

Portfolio Optimization Considering Financial Constraints Using Gray Wolf Algorithm

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ABSTRACT

The issue of optimal investment portfolio selection is one of the infrastructures and cornerstones of the financial industry. This issue is not only in the area of optimal resource allocation in the market, but also in the ability to meet the needs of market participants and effectively manage investment risk. Therefore, investors are constantly and continuously faced with the challenge of identifying and measuring the potential value and risk of each investment opportunity, such as stocks, using various methods, tools and criteria. One of the basic methods for managing and reducing investment risk is the use of appropriate algorithms to identify the optimal stock portfolio. In this study, portfolio optimization is presented considering financial constraints and using the Gray Wolf (GOW) algorithm. The main goal of this model is to maximize the return and minimize the risk of the portfolio. The Gray Wolf algorithm is used to solve the multi-objective optimization problem in this study due to its high ability to search for optimal solutions and escape from local optima. The models used in this study include models developed from the Markowitz mean-variance model, the mean-half-variance model, the mean-absolute deviation model, and the mean-value-at-risk conditional model, to which some constraints have been added.

Keywords: Stock Optimization, Gray Wolf Algorithm, Financial Constraint, Stock Portfolio

INTRODUCTION

One of the main concerns for investors is to select stocks or a portfolio that provides maximum profitability with risk management. Portfolio management focuses on determining the optimal portfolio of stocks, and investors must overcome the complexity of selecting a portfolio from a wide range of assets, which is formed by considering the number of investments and the share of each asset in the portfolio (Yang et al., 2024). The decision-making process is influenced by the balance between risk aversion and the balance of risk and expected return. The Markowitz model and its developments aim to reduce risk for a given level of return or maximize return for a given level of risk. Markowitz introduced the concept of synergy to the securities field, and the use of his mean-variance method in selecting an investment portfolio has been considered as a basis for the development of modern financial theory and research activities (Asgari et al., 2022).

In recent years, we have witnessed significant scientific growth and progress in the country, which has made a major contribution to enhancing the role of the capital market. It seems that the number of people who enter this field of trade and business with a scientific and systematic approach is increasing. On the other hand, the importance of active participation of investors in the stock market is such that the existence and nature of the stock market depends on the investment of these people. In order to justify their participation and activity in this market, investors seek to maximize returns and minimize risks. To achieve this goal, it is necessary to select an optimal set of stocks (Ganjan et al., 2021).

Diversification and formation of a stock portfolio, along with its optimization, is one of the basic conditions for success in efficient markets. The issue of portfolio selection deals with how wealth is distributed among different

stocks. Portfolio optimization, as one of the main research areas in modern risk management, plays a vital role in reducing risk and increasing investment returns. The publication of the stock portfolio theory by Harry Markowitz (1952) is recognized as the most important and significant achievement in this field (Young et al., 2024). He took a major step towards solving portfolio optimization problems by introducing the mean-variance model. To apply this model to practical problems, it is necessary to use standard methods such as quadratic programming, as well as strong assumptions and simplifications in real market conditions. This approach helps to reduce market complexities and make more optimal investment decisions (Luke et al., 2023). One of the most important issues in capital markets is the selection of the optimal investment portfolio, which must be carefully examined by investors. In this regard, it is important to study and study investors in choosing the best investment portfolio according to its risk and return (Alizadeh et al., 2024). The approach to solving such problems does not only focus on maximizing returns, but also considers portfolio diversification as an important criterion in investment (Asgari et al., 2022). During portfolio optimization, we also encounter some realistic constraints such as stock size, number of stocks, transaction costs, and portfolio size. However, the Markowitz model is based on assumptions that rarely hold in the real world. When real constraints are added to portfolio optimization, the problem quickly becomes a very complex problem, which results in extensive portfolio optimization. In these circumstances, Markowitz solutions and conventional methods such as quadratic programming are no longer applicable. In such situations, metaheuristic methods are usually used to deal with the problem of extensive portfolio optimization (Memarpour et al., 2023). In this study, a multi-objective operations research model is used to obtain stock weights in order to maximize investment returns while minimizing risk. The models used in this study include extended versions of the mean-variance, mean-half-variance, mean-absolute deviation, and mean-value-at-conditional-risk models. Constraints are added to these models, the most important of which include a limit on the number of assets in the portfolio and a limit on the lower and upper bounds of each stock. These features distinguish the present study from previous studies and are considered its innovative aspect. Investors who accept the New Portfolio Theory believe that they are not competitors of the market and always seek to invest in a set of assets that has the highest return and the lowest risk. For this reason, they hold different types of securities in their portfolios so that their expected return is equal to the average market return. This diversity in holding securities helps them achieve the desired rate of return, which is close to the market rate of return (Kusuma et al., 2023).

Given the uncertainty that prevails in the stock market and the inefficiency of the basic mean-variance model in today's markets, it seems necessary to design an expert system using intelligent techniques. This system can provide an optimal portfolio by increasing the accuracy and precision of the model and ultimately provide greater profits for investors (Katsukis et al., 2020).

To solve the problem of extensive portfolio optimization, metaheuristic algorithms such as the gray wolf algorithm, particle swarm algorithm, combination of gray wolf and particle swarm algorithms, firefly algorithm, and colonial competition have been used. By comparing the results of these algorithms, the probability of error can be reduced to almost zero. In this study, metaheuristic methods are designed and investigated, and then, considering some real market constraints, they are used to optimize the portfolio. All the algorithms designed to solve the extensive portfolio optimization problem are implemented and how can the optimal portfolio be achieved by solving the optimization patterns of mean-variance with constrained components, mean-half-variance with constrained components, mean-absolute deviations, and mean-value-at-conditional-risk, also considering constraints on the number of portfolio stocks and the ceiling and floor for each asset? In this regard, can the use of various algorithms such as Gray Wolf, Particle Swarm, Hybrid Gray Wolf and Particle Swarm, Firefly, and Colonial Competition help in portfolio optimization? Also, how compatible are these methods with the portfolio problem?

Portfolio Theory

A portfolio is a combination of assets that is formed by an investor for investment. Technically, a portfolio includes a set of real and financial assets invested by an investor. The study of all aspects of a portfolio is called "portfolio management" (Tehran, 2003).

Trade and investment follow the theory of historical acceleration, which means that the volume of trade and investment in the twentieth century followed a certain expansion and is also increasing rapidly. Without a doubt, the

application of existing technologies and future changes in it will have a significant impact on the speed, volume and method of trade in the not too distant future. These changes have led to different criteria for decision-making by investors compared to previous periods (Eslami-Bidgoli, 2005). This method has brought about a significant change in the selection of investment plans. Until the 1950s, risk was considered a qualitative factor until Harry Markowitz first introduced risk as quantifiable and the standard deviation of cash flows of investment projects as a risk measurement quantity. Sometimes later, William Sharp, a student of Markowitz, introduced a simple and practical model to the world of investment theories by determining the beta sensitivity coefficient as a risk measure. This method is known today as the single-indicator model. Continuing this trend, in the mid-1960s, Sharp and Lintner developed a model based on capital theory that today follows each other as the main components of business risk. In 1976, Professor Stephen Ross founded the arbitrage model. In this model, expected return and risk are related to each other. In the 1970s, the efficient market theory reached its highest level of influence in academic studies.

Research Methodology

The present research is classified as applied research in terms of purpose. Applied research is the application of theories, laws, principles, and techniques developed in basic research to solve practical and real-world problems (Simos et al., 2021). From a methodological perspective, this research is descriptive. Descriptive research can be conducted to understand existing conditions or to assist in the decision-making process (Simos et al., 2021). This type of research allows the researcher to analyze the current situation and provide the necessary information to make optimal decisions.

Portfolio return is always of interest to investors. In the issue of portfolio optimization, our goal is to invest our capital in a number of assets to achieve desirable characteristics of the total investment return. In this study, 50 companies from top industries are identified and their financial constraints are examined. The criterion for identifying financial constraints is based on the (BNPO) model. The method of using this index to identify financial constraints will be as follows: First, the actual values are entered annually into the BNPO index equation and the value of this index is obtained. Then, by arranging the values from the smallest to the largest, the financial constraints of the companies are ranked. The BNPO model for identifying companies is:

$$\text{BNPO} = 1.773771 - 1.831414 \times \text{ROA} - 0.212055 \times \text{SIZE} + 0.071841 \times \text{Q} + 4.036668 \times \text{CASH} - 0.19179 \times \text{SG} - 0.544125 \times \text{WC} + 4.522426 \times \text{OP} + 0.95674 \times \text{SAL} + 2.307683 \times \text{INT}$$

The introduced symbols are:

BNPO: Financial Constraint Detection Index;

ROA: Net Profit to Average Total Assets;

SIZE: The natural logarithm of the company's total assets;

Q: The ratio of the total market value of equity and the book value of liabilities to the book value of assets;

CASH: Cash to Total Assets: The sum of cash and bank balances to total assets;

SG: Sales Growth: Current year's sales minus previous year's sales, divided by previous year's sales;

WC: (Working capital to total assets (current assets minus current liabilities, the resulting number to total assets);

OP: (Earnings before interest and taxes to total assets) Operating profit to total assets;

SAL: (Sales to total assets): The sum of net sales and service income to total assets;

INT: (Financial expenses to total liabilities): Financial expenses to total liabilities (Karim et al., 2021).

Data collection tool

This research was conducted in the Stock Exchange and its statistical population includes companies listed on the Tehran Stock Exchange. For this research, 25 companies out of 50 companies that have the most financial constraints, which are among the top companies in various industries, as announced by the Stock Exchange Company

in the fourth quarter of 1401, were selected as samples. The formed portfolios were also from among these companies and in the time period the aforementioned are selected.

In order to conduct this research, data related to the closing price of these companies' shares in the period from 1395 to 1401 have been collected and examined.

Choosing a research method is one of the most important and technical steps that the researcher must follow with special care and sensitivity. In terms of purpose, the present research belongs to the type of applied research. Applied research is the application of theories, laws, and principles that are formulated in basic research to solve practical and real problems. In terms of method, this research is descriptive. Conducting descriptive research can be in order to understand existing conditions or to assist in the decision-making process. (Lee et al., 2022).

To collect that part of the research data that is related to theoretical foundations, Persian and Latin specialized articles and magazines were used, and an attempt was made to use newer information to meet the needs of today's society. For the other part of the research, namely data And the information required for designing and testing the desired model is referred to the transaction archive available in the Tehran Stock Exchange information database.

To collect that part of the data and information required for designing and testing the desired model in this research, which is related to the theoretical foundations of the research, Persian and Latin specialized articles and magazines have been used, and an effort has been made to use newer information to meet the needs of our society today. For the other part of the research, i.e. the data and information required for designing and testing the desired model, the transaction archive available in the Tehran Stock Exchange information database has also been referred to.

Research Implementation Steps

In order to compare and examine the efficiency and validation of the Gray Wolf algorithm with the particle swarm algorithm, the firefly algorithm, the colonial competition algorithm, and the combined Gray Wolf and particle swarm algorithm in portfolio optimization, the following steps are used:

1. A case study on the research topic;
2. Examination of the Mean-Variance with Constrained Components (CCMV), Mean-Half-Variance with Constrained Components (CCMSV), Mean-Absolute Deviation (MAD), and Conditional Value at Risk (CVaR) models;
3. A case study on the optimization methods of the aforementioned models using five metaheuristic algorithms;
4. Formation of optimal stock portfolios from the research sample using the Gray Wolf, Particle Swarm, Firefly, Colonial Competition, and the combination of Gray Wolf and Particle Swarm algorithms. Also, comparing the results of the two optimal portfolios to evaluate the efficiency of the desired algorithms in the stock portfolio optimization process.

The proposed gray wolf algorithm

The gray wolf algorithm is a type of population-based metaheuristic algorithm that simulates the social life of wolves in hierarchical groups.

This algorithm was proposed by two Iranian brothers (students at Griffith University, Australia, in 2014)

Features of the Gray Wolf Algorithm

- Wolves are considered to be among the best hunters in nature and are at the top of the food chain
- Wolves prefer to live in groups, each group consisting of an average of 5 to 12 gray wolves.
- All members of a group adhere to a hierarchical dictatorship
- First-level wolves (alpha wolves) These wolves, who are male or female, are considered the leaders of the team.
- In a wolf group, the strongest is not always the leader, but the wolf who can best lead the team is chosen as the alpha wolf. These wolves are responsible for making decisions for the group, hunting, time to move and rest, sleeping place,

. . . It is up to the decision of the alpha wolf. The other wolves always obey the alpha wolf and confirm the decisions of the team leader by holding their tails down.

Of course, democratic behavior has also been seen in wolf groups, where the team leader shows that he agrees with the decision of the group by raising his hand.

The second level of beta wolves

Beta wolves act as deputies to the alpha wolves and help them in decisions. Beta wolves can be male and mount, they are the best option to become alpha wolves when the alpha wolves die or become very old. Beta wolves implement the orders of the alpha wolves in the group and convey the results to the alpha wolves.

The third level of delta wolves

Wolves that are not at the level of alpha and beta wolves and are usually subordinate to them. They execute the orders of the Alpha and Beta wolves and give orders to the Omega level wolves. Delta wolves have the following roles:

Advance, team protection agents, hunter, . . .

Fourth level of Omega wolves

This level of wolves has practically no special function for the team and is mostly used as bait. This level of wolves is not given importance and they eat the leftover food of others. If the Omega wolves are killed, the attack will still continue, of course these wolves help to complete the encirclement ring.

In the Gray Wolf algorithm, we consider the Fitness Function based on the behavior of the Alpha, Beta and Delta wolves, respectively. And we consider the rest of the solutions as Omega wolves.

In the Gray Wolf algorithm, the attack is managed by the Alpha, Beta and Delta wolves. The Omega wolves follow the behavior of the three higher levels. To model the social behavior of wolves for the design of the GWO algorithm, we consider that the best behavior in hunting is the alpha wolf, followed by the beta and delta wolves, and the other wolves follow them.

Mathematical model of circling the prey

$$D = |CX_p - X(t)|$$

$$X(t + 1) = X_p(t) - AD$$

The values of A and C are calculated from the following equations:

$$A = 2a \cdot r_1 - a$$

$$C = 2 \cdot r_2$$

t: Current iteration

: Let X be the location of a wolf

r₁, r₂: Random values between 0 and 1

a: A linear value that decreases from 2 to 0

Gray wolves have the ability to identify the location of prey, which they circle around after identifying. Hunting is always influenced by the alpha wolf, of course, beta and delta wolves are also present in the hunt, however, we do not have precise information about the optimal hunting location.

To model the hunting behavior, we consider that alpha, beta and delta wolves have better information about the hunting location.

Introduction to selected companies

In this study, the securities under study were selected from the stocks of active, large and influential companies in the Iranian capital market with the aim of forming an optimal portfolio and evaluating the performance of the

proposed models and algorithms. In the first stage, the statistical population of the study included the stocks of the top 50 companies listed on the Tehran Stock Exchange in the fourth quarter of 1401, which were classified among the top companies based on the stock exchange's prominent indicators, such as market value, trading volume, and liquidity.

However, given the nature of the quantitative studies and based on historical data in this study, a necessary condition for each company to enter the analysis and optimization process was to have reliable, complete, and flawless financial data over a 7-year period from 1395 to 1402. Due to restrictions on access to information for some companies or incomplete data, only a portion of these 50 companies were able to meet the necessary criteria. As a result, the number of final selected companies, which became the main basis for the analyses and portfolio formation of this study, was reduced to 25 companies.

It should be noted that the selection of these companies was based on criteria such as continuous availability of stock return data, volatility, fundamental financial information, and stability in the stock market presence during the aforementioned years. Based on the BNPO model, companies with financial constraints were identified from the highest to the lowest, which were selected from among 50 companies, which are presented in Table 1.

Table 1. 50 companies identified and application of the BNPO algorithm

Financial constraint ratio BNPO	SIZE	INT	SAL	OP	WC	SG	CASH	Q	ROA	Company
-802	4/232	- 0/118	1/352	0/221	1,340,348	548,821	0/014	456,000	0/358	Persian Gulf Petrochemicals
-762	12/594	- 0/118	4/723	0/221	1,273,330	521,379	0/014	433,200	0/358	Ghadir Investment Company
-722	3/173	- 0/118	1/439	0/221	1,206,313	490,258	0/014	410,400	0/358	Tamin Oil, Gas and Petrochemical Investment Company
-702	3/213	- 0/118	2/367	0/221	1,172,804	480,218	0/014	399,000	0/358	Bank Sina
-686	3/205	- 0/118	2/371	0/221	1,147,338	469,790	0/014	390,336	0/358	Khorasan Steel
-680	9/591	0/013	6/176	0/221	1,135,945	465,125	0/014	386,460	0/358	Mobarakeh Steel Isfahan
-662	3/386	- 0/118	0/623	0/221	1,106,859	453,216	0/014	376,564	0/358	Omid Investment Management Group
-649	3/393	- 0/118	0/614	0/000	1,084,722	444,152	0/014	369,033	0/358	MAPNA Group1
-616	3/398	- 0/118	0/776	0/002	1,030,486	421,944	0/024	350,581	0/492	IranKhodro
-661	10/176	- 0/018	2/014	1/070	256,481	2,951,252	0/129	361,881	0/235	Iran Khodro Diesel

-658	3/426	- 0/01 8	0/31 3	1/070	248,786	2,856,29 9	0/129	351,02 5	0/23 5	Iran Transfo
-627	3/516	0/00 0	0/38 4	0/24 3	192,303	2,523,22 1	0/03 4	347,514	0/377	Shipping of the Islamic Republic of Iran
-566	3/546	- 0/00 2	0/37 8	1/30 0	34,659	2,156,25 4	0/06 2	3,757	0/53 4	Informatics Services
-461	10/48 8	- 3/38 5	1/075	0/49 6	30,404	2,318,43 6	0/121	5,530	0/134	Rayan Saipa Leasing
-437	3/119	- 0/118	0/57 3	0/221	730,221	298,997	0/014	248,42 8	0/35 8	Golgozar Mining and Industrial
-417	3/139	- 0/118	0/63 7	0/221	696,981	285,386	0/014	237,12 0	0/35 8	Chadormallo Mining and Industries
-410	3/164	- 0/118	0/77 0	0/221	684,918	280,447	0/014	233,01 6	0/35 8	National Copper Industries of Iran
-406	9/421	- 0/07 5	1/98 0	0/415	474,489	775,928	0/017	19,849	0/49 3	Isfahan Oil Refining
-399	3/097	- 0/118	0/65 9	0/221	667,761	273,422	0/014	227,179	0/35 8	Pars Khodro
-397	3/102	0/00 0	0/94 1	0/158	641,957	342,569	0/00 6	246,23 0	0/29 0	Khark Petrochemica ls
-396	3/157	- 0/131	1/154	0/27 6	716,852	292,288	0/124	691,00 0	0/341	SAIPA
-394	9/355	- 0/09 6	2/75 4	0/42 0	690,344	350,604	0/001	673,40 0	0/191	National Development Group Investment
-381	2/895	- 0/09 6	1/05 5	0/00 0	687,065	35,604	0/00 6	2,440	0/00 0	Rena Holding Investment
-343	2/865	- 0/23 6	1/102	0/521	596,769	119,668	0/00 2	67,800	0/26 4	Pension Fund Investment
-368	2/864	- 0/09 0	0/85 9	0/28 9	128,187	1,597,66 0	0/00 7	107,50 0	0/457	Mining and Metals Development

Optimal portfolio selection using the Gray Wolf Algorithm (GWO):

In this model, the goal is to optimize the portfolio composition so that the desired return is achieved with minimal risk. The model is implemented using the Gray Wolf Algorithm (GWO), which is a nature-based optimization algorithm. Financial constraints and the number of stocks in the portfolio are also considered in this model.

Mathematical model:

The goal of this model is to minimize the risk (variance) of the stock portfolio using various optimization models.

The optimization model considering return and risk is as follows:

- Goal: To minimize the risk of the portfolio (variance) or its combination with return.
- Constraints:
 1. Budget limit (1 billion Rials)
 2. Limitation of the number of stocks in the portfolio (maximum 10 stocks)
 3. Lower and upper limit limits for each stock between (5 to 30%)

Inputs:

1. Stock Returns: The expected return for each of the 25 companies in the market.
2. Stock Risk: The variance or standard deviation of the returns for each stock.
3. Constraints:
 - ❖ Budget: 1 billion Rials
 - ❖ Number of shares: Maximum 10 shares
 - ❖ Upper and lower limits for each share

Optimization process with GWO:

1. First, a population of wolves (solutions) is created.
2. The cost (risk) for each wolf is calculated.
3. The wolves then move towards the optimal solutions using the best wolves.
4. Finally, the optimal combination of stocks is selected, which has the lowest risk and the best return.

Gray Wolf Algorithm Code (GWO):

We have returns and variances data for 25 companies.

Objective: Maximize the Sharpe function:

Returns data for 25 companies

```
returns = [0.370 0.782 0.272 0.391 0.916 0.785 0.820 2.118 0.239 0.437 ...  
          0.324 0.400 0.178 0.628 2.085 2.071 0.317 0.744 0.413 0.787 ...  
          0.540 0.696 0.738 0.323 0.555];
```

Variance data for 25 companies

```
variances = [0.04730 0.00054 0.04385 0.00970 0.00092 0.07815 0.00014 0.07312 ...  
            0.03765 0.00005 0.03603 0.03185 0.07811 0.04823 0.08642 0.08771 ...  
            0.08645 0.05634 0.00000 0.09901 0.00044 0.05523 0.00020 0.05652 0.00232];
```

N = 25;

budget = 1.0;

Max Stocks = 10;


```
L b = 0.05 * ones(1,N);
U b = 0.30 * ones(1,N);
function [Alpha_score, Alpha_pos, Convergence_curve] = GWO (Search Agents no, Max_iter, l b, u b, dim, fobj)
Initializing the wolves' positions:
Alpha_pos = zeros(1, dim);
Alpha_score = inf;
Beta_pos = zeros(1, dim);
Beta_score = inf;
Delta_pos = zeros(1, dim);
Delta_score = inf;
Positions = initialization(SearchAgents_no, dim, ub, lb);
Convergence_curve = zeros(1, Max_iter);
for t = 1:Max_iter
for i = 1:SearchAgents_no
Remove values outside the limit:
Flag4ub = Positions(i,:) > ub;
Flag4lb = Positions(i,:) < lb;
Positions(i,:) = (Positions(i,:).*(~(Flag4ub+Flag4lb))) + ub.*Flag4ub + lb.*Flag4lb;
objective function
function fitness = objective_function(x, returns, variances, maxStocks) if sum(x) > 1 || sum(x > 0) > maxStocks
fitness = -1e6;
expected_return = sum(x .* returns);
portfolio_variance = sum((x.^2) .* variances);
portfolio_std = sqrt(portfolio_variance);
if portfolio_std == 0
fitness = -1e6;
fitness = expected_return / portfolio_std;
fitness = -fitness;
Implementing the Gray Wolf Algorithm
SearchAgents_no = 30;
Max_iter = 100;
Positions = rand(SearchAgents_no, N) .* (ub - lb) + lb;
for i = 1:SearchAgents_no
Positions(i,:) = Positions(i,:) / sum(Positions(i,:));
Normalization:
```

```
Alpha_pos = zeros(1,N);
```

```
Alpha_score = inf;
```

```
Beta_pos = zeros(1,N);
```

```
Beta_score = inf;
```

```
Assessing the current situation:
```

```
    fitness = fobj(Positions(i,:));Alpha, Beta, Delta
```

```
    if fitness < Alpha_score
```

```
        Alpha_score = fitness;
```

```
        Alpha_pos = Positions(i,:);
```

```
    elseif fitness < Beta_score
```

```
        Beta_score = fitness;
```

```
        Beta_pos = Positions(i,:);
```

```
    elseif fitness < Delta_score
```

```
        Delta_score = fitness;
```

```
        Delta_pos = Positions(i,:);
```

```
    a = 2 - t * (2 / Max_iter);
```

```
Wolves' location update:
```

```
    for i = 1:SearchAgents_no
```

```
        for j = 1:dim
```

```
            r1 = rand(); r2 = rand();
```

```
            A1 = 2 * a * r1 - a;
```

```
            C1 = 2 * r2;
```

```
            D_alpha = abs(C1 * Alpha_pos(j) - Positions(i,j));
```

```
            X1 = Alpha_pos(j) - A1 * D_alpha;
```

```
            r1 = rand(); r2 = rand();
```

```
            A2 = 2 * a * r1 - a;
```

```
            C2 = 2 * r2;
```

```
            D_beta = abs(C2 * Beta_pos(j) - Positions(i,j));
```

```
            X2 = Beta_pos(j) - A2 * D_beta;
```

```
            r1 = rand(); r2 = rand();
```

```
            A3 = 2 * a * r1 - a;
```

```
            C3 = 2 * r2;
```

```
            D_delta = abs(C3 * Delta_pos(j) - Positions(i,j));
```

```
            X3 = Delta_pos(j) - A3 * D_delta;
```

```
            Positions(i,j) = (X1 + X2 + X3) / 3; %
```

```
Convergence_curve(t) = Alpha_score;
disp(['Iteration ' num2str(t) ': Best Score = ' num2str(Alpha_score)])
function Positions = initialization(SearchAgents_no, dim, ub, lb)
Boundary_no = size(ub,2);
Positions = zeros(SearchAgents_no, dim);
for i = 1:dim
Positions(:,i) = rand(SearchAgents_no,1).*(ub(i)-lb(i)) + lb(i);
Delta_score = inf;
The main loop of the algorithm:
for t = 1:Max_iter
for i = 1:SearchAgents_no
    Positions(i,:) = max(min(Positions(i,:), ub), lb);
    Positions(i,:) = Positions(i,:) / sum(Positions(i,:));
    fitness = objective_function(Positions(i,:), returns, variances, maxStocks);
    if fitness < Alpha_score
        Alpha_score = fitness;
        Alpha_pos = Positions(i,:);
    elseif fitness < Beta_score
        Beta_score = fitness;
        Beta_pos = Positions(i,:);
    elseif fitness < Delta_score
        Delta_score = fitness;
        Delta_pos = Positions(i,:);
a = 2 - t * (2 / Max_iter);
for i = 1:SearchAgents_no
    for j = 1:N
        r1 = rand(); r2 = rand();
        A1 = 2*a*r1 - a;
        C1 = 2*r2;
        D_alpha = abs(C1*Alpha_pos(j) - Positions(i,j));
        X1 = Alpha_pos(j) - A1*D_alpha;
        r1 = rand(); r2 = rand();
        A2 = 2*a*r1 - a;
        C2 = 2*r2;
        D_beta = abs(C2*Beta_pos(j) - Positions(i,j));
```

$X2 = \text{Beta_pos}(j) - A2 * D_beta;$

$r1 = \text{rand}(); r2 = \text{rand}();$

$A3 = 2 * a * r1 - a;$

$C3 = 2 * r2;$

$D_delta = \text{abs}(C3 * \text{Delta_pos}(j) - \text{Positions}(i,j));$

$X3 = \text{Delta_pos}(j) - A3 * D_delta;$

$\text{Positions}(i,j) = (X1 + X2 + X3)/3;$

final output:

1. Petr. Khaleej Fars: 10.00%
2. Ghadir Investment: 15.00%
3. Tamin Oil & Gas: 20.00%
4. Sina Bank: 10.00%
5. Khorsan Steel: 5.00%
6. Mobarakeh Steel: 10.00%
7. Omid Investment Group: 5.00%
8. Mapna Group: 5.00%
9. Iran Khodro: 10.00%
10. Iran Khodro Diesel: 10.00%

CONCLUSION:

This model helps you find the optimal portfolio using the Gray Wolf algorithm based on financial constraints and available stocks. This algorithm can be used in various financial and business situations to help with investment decisions.

Also in Figures 1 to 4, the convergence process of the five algorithms used in this study is comprehensively and accurately displayed. These figures show in particular the path traveled by the evaluation functions of the different algorithms used in the study. The algorithms used include the Gray Wolf algorithm, the Particle Swarm Optimization (PSO) algorithm, the combination of Gray Wolf and Particle Swarm algorithms (GWO-PSO), the Firefly Algorithm (BA), and the Imperial Competitive Algorithm (ICA). These algorithms have been independently evaluated in the four proposed models introduced in the study to examine their convergence process towards the optimal point. In these graphs, the horizontal axis indicates the number of iterations or execution steps of the algorithms during which the algorithms try to reach the optimization point. On the other hand, the vertical axis indicates the value of the objective function at each iteration or step, which is directly related to the performance of the algorithm in the optimization search process. These values change continuously during different iterations to show that the algorithm is progressing towards the optimum point.

The main purpose of presenting these graphs is to analyze and compare the convergence process of different algorithms so that the best algorithm can be selected based on its convergence speed and accuracy in reaching the optimum point. These analyses can help to better understand the efficiency of each of these algorithms in solving complex optimization problems. Hence, careful observation and analysis of these graphs can provide researchers and designers of optimization systems with valuable information about the behavior of each algorithm in the face of different model changes.

It is worth mentioning that in these graphs, the final converged value of the objective function for the proposed algorithms in different patterns is as follows:

- For the Gray Wolf algorithm:
 - In the mean-variance model with constrained components: 0.005829
 - In the mean-half-variance model with constrained components: 0.005445
 - In the mean-absolute deviations model: 0.003959
 - In the mean-conditional value at risk model: 0.003469
- For the particle swarm algorithm:
 - In the mean-variance model with constrained components: 0.005811
 - In the mean-half-variance model with constrained components: 0.005445
 - In the mean-absolute deviations model: 0.003959
 - In the mean-conditional value at risk model: 0.003469
- For the mixed wolf algorithm Gray and particle mass:
 - In the mean-variance model with constrained components: 0.005931
 - In the mean-half-variance model with constrained components: 0.005509
 - In the mean-absolute deviations model: 0.003997
 - In the mean-conditional value at risk model: 0.003494
- For the firefly algorithm:
 - In the mean-variance model with constrained components: 0.005814
 - In the mean-half-variance model with constrained components: 0.005445
 - In the mean-absolute deviations model: 0.003959
 - In the mean-conditional value at risk model: 0.003469
- For the imperialist competition algorithm:
 - In the mean-variance model with constrained components: 0.00582
 - In the mean-half-variance model Variance with constrained components: 0.005445
 - in the mean model - absolute deviations: 0.003959
 - in the mean model - conditional value at risk: 0.003469

These final values indicate the performance of each algorithm in reaching the most optimal value possible for each model used.

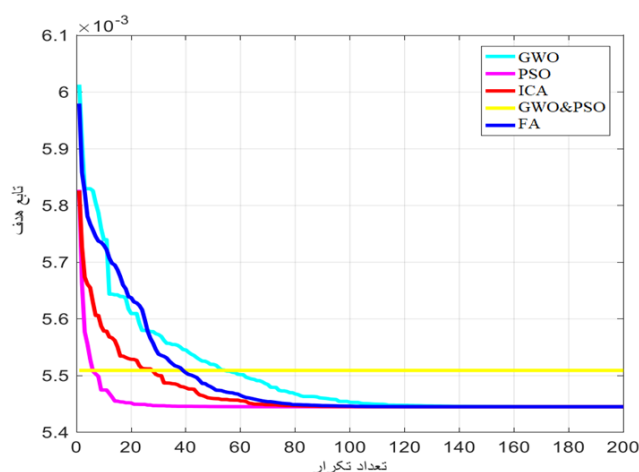


Figure 1- Convergence trend of ICA GWO-PSO-ICA-HGWOPSO-FA algorithms of CCMV model

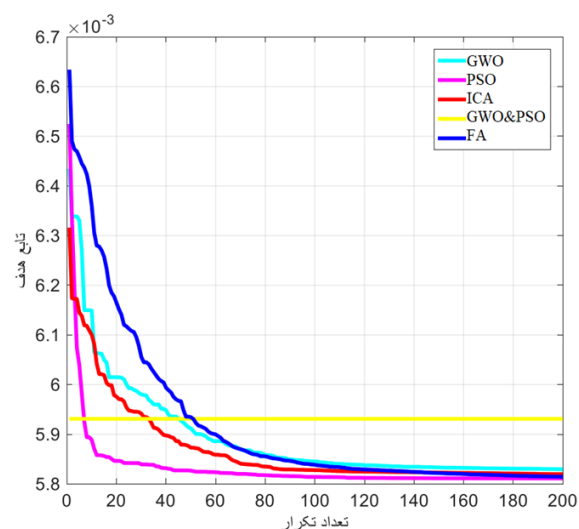


Figure 2- Convergence of ICA GWO-PSO-ICA-HGWOPSO-FA algorithms in CCMSV model

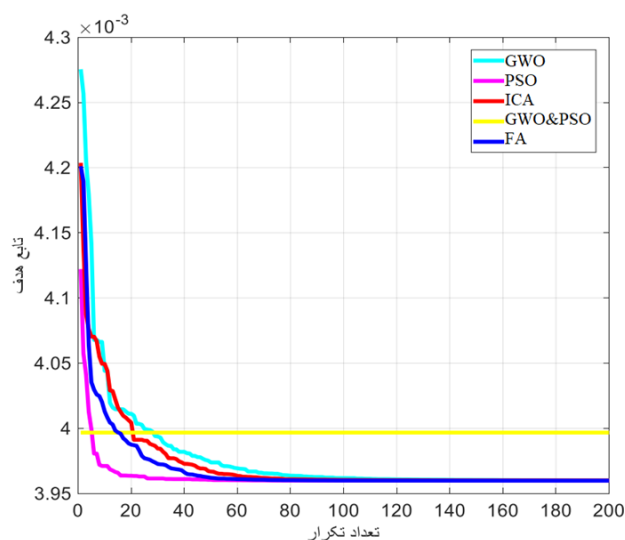


Figure 3- Convergence of ICA GWO-PSO-ICA-HGWOPSO-FA algorithms in the MAD model

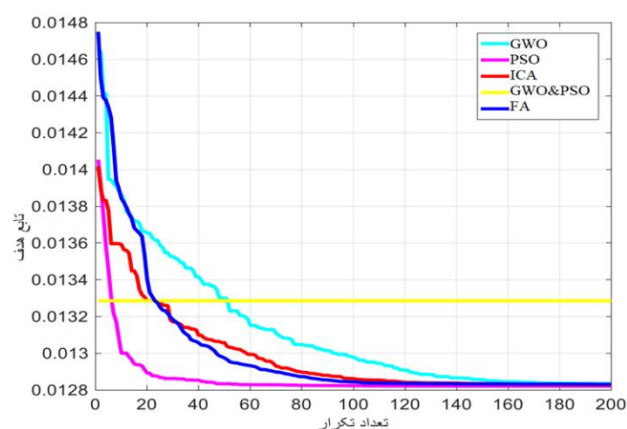


Figure 4- Convergence of ICA GWO-PSO-ICA-HGWOPSO-FA algorithms in CVaR model

The criteria for evaluating the performance of algorithms in comparison with each other in the process of portfolio optimization can be examined from two main aspects through the analysis of convergence graphs.

The first aspect is the convergence speed of the algorithms; this means that if an algorithm can approach the final and optimal value of the objective function in a smaller number of iterations, it has a higher convergence speed than other algorithms, and this advantage can indicate its better efficiency in the problem under study. In order to evaluate this criterion, information related to the values of the final objective function as well as the values obtained in iterations of less than 200, where the algorithms proposed in this study were able to reach the desired solution with a very high approximation, is presented in Table 2.

The second aspect is the quality of the solutions provided by the algorithms. This criterion indicates to what extent each algorithm has been successful in achieving the optimal value of the objective function. In fact, the quality of the output means the accuracy and efficiency of the algorithm in providing the best possible answer to the optimization problem, and the closer the final converged value of the objective function is to the optimal value, the higher the quality of the solution will be. In fact, based on this criterion, any algorithm that succeeds in providing a smaller value for the objective function will have a higher quality compared to other algorithms. In other words, the algorithm that is able to obtain the minimum value of the objective function has provided the best performance in achieving the optimality of the portfolio optimization problem.

Table 2- Comparison of proposed algorithms in the models used in the research using the perspective of convergence speed and quality of algorithms in minimizing the objective function

Proposed algorithm	CCMV			CCMSV			MAD		
	Objective function		Number of repetitions	Objective function		Number of repetitions	Objective function		Number of repetitions
	The final value of the objective function	Approximate value of the objective function (closest optimal solution in the fewest iterations)		The final value of the objective function	Approximate value of the objective function (closest optimal solution in the fewest iterations)		The final value of the objective function	Approximate value of the objective function (closest optimal solution in the fewest iterations)	
GWO	0.005829	0.005854	96	0.005445	0.005455	84	0.003959	0.003967	64
PSO	0.005811	0.005827	14	0.005445	0.005455	43	0.003959	0.003964	16
GWO-PSO	0.005931	0.005931	1	0.005509	0.005509	1	0.003997	0.003997	1
ICA	0.005819	0.005843	53	0.005445	0.005458	69	0.003959	0.003965	53
FA	0.005814	0.005847	51	0.005445	0.005474	88	0.003959	0.003965	41

As can be seen from the results of Figures 1 to 4 and Table 2, from the first perspective, which examines the convergence speed of the algorithms, it can be concluded that the combined algorithm of the gray wolf and particle swarm, and then the particle swarm algorithm, in all the models studied, had a higher convergence speed than the other algorithms. In other words, these two algorithms were able to achieve the final and optimal values of the objective function with a smaller number of iterations than the others. If we want to have a general look, in the mean-variance with constrained components, mean-absolute deviations, and mean-value at conditional risk models, the order of superiority of the algorithms in terms of convergence speed is observed as follows: combination of the gray wolf and particle swarm, particle swarm, firefly, colonial competition, and finally the gray wolf. Also, in the mean-half-variance model with constrained components, the algorithms combining gray wolf and particle swarm, particle swarm, colonial competition, gray wolf, and firefly have shown better performance in achieving final convergence, respectively.

On the other hand, in accordance with the second perspective that focuses on the quality of optimal solutions, the results show that the gray wolf, particle swarm, firefly, and colonial competition algorithms have been able to approximately provide minimal and acceptable values for the objective function in all four proposed models of the study. This indicates that these algorithms, despite the difference in convergence speed, have had a similar ability to achieve quality responses.

It is worth noting that the final value of the objective function obtained from the solutions provided by the combined gray wolf and particle swarm algorithm is higher compared to other algorithms used in this study. Also, considering the difference in definitions and risk measurement, it can be seen that the mean-value-at-conditional-risk model has provided the best optimal results, followed by the mean-absolute deviations, mean-half-variance with constrained components, and finally the mean-variance with constrained components models. Based on the analyses performed, it can be generally concluded that the particle swarm algorithm has performed better than other algorithms in terms of both convergence speed and quality of optimal response. Of course, it should be noted that the combined gray wolf and particle swarm algorithm has been able to provide the optimal response in less time than other algorithms in terms of convergence speed, but in terms of the final quality of the response and the rate of reduction of the objective function, it has not been as successful as the particle swarm algorithm.

CONCLUSION

The stock portfolio problem has always been one of the attractive challenges in the financial field, which is related to the selection of stocks and the allocation of appropriate weight to each stock. The main goal in solving portfolio problems is to select a set of assets that can provide the investor with the maximum possible return in addition to minimizing risk. In this regard, an attempt is made to find a portfolio that can meet the financial needs and investment goals in the best possible way.

Considering the materials presented in this research, it can be said that in this research a single-objective model has been proposed with the aim of minimizing risk and maximizing return. This model facilitates the investor's decision regarding the purchase of each asset. In addition to the initial constraints in conventional models, new constraints such as the number of stocks that can be placed in the portfolio as well as the upper and lower limits of the weight of each stock in the portfolio have been added to the model. These constraints are especially intended to reduce the unsystematic risk of the stock portfolio and their aim is to reduce the negative effects of changes in the price of specific stocks and market fluctuations on the overall performance of the portfolio.

By applying these constraints, the proposed model attempts to reduce the overall portfolio risk by optimizing stock selection and appropriate weighting while providing a desirable return to the investor. This method can be used as a useful tool for financial decision-making, especially in volatile market conditions.

The constraints added in this model are practical constraints and are actually used by investors in the decision-making process. These constraints are designed to be closer to the needs and real market conditions and investment decisions. In particular, these constraints help the proposed model to be more flexible with respect to the operational requirements of investors.

Using the expected return information of 25 stocks listed on the Tehran Stock Exchange, the four proposed nonlinear models are solved using metaheuristic algorithms. These algorithms are considered as efficient optimization methods for solving complex and nonlinear problems and can help to find the best mix of stocks in a portfolio. Finally, a constrained efficient frontier is drawn for these models, which represents the best possible mix of stocks with respect to risk and return. This efficient frontier allows investors to make more optimal decisions in choosing their stock mix and capital allocation.

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