	5	9	9	12	10	13	11	**20**	0.02357	0.0751	1.9663	2.06497	6.51427	3.6491	0.71314
0.08	3	5	5	9	9	12	10	13	11	**25**	0.02357	0.0751	1.93353	2.0322	6.51427	3.6491	0.71916
0.08	3	5	5	9	9	12	10	13	11	**30**	0.02357	0.0751	1.90129	1.99996	6.51427	3.6491	0.72523
0.08	3	5	5	9	9	12	10	13	11	**35**	0.02357	0.0751	1.86956	1.96823	6.51427	3.6491	0.73136

SENSITIVITY ANALYSIS SUMMARY

The model undergoes a comprehensive sensitivity analysis to evaluate the influence of key parameters on system performance. These parameters include time t, arrival rates $\lambda_1(t)$ and $\lambda_2(t)$, and service rates $\mu_1(t)$, $\mu_2(t)$, and $\mu_3(t)$ for the three service stations. The analysis focuses on the following performance metrics:

- Average no. of customers in each queue

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- Service station utilization
- average queueing delay
- Service station throughput

Parameter Variation Methodology

Each parameter is varied across five levels: **-10%, -5%, 0%, +5%, +10%** relative to the baseline values: t=0.2, λ_1=3, γ_1=5, λ_2=5, γ_2=9, α_1=9, β_1=12, α_2=10, β_2=13, α_3=11, β_3=16. The results are detailed in Table 4. Observations from the Sensitivity Analysis as **Time** t: As t increases from -10% to +10%: First and second queues**:** average no. of customers, delay, utilization, and throughput increase. Third queue: average no. of customers, delay, utilization, and throughput decrease.

Arrival Rate λ_1: Increasing λ_1 leads to an increase in the average no. of customers in the first and third queues**.** Utilization, delay, and throughput increase across all service stations**.**

Arrival Parameter γ_1: As γ_1 increases First queue is higher average customer count, increased delay and utilization**.** First station throughput increases**;** third station throughput decreases**.** Second station remains unaffected.

Arrival Rate λ_2: A rise in λ_2 results increased customer counts, delays, utilization, and throughput in the second and third queues**.** First queue remains unchanged.

Arrival Parameter γ_2: As γ_2 increases Second queue shows higher average customer count, delay, utilization, and throughput. First and third queues remain unaffected.

Service Rate α_1: Increasing α_1 causes First queue Customer count decreases. Third queue Customer count increases, along with delay and utilization. First and third throughput rise. Second queue metrics remain constant.

Service Rate β_1: As β_1 increases First and third queues Decline in customer count and utilization. First queue delay decreases. Other metrics remain constant.

Service Rate α2: As α_2 rises Second queue Decrease in average customers**.** Third queue**:** Increase in customer count, delay, and utilization. Second and third throughputs increase**.** First queue metrics remain constant.

Service Rate β_2: Increasing β_2 results in Decrease in customer count, utilization, and delay in the second and third queues**.** Second station throughput increases**;** third station throughput decreases**.** First queue metrics remain unchanged.

Service Rate α_3 and β_3: As α_3 and β_3 increase Third queue**:** Customer count, utilization, and delay decrease**.** First and second queues remain unchanged in all performance metrics.

The sensitivity analysis in Table 4 confirms that performance metrics are highly responsive to time and parameter variations**.** In particular, time, arrival rates**,** and first- and second-stage service rates exert the most pronounced influence. The third queue's performance is especially sensitive to α_3 and β_3.

Table 4 $L_1(t)$, $L_2(t)$, $L_3(t)$, $U_1(t)$, $U_2(t)$, $U_3(t)$, $Thp_1(t)$, $Thp_2(t)$, $Thp_3(t)$, $W_1(t)$, $W_2(t)$, $W_3(t)$ for different values of t, λ_1, γ_1, λ_2, γ_2, α_1, β_1, α_2, β_2, α_3, and β_3.

Parameters	Performance parameter	Percentage variation in parameters				
		-10	-5	0	5	10
t =0.2	$L_1(t)$	0.23406	0.24635	0.25752	0.26767	0.27688
	$L_2(t)$	0.38835	0.40573	0.42142	0.43558	0.44837
	$L_3(t)$	0.14517	0.34842	0.2912	0.24231	0.20067
	$U_1(t)$	0.20869	0.21835	0.22703	0.23484	0.24185
	$U_2(t)$	0.32183	0.33351	0.34388	0.35311	0.36133
	$U_3(t)$	0.33977	0.2942	0.25264	0.21519	0.18182

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	$W_1(t)$	0.1005	0.10002	0.0995	0.09894	0.09835
	$W_2(t)$	0.09779	0.09756	0.09726	0.0969	0.09646
	$W_3(t)$	0.08803	0.08435	0.08117	0.07842	0.07601
	$Thp_1(t)$	2.32896	2.463	2.58818	2.70531	2.81516
	$Thp_2(t)$	3.97132	4.15885	4.33294	4.49511	4.64869
	$Thp_3(t)$	4.71602	4.13055	3.58744	3.09009	2.63996
$\lambda_1 = 3$	$L_1(t)$	0.2372	0.24736	0.25752	0.26768	0.27784
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	0.28832	0.28976	0.2912	0.29264	0.29408
	$U_1(t)$	0.21117	0.21914	0.22703	0.23485	0.24258
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.34388
	$U_3(t)$	0.25048	0.25156	0.25264	0.25371	0.25479
	$W_1(t)$	0.09853	0.09902	0.0995	0.09998	0.9998
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0.08106	0.08112	0.08117	0.08123	0.08128
	$Thp_1(t)$	2.40732	2.49821	2.58818	2.67723	2.76539
	$Thp_2(t)$	4.33294	4.33294	4.33294	4.33294	4.33294
	$Thp_3(t)$	3.55684	3.57215	3.58744	3.60271	3.61796
$\gamma_1=5$	$L_1(t)$	0.25208	0.2548	0.25752	0.26024	0.26295
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	0.2912	0.2912	0.2912	0.2912	0.2912
	$U_1(t)$	0.22282	0.22493	0.22703	0.22913	0.23122
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.3435
	$U_3(t)$	0.25264	0.25264	0.25264	0.25264	0.25264
	$W_1(t)$	0.09924	0.9937	0.0995	0.09963	0.09976
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0.08117	0.08117	0.08117	0.08117	0.08117
	$Thp_1(t)$	2.54015	2.56419	2.58818	2.61209	2.63594
	$Thp_2(t)$	4.33294	4.33294	4.33294	4.33294	4.33294
	$Thp_3(t)$	4.33294	3.5844	3.58744	3.58744	3.58744
	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.38855	0.40499	0.42142	0.43785	0.45428
	$L_3(t)$	0.26496	0.27808	0.2912	0.30432	0.31745
	$U_1(t)$	0.22703	0.22703	0.22703	0.22703	0.22703
	$U_2(t)$	0.32196	0.33301	0.34388	0.35458	0.3651
$\lambda_2 = 5$	$U_3(t)$	0.23277	0.24277	0.25264	0.26238	0.27190
	$W_1(t)$	0.0995	0.0995	0.0995	0.0995	0.0995
	$W_2(t)$	0.09578	0.09652	0.09726	0.0908	0.09875
	$W_3(t)$	0.08016	0.08067	0.08117	0.08168	0.08219
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58818
	$Thp_2(t)$	4.05673	4.19597	4.33294	4.46768	4.60022
	$Thp_3(t)$	3.30528	3.49729	3.58744	3.72578	3.86231
$\gamma_2=9$	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.41214	0.41626	0.42142	0.42606	0.43069
	$L_3(t)$	0.2912	0.2912	0.2912	0.2912	0.2912
	$U_1(t)$	0.22703	0.22703	0.22703	0.22703	0.22703
	$U_2(t)$	0.33777	0.34049	0.34388	0.34692	0.34994
	$U_3(t)$	0.25264	0.25264	0.25264	0.25264	0.25264
	$W_1(t)$	0.0995	0.0995	0.0995	0.0995	0.0995

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	$W_2(t)$	0.09184	0.09703	0.09726	0.09747	0.09768
	$W_3(t)$	0.08117	0.08117	0.08117	0.08117	0.08117
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58818
	$Thp_2(t)$	4.25589	4.29022	4.33294	4.3712	4.40927
	$Thp_3(t)$	3.58744	3.58744	3.58744	3.58744	3.58744
α_1=9	$L_1(t)$	0.26136	0.26001	0.25752	0.25468	0.25143
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	0.24513	0.26246	0.2912	0.33005	0.39727
	$U_1(t)$	0.23	0.22896	0.22703	0.22484	0.22231
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.34388
	$U_3(t)$	0.2174	0.23084	0.25264	0.28111	0.32785
	$W_1(t)$	0.10823	0.10419	0.0995	0.09559	0.09195
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0..7941	0.08007	0.08117	0.08268	0.08533
	$Thp_1(t)$	2.41497	2.49566	2.58818	2.66432	2.73443
	$Thp_2(t)$	4.33294	4.33294	4.33294	4.33294	4.33294
	$Thp_3(t)$	3.08702	3.27793	3.58744	3.99179	4.65547
β_1=12	$L_1(t)$	0.2599	0.25871	0.25752	0.25634	0.25517
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	0.29143	0.29131	0.2912	0.2911	0.29099
	$U_1(t)$	0.22887	0.22795	0.22703	0.22612	0.22521
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.34388
	$U_3(t)$	0.2528	0.25272	0.25264	0.25256	0.25248
	$W_1(t)$	0.10175	0.10061	0.0995	0.09841	0.09734
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0.08118	0.08118	0.08117	0.08117	0.08116
	$Thp_1(t)$	2.5542	2.57127	2.58818	2.60491	2.62149
	$Thp_2(t)$	4.33294	4.35294	4.33294	4.33294	4.33294
	$Thp_3(t)$	3.58983	3.58862	3.58744	3.5863	3.58519
α_2=10	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.43312	0.42767	0.42142	0.41459	0.40883
	$L_3(t)$	0.07649	0.15062	0.2912	0.69945	3.92225
	$U_1(t)$	0.22703	0.22703	0.22703	0.22703	0.22703
	$U_2(t)$	0.35152	0.34798	0.34388	0.33939	0.33557
	$U_3(t)$	0.07364	0.13983	0.25264	0.50314	0.9802
	$W_1(t)$	0.995	0.0995	0.0995	0.0995	0.0995
	$W_2(t)$	0.10622	0.10157	0.09726	0.9325	0.09024
	$W_3(t)$	0.07315	0.07586	0.08117	0.0979	0.28179
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58818
	$Thp_2(t)$	4.07762	4.21051	4.33294	4.44595	4.53019
	$Thp_3(t)$	1.04567	1.98556	3.58744	7.14461	13.91889
β_2=13	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.42568	0.42354	0.42142	0.41931	0.41722
	$L_3(t)$	0.29144	0.29132	0.2912	0.29109	0.29099
	$U_1(t)$	0.22703	0.22703	0.22703	0.22703	0.22703
	$U_2(t)$	0.34667	0.34528	0.34388	0.3425	0.34112
	$U_3(t)$	0.25281	0.25272	0.25264	0.28255	0.25247
	$W_1(t)$	0.0995	0.0995	0.0995	0.0995	0.0995
	$W_2(t)$	0.09951	0.09837	0.09726	0.09617	0.09511

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	$W_3(t)$	0.08118	0.08118	0.08117	0.08117	0.08116
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58818
	$Thp_2(t)$	4.27794	4.30558	4.33294	4.36002	4.38683
	$Thp_3(t)$	3.58994	3.58867	3.58744	3.58626	3.58513
α_3=11	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	1.07556	0.93283	0.2912	0.10243	0.01635
	$U_1(t)$	0.22703	0.22703	0.22703	0.22703	0.22703
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.34388
	$U_3(t)$	0.65889	0.60656	0.25264	0.09736	0.01621
	$W_1(t)$	0.0995	0.0995	0.0995	0.0995	0.0995
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0.12003	0.11267	0.08117	0.07133	0.0659
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58815
	$Thp_2(t)$	4.33294	4.33294	4.33294	4.33294	4.33294
	$Thp_3(t)$	8.96095	8.27956	3.58744	1.4361	0.24808
β_3=16	$L_1(t)$	0.25752	0.25752	0.25752	0.25752	0.25752
	$L_2(t)$	0.42142	0.42142	0.42142	0.42142	0.42142
	$L_3(t)$	0.30284	0.29697	0.2912	0.28553	0.27996
	$U_1(t)$	0.27703	0.22703	0.22703	0.27703	0.22703
	$U_2(t)$	0.34388	0.34388	0.34388	0.34388	0.34388
	$U_3(t)$	0.41226	0.25694	0.25264	0.24839	0.24418
	$W_1(t)$	0.0995	0.0995	0.0995	0.0995	0.0995
	$W_2(t)$	0.09726	0.09726	0.09726	0.09726	0.09726
	$W_3(t)$	0.08351	0.08232	0.08117	0.08005	0.07896
	$Thp_1(t)$	2.58818	2.58818	2.58818	2.58818	2.58818
	$Thp_2(t)$	4.33294	4.33294	4.33294	4.33294	4.33294
	$Thp_3(t)$	3.62664	3.60739	3.58744	3.56682	3.54554

5. COMPARATIVE STUDY

This section provides a comparative evaluation between the proposed queueing model and a standard Poisson service process. The comparison is based on key performance metrics including average no. of customers, service station utilization, average queueing delay, and throughput under varying time values: t=0.20, 0.21, 0.22, 0.23, 0.24. The corresponding results are summarized in Table 5**.**

Table 5 A Performance Comparison Between Time-Invariant and Time-Variant Poisson Service Rate Models

t	Performance Parameter	NHSR	HSR	Discrepancy	Variation Rate (in Percentage)
0.2	$L_1(t)$	0.2575	0.2231	0.0344	15.41909458
	$L_2(t)$	0.4214	0.3647	0.0567	15.54702495
	$L_3(t)$	0.1024	0.1742	0.0718	41.21699196
	$U_1(t)$	0.227	0.2	0.027	13.5
	$U_2(t)$	0.3439	0.3056	0.0383	12.53272251
	$U_3(t)$	0.0974	0.1598	0.0624	39.04881101
	$W_1(t)$	0.0995	0.124	0.0245	19.75806452
	$W_2(t)$	0.0973	0.1193	0.022	18.44090528

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	$W_3(t)$	0.0713	0.0943	0.023	24.3902439
0.21	$L_1(t)$	0.2677	0.2326	0.0351	15.09028375
	$L_2(t)$	0.4356	0.3775	0.0581	15.39072848
	$L_3(t)$	0.0782	0.1446	0.0664	45.9197787
	$U_1(t)$	0.2348	0.2075	0.0273	13.15662651
	$U_2(t)$	0.3531	0.3145	0.0386	12.27344992
	$U_3(t)$	0.0752	0.1346	0.0594	44.1307578
	$W_1(t)$	0.0989	0.1245	0.0256	20.562249
	$W_2(t)$	0.0969	0.1201	0.0232	19.31723564
	$W_3(t)$	0.0697	0.093	0.0233	25.05376344
0.22	$L_1(t)$	0.2769	0.2413	0.0356	14.75341898
	$L_2(t)$	0.4484	0.3892	0.0592	15.21068859
	$L_3(t)$	0.0583	0.1193	0.061	51.13160101
	$U_1(t)$	0.2419	0.2144	0.0275	12.82649254
	$U_2(t)$	0.3613	0.3224	0.0389	12.06575682
	$U_3(t)$	0.0566	0.1124	0.0558	49.64412811
	$W_1(t)$	0.0984	0.1251	0.0267	21.34292566
	$W_2(t)$	0.0965	0.1207	0.0242	20.04971002
	$W_3(t)$	0.0683	0.0918	0.0235	25.59912854
0.23	$L_1(t)$	0.2852	0.2492	0.036	14.44622793
	$L_2(t)$	0.4599	0.3997	0.0602	15.06129597
	$L_3(t)$	0.042	0.0975	0.0555	56.92307692
	$U_1(t)$	0.2482	0.2206	0.0276	12.51133273
	$U_2(t)$	0.3687	0.3295	0.0392	11.89681335
	$U_3(t)$	0.0411	0.0929	0.0518	55.75888052
	$W_1(t)$	0.0977	0.1255	0.0278	22.15139442
	$W_2(t)$	0.096	0.1213	0.0253	20.8573784
	$W_3(t)$	0.067	0.0909	0.0239	26.29262926
0.24	$L_1(t)$	0.2564	0.0364	14.19656786	0.2928
	$L_2(t)$	0.4093	0.061	14.90349377	0.4703
	$L_3(t)$	0.079	0.0501	63.41772152	0.0289
	$U_1(t)$	0.2262	0.0276	12.20159151	0.2538
	$U_2(t)$	0.3359	0.0393	11.69991069	0.3752
	$U_3(t)$	0.0759	0.0474	62.45059289	0.0285
	$W_1(t)$	0.126	0.0289	22.93650794	0.0971
	$W_2(t)$	0.1219	0.0264	21.65709598	0.0955
	$W_3(t)$	0.09	0.0241	26.77777778	0.0659

As time **t** increases, the percentage difference in performance metrics between the two models widens. The use of a NHP service process (as incorporated in the proposed model) leads to significant deviations in predicted system behavior when compared to the traditional homogeneous Poisson model. These deviations highlight the time-sensitive nature of the system, which the developed model captures more effectively. Consequently, the proposed model demonstrates a superior predictive capability, particularly in environments where arrival and service processes vary with time. The analysis confirms that time-dependency and non-homogeneous service assumptions critically impact performance evaluation. The developed model provides a more accurate and robust framework for capturing system dynamics compared to traditional Poisson-based models, especially in non-stationary environments.

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6. CONCLUSION

This study presents the development and implementation of innovative queueing models characterized by time- and state-dependent arrival and service rates, incorporating both parallel and sequential tandem configurations. The proposed model is particularly well-suited for analyzing performance in modern communication networks such as LANs, WANs, MANs, and systems involving data and voice transmission—especially during periods of peak load. The arrival and service mechanisms are governed by non-homogeneous Poisson processes, capturing the dynamic nature of real-world traffic flows. The model assumes that after receiving service at either of the two initial service stations, a customer proceeds to a third queue that is sequentially linked to both, forming what is termed a parallel and sequential queueing system. Comprehensive analytical expressions are derived for key performance metrics, including: The average no. of customers in each queue and in the entire system, The average waiting time per queue and system-wide, Service station throughput, and The variability in queue lengths. The findings demonstrate that both time- and state-dependent behaviors in arrival and service processes exert a profound influence on system performance. Moreover, the model framework is versatile and may be extended to incorporate bulk arrivals, which will be explored in future work.

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