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Research Article

Fuzzy Dominator Coloring

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ARTICLE INFO	ABSTRACT
Received: 30 Dec 2024 Revised: 05 Feb 2025 Accepted: 25 Feb 2025	Finding the right color for a graph while taking into account that each vertex in the graph dominates a whole color class is known as the dominator coloring problem. Fuzzy dominator coloring of certain regular graphs has been discussed here. Fuzzy dominator color and clique number of complete graphs, path graphs, and wheel graphs are related. **Keywords:** Fuzzy graph, antiprism graph, fuzzy dominator coloring, wheel graph, clique number

INTRODUCTION:

A graph is a useful tool for representing information about relationships between items and a basic model of interactions. Vertices and edges represent the item and its relations, respectively. Fuzzy graph models must be created when there is ambiguity in the item's descriptions, their relationships, or both. To solve combinatorial optimization problems including traffic light control, test scheduling, register allocation, and more, fuzzy graph models—of which fuzzy graph coloring is a crucial part—are employed. Fuzzy dominator color plays a key role in these areas.Michelle Gera Ralucca initially introduced fuzzy dominator coloring in 2006 [1]. Concurrently, Eslachi and Onagh [2] defined a fuzzy graph's fuzzy coloring.Jahir Hussain R. and Kanzul Fathima K. S [11] discussed fuzzy dominator coloring and fuzzy dominator chromatic number (FDCN), which is denoted by χfd (G).

Definition 1 [2]: If a) $\vee C = \sigma$ b) $\alpha_i \wedge \alpha_j = 0$ c) for each strong edge uv of $\hat{G}$, $\alpha_i(u) \wedge \alpha_i(v) = 0$ for $1 \le i < k$, then a family $C = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ of fuzzy sets on V is a k-fuzzy coloring of $\hat{G} = (V, \sigma, \mu)$.

Eslahchi and Onagh [2] defined k-fuzzy coloring as a fuzzy graph G is k-fuzzy colorable if it has an appropriate k-fuzzy coloring. The fuzzy chromatic number is the lowest value of k for which a fuzzy graph G is k fuzzy colorable or χ^f(G).

Definition 2[1]: Fuzzy dominator coloring is the process by which G dominates every other vertex in at least one-color class in a fuzzy graph.

Definition 3: In an undirected graph, a clique is a subset X of vertices where each pair of distinct vertices is adjacent. In a graph G, a maximal clique X is a clique of G's vertices; no clique Y of G's vertices can contain all of X plus at least one additional vertex.

Definition 4: The size of the maximal clique is called clique number**.**

Theorem 1: A fuzzy Andrasfai graph with 3n-1 vertices has fuzzy dominator chromatic number

$\boldsymbol{\chi_{fd}(G)=\left\lceil \frac{3n-1}{3} \right\rceil + 2 \ \forall n\geq 2.}$

Proof

Consider the fuzzy Andrasfai graph having vertices U_1, $U_2 \ldots\ldots\ldots\ldots U_{3n-1}$.First color the vertices U_1, $U_3, U_6 \ldots\ldots\ldots\ldots U_{3n-3}$ using the colors $C_1, C_2 \ldots\ldots\ldots C_{\left\lceil \frac{3n-1}{3} \right\rceil}$.The remaining vertices U_2, $U_5 \ldots\ldots\ldots\ldots U_{3n-1}$ are

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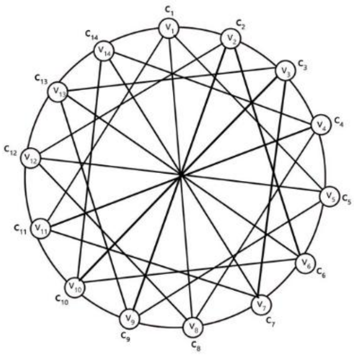
assigned with $\dots..C_{\left\lceil\frac{3n-1}{3}\right\rceil+1}$ and U_4, $U_7 \dots\dots\dots\dots\dots.U_{3n-2}$are colored with $\dots..C_{\left\lceil\frac{3n-1}{3}\right\rceil+2}$. At least one color class is dominated by each vertex of G in this appropriate fuzzy coloring. Therefore, fuzzy dominator chromatic number χ_{fd}(G)=$\left\lceil\frac{3n-1}{3}\right\rceil + 2$ ∀n≥2.

Example



Andrasfai graph A_{14}

Here D={V_1, V_3, V_6, V_9, V_{12}} be the dominating set and colored with the colors C_1, C_2, C_3, C_4, C_5.Then { V_2, V_5, V_8, V_{11}}are assigned with C_6 and the remaining vertices are colored with C_7.It satisfies all the conditions of fuzzy coloring. Therefore, fuzzy dominator chromatic number is 7.

Theorem2: If fuzzy antiprism graph AP_m, m=2n the fuzzy dominator chromatic number

$$\chi_{fd}(\mathbf{AP_m})=\begin{cases}\left\lceil\frac{m}{3}\right\rceil + 2 \ , if\ m = 6k, 6k+2 \\ \left\lceil\frac{m}{3}\right\rceil + 3, if\ m = 6k+4\end{cases}$$

Proof

Let {v_1, v_2.............v_n}and {u_1, u_2..........u_n}be the vertices on inner and outer cycles respectively.

Case (i)m=6k and m=6k+2

The vertices {u_1,u_4,u_7......................}, {v_2,v_5,v_8,.............} be the dominating vertices of the graph. Here we use $C_1, C_2 \dots\dots\dots..C_{\left\lceil\frac{m}{3}\right\rceil}$ colors to color the vertices. The remaining vertices are assigned with $C_{\left\lceil\frac{m}{3}\right\rceil+1}$ and $C_{\left\lceil\frac{m}{3}\right\rceil+2}$ alternatively. In this case, G satisfies all the conditions of a fuzzy dominator coloring. The fuzzy dominator chromatic number, hence

χ_{fd}(G)=$\left\lceil\frac{m}{3}\right\rceil + 2$ ∀n≥2.

Case (ii)m=6k +4

Since the vertices u_1, u_4, u_7,.................v_2,v_5,v_8,.............dominates every other vertices of G .Assign the colors $C_1, C_2 \dots\dots\dots..C_{\left\lceil\frac{m}{3}\right\rceil}$ to these vertices. Use $C_{\left\lceil\frac{m}{3}\right\rceil+1}$, $C_{\left\lceil\frac{m}{3}\right\rceil+2}$, $C_{\left\lceil\frac{m}{3}\right\rceil+3}$ to color the remaining vertices. Hence, fuzzy dominator chromatic number χ_{fd}(G)=$\left\lceil\frac{m}{3}\right\rceil + 3$.

Example

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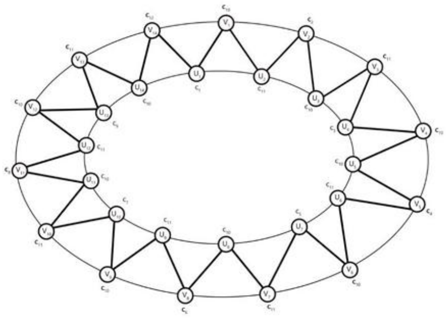


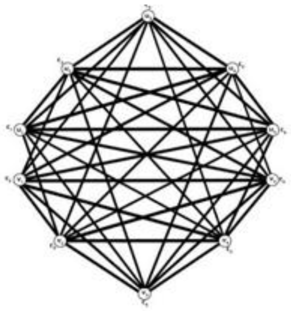
Antiprism graph AP_{14}

D={u_1,u_4,u_7, u_{10}, u_{13}, v_2,v_5,v_8, v_{11}}be the dominating set. We use $K_1, K_2, K_3, K_{4,} K_5, K_6, K_{7,} K_{8,} K_9$ colors. The remaining vertices are colored with K_{10}, K_{11} alternatively. Each vertex in the graph dominates at least one color class in this instance, which is appropriate fuzzy coloring. Hence, fuzzy dominator chromatic number χ_{fd}(G)=11

Theorem 3: The fuzzy dominator chromatic number of a fuzzy cocktail party graph, χ_{fd}(G)=$\frac{k}{2}$

Proof: Separate G's vertices into two sets so that V(G)=$U_1 \cup U_2$, $U_1 \cap U_2 = \emptyset$ .Let U_1={ v_1, v_2.............v_n}, U_2={ u_1, u_2..........u_n}.Here v_i is adjacent to u_j, i≠j, i, j=1,2,3..................n.i.e (v_1, u_1),(v_2, u_2),................(v_n, u_n) are non-adjacent pairs. So, we need $\frac{k}{2}$ colors. Here, G satisfies all the conditions of a fuzzy dominator coloring. So, χ_{fd}(G)= $\frac{k}{2}$

Example



Cocktail party graph CP_{10}

Here (v_1, u_1),(v_2, u_2),................(v_n, u_n) are non-adjacent pairs. So we need 5 colors. Each vertex dominates minimum one-color class. So, the fuzzy dominator chromatic number χ_{fd}(G)= 5.

Theorem 4: The fuzzy complete graph's fuzzy dominator chromatic number and clique number are same.

Proof: Let G have n vertices and be a fuzzy graph. Every vertex is assigned with n $\{C_1, C_2, C_3 .... C_n\}$colors. At least one color class is dominated by each vertex of G. χ_{fd}(G)=n

Assume that G's induced subgraph is H, which is complete. But we know that clique is a subgraph which is complete. Therefore, H is a clique. Also clique number is the size of the maximal clique. But we know that size of maximal clique is n. So, clique number is n.

Therefore, a complete graph's clique number and fuzzy dominator chromatic number are same.

Theorem 5: The fuzzy path's clique number and fuzzy dominator chromatic numbers are same.

Proof: Let G be an n-length fuzzy path. It is evident that each vertex in G dominates both itself and its nearby vertices. Assign color C_1 and C_2 to the alternate vertices of G. It meets every need for fuzzy dominator coloring. Therefore, the fuzzy dominator chromatic number χ_{fd}(G)= 2.

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Consider the complete graph K_2,which is a subgraph of path P_n.Since K_2 is complete, it is a clique.

Also it is the maximal clique. So, the clique number is 2.

Theorem 6: Fuzzy dominator chromatic number and clique number of wheel graph are same

Proof: Let W_n be a fuzzy wheel graph consist of cycle $C_n$with n vertices $\{U_1, U_2 \dots \dots \dots . U_n\}$and each vertex is connected to a central vertex U_0.Give the color C_1 to the vertex U_0.The vertices on the cycles are assigned with color C_2 and C_3alternatively. The central vertex U_0 is dominates the color C_2 and C_3. A minimum of one color class is dominated by each vertex on the cycle. It meets every need for fuzzy dominator coloring. Therefore, chromatic number of fuzzy dominator coloring χ_{fd}(G)= 3.

Consider the complete graph K_3,which is a subgraph of graph W_n.Since K_3 is complete, it is a clique.

Also, it is the maximal clique. So, the clique number is 3.

CONCLUSION:

This paper discusses various graphs' fuzzy dominator chromatic number. Additionally, discussed is the connection between the wheel, path, and full graphs' fuzzy dominator chromatic number and clique number.

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